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Research Article

Time-Variant Reliability Assessment and Its Sensitivity Analysis of Cutting Tool under Invariant Machining Condition Based on Gamma Process

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The time-variant reliability and its sensitivity of cutting tools under both wear deterioration and an invariant machining condition are analyzed. The wear process is modeled by a Gamma process which is a continuous-state and continuous-time stochastic process with the independent and nonnegative increment. The time-variant reliability and its sensitivity of cutting tools under six cases are considered in this paper. For the first two cases, the compensation for the cutting tool wear is not carried out. For the last four cases, the off-line or real-time compensation method is adopted. While the off-line compensation method is used, the machining error of cutting tool is supposed to be stochastic. Whether the detection of the real-time wear is accurate or not is discussed when the real-time compensation method is adopted. The numerical examples are analyzed to demonstrate the idea of how the reliability of cutting tools under the invariant machining condition could be improved according to the methods described in this paper.

1. Introduction

The cutting tool is one of the most important components of machine tools. During manufacturing process, it slides on the surface of the work-piece with a huge friction. Therefore, cutting tool fails due to wear frequently. It has been reported that the downtime due to the cutting tool failure is more than one third of the total down time which is defined as the non-productive lines idling in the manufacturing system [1–3]. Accurate assessment of the cutting tool reliability could result in an optimal replacement strategy for cutting tool, decrease the production cost, and improve the cutting tool reliability.

The reliability assessment of cutting tool has been investigated by many researchers. Klim et al. [4] proposed a reliability model for the quantitative study of the effect of the feed fate variation on the cutting tool wear and life. A deterministic approach based on the Taylor equation was proposed by Nagasaka and Hashimoto to calculate the average cutting tool life in machining stepped parts with varying cutting speeds [5]. The fact is ignored by them that the cutting tool failure is a stochastic phenomenon [6]. The approach was extended by Zhou and Wysk [6] where the stochastic phenomenon of the cutting tool failure was considered. The cutting tool reliability depends not only on the cutting speed but also other machining conditions. Then, Liu and Makis [2] presented an approach to assess the cutting tool reliability under variable machining conditions. Their reliability assessment approach was based on the failure time of cutting tool. This meant that the cutting tool states were classified into two: the fresh and broken state or the success and failure state [3]. The classification method was used in other literature such as [7] where the cutting tool reliability was studied.

However, the performance of cutting tool due to wear is generally subject to progressive deterioration during using [3, 8–10]. Therefore, the multistate classification of the cutting tool deterioration due to wear has been suggested by a few researchers such as [11–14]. But, the reliability assessment based on the multistate classification of the cutting tool wear has not been investigated by them in detail. Then, an approach to reliability assessment was proposed in [3] where the cutting tool deterioration process was modeled as a nonhomogeneous continuous-time Markov process. In fact, the cutting tool deterioration process due to wear is a continuous-time and continuous-state stochastic process. It is also a monotone increasing stochastic process because the wear of cutting tool can not be decreased itself in machining. For the stochastic deterioration process to be monotonic, we can best consider it as a Gamma process [15–17]. Therefore, the Gamma process is employed to model the cutting tool deterioration process in this paper.

A Gamma process is a continuous-time and continuous-state stochastic process with the independent, nonnegative increment having a Gamma distribution with an identical scale parameter. It is suitable to model the gradual damage monotonically accumulating over time in a sequence of tiny increments, such as wear, fatigue, corrosion, crack growth, erosion, consumption, creep, swell, degrading health index, and so forth [17]. It has been used to model the deterioration process in maintenance optimization and other field by many literatures which have been reviewed by van Noortwijk [17]. An approach to reliability assessment based on Gamma process has been presented by the author and his collaborators [18]. It has been validated by comparing the results using the proposed approach with those using traditional approaches in [19]. A method for computing the time-variant reliability of a structural component was proposed by van Noortwijk et al. [20]. In this method, the deterioration process of resistance was modeled as a Gamma process, the stochastic process of loads was generated by a Poisson process, and the variability of the random loads was modeled by a peaks-over- threshold distribution.

The remainder of this paper is organized as follows: the reliability assessment models and their sensitivity analysis under six cases are derived in Section 2, numerical examples are given in Section 3, and the conclusions are drawn finally.

2. Cutting Tool Reliability Model

2.1. Gamma Deterioration Process

Generally, the failure modes of cutting tool include two types: excessive wear and breakage. Often, the breakage of a cutting edge is caused by the incompatible choice of the machining conditions. It is still valid even if the breakage failure is not considered in the comparative analysis of the cutting tool reliability. This has been proved by the tests in [4]. The cutting tool deterioration process due to wear is a continuous-time and continuous-state stochastic process. Moreover, it is also a monotone increasing stochastic process. Therefore, the Gamma process is employed to model the cutting tool deterioration process.

Gamma process is a stochastic process with independent, nonnegative increment having a gamma distribution with an identical scale parameter. It is a continuous-time and continuous-state stochastic process. Let $\{X(t), t \geq 0\}$ be a Gamma process. It is with the following properties [17]:

(1) $X(0) = 0$ with probability one,

(2) $X(\tau) - X(t) \sim G(x \mid v(\tau) - v(t), u)$, for all $\tau > t \geq 0$,

(3) $X(t)$ has independent increments,

where $v(t)$ is the shape function which is a non-decreasing, right-continuous, real-valued function for $t \geq 0$ with $v(0) \equiv 0$, $u > 0$ is the scale parameter, and $G(\cdot)$ is the Gamma distribution.

Let $X(t)$ denote the loss quantity of the cutting tool dimension due to wear (LQCTDW) at time t, $t \geq 0$. In accordance with the definition of the Gamma process, the probability density function of $X(t)$ is given by

$$f_{X(t)}(x) = \frac{u^{v(t)} x^{v(t)-1} \exp(-ux)}{\Gamma(v(t))} I_{[0,\infty)}(x), \tag{2.1}$$

where $\Gamma(\cdot)$ is the Gamma function, $I_A(x) = 1$ for $x \in A$ and $I_A(x) = 0$ for $x \notin A$. Its expectation and variance are, respectively, expressed as

$$E(X(t)) = \frac{v(t)}{u}, \tag{2.2}$$

$$E\left(X(t) - E(X(t))^2\right) = \frac{v(t)}{u^2}. \tag{2.3}$$

Empirical studies show that the expected deterioration at time t is often proportional to the power law [17]:

$$E(X(t)) = \frac{ct^b}{u} = at^b \propto t^b, \tag{2.4}$$

where $a > 0$ (or $c > 0$) and $b > 0$.

