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Research Article

Abstract Description of Internet Traffic of Generalized Cauchy Type

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Self-similar process with long-range dependence (LRD), that is, fractional Gaussian noise (fGn) with LRD is a widely used model of Internet traffic. It is indexed by its Hurst parameter H_{fGn} that linearly relates to its fractal dimension D_{fGn}. Note that, on the one hand, the fractal dimension D of traffic measures local self-similarity. On the other hand, LRD is a global property of traffic, which is characterized by its Hurst parameter H. However, by using fGn, both the self-similarity and the LRD of traffic are measured by H_{fGn} . Therefore, there is a limitation for fGn to accurately model traffic. Recently, the generalized Cauchy (GC) process was introduced to model traffic with the flexibility to separately measure the fractal dimension D_{GC} and the Hurst parameter H_{GC} of traffic. However, there is a fundamental problem whether or not there exists the generality that the GC model is more conformable with real traffic than single parameter models, such as fGn, *irrelevant of traffic traces used in experimental verification*. The solution to that problem remains unknown but is desired for model evaluation in traffic theory or for model selection against specific issues, such as queuing analysis relating to the autocorrelation function (ACF) of arrival traffic. The key contribution of this paper is our solution to that fundamental problem (see Theorem 3.17) with the following features in analysis. (i) Set-valued analysis of the traffic of the fGn type. (ii) Set-valued analysis of the traffic of the GC type. (iii) Revealing the generality previously mentioned by comparing metrics of the traffic of the fGn type to that of the GC type.

1. Introduction

This paper explores the Internet traffic (traffic for short) modeling which plays a role in telecommunications [1]. Let $x[t(i)]$ be an arrival traffic function, implying the number of bytes in the ith packet arriving at $t(i)$ $(i = 0, 1, 2, \ldots)$, where $t(i)$ is the timestamp of the ith packet [2]. To avoid confusion, we use $x(t)$ and $x(i)$ to represent a traffic time series in

the continuous case and the discrete one, respectively, where $x(i)$ implies the size of the ith packet. Note that traffic statistics for $x(i)$ corresponds with the statistics of the traffic time series represented by either byte count or packet size [3].

The pioneer in stochastic modeling of traffic refers to the Danish scientist A. K. Erlang, see Bojkovic et al. [4]. As early as the 1920s, he contributed to his experimental work on the statistics of the traffic in telephony networks and introduced the traffic models of the Poisson type [5, 6]. Erlang's work was so successful in characterizing the old telephony traffic such that it was applied as a law in traffic engineering, see for example, Yue et al. [7], Papoulis [8], Gibson [9], Cooper [10], Pitts and Schormans [11], and McDysan [12]. Note that the autocorrelation function (ACF) of the traffic of the Poisson type, which is Markovian, is exponentially decayed [13]. In fact, the ACF of a Markov process decays exponentially [14]. The Poisson-type models fit in with the traffic in old telephony networks, which are circuit-switched, see for example, [9], Le Gall [15], Lin et al. [16], Manfield and Downs [17], Reiser [18], and Lu [19]. Those types of models, however, fail to effectively characterize the traffic in the Internet, which is packet switched. As a matter of fact, the ACF, the probability density function (PDF), and the power spectrum density (PSD) function of traffic, follow power law, see for example, Resnick [20], Csabai [21], Leland et al. [22], Beran et al. [23], López-Ardao et al. [24], and Cleveland and Sun [25]. Therefore, system responses to the Internet have to take into account the arrival traffic with long-range dependence (LRD), see for example, Tsybakov and Georganas [26], Norros [27], Fishman and Adan [28], Li and Zhao [29], Dahl and Willemain [30], and Kingman [31].

Theoretically, on one hand, Taqqu's Theorem relates a heavy-tailed PDF in power law to a hyperbolically decayed ACF, that is, power law-type ACF [3, 32]. On the other hand, the Fourier transform connects a hyperbolically decayed ACF with $1/f^{\alpha}$ ($\alpha > 0$) noise (power law-type PSD), see for example, Li [33].

Note that, before the Internet's worldwide prevalence, in the seventies of the last century, Tobagi et al. [34] reported a noticeable behavior of traffic, which is called "burstiness" [12]. It implies that there would be no packets transmitted for a while, then flurry of transmission, no transmission for another long period of time, and so on if one observes traffic over a long period of time. This also means that traffic has intermittency. In 1986, Jain and Routhier [35] further described the intermittency or burstiness of traffic using the term "packet trains." They inferred that traffic is neither a Poisson process nor a compound Poisson one [35]. The results in [34, 35] are quite qualitative but they may be considered as early work with respect to fractal-type traffic. The concept of packet train is interesting [36] but we utilize the concept of fractal time series for traffic modeling in this paper.

The early literature quantitatively describing the statistical properties of traffic from a view of fractals refers to Csabai [21], Leland et al. [22], Beran et al. [23], Paxson and Floyd [37], and Crovella and Bestavros [38]. Those scientists revealed some of the main properties of traffic, such as LRD and asymptotic self-similarity. The traffic model described in [22, 23, 37, 39–43], just citing a few, is the fGn that was introduced by Mandelbrot and Van Ness in mathematics [44].

