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Research Article

Unknown Input Observer Design for Fuzzy Bilinear System: An LMI Approach

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A new method to design a fuzzy bilinear observer (FBO) with unknown inputs is developed for a class of nonlinear systems. The nonlinear system is modeled as a fuzzy bilinear model (FBM). This kind of T-S fuzzy model is especially suitable for a nonlinear system with a bilinear term. The proposed fuzzy bilinear observer subject to unknown inputs is developed to ensure the asymptotic convergence of the error dynamic using the Lyapunov method. The proposed design conditions are given in linear matrix inequality (LMI) formulation. The paper studies also the problem of fault detection and isolation. An unknown input fuzzy bilinear fault diagnosis observer design is proposed. This work is given for both continuous and discrete cases of fuzzy bilinear models. Illustrative examples are chosen to provide the effectiveness of the given methodology.

1. Introduction

In the recent past decades, there has been important increasing interest in the state observer design of dynamic systems subjected to unknown inputs that play an essential role in robust model-based fault detection. The case of unknown input linear system has been considered by different authors [1–4], and many types of full order and reduced order unknown input observers (UIOs) are now available.

On the other hand, since bilinear systems present the main advantage of representing an intermediate structure between linear and nonlinear models, a considerable attention has been paid for the study of this class of process [5, 6]. The observer design for bilinear systems with unknown input has been an important research topic during the last years. The works of [7, 8] have considered the design of UIO for bilinear systems in which the error estimation dynamics is linear. Under suitable transformation, the design of UIO for bilinear systems proposed in [9, 10] is equivalent to the design of UIO for linear systems in which the unknown input links with the non measurable states.

However, many physical systems are nonlinear in nature. For such system, the use of the well-known linear techniques may reduce in bad performance and even instability. Generally, analysis for nonlinear systems is a quite involved procedure. In these last decades, a T-S fuzzy approach to represent or approximate a large class of nonlinear systems is developed [11–14]. Then, in the field of stability analysis and stabilization, many works including delay and uncertainty have been developed and applied in a lot of practical situations [15–20]. For the state estimation problems and its application in fault diagnosis for uncertain T-S fuzzy models, robust of fault detection filter is developed in LMI terms [21–25].

It is of importance to design observers for linear or nonlinear systems partially driven by unknown inputs [26–29]. Such a problem arises in systems subject to disturbances and in many applications such as robust control, fault detection and isolation (FDI), system supervision, and fault-tolerant control. The design of observers for nonlinear systems is a challenging problem and has received a considerable amount of attention in the literature. In many approaches, the transform of nonlinear systems to bilinear T-S models provides a better approximation than classical T-S models [30, 31]. Motivated by this, we consider in this work bilinear T-S fuzzy models whose consequent parts are bilinear systems with unknown inputs.

Considering the advantages of bilinear systems and fuzzy control, the fuzzy bilinear system (FBS) based on the T-S fuzzy model with bilinear rule consequence has attracted the interest of researchers [30, 32–35]. For example, robust stabilization for the T-S FBS has studied in [30, 33, 34], and extension to the T-S FBS with time delay is given in [35]. An adaptive fuzzy-bilinear-observer- (FBO-) based synchronization design for generalized Lorenz system (GLS) was also examined in [36], and in [32] an observer is designed using iterative procedure.

In this paper, we propose a novel approach of designing a fuzzy bilinear observer for a class of nonlinear system. The nonlinear system is modeled as a fuzzy bilinear model subject to unknown inputs. This kind of T-S fuzzy model is especially suitable for a nonlinear system with a bilinear term. The considered bilinear observer is obtained by a convex interpolation of unknown input bilinear observers. This interpolation is obtained throughout the same activation functions as the fuzzy bilinear model. Based on Lyapunov theory, the synthesis conditions of the given fuzzy observer are expressed in LMI terms. The design conditions lead to the resolution of linear constraints easy to solve with existing numerical tools. The given observer is then applied for fault detection. These results are provided for both continuous-time and discrete-time T-S bilinear models.

To the best of our knowledge, the FBO synthesis and fault diagnosis for fuzzy bilinear model subjected to unknown input seem not fully addressed in the past works. Moreover, in contrast with previous works, the proposed design is given in LMI formulation solved simultaneously.

This paper is organized as follows. In Section 2 the considered structure of the FBS is presented. In Section 3, the synthesis of fuzzy bilinear observers with unknown input in continuous and discrete cases is presented. Section 4 is devoted to the problem of fault detection by using unknown input fuzzy bilinear fault diagnosis observer. In Section 5, two examples to illustrate the proposed approach are proposed. The practical use of the theoretic study is illustrated by applying the proposed design to an isothermal continuous stirred tank reactor (CSTR).

Notation. In the rest of the paper, the following useful notation is used: $\Re$ denotes the set of real numbers, X^T denotes the transpose of the matrix X, $X \succ 0$ denotes symmetric positive definite matrix, X^{-1} denotes the Moore-Penrose inverse of X, and $\left(\begin{smallmatrix} A & * \\ B & C \end{smallmatrix}\right)$ denotes symmetric matrix where $(*) = B^T$. The operator δ denotes the time derivative for continuous-time models, that is, $\delta(x(t)) = \dot{x}(t)$, and the shift operator for discrete-time models, that is, $\delta(x(t)) = x(t+1)$. For simplicity, in the sequel, we will simply write $h_i(\xi(t)) = h_i(t)$.

