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Research Article

L_∞ Control with Finite-Time Stability for Switched Systems under Asynchronous Switching

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This paper is concerned with the problem of controller design for switched systems under asynchronous switching with exogenous disturbances. The attention is focused on designing the feedback controller that guarantees the finite-time bounded and L_∞ finite-time stability of the dynamic system. Firstly, when there exists asynchronous switching between the controller and the system, a sufficient condition for the existence of stabilizing switching law for the addressed switched system is derived. It is proved that the switched system is finite-time stabilizable under asynchronous switching satisfying the average dwell-time condition. Furthermore, the problem of L_∞ control for switched systems under asynchronous switching is also investigated. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Switched systems are a class of hybrid systems consisting of subsystems and a switching law, which defines a specific subsystem being activated during a certain interval of time. Many real-world processes and systems can be modeled as switched systems such as chemical processes and computer controlled systems. Besides, switched systems are widely applied in many domains, including mechanical systems, automotive industry, aircraft and air traffic control, and many other fields [1–3].

At early time, the issue of stability of switched systems which has attracted most of the attention is one basic research topic. Lyapunov stability theory and its variations or generalizations had played an important role in this research field. Common Lyapunov function method and multiple Lyapunov functions method for switched system are presented by researchers [4–8]. For most switched systems, it is hard to find a common Lyapunov function; however, we can guarantee the switched system is still stable under some properly chosen switching signals which are found by using the multiple Lyapunov functions technique. In addition, more researchers pay attention to average dwell-time control of switched systems [9, 10]. In particular, the average dwell-time approach is employed to deal with the control, observe, and filtering problem of switched delay systems or network control systems [11–14].

As we know, a large number of literatures related to stability of switched systems focus on Lyapunov asymptotic stability, which is defined over an infinite time interval. In many practical applications, however, the main concern is the behavior of the system over a fixed finite-time interval, for instance to avoid saturations or the excitation of nonlinear dynamics. It should be clear that a finite-time stable system may not be Lyapunov asymptotical stable, and a Lyapunov asymptotical stable system may not be finite-time stable since the transient of a system response may exceed the bound. Recently, there have been some literatures discussing the finite-time stability analysis of switched systems [15–17]. In [18], finite-time bounded and finite-time weighted L_2-gain for a class of switched delay systems with time-varying external disturbances is addressed. Reference [19] investigated finite-time control for switched discrete-time system. Considering the potential faults in a system, [20] studied fault-tolerant control with finite-time stability for switched linear systems. Delay-dependent observer-based H_∞ finite-time control for switched systems with time-varying delay was studied in [21]. In [22], the problems of finite-time stability analysis and stabilization for switched nonlinear discrete-time systems are investigated, and then the results are extended to H_∞ finite-time bounded. However, in many applications, external disturbance is always persistent bounded with infinite energy. H_∞ control cannot be employed to deal with a system with persistent bounded disturbance. In this situation, it is more appealing to develop L_∞ control for switched systems with disturbances of this type. So far, however, compared with research results on H_∞ finite-time stability, few results on L_∞ finite-time stability of switched systems have been given in the literature.

Additionally, in actual operation, there inevitably exists asynchronous switching between the controllers and the practical subsystems, that is, the real switching time of controllers exceeds or lags behind that of the practical subsystems, which will deteriorate performance of systems, even makes system out of control. Up to now, there have been a number of literatures on asynchronous switching control research of switched system [23–28]. But it is worth to point that all of these studies focus on designing the controller to guarantee the Lyapunov asymptotical stable or exponential stable of the system. To the best of our knowledge, the finite-time stabilization issue of switched system under asynchronous switching has not been fully investigated, which is quite an important issue for the switched system. This motivates us to carry out present work. In this paper, we deal with the problem of L_∞ finite-time stabilization for switched systems under asynchronous switching.

The main contributions of this paper are that several sufficient conditions ensuring the finite-time bounded and L_∞ finite-time stability are proposed with asynchronous switching between the controllers and the practical subsystems. The result shows that it is unnecessary to guarantee each subsystem can be finite-time stabilizable with L_∞ performance by the designed asynchronous switching controller. During the finite-time interval, the switching frequency only needs to be limited in some value, then the switched system is finite-time stable with L_∞ performance by the designed controller despite of the asynchronous switching between the controllers and the practical subsystems.

This paper is organized as follows. In Section 2, some preliminary definitions are provided, and the problem we deal with is precisely stated. Section 3 provides, the main results of this paper: a sufficient condition for the existence of a state feedback controller guaranteeing the finite-time stability under asynchronous switching between the controllers and the practical subsystems. Moreover, L_∞ control with finite-time stability for switched systems under asynchronous switching is provided in Section 4. Finally, a numerical example is presented by using LMI toolbox to illustrate the efficiency of the proposed method in Section 5. Our conclusions are drawn in Section 6.

Notation. Throughout this paper, A^T denotes transpose of matrix A, L_∞ denotes space of functions with bounded amplitude, $\|x(t)\|$ denotes the usually 2-norm. $\lambda_{\max}(P)$, and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively, I is an identity matrix with appropriate dimension. $S > 0$ denotes S is a positive definite symmetric matrix. Z denotes the integer set and Z^+ denotes the positive integer set.