The model of fGn is characterized by a single parameter H, called the Hurst parameter. Its limitation in accurately modeling traffic was noticed by Paxson and Floyd [37], and Tsybakov and Georganas [39]. Paxson and Floyd noted that "it might be difficult to characterize the correlations over the entire traffic traces with a single Hurst parameter [37, Section 7.4]." They suggested that "further work is required to fully understand the correlational structure of wide-area traffic [37]." Tsybakov and Georganas remarked that "the class of exactly self-similar processes, that is, fGn or fractional Brownian motion (fBm),

is too narrow for modeling actual network traffic [39, Section II]." The authors of [37, 39] qualitatively stated the limitation of fGn in traffic modeling without mentioning how to release the limitation. In this regard, Beran [45, page 101-102] suggested to develop a sufficiently flexible class of parametric correlation models. The key of the Beran's idea implies that the ACF of an LRD series may be fitted by a correlation model with several parameters instead of one, but he did not mention what concrete parametric correlation models are.

Li and Lim recently reported a two-parameter traffic model called the GC process with the demonstrations based on sets of real-traffic traces in [46, 47]. Li [48] discussed its simulation. Nevertheless, whether or not it has the generality to be more agreement with traffic than single parameter models, such as fGn, remains an unsolved problem. Therefore, it may be useful, especially for traffic engineers, to exhibit that generality. Motivated by this, we, in this paper, aim at presenting a solution to it based on the abstract analysis, more precisely, the set-valued analysis in Hilbert spaces, to thoroughly reveal that generality, irrelevant of traces used in experimental verification. To the best of our knowledge, the set-valued analysis of traffic models is rarely seen.

The rest of paper is organized as follows. Related work is explained in Section 2. The set-valued analysis is presented in Section 3. An application case is demonstrated in Section 4. Discussions are provided in Section 5, followed by our conclusions.

2. Related Work

We first respectively brief the ACFs of the fGn and the GC process. Then, fractal dimension and the Hurst parameter are discussed.

2.1. fGn

The continuous fGn is the derivative of the smoothed fractional Brownian motion (fBm) in the sense of the generalized functions over the Schwartz space of test functions, refer to Kanwal [49] for generalized functions.

Denote by $r_{\mathrm{gGn}}(\tau)$ the ACF of the fGn as the increment process of the fBm of the Weyl type. Then, for time lag $\tau \in R$, which is the set of real numbers,

$$r_{\mathrm{gGn}}(\tau) = \frac{\sigma^2 \varepsilon^{2H-2}}{2}\left[\left(\frac{|\tau|}{\varepsilon}+1\right)^{2H} + \left|\frac{|\tau|}{\varepsilon}-1\right|^{2H} - 2\left|\frac{\tau}{\varepsilon}\right|^{2H}\right], \tag{2.1}$$

where $H \in (0,1)$ is the Hurst parameter, $\varepsilon > 0$ is used by smoothing the fBm so that the smoothed fBm is differentiable, and $\sigma^2 = (H\pi)^{-1}\Gamma(1-2H)\cos(H\pi)$ [44]. The PSD of fGn is given by [50]

$$S_{\mathrm{fGn}}(\omega) = \sigma^2 \sin(H\pi)\Gamma(2H+1)|\omega|^{1-2H}, \tag{2.2}$$

where ω is angular frequency.

FGn includes three classes of time series. When $H \in (0.5,1)$, $r_{\mathrm{gGn}}(\tau)$ is positive and finite for all τ. It is nonintegrable and the corresponding series is LRD. For $H \in (0,0.5)$, the integral of $r_{\mathrm{gGn}}(\tau)$ is zero, corresponding series with short-range dependence (SRD). Besides

$r_{\mathrm{gGn}}(\tau)$ for $0 < H < 0.5$ changes its sign and becomes negative for some τ proportional to ε. It reduces to the white noise when $H = 0.5$.

The ACF of fGn in the discrete case is given by

$$r_{\mathrm{fGn}}(k) = 0.5\sigma^2\left[(|k|+1)^{2H} - 2|k|^{2H} + (||k|-1|)^{2H}\right], \tag{2.3}$$

where $k \in I$, where I is the set of integers. To avoid confusion, we often consider ACFs for $k \geq 0$ in the normalized case in what follows as an ACF is an even function. Thus, for $k \geq 0$, one has

$$r_{\mathrm{fGn}}(k) = 0.5\left[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}\right]. \tag{2.4}$$

Considering the right side of (2.4) as the finite 2-order difference of $0.5(k)^{2H}$ and approximating it with the 2-order differential of $0.5(k)^{2H}$ yields the following equation. Its right side is quite accurate to the left for $k > 10$ [51]:

$$0.5\left[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}\right] \approx H(2H-1)(k)^{2H-2}. \tag{2.5}$$

2.2. GC Process

A random function $x(t)$ is called the GC process if it is stationary Gaussian with the ACF given by

$$r_{\mathrm{GC}}(\tau) = E[X(t+\tau)X(t)] = \left(1+|\tau|^{\alpha}\right)^{-\beta/\alpha}, \tag{2.6}$$

where $0 < \alpha \leq 2$ and $\beta > 0$. When $\alpha = \beta = 2$, one gets the usual Cauchy process the ACF of which is expressed by

$$r_{\mathrm{C}}(\tau) = \left(1+|\tau|^{2}\right)^{-1}, \tag{2.7}$$

which is used in geostatistics, see Chilès and Delfiner [52].

