

## Research Article

# Impulsive Biological Pest Control Strategies of the Sugarcane Borer

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We propose an impulsive biological pest control of the sugarcane borer (*Diatraea saccharalis*) by its egg parasitoid *Trichogramma galloi* based on a mathematical model in which the sugarcane borer is represented by the egg and larval stages, and the parasitoid is considered in terms of the parasitized eggs. By using the Floquet theory and the small amplitude perturbation method, we show that there exists a globally asymptotically stable pest-eradication periodic solution when some conditions hold. The numerical simulations show that the impulsive release of parasitoids provides reliable strategies of the biological pest control of the sugarcane borer.

## 1. Introduction

One of the challenges for the improvements in the farming and harvesting of cane is the biological pest control. Biological control is defined as the reduction of pest populations by using their natural enemies: predators, parasitoids, and pathogens [1]. Parasitoids are species which develop within or on the host and ultimately kill it. Thus, parasitoids are commonly reared in laboratories and periodically released in high-density populations as biological control agents of crop pests [2].

The sugarcane borer *Diatraea saccharalis* is reported to be the most important sugarcane pest in the southeast region of Brazil [3]. The sugarcane borer builds internal galleries in the sugarcane plants causing direct damage that results in apical bud death, weight loss, and atrophy. Indirect damage occurs when there is contamination by yeasts that cause red rot in the stalks, either causing contamination or inverting the sugar, increasing yield loss in both sugar and alcohol [4].

There is an important larvae parasitoid of the sugarcane borer, a wasp named *Cotesia flavipes* which is widely used in biological control in Brazil [3]. In spite of this control being considered successful in Brazil, there are some areas where *Cotesia flavipes* does not control the

sugarcane borer efficiently. The using of the egg parasitoid *Trichogramma galloi* is considered an interesting option in this case [5].

Mathematical modelling is an important tool used in studying agricultural problems. Thus, a good strategy of biological pest control, based on mathematical modelling, can increase the ethanol production. The applications of host-parasitoid models for biological control were reviewed in [6].

In [7], a mathematical model of interaction between the sugarcane borer (*Diatraea saccharalis*) and its egg parasitoid *Trichogramma galloi* was proposed which consists of three differential equations

$$\begin{aligned}\frac{dx_1}{dt} &= r\left(1 - \frac{x_1}{K}\right)x_1 - m_1x_1 - n_1x_1 - \beta x_1x_2, \\ \frac{dx_2}{dt} &= \beta x_1x_2 - m_2x_2 - n_2x_2, \\ \frac{dx_3}{dt} &= n_1x_1 - m_3x_3 - n_3x_3,\end{aligned}\tag{1.1}$$

where  $x_1$  is the egg density of the sugarcane borer,  $x_2$  is the density of eggs parasitized by *Trichogramma galloi*, and  $x_3$  is the larvae density of the sugarcane borer;  $r$  is the net reproduction rate;  $K$  is the carrying capacity of the environment;  $m_1$ ,  $m_2$ , and  $m_3$  are mortality rates of the egg, parasitized egg, and larvae populations;  $n_1$  is the fraction of the eggs from which the larvae emerge at time  $t$ ;  $n_2$  is the fraction of the parasitized eggs from which the adult parasitoids emerge at time  $t$ ;  $n_3$  is the fraction of the larvae population which moults into pupal stage at time  $t$ ;  $\beta$  is the rate of parasitism. The dynamics of this model without control was considered in [7].

The proposed model (1.1) is a simplified compartmental one which considers only three easy monitoring stages of the sugarcane borer species: egg stage, parasitized egg stage, and larvae stage. Then, a reducing effect related to the searching efficiency of the adult parasitoid may be ignored by this model. According to [5], the searching time of the parasitoid *Trichogramma galloi* is 1-2 day, and it can cause some augmentation of the parasitoid egg numbers when biological control measures are implemented.

Meanwhile, many authors have investigated the different population models concerning the impulsive pest control [8–15]. The impulsive pest control strategies based on prey-predator models were presented in [8, 11, 13]. The host-parasitoid model with impulsive control was considered in [10]. Impulsive strategies of a pest management for SI epidemic models were proposed in [9, 12]. Pulse vaccination strategies for SIR epidemic models were considered in [14, 15].

In this paper, we suggest impulsive differential equations [16] to model the process of the biological pest control of the sugarcane borer. So we develop (1.1) introducing a periodic releasing of the parasitoids at fixed times

$$\begin{aligned}\frac{dx_1}{dt} &= r\left(1 - \frac{x_1}{K}\right)x_1 - m_1x_1 - n_1x_1 - \beta x_1x_2 \\ \frac{dx_2}{dt} &= \beta x_1x_2 - m_2x_2 - n_2x_2 & t \neq n\tau, n \in Z_+, \\ \frac{dx_3}{dt} &= n_1x_1 - m_3x_3 - n_3x_3 \\ \Delta x_1(t) &= 0 \\ \Delta x_2(t) &= p & t = n\tau, n \in Z_+, \\ \Delta x_3(t) &= 0,\end{aligned}\tag{1.2}$$

where  $p$  is the release amount of the parasitized eggs at  $t = n\tau$ ,  $n \in \mathbb{Z}_+$ ,  $\mathbb{Z}_+ = \{0, 1, 2, \dots\}$ ,  $\tau$  is the period of the impulsive effect.  $\Delta x_i = x_i(t^+) - x_i(t)$ ,  $x_i(t^+) = \lim_{t \rightarrow t^+} x_i(t)$ ,  $i = 1, 2, 3$ . That is, we can use releasing parasitized eggs to eradicate pests or keep the pest population below the economic damage level.