2. Problem Formulation and Preliminary

A switched system is considered as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + G_{\sigma(t)}w(t), \tag{2.1}$$

where $x(t) \in R^n$ is the system state. $u(t) \in R^p$ is the control input, $x(t_0) = x_0$ is the initial state of the system. $w(t) \in R^q$ is the measurement noise over the interval $[t_0, T_f]$, which satisfies $\sup_{t\in[t_0,T_f]}\|w(t)\| < \infty$, $\sigma(t) : Z^+ \to \underline{N} = \{1, 2, \dots, N\}$ is a switching signal which is a piecewise constant function depending on time t or state $x(t)$, and N denotes the number of subsystems. Moreover, $\sigma(t) = i$ means that the ith subsystem is activated. $A_i \in R^{n\times n}$, $B_i \in R^{n\times p}$, $G_i \in R^{n\times q}$ for $i \in \underline{N}$ are real-valued matrices with appropriate dimensions.

Assume that the state of the switched system (2.1) does not jump at the switching instants, that is, the trajectory $x(t)$ is everywhere continuous. The switching law $\sigma(t) : Z^+ \to \underline{N} = \{1, 2, \dots, N\}$ discussed in this paper is time dependent, that is, $\sigma(t) : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \dots, (t_k, \sigma(t_k))\}$, $k \in Z$, where t_0 is the initial switching instant, and t_k denotes the kth switching instant.

Owing to asynchronous switching, the practical switching instant of controller is different from that of systems. For convenience, $\sigma'(t)$ is used to denote the practical switching signal of controller, $\sigma'(t)$ can be written as $\sigma'(t) : \{(t_0 + \Delta_0, \sigma(t_0)), (t_1 + \Delta_1, \sigma(t_1)), \dots, (t_k + \Delta_k, \sigma(t_k))\}$, $k \in Z$, where $|\Delta_k| < \inf_{k\geq 0}(t_{k+1} - t_k)$, $\Delta_k > 0$ (or $|\Delta_k| < \inf_{k\geq 0}(t_k - t_{k-1}), \Delta_k < 0$); Δ_k represents the delayed period of the controller switching (or the exceeded period of the controller switching). In both cases, the period Δ_k is said to be the mismatched period between the controller and the system.

Remark 2.1. Mismatched period Δ_k guarantees that there always exists a period that the controller and the system operate synchronously, which makes it possible to design the stabilizable controller for the system.

Under the asynchronous switching, the switched controller can be written as

$$u(t) = K_{\sigma'(t)}x(t). \tag{2.2}$$

If we substitute the $u(t) = K_{\sigma'(t)}x(t)$ into system (2.1), we can obtain that

$$\dot{x}(t) = \left(A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma'(t)}\right)x(t) + G_{\sigma(t)}w(t). \tag{2.3}$$

The following lemma will be useful for the design of controller.

Lemma 2.2 (see [29]). *If a real scalar function $\varphi(t), v(t)$ satisfies the following differential inequality:*

$$\dot{\varphi}(t) \leq \varsigma\varphi(t) + \kappa v(t), \tag{2.4}$$

then we have

$$\varphi(t) \leq e^{\varsigma(t-t_0)}\varphi(t_0) + \kappa\int_0^{t-t_0} e^{\varsigma\tau}v(t-\tau)d\tau, \tag{2.5}$$

where $\varsigma \in R,\ \kappa \in R,\ t \geq t_0$.

Let us review the definition of average dwell-time, which will be useful in designing the stabilization controller to guarantee the system finite-time stable.

Definition 2.3 (see [30]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1, T_2), if

$$N_\sigma(T_1, T_2) \leq N_0 + \frac{T_2 - T_1}{\tau_a} \tag{2.6}$$

holds for given $N_0 \geq 0,\ \tau_a > 0$. Then the constant τ_a is called the average dwell time, and N_0 is the chatter bound.

For switched system, the general conception of finite-time stability concerns the boundness of continuous state $x(t)$ over finite-time interval $[t_0, T_f]$ with respect to given initial condition x_0. This conception can be formulized through following definition.

Definition 2.4. The switched linear system (2.1) with $G_{\sigma(t)} \equiv 0$ is said to be finite-time stabilizable under the asynchronous switching control mode with respect to $(c_1, c_2, T_f, \sigma(t), \sigma'(t))$ with $c_1 < c_2$ and a given switching signal $\sigma(t)$, if $\|x(t)\| \leq c_2$, for all $t \in [t_0, T_f]$, whenever $\|x_0\| \leq c_1$.

Definition 2.5. Switched system (2.1) is said to be L_∞ finite-time stabilizable with respect to $(c_1, c_2, T_f, \sigma(t), \sigma'(t))$ where $c_1 < c_2$, $\sigma(t)$ is a switching signal of the system, and $\sigma'(t)$ is a switching signal of the controller, the following conditions should be satisfied.

(i) Switched linear system (2.1) with $G_{\sigma(t)} \equiv 0$ is finite-time stabilizable.

(ii) Under zero-initial condition $x(t_0) = 0$, the following inequality holds:

$$\sup_{t\in[t_0,T_f]} \|x(t)\| \leq \gamma \sup_{t\in[t_0,T_f]} \|w(t)\|, \quad \forall w(t) : \sup_{t\in[t_0,T_f]} \|w(t)\| < \infty. \tag{2.7}$$

The main issue in this paper is given as follows.

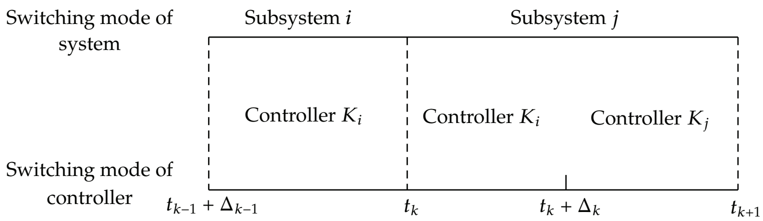

Figure 1: Asynchronous switching mode.