The PSD of the GC process is given by (see [47])

$$\begin{aligned} S_{\mathrm{GC}}(\omega) = {} & \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma((\beta/\alpha)+k)}{\pi\Gamma(\beta/\alpha)\Gamma(1+k)} I_1(\omega) * \mathrm{Sa}(\omega) \\ & + \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma((\beta/\alpha)+k)}{\pi\Gamma(\beta/\alpha)\Gamma(1+k)} [\pi I_2(\omega) - I_2(\omega) * \mathrm{Sa}(\omega)], \end{aligned} \tag{2.8}$$

where $\mathrm{Sa}(\omega) = \sin(\omega)/\omega$ and

$$I_1(\omega) = -2\sin\left(\frac{\alpha k\pi}{2}\right)\Gamma(\alpha k+1)|\omega|^{-\alpha k-1},$$
$$I_2(\omega) = 2\sin\left[\frac{(\beta+\alpha k)\pi}{2}\right]\Gamma\left[1-(\beta+\alpha k)\right]|\omega|^{(\beta+\alpha k)-1}. \tag{2.9}$$

The PSD of the GC process for $\omega \to 0$ is given by (see [53])

$$S_{\mathrm{GC}}(\omega) \sim \frac{1}{\Gamma(\beta)\cos(\beta\pi/2)}|\omega|^{\beta-1}, \quad \omega \longrightarrow 0. \tag{2.10}$$

On the other hand, $S_{\mathrm{GC}}(\omega)$ for $\omega \to \infty$ is given by

$$S_{\mathrm{GC}}(\omega) \sim \frac{\beta\Gamma(1+\alpha)\sin(\alpha\pi/2)}{\pi\alpha}|\omega|^{-(1+\alpha)}, \quad \omega \longrightarrow \infty. \tag{2.11}$$

The above exhibits the power law of $S_{\mathrm{GC}}(\omega)$. The GC process is LRD if $0 < \beta < 1$ and is SRD if $1 < \beta$.

As noted in [53], "the GC process is non-Markovian since $r_{\mathrm{GC}}(t_1, t_2)$ does not satisfy the triangular relation given by

$$r_{\mathrm{GC}}(t_1, t_3) = \frac{r_{\mathrm{GC}}(t_1, t_2)r_{\mathrm{GC}}(t_2, t_3)}{r_{\mathrm{GC}}(t_2, t_2)}, \quad t_1 < t_2 < t_3, \tag{2.12}$$

which is a necessary condition for a Gaussian process to be Markovian, see Todorovic [54]." In fact, up to a multiplicative constant, the Ornstein-Uhlenbeck process is the only stationary Gaussian Markov process, see Lim and Muniandy [55] and Wolpert and Taqqu [56].

2.3. Fractal Dimension and the Hurst Parameter

On the one hand, fractal dimension, denoted by D, of traffic $x(t)$ is a measure to characterize its local self-similarity or irregularity. On the other hand, the Hurst parameter H is used to measure its statistical dependence, see Mandelbrot [57]. Thus, we respectively use D and H to describe the local property and the global property of $x(t)$, see Li and Lim [46, 47] and Li and Zhao [58]. In fact, if the ACF $r_{xx}(\tau)$ is sufficiently smooth on $(0, \infty)$ and if

$$r_{xx}(0) - r_{xx}(\tau) \sim c_1|\tau|^{\alpha} \quad \text{for } |\tau| \longrightarrow 0, \tag{2.13}$$

where c_1 is a constant and α is the fractal index of $x(t)$, D of $x(t)$ is expressed by

$$D = 2 - \frac{\alpha}{2}, \tag{2.14}$$

see, for example, Kent and Wood [59], Hall and Roy [60], Chan et al. [61], and Adler [62]. Applying the binomial series to $r_{\mathrm{fGn}}(\tau)$ yields

$$r_{\mathrm{fGn}}(0) - r_{\mathrm{fGn}}(\tau) \sim c|\tau|^{2H} \quad \text{for } |\tau| \longrightarrow 0. \tag{2.15}$$

Therefore, one has

$$D_{\mathrm{fGn}} = 2 - H_{\mathrm{fGn}}. \tag{2.16}$$

Consequently, the fGn, as the incremental process of the fBm of the Weyl type, is stationary. Its D happens to linearly relate to its H, see [57, page 27] and Gneiting and Schlather [63]. Hence, a single parameter model fails to separately capture the local irregularity and the LRD of traffic.

Recall that a self-similar process $x(t)$ with the self-similarity index κ requires for $a > 0$,

$$x(at) =_d a^{\kappa} x(t), \tag{2.17}$$

where $=_d$ denotes equality in joint finite distribution. It is known that a stationary Gaussian random function $x(t)$ that is not exactly self-similar may satisfy a weaker self-similar property known as local self-similarity. Taking into account the definition of the local self-similarity provided in [59–62], we say that a Gaussian stationary process is locally self-similar of order α if its ACF satisfies for $\tau \to 0$,

$$r_{xx}(\tau) = 1 - \frac{\beta}{\alpha}|\tau|^{\alpha}\left\{1 + O\left(|\tau|^{\alpha}\right)\right\}, \quad \alpha > 0. \tag{2.18}$$

