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Research Article

MHD Boundary Layer Flow due to Exponential Stretching Surface with Radiation and Chemical Reaction

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The effects of radiation and first order homogeneous chemical reaction on hydromagnetic boundary layer flow of a viscous, steady, and incompressible fluid over an exponential stretching sheet have been investigated. The governing system of partial differential equations has been transformed into ordinary differential equations using similarity variables. The dimensionless system of differential equations was then solved numerically by the Runge-Kutta method. The skin-friction coefficient and the rate of heat and mass transfers are presented in tables whilst velocity, temperature, and concentration profiles are illustrated graphically for various varying parameter values. It was found that the rate of heat transfer at the surface decreases with increasing values of the transverse magnetic field parameter and the radiation parameter.

1. Introduction

The effects of radiation on hydromagnetic boundary layer flow of a continuously stretching surface have attracted considerable attention in recent times due to its numerous applications in industry. It occurs frequently in manufacturing involving hot metal rolling, wire drawing, glass-fiber production, paper production, drawing of plastic films, and metal spinning, as well as metal and polymer extrusion processes. Crane [1] was the first to investigate the boundary layer flow caused by a stretching sheet moving with linearly varying velocity from a fixed point whilst the heat transfer aspect of the problem was investigated by Carragher and Crane [2] under the conditions that the temperature difference between the surface and the ambient fluid was proportional to the power of the distance from a fixed point.

Magyari and Keller [3] then investigated the steady boundary layer flow on a stretching continuous surface with exponential temperature distribution while Partha et al. [4] analyzed the effects of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Sajid and Hayat [5] extended the works of Partha et al. [4] to include radiation effects on the flow over exponential stretching sheet and solved the problem analytically using the homotopy analysis method. The numerical solution for the same problem was then presented by Bidin and Nazar [6]. MHD steady flow and heat transfer on the sliding plate have been investigated by Makinde [7] whilst Ibrahim and Makinde [8] analyzed the radiation effects on chemically reacting MHD boundary layer flow of heat and mass transfer through a porous vertical flat plate. Makinde [9] earlier obtained results for the free-convection flow with thermal radiation and mass transfer past a moving vertical porous plate with Makinde and Ogulu [10] reporting on the effects of thermal radiation on heat and mass transfer of a variable viscosity fluid past a vertical porous plate permeated by transverse magnetic field. Magnetohydrodynamics has significant applications in the cooling of nuclear reactors using liquid sodium. Many processes in chemical engineering occur at high temperatures and radiation can be very significant and thus important for the design of pertinent equipment [11].

The present study considers the effect of chemical reaction on MHD boundary layer flow due to an exponential stretching surface in the presence of radiation. The paper is organized as follows: the mathematical model of the problem is described in Section 2 and the numerical method

is described in Section 3. In Section 4, we present both the numerical and graphical results with discussions. The concluding remarks are presented in Section 5.

2. Mathematical Model

Consider a steady two-dimensional flow of an incompressible, viscous, and electrically conducting fluid caused by a stretching surface. Assume that the plate has a surface temperature (T_w) and concentration (C_w) and is placed in a quiescent fluid of uniform ambient temperature (T_∞) and concentration (C_∞) (see Figure 1). A variable magnetic field $B(x)$ is applied normally to the stretching sheet surface and the induced magnetic field is negligible. This can be justified for MHD flow at small magnetic Reynolds number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects are governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2}{\rho c_p}u^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \gamma\left(C - C_\infty\right), \tag{4}$$

where u and v are the velocities in the x- and y-directions, respectively, ρ is the fluid density, ν the kinematic viscosity, k the thermal conductivity, and c_p is the specific heat at constant pressure. T and C represent the fluid temperature and concentration in the boundary layer, respectively, whilst D represents the mass diffusivity, γ the reaction rate parameter, and q_r the radiative heat flux.