Given switched system (2.1), find a sufficient condition ensuring the finite-time stability with respect to $(c_1, c_2, T_f, \sigma(t), \sigma'(t))$ under the asynchronous switching control mode, then the result will be extended to the L_∞ controller design for system (2.1).

3. Finite-Time Stabilization under the Asynchronous Switching

It is assumed that the ith subsystem switched to the jth subsystem at the switching instant t_k. Owing to asynchronous switching, the switching instant of ith controller is $t_k + \Delta_k$, then there exists mismatched period at time interval $[t_k, t_k + \Delta_k)$, $\Delta_k > 0$ (or $(t_k + \Delta_k, t_k)$, $\Delta_k < 0$). In this period, the controller K_i affected the jth subsystem (or the controller K_j affected the ith subsystem).

Remark 3.1. We consider the case of $\Delta_k > 0$, that is to say, the switching time of the controller is lag of the switching time of the system. Figure 1, illustrates the asynchronous switching mode between the controller and the subsystems. From Figure 1, we can see that the controller K_i of the ith subsystem affects the ith subsystem in the matched period $[t_{k-1} + \Delta_{k-1}, t_k)$ and affects the jth subsystem in the mismatched period $[t_k, t_k + \Delta_k)$.

The following theorem presents the finite-time stabilization design method of the system (2.1) under asynchronous switching.

Theorem 3.2. *If there exist matrices $P_i > 0$, $P_{ij} > 0$, K_i and scalars $\mu_1 > 1$, $\mu_2 > 1$, $\lambda^+ > 0$, $\lambda^- > 0$ such that*

$$P_i < \mu_1 P_{ij}, \qquad P_{ij} < \mu_2 P_i, \tag{3.1}$$

$$(A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) < \lambda^- P_i, \tag{3.2}$$

$$(A_j + B_j K_i)^T P_{ij} + P_{ij} (A_j + B_j K_i) < \lambda^+ P_{ij}, \tag{3.3}$$

$$\tau_a > \frac{(T_f - t_0)\ln(\mu_1 \mu_2)}{\ln\Big((\varepsilon^2/\delta^2) \cdot \mathcal{B} \cdot \big(\mu_2/(\mu_1\mu_2)^{N_0}\big)\Big) - \lambda^+ T^+(t_0, T_f) - \lambda^- T^-(t_0, T_f)}, \tag{3.4}$$

where $\mathcal{B}$ denotes $\inf_{i,j \in \underline{N}}\{\lambda_{\min}(P_i), \lambda_{\min}(P_{ij})\}/\sup_{i,j\in\underline{N}}\{\lambda_{\max}(P_i), \lambda_{\max}(P_{ij})\}$, then switched system (2.1) is finite-time stabilizable with respect to $(\delta, \varepsilon, T_f, \sigma(t), \sigma'(t))$ under the feedback controller $u(t) = K_{\sigma'(t)} x(t)$, where $T^-(t_0, T_f)$ and $T^+(t_0, T_f)$ denote the matched period and the mismatched period in finite-time interval $[t_0, T_f]$, respectively.

Proof. Here, we only discuss the situation of $\Delta_k > 0$. For $\Delta_k < 0$, the proof method is similar, and we can reach the same conclusion.

When $t \in [t_{k-1} + \Delta_{k-1}, t_k)$, for the ith subsystem, the state feedback controller $u(t) = K_i x(t)$. So the state equation of closed-loop system can be written as

$$\dot{x}(t) = (A_i + B_i K_i)x(t). \tag{3.5}$$

Choose a switching Lyapunov function as follows:

$$V_i(t) = x^T(t)P_i x(t). \tag{3.6}$$

By (3.2), it implies that

$$\dot{V}_i(t) < \lambda^- V_i(t). \tag{3.7}$$

When $t \in [t_k, t_k + \Delta_k)$, for the jth subsystem, the state feedback controller is still $u(t) = K_i x(t)$. So the closed-loop system can be described as

$$\dot{x}(t) = \left(A_j + B_j K_i\right)x(t). \tag{3.8}$$

Consider the Lyapunov function candidate as follows:

$$V_{ij}(t) = x^T(t)P_{ij}x(t). \tag{3.9}$$

By (3.3), we can obtain that

$$\dot{V}_{ij}(t) < \lambda^+ V_{ij}(t). \tag{3.10}$$

Notice that the Lyapunov function (3.6) and (3.9) can be rewritten as

$$\begin{gathered} V_i(t) = x^T(t)P_i x(t), \quad t \in [t_{k-1} + \Delta_{k-1}, t_k),\ k = 1, 2, \dots, \\ V_i(t) = x^T(t)P_{ij}x(t), \quad t \in [t_k, t_k + \Delta_k),\ k = 0, 1, \dots. \end{gathered} \tag{3.11}$$

Let $t_0 < t_1 < t_2 < \cdots < t_k = T_f$ is the switching time in the period $[t_0, T_f]$, we define the following piecewise Lyapunov function:

$$V(t) = \begin{cases} x^T(t)P_i x(t), & t \in [t_r + \Delta_r, t_{r+1}),\ r = 0, 1, \dots, k-1, \\ x^T(t)P_{ij}x(t), & t \in [t_r, t_r + \Delta_r), \quad r = 0, 1, \dots, k-1. \end{cases} \tag{3.12}$$