The non-stationary Gamma process with parameters c, b, and u is employed to model the deterioration process of cutting tool due to wear under the invariant machining condition. Here, the invariant machining condition means that the cutting speed, feed rate, depth of cut, work-piece material, work-piece geometry, contact angle, and so on [2] are constants in the machining process. c, b, and u can be estimated by the introduced method in [17] when the data of LQCTDW are collected under the identical machining condition. The data are composed of inspection times t_i, $i = 0, 1, \ldots, n$, where $0 = t_0 < t_1 < t_2 < \cdots < t_n$, and corresponding LQCTDW x_i, $i = 0, 1, \ldots, n$, where $0 = x_0 < x_1 < x_2 < \cdots < x_n$.

2.2. Reliability and Sensitivity Analysis without Compensation and Machining Error of Cutting Tool

The cutting tool reliability model under the invariant machining condition is discussed in the first case where the compensation for the cutting tool wear is not carried out during the machining process and cutting tool is manufactured accurately in the section. Let the maximum permissible machining error of the machine tool be noted by δ. δ is a constant and obtained by referring to the technical parameters of the considered machine tool. The time-variant limit state function of cutting tool in the first case is given by

$$g_1(t) = \delta - X(t). \tag{2.5}$$

According to Section 2.1, the cutting tool reliability model is

$$R_1(t) = \int_0^{\delta} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \mathrm{d}x. \tag{2.6}$$

The effect of each parameter in (2.6) on the cutting tool reliability could be found by sensitivity analysis. According to the derivation theorem of integration of variable upper limit, the sensitivity of the cutting tool reliability to the maximum permissible machining error δ is calculated by

$$\frac{\partial R_1(t)}{\partial \delta} = \frac{u^{ct^b} \delta^{ct^b-1} \exp(-u\delta)}{\Gamma(ct^b)}. \tag{2.7}$$

The sensitivity to b is

$$\begin{aligned} \frac{\partial R_1(t)}{\partial b} &= \int_0^{\delta} \frac{\exp(-ux) u^{ct^b} x^{ct^b-1} \ln(t) ct^b (\ln(u) + \ln(x))}{\Gamma(ct^b)} \mathrm{d}x \\ &\quad - \int_0^{\delta} \left(\int_0^{\infty} z^{ct^b-1} \ln(z) \exp(-z) \mathrm{d}z \right) \frac{\exp(-ux) u^{ct^b} x^{ct^b-1} \ln(t) ct^b}{\Gamma^2(ct^b)} \mathrm{d}x. \end{aligned} \tag{2.8}$$

The sensitivity to c can be calculated by

$$\begin{aligned} \frac{\partial R_1(t)}{\partial c} &= \int_0^{\delta} \frac{\exp(-ux) u^{ct^b} x^{ct^b-1} t^b (\ln(u) + \ln(x))}{\Gamma(ct^b)} \mathrm{d}x \\ &\quad - \int_0^{\delta} \left(\int_0^{\infty} z^{ct^b-1} \ln(z) \exp(-z) \mathrm{d}z \right) \frac{\exp(-ux) u^{ct^b} x^{ct^b-1} t^b}{\Gamma^2(ct^b)} \mathrm{d}x. \end{aligned} \tag{2.9}$$

The sensitivity to u is calculated by

$$\frac{\partial R_1(t)}{\partial u} = \frac{\exp(-\delta u)(\delta u)^{ct^b}}{u\Gamma(ct^b)}. \tag{2.10}$$

2.3. Reliability and Sensitivity Analysis with Machining Error of Cutting Tool and without Compensation

In the second case, where cutting tool has the machining error and the compensation for the cutting tool wear is not carried out, the time-variant limit state function of cutting tool under the invariant machining condition is given by

$$g_2(t) = \delta - |X(t) - \delta_d|, \tag{2.11}$$

where δ_d is the machining error cutting tool and equal to the difference between the actual dimension and the ideal one of cutting tool. It is a stochastic real number and follows the normal distribution with expectation $\overline{\delta}_d = 0$ and standard deviation σ_{δ_d}. According to (2.11), $g_2(t) \geq 0$ is equivalent to

$$\delta + \delta_d \geq X(t) \geq -\delta + \delta_d. \tag{2.12}$$

When $\delta_d = y$, the cutting tool reliability is

$$P\{g_2(t) \geq 0 \mid \delta_d = y\} = \int_{\max(-\delta+y,0)}^{\delta+y} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \mathrm{d}x, \tag{2.13}$$

where y must not be more than δ and less than $-\delta$. If y is more than δ or less than $-\delta$, the cutting tool reliability is 0. Therefore, (2.13) can be rewritten by

$$P\{g_2(t) \geq 0 \mid \delta_d = y\} = \int_0^{\delta+y} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \mathrm{d}x. \tag{2.14}$$

Then, the cutting tool reliability model in the second case is assessed by

$$R_2(t) = \int_{-\delta}^{\delta} \int_0^{\delta+y} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \frac{1}{\sigma_{\delta_d}\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \mathrm{d}x\, \mathrm{d}y. \tag{2.15}$$

The sensitivity of (2.15) to σ_{δ_d} can be written by

$$\frac{\partial R_2(t)}{\partial \sigma_{\delta_d}} = \int_{-\delta}^{\delta} \int_0^{\delta+y} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \frac{\left(y^2 - \sigma_{\delta_d}^2\right)}{\sigma_{\delta_d}^4 \sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \mathrm{d}x\, \mathrm{d}y. \tag{2.16}$$