2. General Structure of a Fuzzy Bilinear Model

In this section, fuzzy bilinear systems in the continuous and discrete-time cases are introduced. Indeed, the T-S fuzzy model is described by *if-then* rules and used to present a fuzzy bilinear system. The ith rule of the FBS for nonlinear systems is represented by the following form:

$$R^i : \text{if } \xi_1(t) \text{ is } F_{i1}, \ldots, \xi_g(t) \text{ is } F_{ig}, \tag{2.1}$$

then

$$\begin{gathered} \delta x(t) = A_i x(t) + B_i u(t) + N_i x(t) u(t), \\ y(t) = C x(t), \end{gathered} \tag{2.2}$$

where R^i denotes the ith fuzzy rule for all $i = \{1, \ldots, r\}$, r is the number of *if-then* rules, $\xi_i(t)$ are the premise variables assumed to be measurable, and $F_{ij}(\xi_j(t))$ is the membership degree of $\xi_j(t)$ in the fuzzy set F_{ij}. $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re$ is the control input, and $y(t) \in \Re^p$ is the system output. The matrices $A_i \in \Re^{n\times n}, B_i \in \Re^{n\times 1}$, $N_i \in \Re^{n\times n}$, $C \in \Re^{p\times n}$ are known matrices.

Then, the overall FBS can be described as follows:

$$\begin{gathered} \delta x(t) = \sum_{i=1}^{r} h_i(t)(A_i x(t) + B_i u(t) + N_i x(t) u(t)), \\ y(t) = C x(t) \end{gathered} \tag{2.3}$$

with $h_i(\xi(t)) = \mu_i(\xi(t)) / \sum_{j=1}^{r} \mu_j(\xi(t))$, $\mu_i(\xi(t)) = \prod_{j=1}^{g} F_{ij}(\xi(t))$ and $h_i(\cdot)$ verify the following properties:

$$\left.\begin{aligned} &\sum_{i=1}^{r} h_i(\xi(t)) = 1 \\ &0 \leq h_i(\xi(t)) \leq 1 \end{aligned}\right\} \quad \forall i \in \{1, 2, \ldots, r\}. \tag{2.4}$$

Remark 2.1. Matrices A_i, B_i, N_i, and C can be obtained by using the polytopic transformation [37]. The advantage of this method is (i) to lead to a bilinear transformation of the nonlinear model without any approximation error and (ii) to reduce the number of local models compared to other methods [33].

The following section is dedicated to the state estimation of the FBS (2.3), subject to unknown inputs, where the vector $\xi(t)$ is assumed depending on measurable variables.

3. Design of an Unknown Input Fuzzy Bilinear Observer

Considering an FBS subject to unknown inputs, the FBS (2.3) can be rewritten as follows:

$$\delta x(t) = \sum_{i=1}^{r} h_i(t)(A_i x(t) + B_i u(t) + N_i x(t)u(t) + F_i d(t)),$$
$$y(t) = Cx(t), \tag{3.1}$$

where $d(t) \in \Re^q$ is the unknown inputs vector and $F_i \in \Re^{n\times q}$ is a matrix with full column rank. In order to estimate state of (3.1), we propose a full-order observer of the form

$$\delta z(t) = \sum_{i=1}^{r} h_i(t)\left(H_i z(t) + L_i y(t) + J_i u(t) + M_i y(t)u(t)\right),$$
$$\hat{x}(t) = z(t) - Ey(t), \tag{3.2}$$

where $\hat{x}(t) \in \Re^n$ is the estimated state vector and the activation functions are the same as those used in FBS (3.1). H_i, M_i, L_i, J_i, and E are constant matrices with appropriate dimensions.

Our objective is to design T-S fuzzy bilinear observer of the fuzzy bilinear system (3.1) for system (3.1) subject to unknown input such that the estimation error

$$e(t) = \hat{x}(t) - x(t) \tag{3.3}$$

converges towards zero when $t \to \infty$. Note that estimation error can be rewritten as follows:

$$e(t) = z(t) - Tx(t), \tag{3.4}$$

where $T = I_n + EC$.

The dynamics of the state estimation error is governed by

$$\delta e = \sum_{i=1}^{r} h_i(t)(H_i e + (H_i T + L_i C - TA_i)x + (M_i C - TN_i)xu + (J_i - TB_i)u - TF_i d). \tag{3.5}$$

Hence, if the following constraints are satisfied

$$H_i T + L_i C - TA_i = 0, \tag{3.6}$$

$$M_i C - TN_i = 0, \tag{3.7}$$

$$J_i - TB_i = 0, \tag{3.8}$$

$$TF_i = 0, \tag{3.9}$$

$$T = I_n + EC, \tag{3.10}$$

the estimation error becomes

$$\delta e(t) = \sum_{i=1}^{r} h_i(\xi(t)) H_i e(t). \tag{3.11}$$

The parameter gains H_i, M_i, L_i, J_i, and E should be determined such that the state estimate $\hat{x}(t)$ converges asymptotically to system state $x(t)$.

3.1. Continuous-Time Case

The estimation error for continuous case is given by

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\xi(t)) H_i e(t). \tag{3.12}$$

The following theorem gives sufficient design conditions for the unknown inputs FBS (3.1).