By (3.7) and (3.10), we can obtain that

$$\begin{aligned}
V(t) &< e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})}V(t_{k-1}+\Delta_{k-1}) \\
&< \mu_1 e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})}V\left((t_{k-1}+\Delta_{k-1})^-\right) \\
&< \mu_1 e^{\lambda^+\Delta_{k-1}} e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})}V(t_{k-1}) \\
&< \mu_1\mu_2 e^{\lambda^+\Delta_{k-1}} e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})}V\left(t_{k-1}^-\right) \\
&< \mu_1\mu_2 e^{\lambda^+\Delta_{k-1}} e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})} e^{\lambda^-(t_{k-1}-t_{k-2}-\Delta_{k-2})}V(t_{k-2}+\Delta_{k-2}) \\
&< \mu_1^2\mu_2 e^{\lambda^+\Delta_{k-1}} e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})} e^{\lambda^-(t_{k-1}-t_{k-2}-\Delta_{k-2})}V\left((t_{k-2}+\Delta_{k-2})^-\right) \\
&< \mu_1^2\mu_2 e^{\lambda^+\Delta_{k-1}} e^{\lambda^+\Delta_{k-2}} e^{\lambda^-(t-t_{k-1}-\Delta_{k-1})} e^{\lambda^-(t_{k-1}-t_{k-2}-\Delta_{k-2})}V(t_{k-2}) \\
&= \mu_1^2\mu_2 e^{\lambda^+(\Delta_{k-1}+\Delta_{k-2})+\lambda^-[(t-t_{k-1}-\Delta_{k-1})+(t_{k-1}-t_{k-2}-\Delta_{k-2})]}V(t_{k-2}) \\
&\cdots \\
&< \mu_1^k\mu_2^{k-1} e^{\lambda^+(\Delta_{k-1}+\cdots+\Delta_0)+\lambda^-[(t-t_{k-1}-\Delta_{k-1})+(t_{k-1}-t_{k-2}-\Delta_{k-2})+\cdots+(t_1-t_0-\Delta_0)]}V(t_0) \\
&< \mu_2^{-1}(\mu_1\mu_2)^{k_{[t_0,T_f]}} e^{\lambda^+T^+(t_0,T_f)+\lambda^-T^-(t_0,T_f)}V(t_0),
\end{aligned} \tag{3.13}$$

where $T^+(t_0,T_f)$ denotes the sum of the mismatched period between the controllers and subsystem in (t_0,T_f). $T^-(t_0,T_f)$ denotes the sum of the matched period between the controllers and subsystem in $[t_0,T_f]$.

And from (3.12) we have

$$V(t) \geq \inf_{i,j\in\underline{N}}\left\{\lambda_{\min}(P_i),\lambda_{\min}\left(P_{ij}\right)\right\}\|x(t)\|^2. \tag{3.14}$$

On the other hand, for $i \in \underline{N}$, we have

$$V(t_0) \leq \sup_{i,j\in\underline{N}}\left\{\lambda_{\max}(P_i),\lambda_{\max}\left(P_{ij}\right)\right\}\|x(t_0)\|^2. \tag{3.15}$$

Using the fact

$$\|x(t_0)\| \leq \delta, \tag{3.16}$$

we get

$$V(t_0) \leq \sup_{i,j\in\underline{N}}\left\{\lambda_{\max}(P_i),\lambda_{\max}\left(P_{ij}\right)\right\}\delta^2. \tag{3.17}$$

Altogether (3.13)–(3.17), the following inequality can be derived

$$\|x(t)\|^2 \le \mu_2^{-1}(\mu_1\mu_2)^{k_{[t_0,T_f]}} e^{\lambda^+ T^+(t_0,T_f)+\lambda^- T^-(t_0,T_f)} \frac{\sup_{i,j\in\underline{N}}\{\lambda_{\max}(P_i),\lambda_{\max}(P_{ij})\}}{\inf_{i,j\in\underline{N}}\{\lambda_{\min}(P_i),\lambda_{\min}(P_{ij})\}}\delta^2. \tag{3.18}$$

From the Definition 2.3, we know that $k_{[t_0,T_f]} = N_\sigma$, then we have the relation

$$k_{[t_0,T_f]} \le N_0 + \frac{T_f - t_0}{\tau_a}. \tag{3.19}$$

From (3.4) and (3.19), we get

$$\mu_2^{-1}(\mu_1\mu_2)^{k_{[t_0,T_f]}} e^{\lambda^+ T^+(t_0,T_f)+\lambda^- T^-(t_0,T_f)} \frac{\sup_{i,j\in\underline{N}}\{\lambda_{\max}(P_i),\lambda_{\max}(P_{ij})\}}{\inf_{i,j\in\underline{N}}\{\lambda_{\min}(P_i),\lambda_{\min}(P_{ij})\}}\delta^2 < \varepsilon^2. \tag{3.20}$$

According to (3.18) and (3.20), we have

$$\|x(t)\| < \varepsilon. \tag{3.21}$$

The proof is completed. □

Remark 3.3. From (3.2) and (3.3), we know that for finite-time stabilization issue, the subsystem needs not to be stabilized in finite-time interval, that is to say, the designed asynchronous switching controller needs not to stabilize the subsystem in the matched period and the mismatched period in finite-time interval $[t_0, T_f]$, but the whole system is finite-time stabilizable. Reference [31] gives the exponential stabilization condition under asynchronous switching, which requests that the subsystem can be exponentially stabilized in the matched period. But as to the problem of finite-time stabilization, it is unnecessary to request that the subsystem can be stabilized in the matched period or mismatched period. In particular, when $\lambda^+ = \lambda^- = \lambda$ in (3.2) and (3.3), (3.4) becomes

$$\tau_a > \frac{(T_f - t_0)\ln(\mu_1\mu_2)}{\ln\left((\varepsilon^2/\delta^2)\cdot\mathcal{B}\cdot\left(\mu_2/(\mu_1\mu_2)^{N_0}\right)\right) - \lambda(T_f - t_0)} \tag{3.22}$$

which is independent of $T^+(t_0, T_f)$ and $T^-(t_0, T_f)$.