The boundary conditions for the problem are taken similarly to Ishak [12] given as

$$\begin{gathered} u = U_w = U_0 e^{x/L}, \qquad v = 0, \\ T = T_w = T_\infty + T_0 e^{x/(2L)}, \\ C = C_w = C_\infty + C_0 e^{x/(2L)} \qquad \text{at } y = 0, \\ u \longrightarrow 0, \qquad T \longrightarrow T_\infty, \\ C \longrightarrow C_\infty \qquad \text{as } y \longrightarrow \infty, \end{gathered} \tag{5}$$

where U_0 is the reference velocity, T_0 is the reference temperature, C_0 is the reference concentration, and L is the reference length. Understanding fluid radiations has been based on assumptions of some reasonable simplifications [13, 14]. These simplifications assumed that the fluid is in the optically thin limit and does not absorb its own radiation except those emitted by other boundaries. For an optically thick gas, its self-absorption rises and the situation becomes

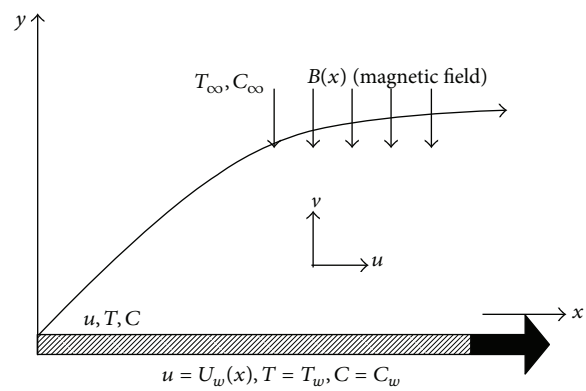

Figure 1: Physical configuration.

difficult. However, the problem can be simplified by using the Rosseland approximation [15–17] which simplifies the radiative heat flux to:

$$q_r = \frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. This approximation is valid at points optically far from the surface and good only for intensive absorption, which is for an optically thick boundary layer, Bataller [18], Siegel and Howell [16], and Sparrow and Cess [17]. It is assumed that the temperature differences within the flow involving the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms result in

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{7}$$

Using (6) and (7) reduces (3) to:

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = {} & \left(\frac{k}{\rho c_p} + \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\right)\frac{\partial^2 T}{\partial y^2} \\ & + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2}{\rho c_p}u^2. \end{aligned} \tag{8}$$

To obtain similarity solutions, it is assumed that the variable magnetic field $B(x)$ is of the form:

$$B(x) = B_0 e^{x/(2L)}, \tag{9}$$

where B_0 is the constant magnetic field.

The continuity equation (1) is satisfied by introducing a stream function ψ defined in the usual form as:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{10}$$

The momentum, energy, and concentration equations were transformed to ordinary differential equations using similarity variables similar to that employed by Ishak [12]:

$$\begin{aligned} u &= U_0 e^{x/L} f'(\eta), \\ v &= -\left(\frac{\nu U_0}{2L}\right)^{1/2} e^{x/(2L)} \left(f(\eta) + \eta f'(\eta)\right), \\ \eta &= y\left(\frac{U_0}{2\nu L}\right)^{1/2} e^{x/(2L)}, \\ T &= T_\infty + T_0 e^{x/(2L)} \theta(\eta), \\ C &= C_\infty + C_0 e^{x/(2L)} \phi(\eta), \end{aligned} \tag{11}$$

where η is the dimensionless similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration, and primes denote differentiation with respect to η.