The sensitivity to δ, b, c and u can be, respectively, expressed by

$$\begin{aligned}\frac{\partial R_2(t)}{\partial \delta} = {} & \frac{\exp\left(-\delta^2/2\sigma_{\delta_d}^2\right)\left(\Gamma\left(ct^b\right)-\Gamma\left(ct^b,2\delta u\right)\right)}{\sqrt{2\pi}\delta\Gamma\left(ct^b\right)} \\ & + \frac{\int_{-\delta}^{\delta}\exp\left(-y^2/2\sigma_{\delta_d}^2-u(\delta+y)\right)u\left(u(\delta+y)\right)^{ct^b-1}\mathrm{d}y}{\sqrt{2\pi}\delta\Gamma\left(ct^b\right)},\end{aligned} \tag{2.17}$$

$$\begin{aligned}\frac{\partial R_2(t)}{\partial b} = {} & \int_{-\delta}^{\delta}\int_0^{\delta+y}\exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}-ux\right)\frac{u^{ct^b}x^{ct^b-1}\ln(t)ct^b(\ln(u)+\ln(x))}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma\left(ct^b\right)}\mathrm{d}x\,\mathrm{d}y \\ & -\int_{-\delta}^{\delta}\int_0^{\delta+y}\left(\int_0^{\infty}z^{ct^b-1}\ln(z)\exp(-z)\mathrm{d}z\right)\exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}-ux\right)\frac{u^{ct^b}x^{ct^b-1}\ln(t)ct^b}{\Gamma^2\left(ct^b\right)\sigma_{\delta_d}\sqrt{2\pi}}\mathrm{d}x\,\mathrm{d}y,\end{aligned} \tag{2.18}$$

$$\begin{aligned}\frac{\partial R_2(t)}{\partial c} = {} & \int_{-\delta}^{\delta}\int_0^{\delta+y}\exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right)\frac{1}{\sigma_{\delta_d}\sqrt{2\pi}}\frac{\exp(-ux)u^{ct^b}x^{ct^b-1}t^b(\ln(u)+\ln(x))}{\Gamma\left(ct^b\right)}\mathrm{d}x\,\mathrm{d}y \\ & -\int_{-\delta}^{\delta}\int_0^{\delta}\left(\int_0^{\infty}z^{ct^b-1}\ln(z)\exp(-z)\mathrm{d}z\right)\exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \\ & \times\frac{1}{\sigma_{\delta_d}\sqrt{2\pi}}\frac{\exp(-ux)u^{ct^b}x^{ct^b-1}t^b}{\Gamma^2\left(ct^b\right)}\mathrm{d}x\,\mathrm{d}y,\end{aligned} \tag{2.19}$$

$$\frac{\partial R_2(t)}{\partial u} = \int_{-\delta}^{\delta}\frac{\exp\left(-y^2/2\sigma_{\delta_d}^2-u(\delta+y)\right)\left(u(\delta+y)\right)^{ct^b}}{\sqrt{2\pi}u\sigma_{\delta_d}\Gamma\left(ct^b\right)}\mathrm{d}y, \tag{2.20}$$

where $\Gamma(ct^b,2\delta u)=\int_{2\delta u}^{\infty}z^{ct^b-1}\exp(-z)\mathrm{d}z$ is the incomplete Gamma function.

2.4. Reliability and Sensitivity Analysis with Compensation for Cutting Tool Wear

Nowadays, there are three methods to compensate the cutting tool wear. The first is the off-line compensation method such as [21–25], where the compensation quantity at time t for LQCTDW is estimated by a compensation function prior to machining. The second is the on-line compensation method such as [26–29], where the compensation quantity for LQCTDW is determined according to the actual LQCTDW which is measured by the direct or indirect method during machining. This kind of method could be classified into two types. One is the regular compensation method where the actual LQCTDW is measured and then it is compensated periodically in machining process, such as [27, 30–32]. The other is the real-time compensation method where the actual LQCTDW is estimated and then compensated real-timely and continuously, such as [26, 28, 29]. The third is the combination compensation method where two or more compensation methods are combined to decrease the machining error due to LQCTDW, such as [22, 26, 33].

2.4.1. Reliability and Sensitivity Analysis Using Off-Line Compensation Method

Let the compensation function be denoted by $h(t)$ in the off-line compensation method, where $h(t)$ is a continuous real function and $h(t) \in [0, +\infty)$. Then, the time-variant limit state function of cutting tool in the third case where the dimension of cutting tool before working is stochastic and the off-line compensation method used is given by

$$g_3(t) = \delta - |X(t) - h(t) - \delta_d|. \tag{2.21}$$

$g_3(t) \geq 0$ is equivalent to

$$\delta + \delta_d + h(t) \geq X(t) \geq -\delta + \delta_d + h(t). \tag{2.22}$$

Therefore, the reliability model of cutting tool in the third case could be written by

$$R_3(t) = \int_{-\delta}^{\delta} \int_{\max(-\delta+y+h(t),0)}^{\delta+y+h(t)} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \frac{1}{\sigma_{\delta_d}\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \mathrm{d}x\, \mathrm{d}y. \tag{2.23}$$

When $h(t) \geq 2\delta$, (2.23) is transformed into

$$R_3(t) = \int_{-\delta}^{\delta} \int_{-\delta+y+h(t)}^{\delta+y+h(t)} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \frac{1}{\sigma_{\delta_d}\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \mathrm{d}x\, \mathrm{d}y. \tag{2.24}$$

Its sensitivity to $h(t)$ and δ can be written, respectively, by

$$\begin{aligned} \frac{\partial R_3(t)}{\partial h(t)} = {} & \int_{-\delta}^{\delta} \frac{u^{ct^b} (\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(\delta + y + h(t))\right) \mathrm{d}y \\ & - \int_{-\delta}^{\delta} \frac{u^{ct^b} (-\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(-\delta + y + h(t))\right) \mathrm{d}y, \end{aligned} \tag{2.25}$$

$$\begin{aligned} \frac{\partial R_3(t)}{\partial \delta} = {} & \int_{-2\delta+h(t)}^{2\delta+h(t)} \frac{u^{ct^b} x^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{\delta^2}{2\sigma_{\delta_d}^2} - ux\right) \mathrm{d}x \\ & + \int_{-\delta}^{\delta} \frac{u^{ct^b} (\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(\delta + y + h(t))\right) \mathrm{d}y \\ & + \int_{-\delta}^{\delta} \frac{u^{ct^b} (-\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(-\delta + y + h(t))\right) \mathrm{d}y. \end{aligned} \tag{2.26}$$

When $h(t) < 2\delta$, (2.23) is transformed into

$$\begin{aligned} R_3(t) = & \int_{-\delta}^{\delta-h(t)} \int_0^{\delta+y+h(t)} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \frac{1}{\sigma_{\delta_d}\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \mathrm{d}x\, \mathrm{d}y \\ & + \int_{\delta-h(t)}^{\delta} \int_{-\delta+y+h(t)}^{\delta+y+h(t)} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \frac{1}{\sigma_{\delta_d}\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) \mathrm{d}x\, \mathrm{d}y. \end{aligned} \tag{2.27}$$