Theorem 3.1. *If there exist a symmetric definite positive matrix P, matrices W_i, V_i, S, R_i such that the following linear conditions hold for all $i = 1, \ldots, r$:*

$$((P + SC)A_i - W_i C)^T + ((P + SC)A_i - W_i C) < 0, \tag{3.13}$$

$$R_i = (P + SC)B_i, \tag{3.14}$$

$$V_i C = (P + SC)N_i, \tag{3.15}$$

$$(P + SC)F_i = 0, \tag{3.16}$$

then the state estimation of the CFBO (3.2) *converges globally asymptotically to the state of the CFBS* (3.1). *The observer gains are determined by*

$$\begin{aligned} E &= P^{-1}S, \\ J_i &= P^{-1}R_i, \\ M_i &= P^{-1}V_i, \\ H_i &= (I_n + EC)A_i - P^{-1}W_iC, \\ L_i &= P^{-1}W_i - H_iE. \end{aligned} \tag{3.17}$$

Proof. In order to establish the stability of the estimation error $e(t)$, let us consider the following Lyapunov function:

$$V(t) = e^T(t)Pe(t), \quad P = P^T > 0. \tag{3.18}$$

Using (3.12), the derivative of the Lyapunov function (3.18) is given by

$$\dot{V}(t) = \sum_{i=1}^{r} h_i(t)\left(e^T(t)\left(H_i^T P + PH_i\right)e(t)\right). \tag{3.19}$$

From (3.6) and using (3.10), we get

$$H_i = TA_i - K_iC \tag{3.20}$$

with

$$K_i = H_iE + L_i. \tag{3.21}$$

Then, the derivative of the Lyapunov function is negative if

$$(TA_i - K_iC)^T P + P(TA_i - K_iC) < 0. \tag{3.22}$$

Taking into account (3.10) and considering the variable change:

$$S = PE, \tag{3.23}$$

$$W_i = PK_i, \tag{3.24}$$

we get the LMI (3.13). Taking account (3.10) and (3.23), equality (3.16) is derived from (3.9).

Similarly, using the following variable change:

$$R_i = PJ_i, \qquad V_i = PM_i, \tag{3.25}$$

we get equalities (3.14) and (3.15) from (3.8) and (3.7), respectively, which ends the proof. □

Remark 3.2. Classical numerical tools may be used for solving the LMI problem (3.13) subject to linear equality constraints (3.14)–(3.16). Solving this linear problem allows to deduce the observer parameters from P, W_i, V_i, S, and R_i as mentioned by (3.17).

3.2. Discrete-Time Case

For discrete-time case, the estimation error is given by

$$e(t+1) = \sum_{i=1}^{r} h_i(t) H_i e(t). \tag{3.26}$$

The following result gives linear conditions to design discrete-time unknown inputs DFBS (3.1).

Theorem 3.3. *If there exists a symmetric definite positive matrix P, matrices W_i, V_i, S, R_i such that the following linear conditions hold for all $i = 1, \ldots, r$*

$$\begin{bmatrix} P & * \\ (P+SC)A_i - W_i C & P \end{bmatrix} \succ 0, \tag{3.27}$$

$$R_i = (P+SC)B_i, \tag{3.28}$$

$$V_i C = (P+SC)N_i, \tag{3.29}$$

$$(P+SC)F_i = 0, \tag{3.30}$$

then the state estimation of the DFBO (3.2) *converges globally asymptotically to the state of the DFBS* (3.1)*. The observer gains are determined by* (3.17).

Proof. To prove the asymptotic convergence of the DFBS (3.1), sufficient conditions are derived using quadratic Lyapunov function (3.18). Indeed, the variation $\Delta V(t) = V(t+1) - V(t)$ along the solution of (3.26) is

$$\Delta V(t) = e^T(t)\left(H_i^T P H_i - P\right)e(t). \tag{3.31}$$

Sufficient conditions for the negativity of (3.31) are

$$H_i^T P H_i - P \prec 0. \tag{3.32}$$

Substituting (3.20) in (3.32), we obtain

$$(TA_i - K_iC)^T P(TA_i - K_iC) - P \prec 0. \tag{3.33}$$

Thus by taking account the expression of T (3.10), introducing the same variables change (3.23)-(3.24), and then applying Schur complement to (3.33), we get LMI conditions (3.27). Equality constraints (3.28)–(3.30) are obtained as previously mentioned which ends the proof. □

3.3. *Design Algorithm*

A design procedure to design FBO for both continuous and discrete cases is summarized as follows.

(1) Solve linear constraints (3.13)–(3.16) for continuous-time (or (3.27)–(3.30) for discrete-time) case to get W_i, V_i, S, R_i and $P > 0$.

(2) Deduce $K_i = P^{-1}W_i$.

(3) Knowing that $T = I + EC$, the observer gains are computed as follows:

$$\begin{aligned} E &= P^{-1}S, \\ H_i &= TA_i - K_iC, \\ L_i &= K_i - H_iE, \\ J_i &= TB_i, \\ M_i &= P^{-1}V_i. \end{aligned} \tag{3.34}$$

In the following, the proposed observer is used for fault detection and isolation of actuator fault.

4. Fault Detection and Isolation for Fuzzy Bilinear System

The fault detection and isolation problem for nonlinear systems is far more complicated. In this section, an unknown input fuzzy bilinear fault diagnosis observer is considered for nonlinear model in T-S fuzzy modeling. Based on proposed unknown inputs fuzzy bilinear observer, a fuzzy bilinear system affected by an actuator fault vector $f(t) \in \Re^{nf}$ is considered. In this section, a residual generation is considered in order to be sensitive to fault vector $f(t)$ and insensitive to the unknown inputs $d(t)$. Then, the considered system is as follows:

$$\begin{aligned} \delta x(t) &= \sum_{i=1}^{r} h_i(t) \begin{pmatrix} A_i x(t) + B_i u(t) + N_i x(t) u(t) \\ +F_i d(t) + G_i f(t) \end{pmatrix} \\ y(t) &= Cx(t), \end{aligned} \tag{4.1}$$

where $f(t)$ represents the vector of faults and the G_i represents matrix with appropriate dimensions.