Remark 3.4. In fact, (3.4) in Theorem 3.2 implies that if switching sequence $\sigma(t) : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k))\}$ of the system can be prespecified, that is, τ_a is a known constant, the matched period $T^-(t_0, T_f)$ and the mismatched period $T^+(t_0, T_f)$ should satisfy the following relation:

$$\lambda^+ T^+(t_0, T_f) + \lambda^- T^-(t_0, T_f) < \ln\left(\frac{\varepsilon^2}{\delta^2} \cdot \frac{\inf_{i,j\in \underline{N}}\{\lambda_{\min}(P_i), \lambda_{\min}(P_{ij})\}}{\sup_{i,j\in \underline{N}}\{\lambda_{\max}(P_i), \lambda_{\max}(P_{ij})\}} \cdot \frac{\mu_2}{(\mu_1\mu_2)^{N_0}}\right) - \frac{(T_f - t_0)\ln(\mu_1\mu_2)}{\tau_a}. \tag{3.23}$$

Remark 3.5. Reference [31] gives the design method of exponential stabilization controller under asynchronous switching. The condition implies that the ratio of the mismatched period and the matched period should be less than some value which means that the matched period should be large enough to stabilize the subsystem. However, from the condition of Theorem 3.2, we know that when the switching sequence is unknown, the ratio of the mismatched period and the matched period can be designed freely to guarantee the finite-time stability of the system by the asynchronous switched controller. But if switching sequence of the system is prespecified, the ratio of the mismatched period and the matched period may need to be limited. On the other hand, the average dwell-time scheme with Lyapunov stability limits the dwell-time τ_a and the ratio of $T^+(t_0, T_f)$ and $T^-(t_0, T_f)$ to satisfy the proposed condition in [31] at the same time. But for the average dwell-time scheme with finite-time stability, we can predetermine one value among two parameters of the dwell-time τ_a and the ratio of $T^+(t_0, T_f)$ and $T^-(t_0, T_f)$, then the other value can be determined by the condition (3.4).

Remark 3.6. In order to get the solution of the asynchronous switched controller K_i, we denote $X_i = P_i^{-1}$, $X_{ij} = P_{ij}^{-1}$, $W_i = K_i P_i^{-1}$, then (3.1) to (3.3) can be written as

$$\mu_1 X_i > X_{ij}, \qquad \mu_2 X_{ij} > X_i, \tag{3.24}$$

$$(A_i X_i + B_i W_i)^T + (A_i X_i + B_i W_i) < \lambda^- X_i, \tag{3.25}$$

$$X_{ij}\left(A_j + B_j W_i X_i^{-1}\right)^T + \left(A_j + B_j W_i X_i^{-1}\right) X_{ij} < \lambda^+ X_{ij}. \tag{3.26}$$

It is noticed that the matrix inequalities (3.24), (3.25), and(3.26) are coupled. Therefore, we can firstly solve the linear matrix inequality (3.25) to obtain the solution to matrices X_i and W_i. Then we solve the matrix inequality (3.24), (3.26) by substituting X_i and W_i into (3.24), (3.26). By adjusting the parameter μ_1, μ_2, and λ^+ appropriately, we seek the feasible solutions X_i, W_i and X_{ij} such that the matrix inequalities (3.24) and (3.26) hold. If the chosen parameters μ_1, μ_2, and λ^+ have no feasible solution, we can adjust μ_1, μ_2, or λ^+ to be larger. Following this guideline, the solution to the matrix inequalities (3.24)to (3.26) will be found.

4. L_∞ Finite-Time Stabilization under the Asynchronous Switching

Now, we are in a position to investigate L_∞ finite-time stabilization design method of the system (2.1) under asynchronous switching.

Theorem 4.1. *If there exist matrices $P_i > 0$, $P_{ij} > 0$, K_i and scalars $\mu_1 > 1$, $\mu_2 > 1$, $\lambda^+ > 0$, $\lambda^- > 0$ such that*

$$P_i < \mu_1 P_{ij}, \qquad P_{ij} < \mu_2 P_i, \tag{4.1}$$

$$\begin{bmatrix} (A_i + B_i K_i)^T P_i + P_i (A_i + B_i K_i) - \lambda^- P_i & P_i G_i \\ G_i^T P_i & -\varepsilon_i I \end{bmatrix} < 0, \tag{4.2}$$

$$\begin{bmatrix} \left(A_j + B_j K_i\right)^T P_{ij} + P_{ij}\left(A_j + B_j K_i\right) - \lambda^+ P_{ij} & P_{ij} G_j \\ G_j^T P_{ij} & -\varepsilon_{ij} I \end{bmatrix} < 0, \tag{4.3}$$

$$\tau_a > \frac{\left(T_f - t_0\right) \ln\left(\mu_1 \mu_2\right)}{\ln\left(\left(\varepsilon^2/\delta^2\right) \cdot \mathcal{B} \cdot \left(\mu_2 / \left(\mu_1 \mu_2\right)^{N_0}\right)\right) - \lambda^+ T^+\left(t_0, T_f\right) - \lambda^- T^-\left(t_0, T_f\right)}. \tag{4.4}$$

L_∞ disturbance attenuation performance $\gamma^- \leq \sqrt{\varepsilon_i (e^{\lambda^- T^-(t_0, T_f)} - 1)/\lambda^- \lambda_{\min}(P_i)}$ during the matched period and $\gamma^+ \leq \sqrt{\varepsilon_{ij} (e^{\lambda^+ T^+(t_0, T_f)} - 1)/\lambda^+ \lambda_{\min}(P_{ij})}$ during the mismatched period, then switched system (2.1) is finite-time stabilizable of L_∞ disturbance attenuation performance with respect to $(\delta, \varepsilon, T_f, \sigma(t), \sigma'(t))$ under the feedback controller $u(t) = K_{\sigma'(t)} x(t)$, where $T^-(t_0, T_f)$ and $T^+(t_0, T_f)$ denote the matched period and the mismatched period in finite-time interval $[t_0, T_f]$, respectively.