The transformed ordinary differential equations are

$$f''' + ff'' - 2f'^2 - Mf' = 0, \tag{12}$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + \Pr f\theta' - \Pr f'\theta + \Pr M\mathrm{Ec} f'^2 + \Pr \mathrm{Ec} f''^2 = 0, \tag{13}$$

$$\phi'' + \mathrm{Sc} f\phi' - \mathrm{Sc} f'\phi - \mathrm{Sc}\beta\phi = 0, \tag{14}$$

in which $M = 2\sigma B_0^2 L/(\rho U_0)$ is the magnetic parameter, $K = 4\sigma^* T_\infty^3/(k^* k)$ is the radiation parameter, $\Pr = \rho\nu c_p/k$ is the Prandtl number, $\mathrm{Ec} = U_0^2 e^{2x/L}/(c_p(T_w - T_\infty))$ is the Eckert number, and $\mathrm{Sc} = \nu/D$ is the Schmidt number while $\beta = 2L\gamma/U_w$ is the reaction rate parameter.

The transformed boundary conditions are

$$\begin{gathered} f'(0) = 1, \qquad f(0) = 0, \\ \theta(0) = 1, \qquad \phi(0) = 1, \\ f'(\eta) \longrightarrow 0, \quad \theta(\eta) \longrightarrow 0, \quad \phi(\eta) \longrightarrow 0 \\ \text{as } \eta \longrightarrow \infty. \end{gathered} \tag{15}$$

3. Numerical Procedure

The nonlinear differential equations (12), (13), and (14) with the boundary conditions (15) have been solved numerically using the fourth order Runge-Kutta integration scheme with a modified version of the Newton-Raphson algorithm.

We let

$$\begin{gathered} f = x_1, \qquad f' = x_2, \qquad f'' = x_3, \\ \theta = x_4, \qquad \theta' = x_5, \\ \phi = x_6, \qquad \phi' = x_7. \end{gathered} \tag{16}$$

Equations (12), (13), and (14) are then reduced to systems of first order differential equations as

$$\begin{aligned} f' &= x_1' = x_2, \\ f'' &= x_2' = x_3, \\ f''' &= x_3' = -x_1 x_3 + 2x_2^2 + Mx_2, \\ \theta' &= x_4' = x_5, \\ \theta'' &= x_5' \\ &= \frac{\left(-\Pr x_1 x_5 + \Pr x_2 x_4 - \Pr M\mathrm{Ec}\, x_2^2 - \Pr \mathrm{Ec}\, x_3^2\right)}{(1 + 4K/3)}, \\ \phi' &= x_6' = x_7, \\ \phi'' &= -\mathrm{Sc}\left(x_1 x_7 - x_2 x_6 - \beta x_6\right) \end{aligned} \tag{17}$$

subject to the following initial conditions:

$$\begin{gathered} x_1(0) = 0, \qquad x_2(0) = 1, \qquad x_3(0) = s_1, \\ x_4(0) = 1, \qquad x_5(0) = s_2, \\ x_6(0) = 1, \qquad x_7(0) = s_3. \end{gathered} \tag{18}$$

In the shooting method, the unspecified initial conditions s_1, s_2, and s_3 in (18) are assumed and (17) integrated numerically as an initial valued problem to a given terminal point. The accuracy of the assumed missing initial conditions was checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If differences exist, improved values of the missing initial conditions are obtained and the process repeated. The computations were done by a written program which uses a symbolic and computational computer language (maple) with a step size of $\Delta\eta = 0.001$ selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The maximum value of η_∞ to each group of parameters was determined when the values of the unknown boundary conditions at $\eta = 0$ do not change to successful loop with error less than 10^{-7}.

4. Results and Discussion

The system of ordinary differential equations (12), (13), and (14) has been solved numerically using shooting technique together with the fourth order Runge-Kutta method and a modified version of the Newton-Raphson algorithm to tackle the problem [8, 20–22]. From the process of numerical computation, the main physical quantities of interest, namely, the local skin friction coefficient, the local Nusselt number and the local Sherwood numbers, which are, respectively, proportional to $-f''(0)$, $-\theta'(0)$, and $-\phi'(0)$, were worked out and their numerical results presented in Table 1. It is observed that increasing the radiation parameter (K) increases the rate of heat transfer at the surface $-\theta'(0)$. However, the skin friction coefficient and the rate of mass transfer are not

Table 1: Values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for varying values of K, M, Pr, Sc, Ec and β.