The fractal dimension D of a locally self-similar process of order α is given by (2.14). Therefore, we have the asymptotic expressions given by

$$\begin{aligned} r_{\mathrm{GC}}(\tau) &\sim |\tau|^{\alpha}, \quad \tau \longrightarrow 0, \\ r_{\mathrm{GC}}(\tau) &\sim |\tau|^{-\beta}, \quad \tau \longrightarrow \infty. \end{aligned} \tag{2.19}$$

Note that traffic $x(t)$ is LRD if its ACF $r_{xx}(\tau)$ satisfies

$$r_{xx}(\tau) \sim |\tau|^{-b}, \quad \tau \longrightarrow \infty, \tag{2.20}$$

where $0 < b < 1$. Denote by D_{GC} and H_{GC} the fractal dimension and the Hurst parameter of traffic of the GC type, respectively. Then, according to (2.19), one has

$$\begin{aligned} D_{\mathrm{GC}} &= 2 - \frac{\alpha}{2}, \\ H_{\mathrm{GC}} &= 1 - \frac{\beta}{2}. \end{aligned} \tag{2.21}$$

Replacing α and β, respectively, by D_{GC} and H_{GC} according to (2.21), we have

$$r_{\mathrm{GC}}(\tau) = \left(1 + |\tau|^{4-2D_{\mathrm{GC}}}\right)^{-(1-H_{\mathrm{GC}})/(2-D_{\mathrm{GC}})}, \tag{2.22}$$

where D_{GC} is independent of H_{GC}. Thus,

$$D_{GC} \neq D_{\mathrm{fGn}}. \tag{2.23}$$

3. Set-Valued Analysis

A physically measured traffic trace has single history with finite length. Without losing generality, the maximum possible length of a traffic series is assumed as $N \in I_+(= 1, 2, \ldots)$. Let l_N^2 be a space containing all ACFs, including ACFs of real traffic. Let r be an ACF of a real-traffic series. Define the norm of r as an inner product given by

$$\|r\| = \sqrt{\langle r, r\rangle} = \sqrt{\sum_{k=0}^{N-1} |r|^2}. \tag{3.1}$$

Then, the inner space given by

$$l_N^2 = \left\{ r; \sqrt{\sum_{k=0}^{N-1} |r|^2} < \infty \right\} \tag{3.2}$$

is a Hilbert space when all limits are included [64, 65].

Remark 3.1. l_N^2 is a finite-dimensional normed space.

Now, we consider the following consequences of a linear normed space with finite dimensions.

Lemma 3.2. *In a linear finite-dimensional space, all norms are equivalent [66].*

Lemma 3.3. *Every finite-dimensional subspace of a linear normed space is closed [67].*

Lemma 3.4. *Let $\mathcal{H}$ be a Hilbert space and $\mathcal{M}$ be a closed subspace of $\mathcal{H}$. Let $x \in \mathcal{H}$, $x \notin \mathcal{M}$. Then there exists a unique element $\widehat{x} \in \mathcal{M}$ satisfying $\|x - \widehat{x}\| = \inf_{y\in\mathcal{M}}\|x - y\|$ [66, 67], Aubin [68].*

From the above, we obtain the following theorem. Its proof is straightforward according to Lemmas 3.2–3.4.

Theorem 3.5. *Let $r \in l_N^2$ be an ACF of a real-traffic series. Let $\mathcal{S}$ be a closed subspace of l_N^2. Then, there exists a unique $R \in \mathcal{S}$ such that $\|r - R\| = \inf_{s\in\mathcal{S}}\|r - s\|$ [64, 65].*

Let $e = R - r$ be the error. Its norm is defined by

$$\|e\| = \sqrt{\langle e, e\rangle} = \frac{1}{N}\sum_{k=0}^{N-1}|e|^2. \tag{3.3}$$

Let the functional of e be $F(e) = \|e\|$. Then, $F(e)$ is convex. Thus, the optimal approximation of r in $\mathcal{S}$ can be expressed by

$$R = \arg\min F(e), \quad r \in l_N^2,\ R \in \mathcal{S}. \tag{3.4}$$

Suppose R has m parameters such that

$$R(k) = R(k; a_1, a_2, \ldots, a_m). \tag{3.5}$$

Then, the error by taking the approximation (3.5) as a traffic model is a function of a_j ($j = 1, 2, \ldots, m$). To clarify this point, we utilize the cost function of m dimensions expressed by

$$J(a_1, a_2, \ldots, a_m) = \frac{1}{N}\sum_k [R(k) - r(k)]^2. \tag{3.6}$$

The partial derivative of J with respect to m parameters, which will be zero at the J minimum, yields [69]

$$\frac{\partial J}{\partial a_j} = \frac{2}{N}\sum_k (R - r)\frac{\partial R}{\partial a_j}, \quad j = 1, 2, \ldots, m. \tag{3.7}$$

Let $(a_{10}, a_{20}, \ldots, a_{m0})$ be the solution of $\partial J/\partial a_j = 0$. Then, $R(k; a_{10}, a_{20}, \ldots, a_{m0})$ is the optimal approximation of r in $\mathcal{S}$.

The above discussions draw attention to the fact that an optimal approximation of r in $\mathcal{S}$ may have m parameters. Obviously, an approximation of r is related to a subspace of l_N^2 as can be seen from Theorem 3.5. For this reason, we, below, consider the extensions of the fGn's ACF towards constructing the ACF of the GC process.