## 2. Preliminary

In this section, we will give some definitions, notations, and some lemmas which will be useful for our main results.

Let  $R_+ = [0, \infty)$ ,  $R_+^3 = \{x \in R^3 : x > 0\}$ . Denote  $f = (f_1, f_2, f_3)^T$ , the map defined by the right-hand side of the first three equations of the system (1.2). Let  $V_0 = \{V : R_+ \times R_+^3 \mapsto R_+\}$  be continuous on  $(n\tau, (n+1)\tau] \times R_+^3$ ,  $\lim_{(t,y) \rightarrow (n\tau^+, x)} V(t, y) = V(n\tau^+, x)$  exist and  $V$  is locally Lipschitzian in  $x$ .

*Definition 2.1.*  $V \in V_0$ , then for  $(t, x) \in (n\tau, (n+1)\tau] \times R_+^3$ , the upper right derivative of  $V(t, x)$  with respect to the impulsive differential system (1.2) is defined as

$$D^+V(t, x) = \limsup_{h \rightarrow 0} \frac{1}{h} [V(t+h, x+hf(t, x)) - V(t, x)]. \quad (2.1)$$

The solution of system (1.2), denoted by  $x(t) : R_+ \mapsto R_+^3$ , is continuously differentiable on  $(n\tau, (n+1)\tau] \times R_+^3$ . Obviously, the global existence and uniqueness of solution of system (1.2) is guaranteed by the smoothness properties of  $f$ , for details see [16].

We will use a basic comparison result from impulsive differential equations.

**Lemma 2.2** (see [16]). *Let  $V \in V_0$ , assume that*

$$\begin{aligned} D^+V(t, x) &\leq g(t, V(t, x)), \quad t \neq n\tau, \\ V(t, x(t^+)) &\leq \varphi_n(V(t, x(t))), \quad t = n\tau, \end{aligned} \quad (2.2)$$

where  $g : R_+ \times R_+^3 \mapsto R_+$  is continuous on  $(n\tau, (n+1)\tau] \times R_+$  and  $\varphi_n : R_+ \mapsto R_+$  is nondecreasing. Let  $R(t)$  be the maximal solution of the scalar impulsive differential equation

$$\begin{aligned} \dot{u}(t, x) &= g(t, u(t)), \quad t \neq n\tau, \\ u(t^+) &= \varphi_n(u(t)), \quad t = n\tau, \\ u(0^+) &= u_0 \end{aligned} \quad (2.3)$$

existing on  $[0, \infty)$ . Then  $V(0^+, x_0) \leq u_0$  implies that  $V(t, x(t)) \leq R(t)$ ,  $t \geq 0$ , where  $x(t)$  is any solution of (1.2), similar results can be obtained when all the directions of the inequalities in the lemma are reversed and  $\varphi_n$  is nonincreasing. Note that if one has some smoothness conditions of  $g$  to guarantee the existence and uniqueness of solutions for (2.3), then  $R(t)$  is exactly the unique solution of (2.3).

Next, we consider the following system:

$$\begin{aligned} \frac{du_2}{dt} &= a - m_2 u_2 - n_2 u_2, \quad t \neq n\tau, \\ \Delta u_2(t) &= b, \quad t = n\tau \\ u_2(0^+) &= u_{20} \geq 0. \end{aligned} \quad (2.4)$$

**Lemma 2.3.** *System (2.4) has a unique positive periodic solution  $\tilde{u}_2(t)$  with period  $\tau$  and for every solution  $u_2(t)$  of (2.4)  $|u_2(t) - \tilde{u}_2(t)| \rightarrow 0$  as  $t \rightarrow \infty$ , where*

$$\begin{aligned} \tilde{u}_2(t) &= \frac{a}{m_2 + n_2} + \frac{be^{-(m_2+n_2)(t-n\tau)}}{1 - e^{-(m_2+n_2)\tau}}, \quad t \in (n\tau, (n+1)\tau], \quad n \in Z_+, \\ \tilde{u}_2(0^+) &= \frac{a}{m_2 + n_2} + \frac{p}{1 - e^{-(m_2+n_2)\tau}}. \end{aligned} \quad (2.5)$$

*Proof.* Integrating and solving the first equation of (2.4) between pulses, we get

$$u_2(t) = \frac{a}{m_2 + n_2} + u_2(n\tau^+)e^{-(m_2+n_2)(t-n\tau)}, \quad t \in (n\tau, (n+1)\tau]. \quad (2.6)$$

After each successive pulse, we can deduce the following map of system (2.6):

$$u_2((n+1)\tau^+) = \frac{a}{m_2 + n_2} + \left[ u_2(n\tau^+) - \frac{a}{m_2 + n_2} \right] e^{-(m_2+n_2)\tau} + p, \quad t \in (n\tau, (n+1)\tau]. \quad (2.7)$$