The following unknown input fuzzy bilinear fault detection observer is proposed:

$$\delta z(t) = \sum_{i=1}^{r} h_i(t)\big(H_i z(t) + L_i y(t) + J_i u(t) + M_i y(t)u(t)\big),$$
$$r(t) = E_1 z(t) + E_2 y(t), \tag{4.2}$$

where $z(t)$ represents the estimated vector and $r(t)$ is the output signal called the residual.

The determination of gain matrices in (4.2) will be determined to ensure the convergence of the estimated errors. In order to describe the dynamic of unknown input fuzzy bilinear fault detection observer (4.2), the state estimation error is defined by $e(t) = z(t) - \overline{T}x(t)$. Then, from (4.1) and (4.2), we have

$$\delta e(t) = \sum_{i=1}^{r} h_i(t)\begin{pmatrix} H_i e(t) + \left(H_i\overline{T} + L_i C - \overline{T}A_i\right)x(t) - \overline{T}F_i d(t) \\ \left(M_i C - \overline{T}N_i\right)x(t)u(t) + \left(J_i - \overline{T}B_i\right)u(t) - \overline{T}G_i f(t) \end{pmatrix},$$
$$r(t) = E_1 e(t) + \left(E_1\overline{T} + E_2 C\right)x(t). \tag{4.3}$$

If the following conditions are satisfied:

$$H_i\overline{T} + L_i C - \overline{T}A_i = 0, \tag{4.4}$$

$$M_i C - \overline{T}N_i = 0, \tag{4.5}$$

$$J_i - \overline{T}B_i = 0, \tag{4.6}$$

$$\overline{T}F_i = 0, \tag{4.7}$$

$$E_1\overline{T} + E_2 C = 0, \tag{4.8}$$

we get

$$\delta e(t) = \sum_{i=1}^{r} h_i(\xi(t))\left(H_i e(t) - \overline{T}G_i f(t)\right),$$
$$r(t) = E_1 e(t). \tag{4.9}$$

Multiplying (4.8) by F_i we get

$$E_1\overline{T}F_i + E_2 C F_i = 0. \tag{4.10}$$

Taking into account the constraint (4.7), (4.10) becomes

$$E_2 C F_i = 0, \tag{4.11}$$

or equivalently

$$E_2 C\mathcal{F} = 0, \quad \mathcal{F} = [F_1, F_2, \ldots, F_r]. \tag{4.12}$$

Then we get

$$E_2 = \Omega\left(I_p - C\mathcal{F}(C\mathcal{F})^+\right), \tag{4.13}$$

where $(C\mathcal{F})^+$ is the pseudoinverse of $C\mathcal{F}$ and Ω is an arbitrary matrix. Substituting (4.13) into (4.8) leads to

$$E_1 T + \Omega C\left(I_n - \mathcal{F}(C\mathcal{F})^+ C\right) = 0. \tag{4.14}$$

A suitable choice of E_1 and $\overline{T}$ satisfying the relation (4.14) is

$$E_1 = -\Omega C, \tag{4.15}$$

$$\overline{T} = I_n - \mathcal{F}(C\mathcal{F})^+ C. \tag{4.16}$$

Then, the observer gains are obtained by the following result.

Theorem 4.1. *If there exist a symmetric definite positive matrix P, matrices Z_i, V_i, U_i such that the following linear conditions hold for all $i = 1, \ldots, r$*

$$Z_i^T + Z_i \prec 0 \ (\textit{for continuous case}), \tag{4.17}$$

or

$$\begin{gathered}\begin{bmatrix} P & * \\ Z_i & P \end{bmatrix} \succ 0 \ (\textit{for discrete case}), \\ Z_i\overline{T} + U_i C - P\overline{T}A_i = 0, \\ V_i C - P\overline{T}N_i = 0, \end{gathered} \tag{4.18}$$

then the state estimation of the FBO (4.2) *converges globally asymptotically to the state of the FBS* (4.1)*. The observer gains are determined by*

$$\begin{gathered} H_i = P^{-1}Z_i, \\ L_i = P^{-1}U_i, \\ M_i = P^{-1}V_i, \\ J_i = \overline{T}B_i, \end{gathered} \tag{4.19}$$

where $\overline{T}$, E_1, and E_2 are given in (4.16), (4.15), *and* (4.13)*, respectively.*

Proof. The proof of this result is similar to the one of Theorem 3.1. □

To illustrate the theoretical development and the design algorithm, numerical examples are proposed in the following section.

5. Simulation Examples

In this section, we consider two examples: the first is an academic example in discrete-time case, and the second is a physical model of an isothermal continuous stirred tank reactor (CSTR) for the Van de Vusse reactor system.

5.1. Example 1: Synthesis of a Discrete Fuzzy Bilinear Observer

Let us consider now the following discrete system defined by

$$\begin{gathered} x_1(t+1) = -0.2x_2 + 0.5x_3^3 + \left(0.5x_3^2 - 0.7x_1\right)u + 0.1d, \\ x_2(t+1) = (0.3 - x_3)x_1 - 0.1x_2 + (0.3 - 0.1x_2)u + 0.2d, \\ x_3(t+1) = (0.3 - x_3)x_2 - 0.6x_3 + (0.4 - x_1)u + 0.3d, \\ y(t) = Cx(t), \end{gathered} \tag{5.1}$$

where

$$C = \begin{bmatrix} 0 & 0 & 0.1 \\ -0.6 & -0.1 & -0.4 \end{bmatrix}. \tag{5.2}$$