Definition 3.6. Let $\mathcal{H}$ be a Hilbert space equipped with a distance d. When K is a subset of $\mathcal{H}$, the distance from r to K is denoted by $d(r, K) = \inf_{s\in K} d(r, s)$ [70, 71].

Definition 3.7 (see [70]). Let $\{K_n\}(n \in I_+)$ be a sequence of subspaces of a Hilbert space $\mathcal{H}$. Then, the subset

$$\limsup_{n\to\infty} K_n = \left\{ r \in \mathcal{H} : \liminf_{n\to\infty} d(r, K_n) = 0 \right\} \tag{3.8}$$

is the upper limit of the sequence K_n. Besides, the subset

$$\liminf_{n\to\infty} K_n = \left\{ r \in \mathcal{H} : \lim_{n\to\infty} d(r, K_n) = 0 \right\} \tag{3.9}$$

is the lower limit of K_n. A subset K is said to be the limit or the set limit of K_n if

$$K = \liminf_{n\to\infty} K_n = \limsup_{n\to\infty} K_n = \lim_{n\to\infty} K_n. \tag{3.10}$$

Considering the above terms, one has the lemma below.

Lemma 3.8. *Any monotone sequence of subsets K_n has a limit [70].*

According to Lemma 3.8, therefore, the following holds.

Corollary 3.9 (see [70, 71]). *Let $\{K_n\}$ $(n \in I_+)$ be a family of increasing closed subspaces of a Hilbert space $\mathscr{H}$: $K_0 \subset K_1 \subset K_2 \subset \cdots$. Then,*

$$\begin{gathered} d(r;K_0) \geq d(r;K_1) \geq d(r;K_2) \geq \cdots, \\ \lim_{n\to\infty} d(r,K_n) = 0. \end{gathered} \tag{3.11}$$

We now turn to constructing the ACF of the GC process.

Corollary 3.10. $(c/2H(2H-1))[(\tau+1)^{2H} - 2\tau^{2H} + (\tau-1)^{2H}] \approx c(\tau+1)^{2H-2}$.

Proof. According to (2.5), this corollary results. □

Let

$$\mathcal{G} = \left\{ r; r = r_{\mathrm{fGn}},\ \|r\| = \sqrt{\sum_{k=0}^{N-1} |r|^2} < \infty \right\}, \qquad c > 0. \tag{3.12}$$

Then, $\mathcal{G}$ is the set containing the ACF of fGn. Therefore, we have the following remark.

Remark 3.11. $\mathcal{G} \subset l_N^2$. Besides, it is closed according to Lemma 3.3.

We now construct the second space. Let $\mathcal{GA}$ be the set containing ACFs of traffic in the form $c(|\tau|+1)^{2H-2}$ for $c > 0$. Then,

$$\mathcal{GA} = \left\{ r; r = c(|\tau|+1)^{2H-2}, \quad \|r\| = \sqrt{\sum_{k=0}^{N-1} |r|^2} < \infty \right\}. \tag{3.13}$$

According to Corollary 3.10, element in $\mathcal{GA}$ is an approximation of the ACF of fGn. Hence, we have $d(r;\mathcal{GA}) \approx d(r;\mathcal{G})$. Based on $\mathcal{GA}$, we further construct a space as follows.

Proposition 3.12. *The following $\mathcal{G}_{a_1}$ is an extension of $\mathcal{GA}$, where $c > 0$;*

$$\mathcal{G}_{a_1} = \left\{ r;\ r = c\left(|\tau|^{a_2}+1\right)^{2H-2},\ a_2 \in (0,1),\ \|r\| = \sqrt{\sum_{k=0}^{N-1} |r|^2} < \infty \right\}. \tag{3.14}$$

Proof. $(|\tau|^{a_2}+1)^{2H-2}$ equals to $(|\tau|+1)^{2H-2}$ for $a_2 = 1$, meaning $\mathcal{G}_{a_1} \supset \mathcal{GA}$. Thus, this proposition results. □

Remark 3.13. $(|\tau|^{a_2}+1)^{2H-2}$ is nonintegrable for $a_2(2-2H) \in (0,1)$ because $(|\tau|^{a_2}+1)^{2H-2} \sim |\tau|^{a_2(2H-2)}(\tau \to \infty)$. Clearly, $\mathcal{G}_{a_1} \subset l_N^2$. In addition, it is closed according to Lemma 3.3.

The space $\mathcal{G}_{a_1}$ can be further extended into the following.

Proposition 3.14. *The following $\mathcal{G}_{a_2}$ is an extension of $\mathcal{G}_{a_1}$;*

$$\mathcal{G}_{a_2} = \left\{ r; r = r_{GC},\ \|r\| = \sqrt{\sum_{k=0}^{N-1} |r|^2} < \infty \right\}, \tag{3.15}$$

where $r_{GC} = (|\tau|^{a_2}+1)^{-a_1}$, $a_1 > 0$, $a_2 \in (0,1)$, $a_1 a_2 \in (0,1)$.

Proof. $(|\tau|^{a_2}+1)^{2H-2}$ is a special case of $(|\tau|^{a_2}+1)^{-a_1}$ for $a_1 = 2-2H$, implying $\mathcal{G}_{a_2} \supset \mathcal{G}_{a_1}$. Thus, Proposition 3.14 holds. □

According to Proposition 3.14, therefore, we have the remarks below.