Equation (2.7) has a unique fixed point  $u_2^* = a/(m_2 + n_2) + p/(1 - e^{-(m_2+n_2)\tau})$ , it corresponds to the unique positive periodic solution  $\tilde{u}_2(t)$  of system (2.4) with the initial value  $\tilde{u}_2(0^+) = a/(m_2 + n_2) + p/(1 - e^{-(m_2+n_2)\tau})$ . The fixed point  $u_2^*$  of map (2.7) implies that there is a corresponding cycle of period  $\tau$  in  $u_2(t)$ , that is,  $\tilde{u}_2(t) = (a/m_2 + n_2) + (pe^{-(m_2+n_2)(t-n\tau)})/(1 - e^{-(m_2+n_2)\tau})$ ,  $t \in (n\tau, (n+1)\tau]$ ,  $n \in Z_+$ . From (2.7) we obtain

$$u_2(n\tau^+) = u_2(0^+)e^{-n(m_2+n_2)\tau} + \left( p + \frac{a}{m_2 + n_2} (1 - e^{-(m_2+n_2)\tau}) \right) \frac{(1 - e^{-n(m_2+n_2)\tau})}{1 - e^{-(m_2+n_2)\tau}}, \quad (2.8)$$

thus,  $u_2(n\tau^+) \rightarrow u_2^*$  as  $t \rightarrow \infty$ , so  $\tilde{u}_2(t)$  is globally asymptotically stable. Thus, we have  $u_2(t) = (u_2(0^+) - \tilde{u}_2(0^+))e^{-(m_2+n_2)t} + \tilde{u}_2(t)$ .  $\square$

Consequently,  $u_2(t) \rightarrow \tilde{u}_2(t)$  as  $t \rightarrow \infty$ , that is,  $|u_2(t) - \tilde{u}_2(t)| \rightarrow 0$  as  $t \rightarrow \infty$ .

If  $a = 0$ , the system (2.4) has a unique positive periodic solution  $\tilde{u}_2(t) = (pe^{-(m_2+n_2)(t-n\tau)})/(1 - e^{-(m_2+n_2)\tau})$  with initial condition  $\tilde{u}_2(0^+) = p/(1 - e^{-(m_2+n_2)\tau})$  and  $\tilde{u}_2(t)$  is globally asymptotically stable. This completes the proof.

Therefore, system (1.2) has a pest-eradication periodic solution  $(0, \tilde{x}_2(t), 0)$ , where  $\tilde{x}_2(t) = pe^{-(m_2+n_2)(t-n\tau)}/(1 - e^{-(m_2+n_2)\tau})$ .

To study the stability of the pest-eradication periodic solution of (1.2), we present the Floquet theory for a linear  $\tau$  periodic impulsive equation

$$\begin{aligned} \frac{dx}{dt} &= A(t)x, \quad t \neq \tau_k, \quad t \in \mathbb{R}, \\ x(t^+) &= x(t) + B_k x(t), \quad t = \tau_k, \quad k \in \mathbb{Z}_+. \end{aligned} \quad (2.9)$$

Then, we introduce the following conditions:

- (H<sub>1</sub>)  $A(\cdot) \in PC(\mathbb{R}, C^{n \times n})$  and  $A(t + \tau) = A(t)$  ( $t \in \mathbb{R}$ ), where  $PC(\mathbb{R}, C^{n \times n})$  is the set of all piecewise continuous matrix functions which is left continuous at  $t = \tau_k$ , and  $C^{n \times n}$  is the set of all  $n \times n$  matrices.
- (H<sub>2</sub>)  $B_k \in C^{n \times n}$ ,  $\det(E + B_k) \neq 0$ ,  $\tau_k < \tau_{k+1}$  ( $k \in \mathbb{Z}_+$ ).
- (H<sub>3</sub>) There exist a  $h \in \mathbb{Z}_+$ , such that  $B_{k+h} = B_k$ ,  $\tau_{k+h} = \tau_k + \tau$  ( $k \in \mathbb{Z}_+$ ).

Let  $\Phi(t)$  be the fundamental matrix of (2.9), then there exists a unique nonsingular matrix  $M \in C^{n \times n}$  such that

$$\Phi(t + \tau) = \Phi(t) M. \quad (2.10)$$

By equality (2.10) there corresponds to the fundamental matrix  $\Phi(t)$  the constant matrix  $M$  which is called monodromy matrix of (2.9). All monodromy matrices of (2.9) are similar and have the same eigenvalues. The eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_n$  of the monodromy matrices are called the Floquet multipliers of (2.9).

**Lemma 2.4** (Floquet theory [16]). *Let conditions (H<sub>1</sub> – H<sub>3</sub>) hold. Then the linear  $\tau$  periodic impulsive system (2.9) is as follows:*

- (a) *stable if and only if all multipliers  $\lambda_i$  ( $i = 1, 2, \dots, n$ ) of equation (2.9) satisfy the inequality  $|\lambda_i| \leq 1$ ,*
- (b) *asymptotically stable if and only if all multipliers  $\lambda_i$  ( $i = 1, 2, \dots, n$ ) of equation (2.9) satisfy the inequality  $|\lambda_i| < 1$ ,*
- (c) *unstable if  $|\lambda_i| > 1$  for some  $i = 1, 2, \dots, n$ .*

### 3. Stability of the Pest-Eradication Periodic Solution

In this section, we study the stability of the pest-eradication periodic solution  $(0, \tilde{x}_2(t), 0)$  of the system (1.2). Next, we present an important result, concerning a condition that guarantees the global stability of this solution.

**Theorem 3.1.** *The pest-eradication periodic solution  $(0, \tilde{x}_2(t), 0)$  of the system (1.2) is globally asymptotically stable provided that inequality*

$$p > \frac{(r - m_1 - n_1)(m_2 + n_2) \tau}{\beta} \quad (3.1)$$

*holds.*

*Proof.* The local stability of a periodic solution  $(0, \tilde{x}_2(t), 0)$  of system (1.2) may be determined by considering the behavior of small-amplitude perturbations  $(y_1(t), y_2(t), y_3(t))$  of the solution.