K	M	Pr	Sc	Ec	β	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	1	0.71	0.24	1	1	1.629178	−0.006338	0.561835
0.1	1	0.71	0.24	1	1	1.629178	0.0069647	0.561835
0.5	1	0.71	0.24	1	1	1.629178	0.0357547	0.561835
0.1	2	0.71	0.24	1	1	1.912620	−0.276418	0.554247
0.1	5	0.71	0.24	1	1	2.581130	−0.874464	0.541547
0.1	10	0.71	0.24	1	1	3.415289	−1.536591	0.531405
0.1	1	2.14	0.24	1	1	1.629178	−0.268846	0.561835
0.1	1	5.71	0.24	1	1	1.629178	−1.153452	0.561835
0.1	1	7.10	0.24	1	1	1.629178	−1.499348	0.561835
0.1	1	0.71	1	1	1	1.629178	0.0069647	1.399541
0.1	1	0.71	2	1	1	1.629178	0.0069647	2.394415
0.1	1	0.71	2.64	1	1	1.629178	0.0069647	3.027351
0.1	1	0.71	0.24	2	1	1.629178	−0.598521	0.561835
0.1	1	0.71	0.24	3	1	1.629178	−1.204006	0.561835
0.1	1	0.71	0.24	1	2	1.629178	0.0069647	0.754006
0.1	1	0.71	0.24	1	3	1.629178	0.0069647	0.903556

Table 2: Values of $\theta'(0)$ for different values of K, M and Pr compared to previous results.

K	M	Pr	Magyari and Keller [3]	El-Aziz [19]	Bidin and Nazar [6]	Ishak [12]	Present study
0	0	1	−0.954782	−0.954785	−0.9548	−0.9548	−0.954811
		2			−1.4714	−1.4715	−1.471454
		3	−1.869075	−1.869074	−1.8691	−1.8691	−1.869069
		5	−2.500132	−2.500132		−2.5001	−2.500128
		10	−3.660379	−3.660372		−3.6604	−3.660369
	1	1				−0.8611	−0.861509

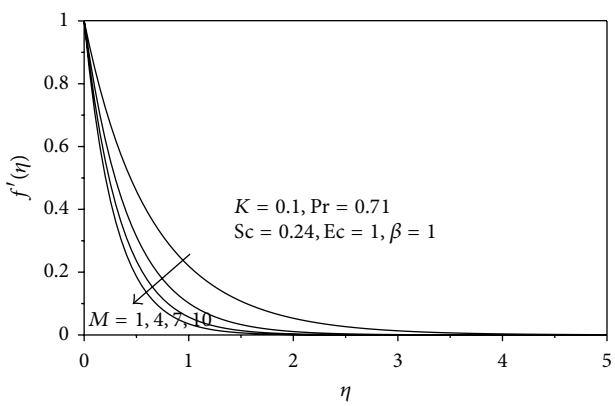

Figure 2: Velocity profiles for varying values of magnetic parameter (M).

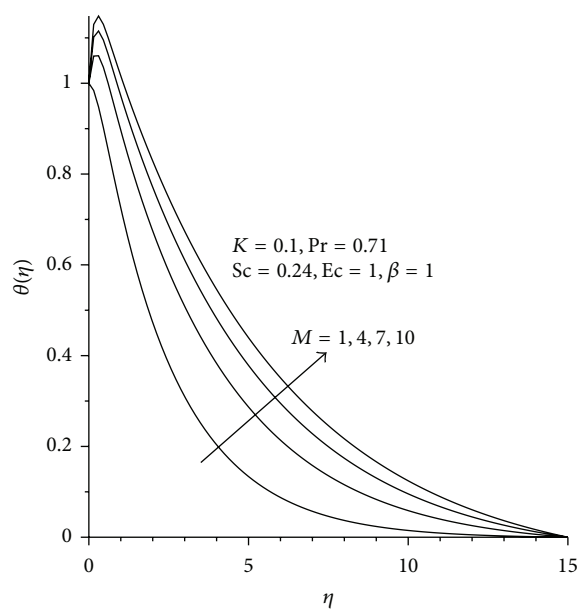

Figure 3: Temperature profiles for varying values of magnetic parameter (M).