Proof. It can be concluded from Theorem 4.1 that system (2.1) is finite-time stable under the feedback controller $u(t) = K_{\sigma'(t)} x(t)$.

When $t \in [t_{k-1} + \Delta_{k-1}, t_k)$, for the ith subsystem, the state feedback controller $u(t) = K_i x(t)$. So the state equation of closed-loop system can be written as

$$\dot{x}(t) = (A_i + B_i K_i) x(t) + G_i w(t). \tag{4.5}$$

Choose a switching Lyapunov function as follows:

$$V_i(t) = x^T(t) P_i x(t), \quad t \in [t_{k-1} + \Delta_{k-1}, t_k), \; k = 1, 2, \ldots. \tag{4.6}$$

By (4.2), it implies that

$$\dot{V}_i(t) \leq \lambda^- V_i(t) + \varepsilon_i w^T(t) w(t). \tag{4.7}$$

With zero initial conditions, by Lemma 2.2, we have

$$V_i(t) \leq \varepsilon_i \int_0^{t - t_{k-1} - \Delta_{k-1}} e^{\lambda^- \tau} w^T(t - \tau) w(t - \tau) d\tau. \tag{4.8}$$

Note that

$$V_i(t) \geq \lambda_{\min}(P_i) \|x(t)\|^2. \tag{4.9}$$

From (4.8) and (4.9), we can obtain

$$\lambda_{\min}(P_i) \sup_{t\in[t_{k-1}+\Delta_{k-1},t_k)} \|x(t)\|^2 \le \frac{\varepsilon_i\left(e^{\lambda^- T^-(t_0,T_f)} - 1\right)}{\lambda^-} \sup_{t\in[t_{k-1}+\Delta_{k-1},t_k)} \|w(t)\|^2. \tag{4.10}$$

From (4.10), we have

$$\frac{\sup_{t\in[t_{k-1}+\Delta_{k-1},t_k)}\|x(t)\|}{\sup_{t\in[t_{k-1}+\Delta_{k-1},t_k)}\|w(t)\|} \le \sqrt{\frac{\varepsilon_i\left(e^{\lambda^- T^-(t_0,T_f)} - 1\right)}{\lambda^-\lambda_{\min}(P_i)}}. \tag{4.11}$$

When $t \in [t_k, t_k + \Delta_k)$, for the jth subsystem, the state feedback controller is still $u(t) = K_i x(t)$. So the closed-loop system can be described as

$$\dot{x}(t) = \left(A_j + B_j K_i\right)x(t) + G_j w(t). \tag{4.12}$$

Consider the Lyapunov function candidate as follows:

$$V_{ij}(t) = x^T(t)P_{ij}x(t), \quad t \in [t_k, t_k + \Delta_k),\ k = 0, 1, \ldots. \tag{4.13}$$

By (4.3), it implies that

$$\dot{V}_{ij}(t) \le \lambda^+ V_{ij}(t) + \varepsilon_{ij} w^T(t)w(t). \tag{4.14}$$

With zero initial conditions, by Lemma 2.2, we have

$$V_{ij}(t) \le \varepsilon_{ij}\int_0^{t-t_k} e^{\lambda^+\tau} w^T(t-\tau)w(t-\tau)d\tau. \tag{4.15}$$

Notice that

$$V_{ij}(t) \ge \lambda_{\min}\left(P_{ij}\right)\|x(t)\|^2 . \tag{4.16}$$

From (4.15) and (4.16), we can obtain

$$\lambda_{\min}\left(P_{ij}\right) \sup_{t\in[t_k,t_k+\Delta_k)} \|x(t)\|^2 \le \frac{\varepsilon_{ij}\left(e^{\lambda^+ T^+(t_0,T_f)} - 1\right)}{\lambda^+} \sup_{t\in[t_k,t_k+\Delta_k)} \|w(t)\|^2. \tag{4.17}$$

From (4.17), we have

$$\frac{\sup_{t\in[t_k,t_k+\Delta_k)}\|x(t)\|}{\sup_{t\in[t_k,t_k+\Delta_k)}\|w(t)\|} \le \sqrt{\frac{\varepsilon_{ij}\left(e^{\lambda^+ T^+(t_0,T_f)} - 1\right)}{\lambda^+\lambda_{\min}\left(P_{ij}\right)}} . \tag{4.18}$$

By (4.11) and (4.18), during the finite-time $[t_0, T_f] = \bigcup_{r=0}^{k-1}[t_r, t_r + \Delta_r) \cup [t_r + \Delta_r, t_{r+1})$, we can obtain

$$\frac{\sup_{t\in[t_0,T_f)}\|x(t)\|}{\sup_{t\in[t_0,T_f)}\|w(t)\|} \leq \max\left(\sqrt{\frac{e^{\lambda^- T^-(t_0,T_f)}-1}{\lambda^-}\max_{i\in\underline{N}}\left(\frac{\varepsilon_i}{\lambda_{\min}(P_i)}\right)}, \sqrt{\frac{e^{\lambda^+ T^+(t_0,T_f)}-1}{\lambda^+}\max_{i,j\in\underline{N}}\left(\frac{\varepsilon_{ij}}{\lambda_{\min}(P_{ij})}\right)}\right) \tag{4.19}$$