Remark 3.15. $(|\tau|^{a_2}+1)^{-a_1}$ is nonintegrable for $a_1 a_2 \in (0,1)$ because $(|\tau|^{a_2}+1)^{-a_1} \sim |\tau|^{a_1 a_2}$ $(\tau \to \infty)$. Clearly, $\mathcal{G}_{a_2} \subset l_N^2$. It is closed according to Lemma 3.3.

Remark 3.16. Proposition 3.14 presents a class of parametric ACF structures.

From the above, we have the theorem below.

Theorem 3.17. *Let $r \in l_N^2$ be an ACF of real traffic. Then,*

$$d(r;\mathcal{G}_{a_2}) \le d(r;\mathcal{G}_{a_1}) \le d(r;\mathcal{GA}). \tag{3.16}$$

Proof. Because $\mathcal{G}_{a_2} \supset \mathcal{G}_{a_1} \supset \mathcal{GA}$, Theorem 3.17 holds according to Corollary 3.9. □

Theorem 3.17 exhibits the generality of the GC process in accurate modeling of traffic. In what follows, we let $a_2 = \alpha$ and $a_1 = \beta/\alpha$ so as to be consistent with (2.6) in computations.

In the end of this section, we note that the purpose for using the abstract expression of m-parameter model (3.5) as well as (3.6) and (3.7) is simply to mention the concept of multiparameter model of ACF. For traffic, the GC model equipped with two parameters can be well explained because one parameter is the fractal index for local property and the other the LRD index for global one.

4. Application of Theorem 3.17 to Traffic Modeling

As an application of Theorem 3.17, we show the ACF modeling of $x(i)$ of real-traffic trace named by AMP-1131669938-1.psize, which was collected by the US National Laboratory for Applied Network Research (NLANR) in November 2005 [73]. We first model it in $\mathcal{G}_{a_2}$. Then, we compare it with that in $\mathcal{G}$ (i.e., fGn model). Because $d(r;\mathcal{GA}) \approx d(r;\mathcal{G})$, we use $d(r;\ \mathcal{G})$ in this section.

Denote the measured ACF of $x(i)$ by $r(k)$. Denote by $R\mathrm{gc}(k)$ and $R\mathrm{fgn}(k)$ the modeled ACFs in $\mathcal{G}_{a_2}$ and $\mathcal{G}$, respectively. Let $M^2(R\mathrm{gc}) = E[(R\mathrm{gc}-r)^2]$ be the mean square error (MSE) by using $R\mathrm{gc}(k)$ and $M^2(R\mathrm{fgn}) = E[(R\mathrm{fgn}-r)^2]$ be the MSE by using $R\mathrm{fgn}(k)$. For the sake of demonstration, we use (4.1) for the MSE in $\mathcal{G}_{a_2}$ and (4.2) for that in $\mathcal{G}$;

$$J1(\alpha,\beta) = \frac{1}{N}\sum_k \left[R\mathrm{gc}(k) - r(k)\right]^2, \tag{4.1}$$

$$J2(H) = \frac{1}{N}\sum_k \left[R\mathrm{fgn}(k) - r(k)\right]^2. \tag{4.2}$$

Figure 1(a) shows the first 2048 points of AMP-1131669938-1.psize. Figure 1(b) is the right part, that is, the part for $k \geq 0$, of the measured ACF $r(k)$ with the block size $L = 2048$ and average count = 30. By least squares fitting, we obtain the estimates $(\alpha_0, \beta_0) = (0.020, 0.028)$. Thus, we have

$$R\mathrm{gc}(k) = \left(k^{0.020} + 1\right)^{-0.028/0.020}, \tag{4.3}$$

with $M^2(R\mathrm{gc}) = 5.157 \times 10^{-5}$. Figure 1(c) shows $R\mathrm{gc}(k)$ and Figure 1(d) indicates that $R\mathrm{gc}(k)$ fits well with $r(k)$. Figure 2 illustrates the estimates of α and β. According to (2.21), H_{GC} and D_{GC} of that series equal to 0.986 and 1.990, respectively.

With least squares fitting in $\mathcal{G}$, however, we have

$$R\mathrm{fgn}(k) = 0.5\left[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}\right]_{H=0.930}, \tag{4.4}$$

with $M^2(R\mathrm{fgn}) = 1.347 \times 10^{-3}$. Figure 3(a) plots the $R\mathrm{fgn}(k)$ and Figure 3(b) shows the data fitting in $\mathcal{G}$. Figures 1(d) and 3(b) exhibit an application case of (3.16) in Theorem 3.17. Judging from them, it is obvious that the GC process is more effective with that trace for both short-term and long-term lags.

Purely from a view of curve fitting, the fitting accuracy of 10^{-3} in $\mathcal{G}$ may not be too large. The unsatisfactory point of the modeling in $\mathcal{G}$ is in two aspects. One is that $R\mathrm{fgn}(k)$ may overestimate autocorrelations of traffic for small lags (around the knee of the ACF curve). The other is that it may underestimate autocorrelations for large lags as evidenced by Figure 3(b), refer to Li and Lim [46] for more cases regarding modeling real-traffic traces in $\mathcal{G}_{a_2}$.