Its sensitivity to $h(t)$ and δ can be written by

$$\begin{aligned} \frac{\partial R_3(t)}{\partial h(t)} = & \int_{-\delta}^{\delta} \frac{u^{ct^b} (\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(\delta + y + h(t))\right) \mathrm{d}y \\ & - \int_{\delta-h(t)}^{\delta} \frac{u^{ct^b} (-\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(-\delta + y + h(t))\right) \mathrm{d}y, \\ \frac{\partial R_3(t)}{\partial \delta} = & \int_0^{2\delta+h(t)} \frac{u^{ct^b} x^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{\delta^2}{2\sigma_{\delta_d}^2} - ux\right) \mathrm{d}x \\ & + \int_{-\delta}^{\delta} \frac{u^{ct^b} (\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(\delta + y + h(t))\right) \mathrm{d}y \\ & + \int_{\delta-h(t)}^{\delta} \frac{u^{ct^b} (-\delta + y + h(t))^{ct^b-1}}{\sigma_{\delta_d}\sqrt{2\pi}\Gamma(ct^b)} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2} - u(-\delta + y + h(t))\right) \mathrm{d}y. \end{aligned} \tag{2.28}$$

The sensitivity of the cutting tool reliability in the third case to σ_{δ_d}, b, c, and u can be, respectively, expressed using (2.16), (2.18), (2.19), and (2.20), where the integral upper limit $\delta + y$ and the integral under limit 0 are only replaced by $\delta + y + h(t)$ and $\max(-\delta + y + h(t), 0)$.

In the fourth case, there are two assumptions. One is cutting tool is manufactured accurately or σ_{δ_d} of the machining error is close to zero. The other is the off-line compensation method is used. The reliability model of cutting tool under the invariant machining condition could be written by

$$R_4(t) = \int_{\max(-\delta+h(t),0)}^{\delta+h(t)} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \mathrm{d}x. \tag{2.29}$$

When $h(t) \geq \delta$, (2.29) is transformed into

$$R_4(t) = \int_{-\delta+h(t)}^{\delta+h(t)} \frac{u^{ct^b} x^{ct^b-1} \exp(-ux)}{\Gamma(ct^b)} \mathrm{d}x. \tag{2.30}$$

Its sensitivity to $h(t)$ and δ can be calculated by

$$\frac{\partial R_4(t)}{\partial h(t)} = \frac{u^{ct^b}(\delta + h(t))^{ct^b-1}\exp(-u(\delta + h(t)))}{\Gamma(ct^b)} - \frac{u^{ct^b}(-\delta + h(t))^{ct^b-1}\exp(-u(-\delta + h(t)))}{\Gamma(ct^b)},$$
$$\frac{\partial R_4(t)}{\partial \delta} = \frac{u^{ct^b}(\delta + h(t))^{ct^b-1}\exp(-u(\delta + h(t)))}{\Gamma(ct^b)} + \frac{u^{ct^b}(-\delta + h(t))^{ct^b-1}\exp(-u(-\delta + h(t)))}{\Gamma(ct^b)}. \tag{2.31}$$

When $h(t) < \delta$, (2.29) is transformed into

$$R_4(t) = \int_0^{\delta+h(t)} \frac{u^{ct^b} x^{ct^b-1}\exp(-ux)}{\Gamma(ct^b)} dx. \tag{2.32}$$

Its sensitivity to $h(t)$ and δ can be calculated by

$$\frac{\partial R_4(t)}{\partial h(t)} = \frac{\partial R_4(t)}{\partial \delta} = \frac{u^{ct^b}(\delta + h(t))^{ct^b-1}\exp(-u(\delta + h(t)))}{\Gamma(ct^b)}. \tag{2.33}$$

The sensitivity of (2.29) to b, c and u can be calculated by (2.8), (2.9), and (2.10), where the integral upper limit δ and the integral under limit 0 are only replaced by $\delta + h(t)$ and $\max(-\delta + h(t), 0)$.

2.4.2. Reliability and Sensitivity Analysis Using Real-Time Compensation Method

When the real-time method is employed to compensate LQCTDW, the time-variant limit state function of cutting tool still can be expressed by (2.21) but $h(t)$ is the real-time compensation function. $h(t)$ is determined by measuring LQCTDW. In the fifth case, where the measurement of LQCTDW is accurate, $X(t) - h(t)$ in (2.21) is identically equal to 0 and then the reliability model of cutting tool is

$$R_5(t) = \int_{-\delta}^{\delta} \frac{1}{\sigma_{\delta_d}\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma_{\delta_d}^2}\right) dy. \tag{2.34}$$

The cutting tool reliability is determined by only two parameters σ_{δ_d} and δ. The sensitivity of (2.34) to them are formulated, respectively, by

$$\frac{\partial R_5(t)}{\partial \sigma_{\delta_d}} = -\sqrt{\frac{2}{\pi}} \frac{\delta}{\sigma_{\delta_d}^2} \exp\left(-\frac{\delta^2}{2\sigma_{\delta_d}^2}\right), \tag{2.35}$$

$$\frac{\partial R_5(t)}{\partial \delta} = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_{\delta_d}} \exp\left(-\frac{\delta^2}{2\sigma_{\delta_d}^2}\right). \tag{2.36}$$

In the sixth case where the measurement of LQCTDW is not accurate, $r(t) = X(t) - h(t)$ is not identically equal to 0 but a stochastic process which is assumed to follow a normal distribution with expectation $\overline{r}(t) = 0$ and standard deviation σ_r at any time t. σ_r could be estimated by the historical data which are collected by the adopted real-time measuring system. Then, the time-variant limit state function of cutting tool in the sixth case is given by

$$g_6(t) = \delta - |X(t) - h(t) - \delta_d|, \tag{2.37}$$

where δ_d and $r(t)$ are independent. $g_6(t) \geq 0$ is equivalent to

$$\delta \geq r(t) - \delta_d \geq -\delta, \tag{2.38}$$

where $Y = r(t) - \delta_d$ follows a normal distribution with expectation $\overline{Y} = \overline{r}(t) - \overline{\delta}_d = 0$ and standard deviation $\sigma_Y = \sqrt{\sigma_r^2 + \sigma_{\delta_d}^2}$. Then, the reliability model of cutting tool is formulated by

$$R_6(t) = \Phi\left(\frac{\delta}{\sqrt{\sigma_r^2 + \sigma_{\delta_d}^2}}\right) - \Phi\left(-\frac{\delta}{\sqrt{\sigma_r^2 + \sigma_{\delta_d}^2}}\right), \tag{2.39}$$

where $\Phi(\cdot)$ is the cumulative function of standard normal distribution.