By the definition of L_∞ finite-time stabilization, we can obtain that the designed controller $u(t) = K_{\sigma'(t)}x(t)$ can guarantee the finite-time stability of L_∞ disturbance attenuation performance. This completes the proof. □

Remark 4.2. Theorem 4.1 represents that if each subsystem satisfies L_∞ disturbance attenuation performance during the mismatched period and the matched period, the designed asynchronous switched controller $u(t) = K_{\sigma'(t)}x(t)$ can guarantee the whole system has L_∞ disturbance attenuation performance. However, the condition of each subsystem satisfying L_∞ disturbance attenuation performance during the mismatched period and the matched period seems to be more conservative, and in fact through the following theorem, this condition is not essential.

Remark 4.3. Although Theorem 4.1 gives the method of finite-time stabilization with L_∞ disturbance attenuation performance, the matched period $T^-(t_0, T_f)$ and the mismatched period $T^+(t_0, T_f)$ need to be prespecified in order to obtain L_∞ disturbance attenuation performance of the system. However, in practical engineering it is difficult to obtain the matched period $T^-(t_0, T_f)$ and the mismatched period $T^+(t_0, T_f)$ before designing the controller. Based on these, the following result can be derived.

Theorem 4.4. *If there exist matrices $P_i > 0$, $P_{ij} > 0$, K_i and scalars $\mu_1 > 1$, $\mu_2 > 1$, $\lambda^+ > 0$, $\lambda^- > 0$ such that*

$$P_i < \mu_1 P_{ij}, \qquad P_{ij} < \mu_2 P_i, \tag{4.20}$$

$$\begin{bmatrix} (A_i + B_iK_i)^T P_i + P_i(A_i + B_iK_i) - \lambda^- P_i & P_iG_i \\ G_i^T P_i & -\varepsilon_i I \end{bmatrix} < 0, \tag{4.21}$$

$$\begin{bmatrix} (A_j + B_jK_i)^T P_{ij} + P_{ij}(A_j + B_jK_i) - \lambda^+ P_{ij} & P_{ij}G_j \\ G_j^T P_{ij} & -\varepsilon_{ij} I \end{bmatrix} < 0, \tag{4.22}$$

$$\tau_a > \frac{(T_f - t_0)\ln(\mu_1\mu_2)}{\ln\left((\varepsilon^2/\delta^2)\cdot\mathcal{B}\cdot\left(\mu_2/(\mu_1\mu_2)^{N_0}\right)\right) - \lambda^+ T^+(t_0,T_f) - \lambda^- T^-(t_0,T_f)} \tag{4.23}$$

and in finite-time interval $[t_0, T_f]$ the measurement noise $w(t)$ satisfies $\sup_{t\in[t_0,T_f]}\|w(t)\| < \infty$, then switched system (2.1) *is finite-time stabilizable of L_∞ disturbance attenuation performance $\gamma = \sqrt{\max_{i,j\in\underline{N}}(\varepsilon_i, \varepsilon_{ij})(e^{\max(\lambda^+,\lambda^-)(T_f-t_0)} - 1)/\max(\lambda^+,\lambda^-)\min_{i,j\in\underline{N}}(\lambda_{\min}(P_i), \lambda_{\min}(P_{ij}))}$ with respect to $(\delta, \varepsilon, T_f, \sigma(t), \sigma'(t))$ under the feedback controller $u(t) = K_{\sigma'(t)}x(t)$, where $T^-(t_0, T_f)$ and $T^+(t_0, T_f)$ denote the matched period and the mismatched period in finite-time interval $[t_0, T_f]$, respectively.*

Proof. At first, from Theorem 4.4, system (2.1) is finite-time stable under the feedback controller $u(t) = K_{\sigma'(t)}x(t)$.

Then following the proof line of Theorem 4.1 and considering (4.6) and (4.13), we can define piecewise Lyapunov function

$$V(t) = \begin{cases} x^T(t)P_i x(t), & t \in [t_r + \Delta_r, t_{r+1}),\ r = 0, 1, \ldots, k-1, \\ x^T(t)P_{ij}x(t), & t \in [t_r, t_r + \Delta_r), \quad r = 0, 1, \ldots, k-1. \end{cases} \tag{4.24}$$

By (4.21) and (4.22), it implies that

$$\dot{V}(t) \leq \max(\lambda^+, \lambda^-)V(t) + \max_{i,j\in \underline{N}}(\varepsilon_i, \varepsilon_{ij})w^T(t)w(t). \tag{4.25}$$

With zero initial conditions, by Lemma 2.2, we have

$$V(t) \leq \max_{i,j\in \underline{N}}(\varepsilon_i, \varepsilon_{ij}) \int_0^{T_f - t_0} e^{\max(\lambda^+,\lambda^-)\tau} w^T(t-\tau)w(t-\tau)d\tau. \tag{4.26}$$

Notice that

$$V(t) \geq \min_{i,j\in \underline{N}}(\lambda_{\min}(P_i), \lambda_{\min}(P_{ij}))\|x(t)\|^2 . \tag{4.27}$$

From (4.26) and (4.27), we can obtain

$$\min_{i,j\in \underline{N}}(\lambda_{\min}(P_i), \lambda_{\min}(P_{ij})) \sup_{t\in[t_0,T_f]} \|x(t)\|^2 \leq \frac{\max_{i,j\in \underline{N}}(\varepsilon_i, \varepsilon_{ij})\left(e^{\max(\lambda^+,\lambda^-)(T_f - t_0)} - 1\right)}{\max(\lambda^+, \lambda^-)} \sup_{t\in\left[t_0,T_f\right]} \|w(t)\|^2. \tag{4.28}$$