Define

$$x_1(t) = y_1(t), \quad x_2(t) = \tilde{x}_2(t) + y_2(t), \quad x_3(t) = y_3(t), \quad (3.2)$$

where  $y_1(t), y_2(t), y_3(t)$  are small perturbations.

Linearizing the system (1.2), we have the following linear  $\tau$  periodic impulsive system:

$$\begin{aligned} \frac{dy_1}{dt} &= r y_1 - m_1 y_1 - n_1 y_1 - \beta \tilde{x}_2 y_1 \\ \frac{dy_2}{dt} &= \beta \tilde{x}_2 y_1 - m_2 y_2 - n_2 y_2 \quad t \neq n\tau, \quad n \in Z_+, \\ \frac{dy_3}{dt} &= n_1 y_1 - m_3 y_3 - n_3 y_3 \\ y_1(t^+) &= y_1(t) \\ y_2(t^+) &= y_2(t) \quad t = n\tau, \quad n \in Z_+, \\ y_3(t^+) &= y_3(t). \end{aligned} \quad (3.3)$$

Let  $\Phi(t)$  be the fundamental matrix of (3.3). Then we have

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \Phi(t) \begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix}, \quad (3.4)$$

where  $\Phi(t)$  must satisfy the following equation:

$$\frac{d\Phi(t)}{dt} = \begin{bmatrix} r - m_1 - n_1 - \beta \tilde{x}_2 & 0 & 0 \\ \beta \tilde{x}_2 & -m_2 - n_2 & 0 \\ n_1 & 0 & -m_3 - n_3 \end{bmatrix} \Phi(t), \quad (3.5)$$

and initial condition

$$\Phi(t) = I, \quad (3.6)$$

where  $I$  is the identity matrix.

The solution of (3.5) is

$$\Phi(t) = \begin{bmatrix} \exp\left(\int_0^t (r - m_1 - n_1 - \beta \tilde{x}_2(s)) ds\right) & 0 & 0 \\ * & \exp(-(m_2 + n_2)\tau) & 0 \\ * & * & \exp(-(m_3 + n_3)\tau) \end{bmatrix}. \quad (3.7)$$

There is no need to calculate the exact form of (\*) as it is not required in the analysis that follows. The resetting impulsive condition of (3.3) becomes

$$\begin{bmatrix} y_1(n\tau^+) \\ y_2(n\tau^+) \\ y_3(n\tau^+) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1(n\tau) \\ y_2(n\tau) \\ y_3(n\tau) \end{bmatrix}. \quad (3.8)$$

Hence, if absolute values of all eigenvalues of

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Phi(\tau) = \Phi(\tau) \quad (3.9)$$

are less than one, the  $\tau$  periodic solution is locally stable. Then the eigenvalues of  $M$  are the following:

$$\begin{aligned} \lambda_1 &= \exp\left(\int_0^t (r - m_1 - n_1 - \beta \tilde{x}_2(s)) ds\right) \\ \lambda_2 &= \exp(-(m_2 + n_2)\tau) < 1 \\ \lambda_3 &= \exp(-(m_3 + n_3)\tau) < 1. \end{aligned} \quad (3.10)$$

From (3.10), one can see that  $|\lambda_1| < 1$  if and only if condition (3.1) holds true. According to Lemma 2.4, the pest-eradication periodic solution  $(0, \tilde{x}_2(t), 0)$  is locally asymptotically stable.

In the following, we prove the global attractivity. Choose sufficiently small  $\varepsilon > 0$  such that

$$\delta = \exp\left(\int_0^\tau (r - m_1 - n_1 - \beta(\tilde{x}_2(t) - \varepsilon)) dt\right) < 1. \quad (3.11)$$

From the second equation of system (1.2), noting that  $((dx_2)/dt) \geq -(m_2 + n_2)x_2$ , we consider the following impulsive differential equation

$$\begin{aligned} \frac{du_2}{dt} &= -(m_2 + n_2)u_2, \quad t \neq n\tau \\ \Delta u_2(t) &= p, \quad t = n\tau, \\ u_2(0^+) &= x_2(0^+). \end{aligned} \quad (3.12)$$

From Lemma 2.3, system (3.12) has a globally asymptotically stable positive periodic solution

$$\tilde{u}_2(t) = \frac{pe^{-(m_2+n_2)(t-n\tau)}}{1 - e^{-(m_2+n_2)\tau}} = \tilde{x}_2, \quad t \in (n\tau, (n+1)\tau], \quad n \in \mathbb{Z}_+. \quad (3.13)$$

So by Lemma 2.2, we get

$$x_2(t) \geq u_2(t) \geq \tilde{x}_2(t) - \varepsilon, \quad (3.14)$$

for all  $t$  large enough.