5. Discussion

A conventional method to assess whether a model is appropriate is goodness-of-fit test in statistics ([3, 69], Press et al. [74]). However, it still needs sets of traffic data involved in the test. In fact, experimental processing of specific sets of real traffic, no matter how many traces are involved in experimental verification or goodness-of-fit test, may not deterministically infer the generality of the GC process expressed by Theorem 3.17, theoretically speaking.

Recall that an ACF of arrival traffic has a considerable impact on queuing systems, see, for example, Hajek and He [75], Livny et al. [72], Li and Hwang [76, 77], Wittevrongel and Bruneel [78], and Geist and Westall [79]. Therefore, using the ACF of the arrival traffic

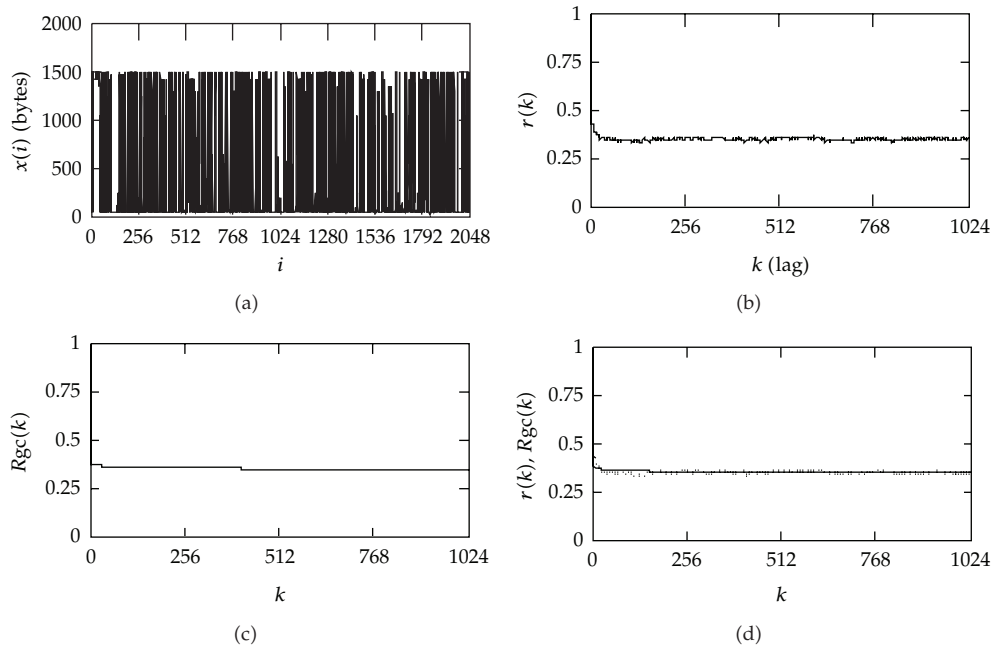

Figure 1: Modeling procedure using the GC process. (a) Traffic series: AMP-1131669938-1.psize. (b) Measured ACF $r(k)$. (c) Modeled ACF $R\text{gc}(k)$. (d) Fitting the data: dot line, $r(k)$; solid line, $R\text{gc}(k)$.

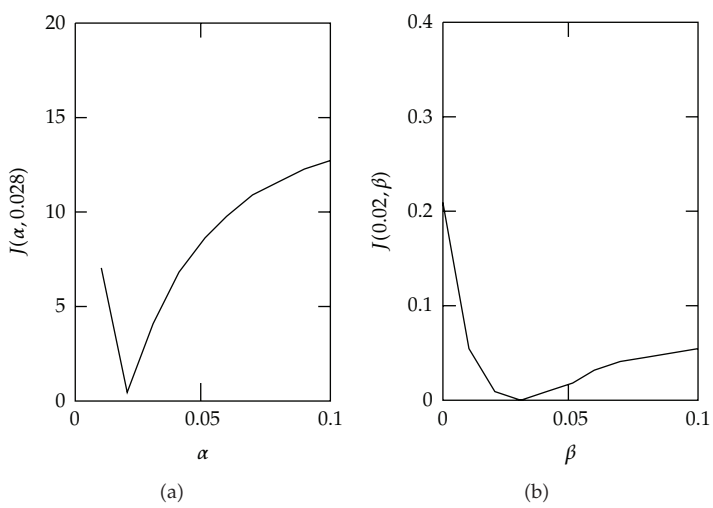

Figure 2: Estimations of α and β for $R\text{gc}(k)$ of AMP-1131669938-1.psize.

of the GC type may bring in considerable advances in practice, such as system analysis or evaluation, which we will work on in the future.

The GC model has one significance to separately characterize the local self-similarity and the LRD. In the case study in the previous section, we have $H_{\text{GC}} = 0.986$ and $D_{\text{GC}} = 1.990$ for AMP-1131669938-1.psize. Both H_{GC} and D_{GC} are of large value for this trace since $H_{\text{GC}} \in (0.5, 1)$ for LRD and $D_{\text{GC}} \in (1, 2)$. Note that a large value of H corresponds to strong