This system can be written as

$$x(t+1) = A(x(t))x(t) + B(x(t))u(t) + Nx(t)u(t) + Fd(t) \tag{5.3}$$

with

$$\begin{gathered} A(x(t)) = \begin{bmatrix} 0 & -0.2 & 0.5x_3^2 \\ 0.3 - x_3 & -0.1 & 0 \\ 0 & 0.3 - x_3 & -0.6 \end{bmatrix}, \\ B(x(t)) = \begin{bmatrix} 0.5x_3^2 \\ 0.3 \\ 0.4 \end{bmatrix}, \qquad N = \begin{bmatrix} -0.7 & 0 & 0 \\ 0 & -0.1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \qquad F = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}. \end{gathered} \tag{5.4}$$

Using the polytopic transformation [37] with $-0.2 < x_3(t) < 0.2$ and $0 < x_3^2(t) < 0.33$, the DFBS can be described as (2.3)

$$\begin{gathered} x(t+1) = \sum_{i=1}^{4} h_i(x(t))(A_i x(t) + B_i u(t) + N_i x(t) u(t) + F_i d(t)); \\ y(t) = Cx(t), \end{gathered} \tag{5.5}$$

where

$$\begin{gathered} A_1 = \begin{bmatrix} 0 & -0.2 & 0.165 \\ 0.1 & -0.1 & 0 \\ 0 & 0.1 & -0.6 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -0.2 & 0 \\ 0.1 & -0.1 & 0 \\ 0 & 0.1 & -0.6 \end{bmatrix}, \\ A_3 = \begin{bmatrix} 0 & -0.2 & 0.165 \\ 0.5 & -0.1 & 0 \\ 0 & 0.5 & -0.6 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 0 & -0.2 & 0 \\ 0.5 & -0.1 & 0 \\ 0 & 0.5 & -0.6 \end{bmatrix}, \\ B_1 = B_3 = \begin{bmatrix} 0.165 \\ 0.3 \\ 0.4 \end{bmatrix}, \qquad B_2 = B_4 = \begin{bmatrix} 0 \\ 0.3 \\ 0.4 \end{bmatrix}, \\ N_1 = N_2 = N_3 = N_4 = \begin{bmatrix} -0.7 & 0 & 0 \\ 0 & -0.1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \\ F_1 = F_2 = F_3 = F_4 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}, \end{gathered} \tag{5.6}$$

$$\begin{gathered} h_1(x(t)) = F_{11}(x_3) \cdot F_{21}(x_3), \qquad h_2(x(t)) = F_{12}(x_3) \cdot F_{21}(x_3), \\ h_3(x(t)) = F_{11}(x_3) \cdot F_{22}(x_3), \qquad h_4(x(t)) = F_{12}(x_3) \cdot F_{22}(x_3) \end{gathered}$$

with

$$\begin{gathered} F_{11}(x_3) = \frac{x_3 + 1}{2}, \qquad F_{12}(x_3) = \frac{1 - x_3}{2}, \\ F_{21}(x_3) = \frac{x_3^2}{0.33}, \qquad F_{22}(x_3) = 1 - \frac{x_3^2}{0.33}. \end{gathered} \tag{5.7}$$

$u(t)$ is the input signal given in Figure 1, and $d(t)$ is the unknown input taken as a sine wave signal of amplitude 0.1 and frequency 50 rad/s.

Solving the design conditions (3.27)–(3.30), we get

$$P = \begin{bmatrix} 85.457 & 14.243 & 0 \\ 14.243 & 2.374 & 0 \\ 0 & 0 & 87.83 \end{bmatrix}. \tag{5.8}$$

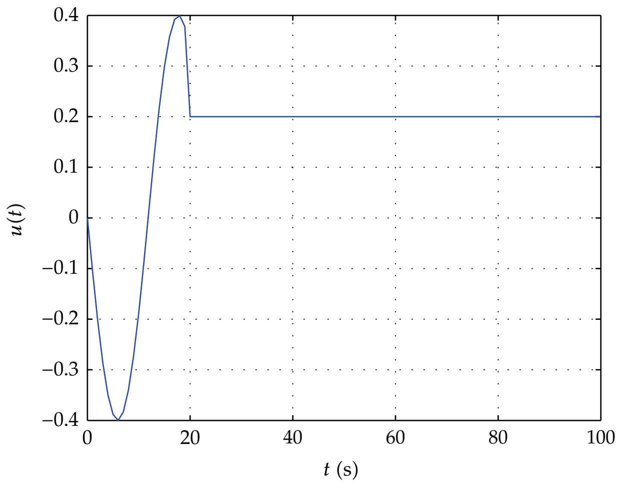


Figure 1: Input signal.

Therefore, the observer gains are computed from (3.17) as follows:

$$H_1 = H_2 = \begin{bmatrix} -0.018 & 0.07 & 0 \\ 0.107 & -0.418 & -0.001 \\ 0 & 0 & 0 \end{bmatrix},$$

$$H_3 = H_4 = \begin{bmatrix} -0.035 & 0.134 & 0 \\ 0.21 & -0.805 & -0.002 \\ 0 & 0 & 0 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.135 & 0.164 \\ -0.808 & -0.983 \\ 0 & 0 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.432 & 0.164 \\ -2.595 & -0.983 \\ 0 & 0 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 1.694 & 0.417 \\ -10.164 & -2.5 \\ 0 & 0 \end{bmatrix}, \qquad L_4 = \begin{bmatrix} 1.992 & 0.417 \\ -11.951 & -2.499 \\ 0 & 0 \end{bmatrix}, \tag{5.9}$$

$$J_1 = J_3 = \begin{bmatrix} -0.035 \\ 0.207 \\ 0 \end{bmatrix}, \qquad J_2 = J_4 = \begin{bmatrix} -0.005 \\ 0.029 \\ 0 \end{bmatrix},$$

$$M_1 = M_3 = \begin{bmatrix} 0.024 & 0.006 \\ -0.143 & -0.036 \\ 0 & 0 \end{bmatrix}, \qquad M_2 = M_4 = \begin{bmatrix} 0.024 & 0.006 \\ -0.143 & -0.036 \\ 0 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 9.226 & 1.967 \\ -15.357 & -1.805 \\ -10 & 0 \end{bmatrix}.$$