From system, (1.2) and (3.14), we obtain that

$$\begin{aligned} \frac{dx_1}{dt} &\leq r \left(1 - \frac{x_1}{K}\right) x_1 - m_1 x_1 - n_1 x_1 - \beta x_1 (\tilde{x}_2 - \varepsilon), \quad t \neq n\tau, \\ \Delta x_1(t) &= 0, \quad t = n\tau. \end{aligned} \quad (3.15)$$

Integrating (3.15) on  $(n\tau, (n+1)\tau]$ , we get

$$x_1((n+1)\tau^+) \leq x_1(n\tau) \exp\left(\int_0^\tau (r - m_1 - n_1 - \beta(\tilde{x}_2(t) - \varepsilon)) dt\right) = x_1(n\tau)\delta. \quad (3.16)$$

Thus,  $x_1(n\tau) \leq x_1(0^+) \delta^n$  and  $x_1(n\tau) \rightarrow 0$  as  $n \rightarrow \infty$ . Therefore,  $x_1(t) \rightarrow 0$  as  $n \rightarrow \infty$ , since  $0 < x_1(t) \leq x_1(n\tau)$  for  $t \in (n\tau, (n+1)\tau]$ ,  $n \in \mathbb{Z}_+$ .

Next, we prove that  $x_2(t) \rightarrow \tilde{x}_2(t)$  as  $t \rightarrow \infty$ . For  $0 < \varepsilon_1 \leq m_2 + n_2$ , there must exist a  $t_0 > 0$  such that  $0 < x_1(t) \leq \varepsilon_1$  for all  $t \geq t_0$ . Without loss of generality, we may assume that  $0 < x_1(t) \leq \varepsilon_1$  for all  $t \geq 0$ , from system (1.2) we have

$$\frac{dx_2}{dt} \leq [\varepsilon_1 - (m_2 + n_2)]x_2. \quad (3.17)$$

Then, we have  $x_2(t) \leq v_2(t)$ , while  $v_2(t)$  is the solution of

$$\begin{aligned} \frac{dv_2}{dt} &= [\varepsilon_1 - (m_2 + n_2)]v_2, \quad t \neq n\tau, \\ \Delta v_2(t) &= p, \quad t = n\tau, \\ v_2(0^+) &= x_2(0^+). \end{aligned} \quad (3.18)$$

By Lemma 2.3, system (3.18) has a positive periodic solution

$$\tilde{v}_2(t) = \frac{pe^{[\varepsilon_1 - (m_2 + n_2)](t - n\tau)}}{1 - e^{[\varepsilon_1 - (m_2 + n_2)]\tau}}, \quad t \in (n\tau, (n+1)\tau], \quad n \in \mathbb{Z}_+. \quad (3.19)$$

Therefore, for any  $\varepsilon_2 > 0$ , there exists a  $t_1$ ,  $t > t_1$  such that

$$x_2(t) \leq v_2(t) < \tilde{v}_2(t) + \varepsilon_2. \quad (3.20)$$

Combining (3.14) and (3.20), we obtain

$$\tilde{x}_2(t) - \varepsilon \leq x_2(t) < \tilde{v}_2(t) + \varepsilon_2, \quad (3.21)$$

for  $t$  large enough. Let  $\varepsilon_1, \varepsilon_2 \rightarrow 0$ , we get  $\tilde{v}_2(t) \rightarrow \tilde{x}_2(t)$ , then  $x_2(t) \rightarrow \tilde{x}_2(t)$  as  $t \rightarrow \infty$ .

Assuming that  $0 < x_1(t) \leq \varepsilon_1$  for all  $t \geq 0$ , from system (1.2) we have

$$\frac{dx_3}{dt} \leq n_1 \varepsilon_1 - (m_3 + n_3)x_3. \quad (3.22)$$

Then, we have  $x_3(t) \leq v_3(t)$ , while  $v_3(t)$  is the solution of the following system:

$$\begin{aligned} \frac{dv_3}{dt} &= n_1 \varepsilon_1 - m_3 v_3 - n_3 v_3, \quad t \neq n\tau, \\ \Delta v_3(t) &= 0, \quad t = n\tau, \\ v_3(0^+) &= v_{30} \geq 0. \end{aligned} \quad (3.23)$$

By Lemma 2.3, system (3.23) has a positive solution

$$\tilde{v}_3 = \frac{n_1 \varepsilon_1}{m_3 + n_3}. \quad (3.24)$$

Thus, for any  $\varepsilon_3 > 0$ , there exists a  $t_1$ ,  $t > t_1$  such that