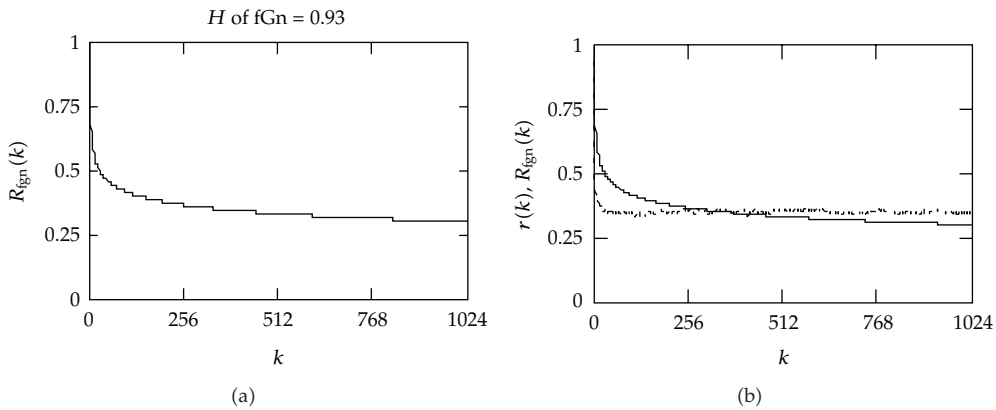

Figure 3: Modeling ACF of AMP-1131669938-1.psize in $\mathcal{G}$. (a) Modeled ACF in $\mathcal{G}$. (b) Fitting the data: dot line, $r(k)$; solid line, $R\text{fgn}(k)$.

LRD while a large value of D implies highly local irregularity. The phenomenon of traffic like this was demonstrated with more real-traffic traces in [46]. This phenomenon may not be satisfactorily observed using single parameter models, that is, fGn due to the restrictive relationship $D_{\text{fGn}} = 2 - H_{\text{fGn}}$.

The GC model has another significance to explain the complicated phenomenon of traffic, which was observed by Paxosn and Floyd [37] and Feldmann et al. [80], and which was stated like this. Traffic has robust long-term persistence at large time scales but high irregularity at small time scales. This phenomenon may be described by $\text{Var}[D(n)] > \text{Var}[H(n)]$, where $D(n)$ and $H(n)$ are the fractal dimension and the Hurst parameter of traffic in the nth interval on an interval-by-interval basis for $n = 1, 2, \ldots$, respectively. This complicated phenomenon of traffic can be well characterized by the GC model because H_{GC} is independent of D_{GC}, refer to [46] for the demonstrations of this phenomenon with real traffic. Again, we note that it may not be described by single parameter models, such as fGn. In fact, $\text{Var}[H_{\text{fGn}}(n)] = \text{Var}[D_{\text{fGn}}(n)]$ because D_{fGn} and H_{fGn} are restricted by $D_{\text{fGn}} = 2 - H_{\text{fGn}}$.

The third significance of the traffic model of the GC type can be briefed as follows. It is well known that the amount of traffic accumulated in the interval $[0, t]$ is upper bounded by

$$\int_0^t x(u)du \leq \sigma + \rho t, \tag{5.1}$$

where σ and ρ are constants and $t > 0$, see Cruz [81]. It is obviously that a tightened bound of $\int_0^t x(u)du$ is particularly desired in practice, such as delay computations. By applying the GC model to the traffic bound, we have the tightened bound expressed by

$$\int_0^t x(u)du \leq r^{2D-5}\sigma[u(t) - u(t-\varepsilon)] + a^{-H}\rho u(t-\varepsilon)t, \tag{5.2}$$

where $r > 0$ is a small-scale factor, $a > 0$ is a large-scale factor, and $\varepsilon > 0$, $u(t)$ is the unit step function, see Li and Zhao [58] for details. For instance, if we let $D = 1.8$, $H = 0.9$, $r = 1.5$, and $a = 10$, then we have a tightened bound given by

$$\int_0^t x(u)du \leq 0.567\sigma[u(t) - u(t-\varepsilon)] + 0.126\rho u(t-\varepsilon)t. \tag{5.3}$$

The conventional traffic bound, that is, the right side of (5.1), is a special case of (5.2) for $r = a = 1$. We should emphasize that the fractal dimension D and the Hurst parameter H in (5.2) have to be considered in the sense of the GC model of traffic [58].

Our future work is in two ways. One is to explore more specific significances of the GC model of traffic in practical issues, for example, queuing. The other is to study whether the GC model of random processes may provide new explanation for the random phenomena in nonlinear time-varying systems or complex systems discussed by Dong et al. [82–84], and Shen et al. [85–87], Chen et al. [88], and Sheng et al. [89, 90].

6. Conclusions

FGn, which is a self-similar process with LRD for $H \in (0.5, 1)$ and a widely used model in traffic engineering, was proposed as a traffic model by Leland et al. [22], Beran et al. [23], and Paxson and Floyd [37], based on their data processing of sets of real-traffic traces. The GC process, which is a locally self-similar process with LRD for $\beta \in (0, 1)$, was recently reported by Li and Lim [46], also based on their processing the same sets of traffic traces as those in [22, 37]. However, experimental processing of real traffic relying on selected sample records of traffic may be limited, in methodology, to be used to abstractly evaluate which is more conformable with real traffic without relating to the selected sample records of traffic. The theoretical significance of this paper is to provide us with the abstract assessment in terms of the generality described by (3.16) in Theorem 3.17 that the GC model is more conformable with real traffic than single parameter models, for example, fGn, regardless of any sample records of traffic, which may yet be a theoretical supplement with respect to the traffic model of the GC type. In addition, we have given our construction procedure of the ACF of the GC process in Hilbert spaces with the technique of extensions based on fGn.

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