These parameters define completely the observer

$$z(t+1) = \sum_{i=1}^{4} h_i(x_3)\left(H_i z(t) + L_i y(t) + J_i u(t) + M_i y(t)u(t)\right),$$
$$\hat{x}(t) = z(t) - Ey(t). \tag{5.10}$$

To show the effectiveness of the designed observer, simulation results are presented in Figures 2, 3, and 4 for initial conditions given by $x_0 = [0.5\ \ 0.5\ \ 0.5]^T$ and $\hat{x}_0 = [1\ \ 1\ \ 1]^T$.

It can be deduced from Figures 2, 3, and 4 that the proposed observer succeeds to track the system trajectories in spite of the presence of the unknown input.

5.2. Example 2: Synthesis of a Continuous Fuzzy Bilinear Observer

In this subsection, we intend to apply the proposed design to an isothermal continuous stirred tank reactor (CSTR) (see, e.g., [38–40]).

The dynamics of CSTR for the Van de Vusse reactor can be described by the following nonlinear second order system:

$$\dot{x}_1 = -k_1 x_1 - k_3 x_1^2 + u(C_{A0} - x_1) + 0.6d,$$
$$\dot{x}_2 = k_1 x_1 - k_2 x_2 + u(-x_2) + d, \tag{5.11}$$
$$y = x_1 + x_2,$$

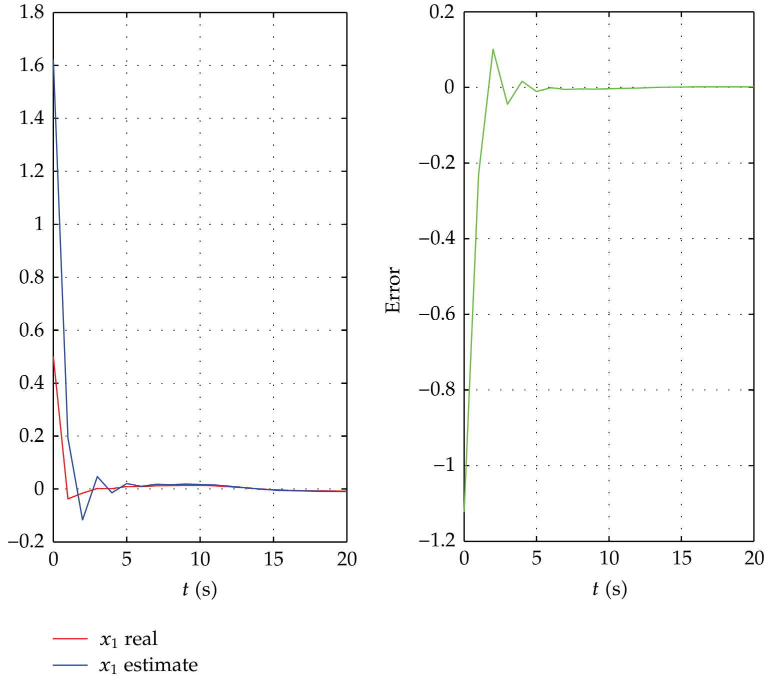
where the state x_1 represents the concentration of the reactant inside the reactor (mol/L) and the state x_2 is the concentration of the product in the CSTR output stream (mol/L). The output y determines the grade of the final product. The input-feed stream to the CSTR consists of a reactant with concentration C_{A0}, and the controlled input is the dilution rate $u = F/V(\mathrm{h}^{-1})$, where F is the input flow rate to the reactor (L/h) and V is the constant volume of the CSTR (liters). In all the following discussions, the kinetic parameters are chosen to be $k_1 = 50\,\mathrm{h}^{-1}$, $k_2 = 100\,\mathrm{h}^{-1}$, $k_3 = 10\,\mathrm{L/(molh)}$, $C_{A0} = 10\,\mathrm{mol/L}$, and $V = 1\,\mathrm{L}$ as in [39].