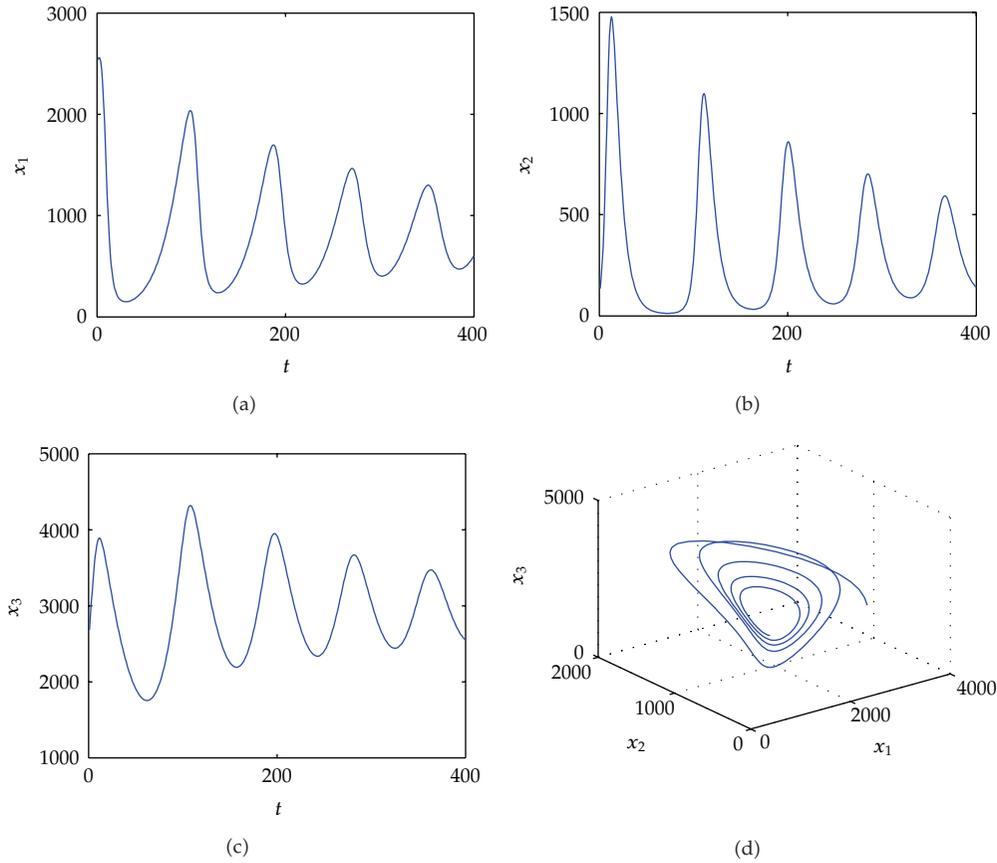
$$x_3(t) \leq v_3(t) < \tilde{v}_3(t) + \varepsilon_3. \quad (3.25)$$

Let  $\varepsilon_1, \varepsilon_3 \rightarrow 0$ , we get  $\tilde{v}_3(t) \rightarrow 0$ , then  $x_3(t) \rightarrow 0$  as  $t \rightarrow \infty$ . This completes the proof.  $\square$

#### 4. Numerical Simulations of the Impulsive Biological Control

For numerical simulations of interactions between the sugarcane borer and its parasitoid the following values of model coefficients were used:  $n_1 = 0.1$ ,  $n_2 = 0.1$ ,  $n_3 = 0.02439$ ,  $m_1 = 0.03566$ ,  $m_2 = 0.03566$ ,  $m_3 = 0.00256$ ,  $K = 25000$ . These values were obtained based on data published about the use of the egg parasitoid *Trichogramma galloi* against the sugarcane borer *Diatraea saccharalis* [3, 5, 7]. Figure 1 shows the population oscillations for  $r = 0.1908$  and  $\beta = 0.0001723$  without control.

According to [10], economic injury level (EIL) cause economic damage. Economic threshold (ET) is population density at which control measures should be determined to prevent an increasing pest population from reaching the economic injury level. One can see from Figure 1 that the sugarcane borer larvae density  $x_3$  takes on values more than the EIL for this pest  $x_{\text{EIL}} = 2500$  numbers/ha [3]. In this case, it is necessary to apply the biological control.

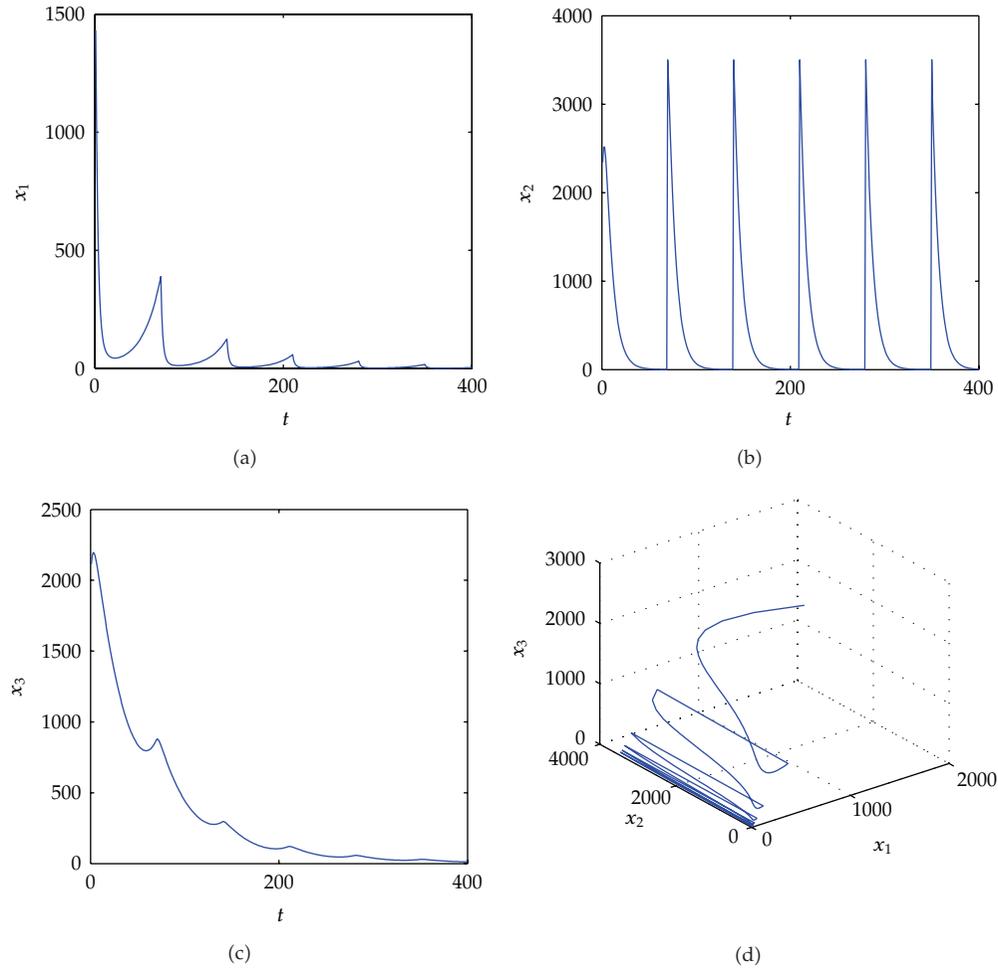


**Figure 1:** Evolution of the egg (a), parasitized egg (b), larvae populations (c), and phase portraits (d) of system (1.2) without control.