The sensitivity of (2.39) to δ, σ_r, and σ_{δ_d} are expressed, respectively, by

$$\frac{\partial R_6(t)}{\partial \delta} = \frac{2}{\sqrt{2\pi\left(\sigma_r^2 + \sigma_{\delta_d}^2\right)}} \exp\left(-\frac{\delta^2}{2\left(\sigma_r^2 + \sigma_{\delta_d}^2\right)}\right), \tag{2.40}$$

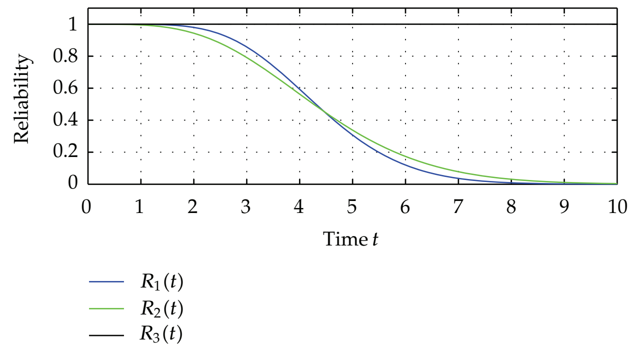
$$\frac{\partial R_6(t)}{\partial \sigma_r} = -\frac{2\delta\sigma_r}{\sqrt{2\pi}\left(\sigma_r^2 + \sigma_{\delta_d}^2\right)^{3/2}} \exp\left(-\frac{\delta^2}{2\left(\sigma_r^2 + \sigma_{\delta_d}^2\right)}\right), \tag{2.41}$$

$$\frac{\partial R_6(t)}{\partial \sigma_{\delta_d}} = -\frac{2\delta\sigma_{\delta_d}}{\sqrt{2\pi}\left(\sigma_r^2 + \sigma_{\delta_d}^2\right)^{3/2}} \exp\left(-\frac{\delta^2}{2\left(\sigma_r^2 + \sigma_{\delta_d}^2\right)}\right). \tag{2.42}$$

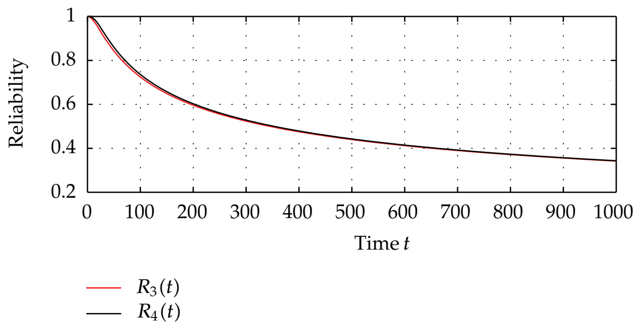
3. Numerical Examples and Discussion

This section will show how the proposed reliability assessment method for cutting tool is applied and how the cutting tool reliability is improved using the proposed reliability model and its sensitivity analysis when cutting tool suffers from the failure due to wear under the invariant machining condition by numerical examples. Let $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$, $\sigma_r = 0.8$, $\sigma_{\delta_d} = 1.5$. The off-line compensation function $h(t)$ is assumed to be equal to the expectation function of the cutting tool wear process ct^b/u.

The reliability curves of cutting tool $R_1(t)$, $R_2(t)$, $R_3(t)$, and $R_4(t)$ are shown in Figure 1. $R_5(t)$ and $R_6(t)$ are identically equal to 0.99999942669686 and 0.99998974684970,



(a)



(b)

Figure 1: Reliability curves of cutting tool under invariant machining condition with $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$, $\sigma_{\delta_d} = 1.5$.

respectively. From Figure 1, it can be observed that $R_3(t)$ or $R_4(t)$ is much larger than $R_1(t)$ and $R_2(t)$ with the increasing of t, and $R_4(t)$ is slightly more than $R_3(t)$ at any time. This implies that the off-line compensation method could improve the reliability of cutting tool greatly when the compensation function is close to the actual wear of cutting tool and the machining error of cutting tool could decrease the cutting tool reliability when the off-line compensation method is used. According to the calculation results, it can be seen that the reliability of cutting tool always could be kept at very high level within the considered time range and the measurement error of the wear decrease the reliability of cutting tool when the real-time compensation method is adopted. Moreover, it is obvious that the real-time compensation method could improve the reliability of cutting tool more effectively than the off-line compensation method.

According to Figure 1, it can be obtained that $R_1(t)$ is less than $R_2(t)$ when t is more than one certain value. It implies that the machining error of cutting tool could increases the reliability when t is more than one certain value. $R_1(t)$ is compared with $R_2(t)$ with the different σ_{δ_d} in Figure 2. Figure 2 shows that the added value of the reliability is larger and larger but the reliability in the early phase is decreased greatly with the increasing of the machining error standard deviation of cutting tool when t is more than one certain value.

The sensitivity curves of the cutting tool reliability in the first case $R_1(t)$ to δ, b, c, and u are shown in Figures 3, 4, 5, and 6, respectively, and that of $R_2(t)$, $R_3(t)$, and $R_4(t)$ are similar. Here, the considered parameter (one of δ, b, c, and u) is the only variable. For example, δ

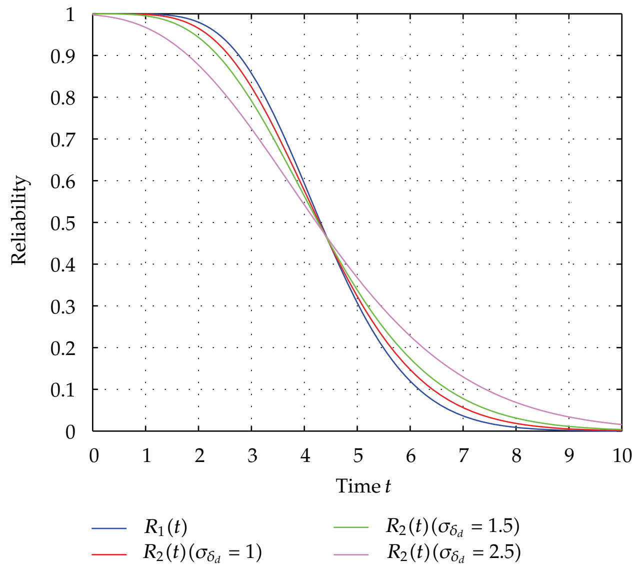


Figure 2: Comparison of reliability of cutting tool $R_1(t)$ and $R_2(t)$ with the different σ_{δ_d} when $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$.