From (4.28), we have

$$\frac{\sup_{t\in[t_0,T_f]}\|x(t)\|}{\sup_{t\in[t_0,T_f]}\|w(t)\|} \leq \sqrt{\frac{\max_{i,j\in \underline{N}}(\varepsilon_i, \varepsilon_{ij})\left(e^{\max(\lambda^+,\lambda^-)(T_f - t_0)} - 1\right)}{\max(\lambda^+, \lambda^-)\min_{i,j\in \underline{N}}(\lambda_{\min}(P_i), \lambda_{\min}(P_{ij}))}}. \tag{4.29}$$

By the definition of L_∞ finite-time stabilization, we can obtain that the designed controller $u(t) = K_{\sigma'(t)}x(t)$ can guarantee the finite-time stability of L_∞ disturbance attenuation performance. This completes the proof. □

Remark 4.5. It should be pointed out that the conditions in Theorems 4.4 are not standard LMIs conditions. However, through the variable substitution, (4.20) to (4.22) can be solved following the method proposed in Remark 3.6.

Remark 4.6. Theorem 4.4 presents that if the measurement noise $w(t)$ is magnitude bounded during finite-time interval $[t_0, T_f]$, then we can design the asynchronous switching controller such that the system has L_∞ disturbance attenuation performance. However, it is unnecessary to guarantee L_∞ disturbance attenuation performance during the mismatched period and the matched period by the designed controller which is less conservative than Theorem 4.1.

5. Numerical Example

We consider an example to illustrate the main result. Consider the switched linear system given by the system (2.1) with $u(t) = K_{\sigma'(t)}x(t)$,

$$\dot{x}(t) = \left(A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma'(t)}\right)x(t) + G_{\sigma(t)}w(t), \tag{5.1}$$

where $A_1 = \left[\begin{smallmatrix}-1 & 0\\ 0 & 0.1\end{smallmatrix}\right]$, $A_2 = \left[\begin{smallmatrix}2.1 & 1\\ 0 & 0.3\end{smallmatrix}\right]$, $B_1 = \left[\begin{smallmatrix}0.2 & 0.14\\ 0 & 2\end{smallmatrix}\right]$, $B_2 = \left[\begin{smallmatrix}1 & 0\\ 0.3 & 0.1\end{smallmatrix}\right]$, $G_1 = \left[\begin{smallmatrix}0.2 & 0\\ 0.3 & 0.1\end{smallmatrix}\right]$, $G_2 = \left[\begin{smallmatrix}0.1 & 0.2\\ 0.4 & 0\end{smallmatrix}\right]$.

Applying Theorem 4.4 and solving corresponding matrix inequalities lead to feasible solutions, when $\delta = 0.1,\ \varepsilon = 10,\ \varepsilon_1 = \varepsilon_2 = 100,\ \varepsilon_{12} = \varepsilon_{21} = 10,\ \mu_1 = \mu_2 = 20,\ \lambda^+ = 100,\ \lambda^- = 10,\ T_f = 0.005,\ t_0 = 0, N_0 = 0,\ \tau_a = 0.00375$.

$$\begin{aligned} K_1 &= \begin{bmatrix} 9.6364 & 1.4424\\ -10.3539 & 0.4207\end{bmatrix}, & K_2 &= \begin{bmatrix} 2.1337 & 0.4083\\ -0.6623 & 3.5807\end{bmatrix},\\ X_1 &= \begin{bmatrix} 8.2146 & -14.6028\\ -14.6028 & 86.9322\end{bmatrix}, & X_2 &= \begin{bmatrix} 92.6569 & 14.6028\\ 14.6028 & 13.9393\end{bmatrix},\\ X_{12} &= \begin{bmatrix} 7.9844 & -0.3851\\ -0.3851 & 9.9854\end{bmatrix}, & X_{21} &= \begin{bmatrix} 10.1766 & 0.1461\\ 0.1461 & 8.7611\end{bmatrix}. \end{aligned} \tag{5.2}$$

Then from (3.23), we know that the matched period $T^-(t_0, T_f)$ and the mismatched period $T^+(t_0, T_f)$ satisfy the following relation:

$$100T^+\left(t_0, T_f\right) + 10T^-\left(t_0, T_f\right) < 0.36. \tag{5.3}$$

Notice that $T^+(t_0, T_f) + T^-(t_0, T_f) = 0.005$, then we have

$$\begin{gathered} T^+\left(t_0, T_f\right) < 0.003,\\ 0.003 < T^-\left(t_0, T_f\right) < 0.005. \end{gathered} \tag{5.4}$$

the L_∞ state feedback controller K_1, K_2 can guarantee that system (5.1) is finite-time stabilizable with respect to $(0.1, 10, 0.005, \sigma(t), \sigma'(t))$ under the asynchronous switching where L_∞ disturbance attenuation performance $\gamma = 7.8$.

6. Conclusions

The L_∞ finite-time stabilization problems for switched linear system are addressed in this paper. When there exists asynchronous switching between the controller and the system, a sufficient condition for the existence of stabilizing switching law for the addressed switched system is derived. It is proved that the switched system is finite-time stabilizable under asynchronous switching satisfying the average dwell-time condition. Furthermore, the problem of L_∞ control for switched systems under asynchronous switching is also investigated. At last, a numerical example is given to illustrate the effectiveness of the proposed method.

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