From Theorem 3.1, we have shown that the pest-eradication periodic solution  $(0, \tilde{x}_2(t), 0)$  of the system (1.2) is globally asymptotically stable if the condition (3.1) holds

$$p > p_{\min} = \frac{(r - m_1 - n_1)(m_2 + n_2)\tau}{\beta}. \quad (4.1)$$

Choosing  $\tau = 70$  days from (4.1), we derive that when  $p > p_{\min} = 3027$  parasitoids/ha, the pest-eradication periodic solution of the host-parasitoid system is asymptotically stable. Dynamical behavior of the system with impulsive control  $p = 3500$  parasitoids/ha and with economic threshold  $x_{ET} = 2000$  is shown in Figure 2. We can conclude that this control strategy seems successful because the larvae population of the sugarcane borer goes to extinction. But the aim of the biological control is not to eliminate all larvae population. The aim of the biological control of the sugarcane borer is to keep the larvae population at an acceptable low level (below the EIL) that indicates the pest densities at which applied biological control is economically justified.



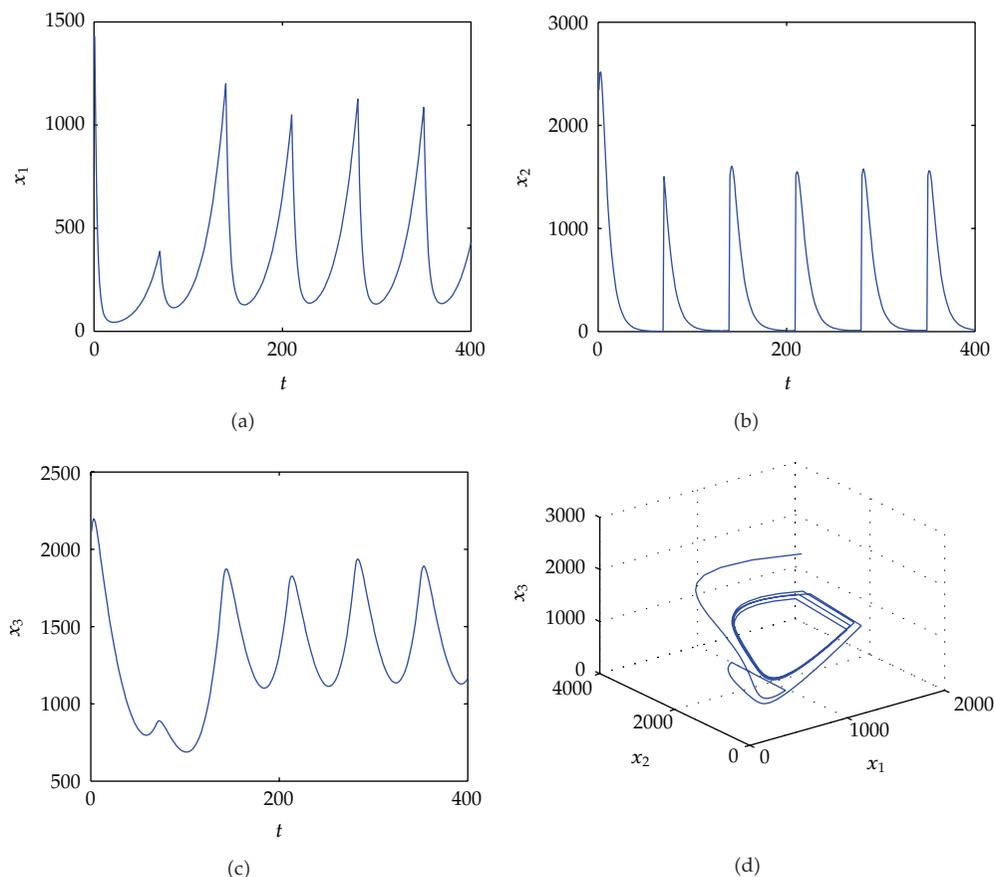
**Figure 2:** Evolution of the egg (a), parasitized egg (b), larvae populations (c), and phase portraits (d) of system (1.2) for  $p = 3500$  parasitoids/ha.

Choosing the release amount  $p = 1500$  parasitoids/ha, we can control the larvae population and keep it below the EIL (see Figure 3). It is obvious that the cost of the control strategy  $p = 1500$  is less than the cost of  $p = 3500$ .

Applying the control strategy  $p = 1000$ , we can see that the number of larvae individuals exceed  $x_{EIL}$  at some time (see Figure 4).

## 5. Discussion and Conclusion

In this paper, we suggest a system of impulsive differential equations to model the process of the biological control of the sugarcane borer by periodically releasing its parasitoids. By using the Floquet theory and small amplitude perturbation method, we have proved that for any fixed period  $\tau$  there exists a globally asymptotically stable pest-eradication periodic solution



**Figure 3:** Evolution of the egg (a), parasitized egg (b), larvae populations (c), and phase portraits (d) of system (1.2) for  $p = 1500$  parasitoids/ha.