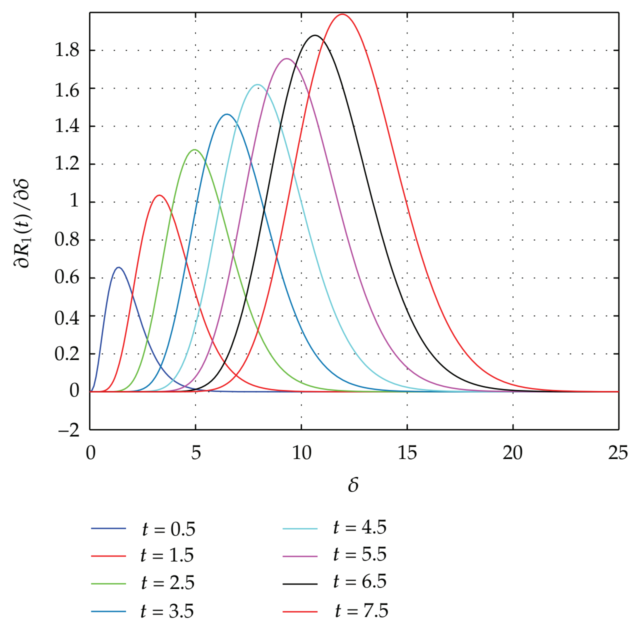


Figure 3: Sensitivity curves of $R_1(t)$ to δ with the different machining time t when $b = 0.8$, $u = 2.1$, $c = 5.0$.

is the only variable in $\partial R_1(t)/\partial\delta$ and the curves are shown in Figure 3. When t is the only variable, $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, and $c = 5.0$, the sensitivity curves of $R_1(t)$ to δ, b, c, and u are similar to Figure 8. From Figures 3, 4, 5, and 6, it can bee seen that the sensitivity of $R_1(t)$ to any one of δ, b, c, and u has the maximum or minimum when the considered parameter changes within its domain and it tends towards zero gradually with the increasing of the

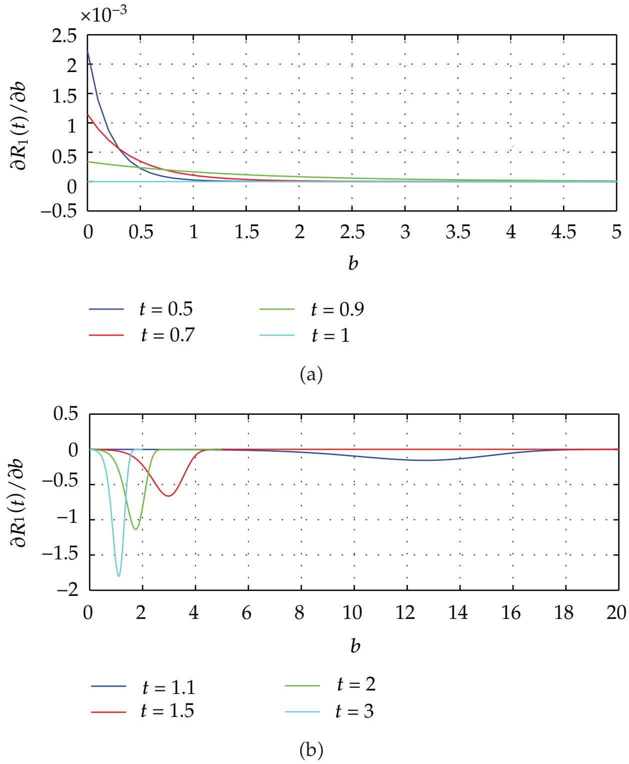


Figure 4: Sensitivity curves of $R_1(t)$ to b with the different machining time t when $\delta = 7.5\,\mu$m, $u = 2.1$, $c = 5.0$.

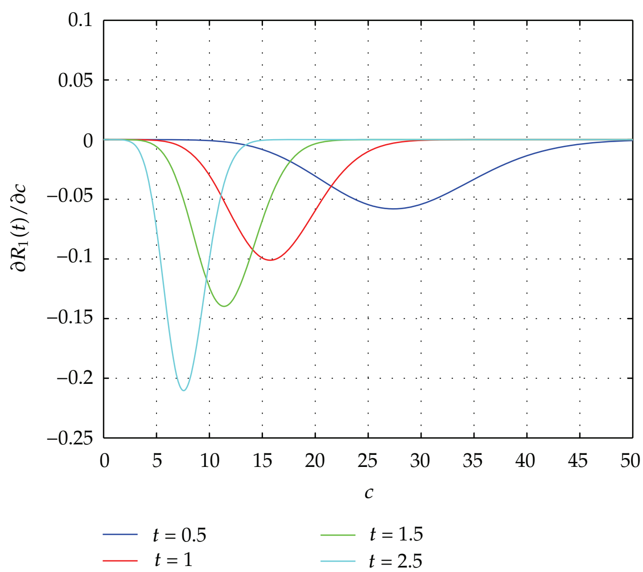


Figure 5: Sensitivity curves of $R_1(t)$ to c with the different machining time t when $\delta = 7.5\,\mu$m, $u = 2.1$, $b = 0.8$.