The system (5.11) can be written as

$$\dot{x}(t) = A(x(t))x(t) + Bu(t) + Nx(t)u(t) + Fd(t), \tag{5.12}$$

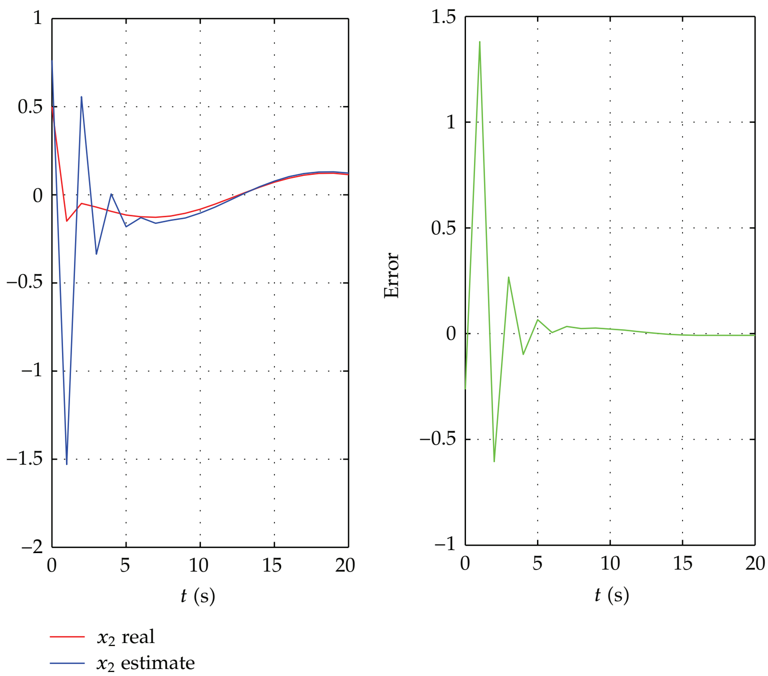
where

$$A(x(t)) = \begin{bmatrix} -k_1 - k_3 x_1 & 0 \\ k_1 & -k_2 \end{bmatrix}, \qquad B = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$
$$N = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad F = \begin{bmatrix} 0.6 \\ 1 \end{bmatrix} \tag{5.13}$$



(a) (b)

Figure 2: Trajectories of (a) x_1 and $\hat{x}_1$, (b) $\hat{x}_1 - x_1$.



(a) (b)

Figure 3: Trajectories of (a) x_2 and $\hat{x}_2$, (b) $\hat{x}_2 - x_2$.

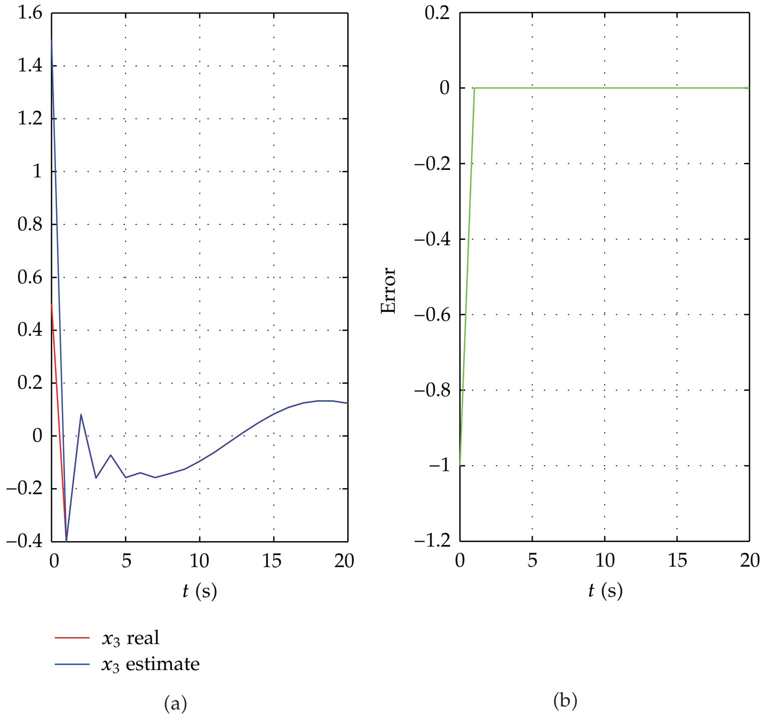


Figure 4: Trajectories of (a) x_3 and $\hat{x}_3$, (b) $\hat{x}_3 - x_3$.

with $x_1(t) \in [1, -1]$. This system can be represented using the polytopic transformation [37] as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(x_1(t))(A_i x(t) + B_i u(t) + N_i x(t) u(t) + F_i d(t))$$
$$y(t) = Cx(t), \tag{5.14}$$

where

$$A_1 = \begin{bmatrix} -60 & 0 \\ 50 & -100 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -40 & 0 \\ 50 & -100 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 10 \\ 0 \end{bmatrix}, \qquad N_1 = N_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{5.15}$$
$$F_1 = F_2 = \begin{bmatrix} 0.6 \\ 1 \end{bmatrix}, \qquad C = [1 \ \ 1],$$

$h_1(x_1(t)) = (1 - x_1(t))/2$, $h_2(x_1(t)) = (x_1(t) + 1)/2$.

5.2.1. Observer Design

The observer gains are obtained by solving design conditions (3.13)–(3.16), which lead to the following parameters:

$$P = \begin{bmatrix} 72.215 & 72.215 \\ 72.215 & 72.215 \end{bmatrix}. \tag{5.16}$$

Then the observer is completely defined from (3.17) by

$$\begin{gathered} H_1 = \begin{bmatrix} -57.470 & 56.231 \\ 56.970 & -56.732 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} -52.743 & 52.892 \\ 52.243 & -53.392 \end{bmatrix}, \\ L_1 = \begin{bmatrix} -8.175 \\ 8.175 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} -3.362 \\ 3.362 \end{bmatrix}, \\ J_1 = \begin{bmatrix} 4.063 \\ -4.063 \end{bmatrix}, \qquad J_2 = \begin{bmatrix} 4.061 \\ -4.061 \end{bmatrix}, \\ M_1 = \begin{bmatrix} 0.093 \\ -0.093 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 0.092 \\ -0.092 \end{bmatrix}, \\ E = \begin{bmatrix} -0.597 \\ -0.403 \end{bmatrix}. \end{gathered} \tag{5.17}$$

Then, the fuzzy bilinear state and their estimation are given in the following figures.

Figures 5 and 6 show, respectively, the evolution of the state variables x_1 and x_2 of the considered system and their corresponding observer estimation $\hat{x}_1$ and $\hat{x}_2$ with the input signal $u(t) = 4.5\sin(0.5\pi t)$ and the initial conditions $x_0 = [1 \ \ 1]^T$ and $\hat{x}_0 = [0.5 \ \ 0.5]^T$.