$(0, \tilde{x}_2(t), 0)$  of the system (1.2) if the number of the parasitoids in periodic releases is greater than some critical value  $p_{\min}$ .

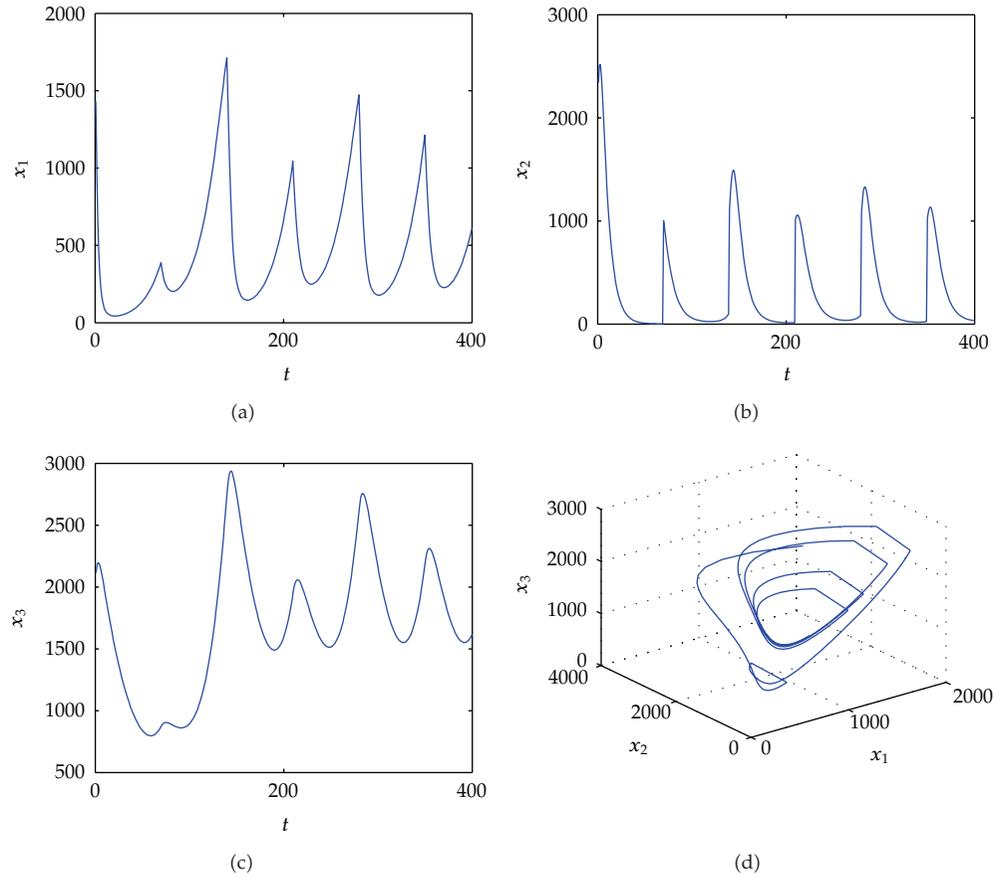
When the stability of the pest-eradication periodic solution is lost, the numerical results show that the system (1.2) has rich dynamics.

If we choose the biological control strategy by periodical releases of the constant amount of parasitoids, the results of Theorem 3.1 can help in designing the control strategy by informing decisions on the timing of parasitoid releases. In this case, from (3.1) we have

$$\tau < \tau_{\max} = \frac{\beta p}{(r - m_1 - n_1)(m_2 + n_2)}. \quad (5.1)$$

From (5.1) we can conclude that there exists a globally asymptotically stable pest-eradication periodic solution  $(0, \tilde{x}_2(t), 0)$  of the system (1.2) if the impulsive period is less than some critical value  $\tau_{\max}$ .

It is interesting to discuss the result of Theorem 3.1, comparing the condition of the pest-free globally stable solution (3.1) with the similar result presented in [10]. In their paper,



**Figure 4:** Evolution of the egg (a), parasitized egg (b), larvae populations (c), and phase portraits (d) of system (1.2) for  $p = 1000$  parasitoids/ha.

the authors considered integrated pest management strategies based on discrete-time host-parasitoid models. Integrated pest management (IPM) is a long-term control strategy that combines biological, cultural, and chemical tactics to reduce pest populations to tolerable levels when the pests reach the ET [17]. IPM control strategy is not used in sugarcane crops because it is impossible to kill the sugarcane borer larvae by insecticides when it builds internal galleries in the sugarcane plants. The biological control is unique strategy of the pest control in this case.

The resetting impulsive condition (3.1) which guarantees the global stability of the host-eradication periodic solution becomes

$$r - m_1 - n_1 < \frac{\beta p}{\tau (m_2 + n_2)}, \quad (5.2)$$

which means that if the intrinsic growth rate is less than the mean parasitization rate over period  $\tau$ , then the host population will become extinct eventually. This is the same conclusion which Tang et al. presented in [10] based on the inequality (3.9) from [10]. Then,

the inequality (5.2) for the continuous-time model (1.2) and the inequality (3.9) for the discrete-time host-parasitoid model from [10] lead to similar results.

Thus, the results of the present study show that the impulsive release of the parasitoids provides reliable strategies of the biological pest control of the sugarcane borer.

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