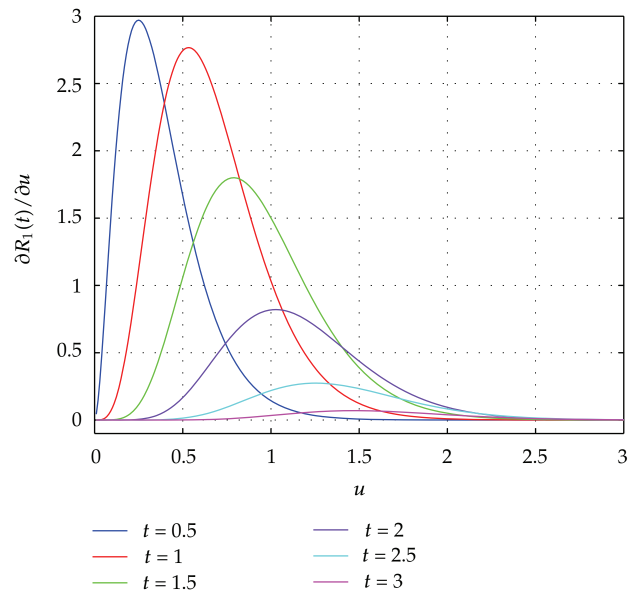


Figure 6: Sensitivity curves of $R_1(t)$ with respect to u with the different machining time t when $\delta = 7.5\,\mu$m, $c = 5.0$, $b = 0.8$.

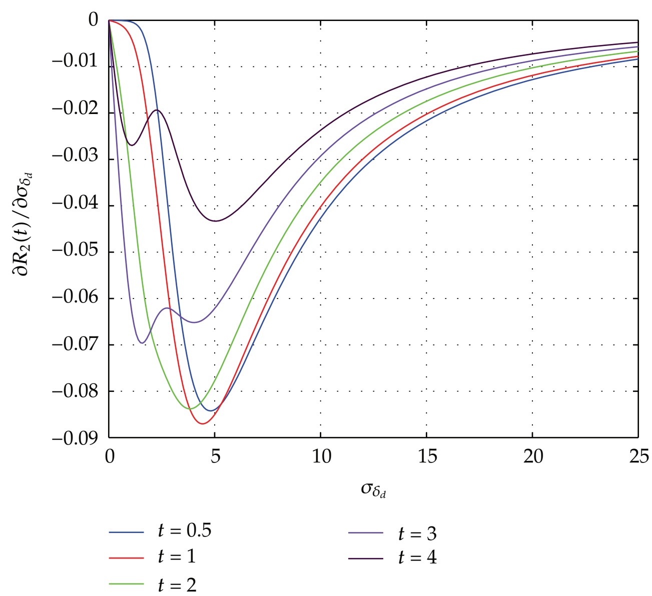


Figure 7: Sensitivity curves of $R_2(t)$ to σ_{δ_d} with the different machining time t when $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$.

considered parameter at the time t. $R_1(t)$ is more sensitive to b and u than δ and c according to Figure 8.

The sensitivity curves of the cutting tool reliability in the second case $R_2(t)$ to σ_{δ_d} are shown in Figure 7, where σ_{δ_d} is the only variable and that to δ is also similar to Figure 6. The sensitivity curves of $R_2(t)$ to σ_{δ_d}, δ, b, c, and u are shown in Figure 8 simultaneously where

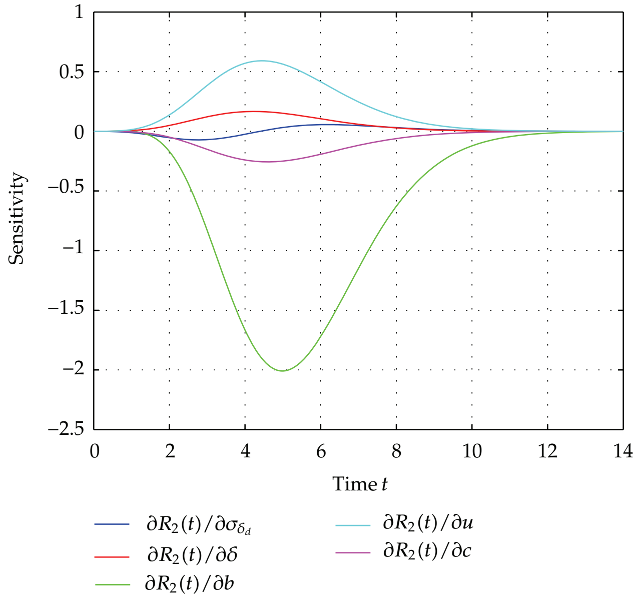


Figure 8: Sensitivity curves of $R_2(t)$ to σ_{δ_d}, δ, b, c, u with $\delta = 7.5\,\mu$m, $c = 5.0$, $b = 0.8$, $u = 2.1$, $\sigma_{\delta_d} = 1.5$.

t is the only variable, $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$, and $\sigma_{\delta_d} = 1.5$. $R_2(t)$ is the most sensitive to b among all parameters according to Figure 8.

On the basis of Figure 3 to Figure 8, the sensitivity of $R_1(t)$ and $R_2(t)$ to δ and u are always more than zero but that to other parameters are less than or close to zero. Therefore, $R_1(t)$ and $R_2(t)$ could be increased by increasing δ or u or by decreasing b, c, or σ_{δ_d} in the numerical example.

The sensitivity curves of the cutting tool reliability in the third case $R_3(t)$ to $h(t)$ is shown in Figure 9 where $h(t)$ is the only variable and that to δ, σ_{δ_d}, b, c, and u are not given because they have the similar law to the sensitivity curves of $R_2(t)$. The sensitivity curves of $R_3(t)$ to $h(t)$, σ_{δ_d}, δ, b, c, and u are shown in Figure 10 simultaneously where t is the only variable, $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$, $\sigma_{\delta_d} = 1.5$, and $h(t) = ct^b/u$.

On the basis of Figure 9, the sensitivity of $R_3(t)$ to $h(t)$ is more than zero when $h(t)$ is less than one certain positive number and it is less than zero when $h(t)$ is more than this positive number. The sensitivity of $R_3(t)$ to δ is more than zero and has the maximum when δ is the only variable and changes during its domain from Figure 6. $R_3(t)$ is the most sensitive to b among all parameters according to Figure 10.

The sensitivity curves of $R_4(t)$ to δ and $h(t)$ are similar to those shown in Figures 6 and 9, respectively. When t is the only variable, $\delta = 7.5\,\mu$m, $b = 0.8$, $u = 2.1$, $c = 5.0$, and $h(t) = ct^b/u$, the sensitivity curves of $R_4(t)$ to $h(t)$, δ, b, c, and u are similar to those in Figure 10. From Figure 9, it can be obtained that the reliability of cutting tool can be improved by using the off-line compensation method only if $h(t)$ is assigned properly.