5.2.2. Residual Generation

In this paragraph, we will consider the same system of isothermal stirred tank reactor subject to actuator fault:

$$\begin{gathered} \dot{x}(t) = \sum_{i=1}^{2} h_i(t)\left(A_i x(t) + B_i u(t) + N_i x(t) u(t) + F_i d(t) + G_i f(t)\right) \\ y(t) = Cx(t) \end{gathered} \tag{5.18}$$

with A_i, B_i, N_i, F_i, and C being the same previous matrices and

$$\begin{gathered} G_1 = G_2 = [0.5 \ \ 0.5], \\ f(t) = \begin{cases} 50\sin\pi t, & \text{for } t \in [6 \ \ 8], \\ 0, & \text{elsewhere.} \end{cases} \end{gathered} \tag{5.19}$$

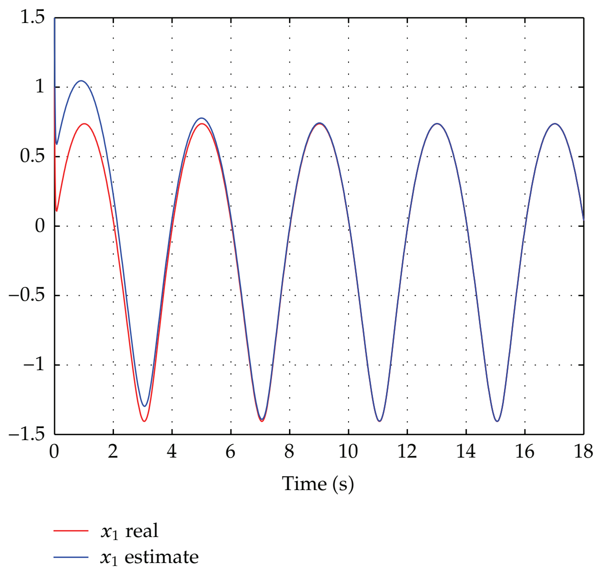


Figure 5: The state x_1 and its estimate.

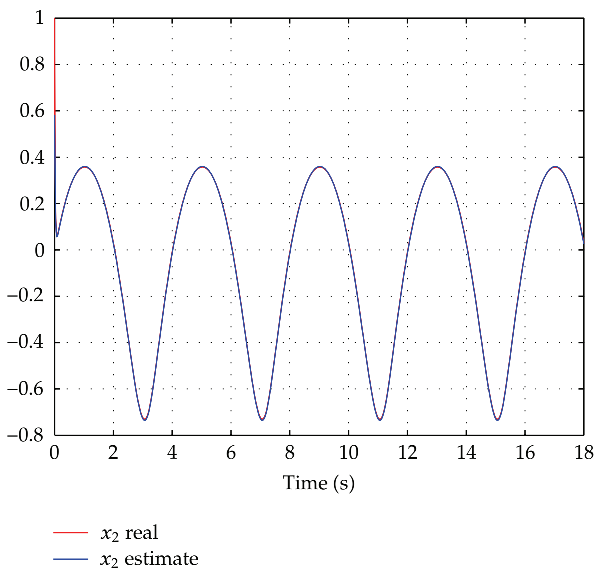


Figure 6: The state x_2 and its estimate.

Figure 7 displays the convergence of the residual corresponding to the actuator fault signal. One can see that the residual is almost zero throughout the time simulation run except at time $t = 6\,\mathrm{s}$ where it appears at the actuator fault and disappears at $t = 8\,\mathrm{s}$. Figure 7 shows that the residual $r(t)$ is sensitive to $f(t)$ and insensitive to $d(t)$. So the designed unknown input fuzzy bilinear fault diagnosis observer can be efficiently used to detect faults.

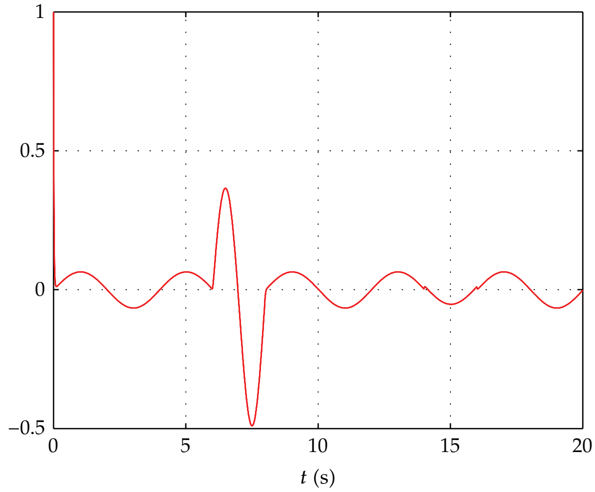


Figure 7: Residual signal $r(t)$.

6. Conclusion

In this paper, a bilinear observer design is proposed for a class of unknown inputs nonlinear system. Such design is based on a T-S fuzzy bilinear model representation, particularly suitable for a nonlinear system with a bilinear term. The proposed results are developed for both continuous-time and discrete-time cases. The synthesis conditions lead to the resolution of linear constraints easy to solve with existing numerical tools. The proposed unknown input bilinear observer structure is applied for fault detection. Two illustrative examples are also given.

Based on the results in the paper, interesting future studies may be extended the proposed technique to uncertain fuzzy bilinear systems or fuzzy bilinear systems with time-delay, and can also be considered the problem of pole placements to improve the performance of the proposed fuzzy bilinear observer.

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