The sensitivity curves of the cutting tool reliability in the fifth case $R_5(t)$ to δ is shown in Figure 11 where δ is the only variable. The sensitivity curves of $R_5(t)$ to σ_{δ_d} and $R_6(t)$ to σ_r and σ_{δ_d} is similar to $\partial R_3(t)/\partial\sigma_{\delta_d}$ in Figure 10. The sensitivity curves of $R_6(t)$ to δ are similar to those in Figure 11.

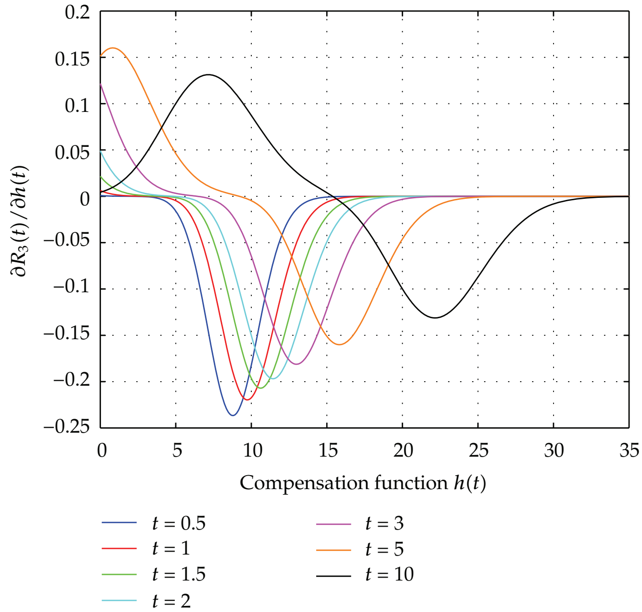


Figure 9: Sensitivity curves of $R_3(t)$ to $h(t)$ with the different machining time t when $b = 0.8$, $c = 5.0$, $\delta = 7.5\,\mu$m, $\sigma_{\delta_d} = 1.5$, $u = 2.1$.

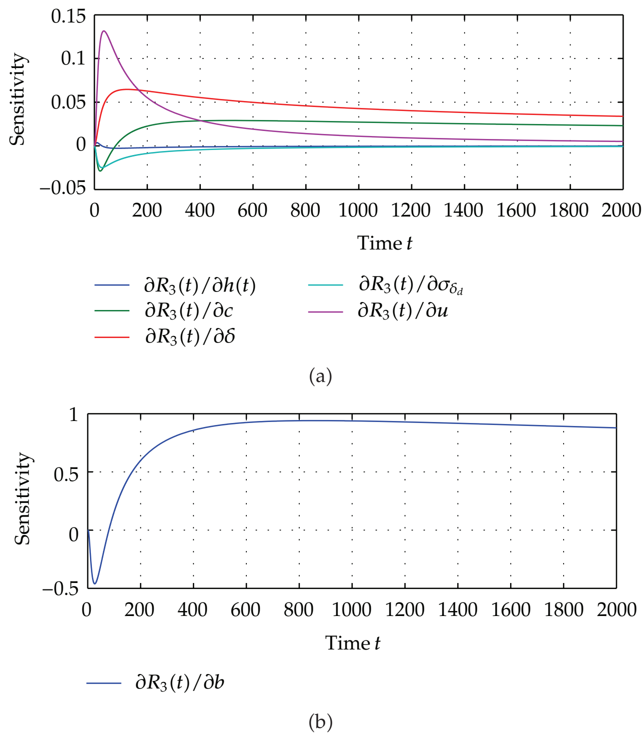


Figure 10: Sensitivity curves of $R_3(t)$ to σ_{δ_d}, $h(t)$, δ, b, c, u with $\delta = 7.5\,\mu$m, $c = 5.0$, $b = 0.8$, $u = 2.1$, $\sigma_{\delta_d} = 1.5$, $h(t) = ct^b/u$.

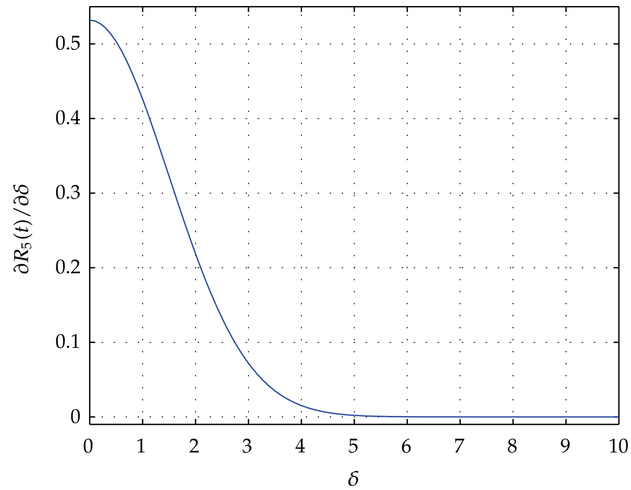


Figure 11: Sensitivity curves of $R_5(t)$ to σ_{δ_d} with $\sigma_r = 0.8$ and $\delta = 7.5\,\mu$m.

According to Figures 10 and 11, it can be observed that $R_5(t)$ and $R_6(t)$ are more sensitive to δ than other parameters, they can be improved by increasing δ, $R_5(t)$ could be decreased when σ_{δ_d} increases and $R_6(t)$ will be decreased with the increasing of σ_r or σ_{δ_d}.

4. Conclusions

The cutting tool reliability assessment and its sensitivity analysis under the invariant machining condition are presented in this paper. Here, cutting tool suffers from the failure due to wear and the wear process is modeled by a Gamma process. The deterioration of cutting tool is assumed to be continuous. Therefore, the reliability assessment method for cutting tool is practical.

The sensitivity analysis of the cutting tool reliability offers the approach to improve the reliability under six cases when the machining condition is invariant.

Notations

$X(t)$: Loss quantity of the dimension of cutting tool due to wear
u: Scale parameter of Gamma process
$\Gamma(\cdot)$: Gamma function
$g(\cdot)$: Limit state function
x_i: Measurement value of $X(t)$
δ_d: Machining error of cutting tool
$h(t)$: Compensation function of the cutting tool wear
σ_r: Standard deviation of $r(t)$
δ: Maximum permissible machining error of the machine tool
$G(\cdot)$: Gamma distribution
a, c, b: Parameters of shape function
$v(t)$: Shape function of Gamma process

$R(t)$: Reliability function
σ_{δ_d}: Standard deviation of δ_d
$r(t)$: Measurement error of $X(t)$.

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