affected by the radiation parameter. The magnetic parameter is observed to increase the skin friction coefficient at the surface due to the presence of the Lorenz force. It however reduces the rate of both heat and mass transfers at the boundary for obvious reasons. Conversely, the rate of mass transfers at the surface decreases with increasing values of reaction rate and Schmidt numbers whilst the rate of heat transfer decreases with increasing values of the Eckert number. Comparison with earlier results from the literature showed a perfect agreement (see Table 2).

The velocity profiles for increasing values of the magnetic field parameter (M) shown in Figure 2 indicate that the rate of flow is considerably reduced. This clearly reveals that the transverse magnetic field opposes the fluid transport due to increasing Lorentz force associated with increasing magnetic parameter. Figures 3 and 4 illustrate the effect of increasing the magnetic parameter on the temperature and concentration profiles, respectively. Whilst the temperature profiles

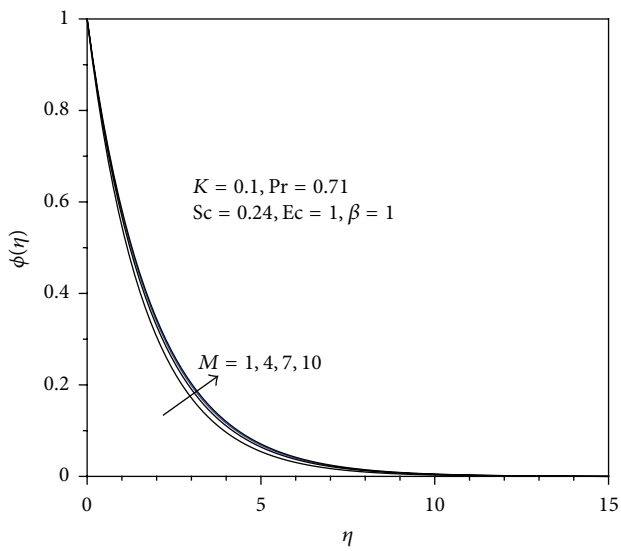

Figure 4: Concentration profiles for varying values of the magnetic parameter (M).

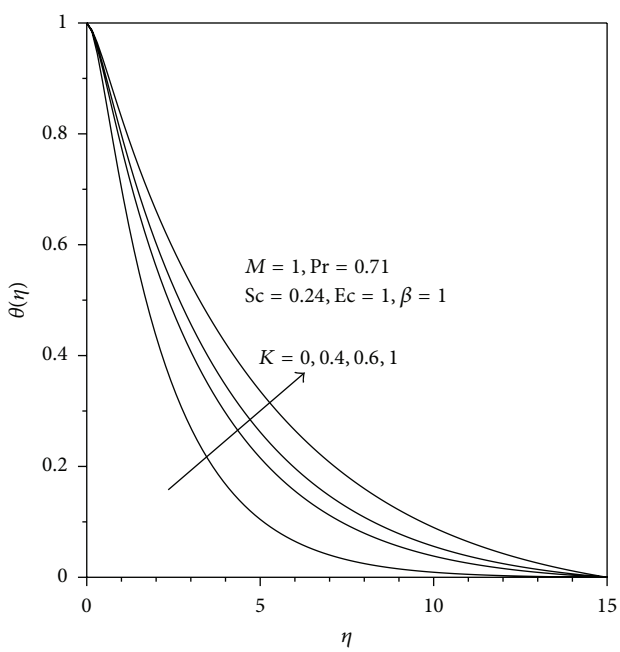

Figure 5: Temperature profiles for varying values of radiation parameter (K).

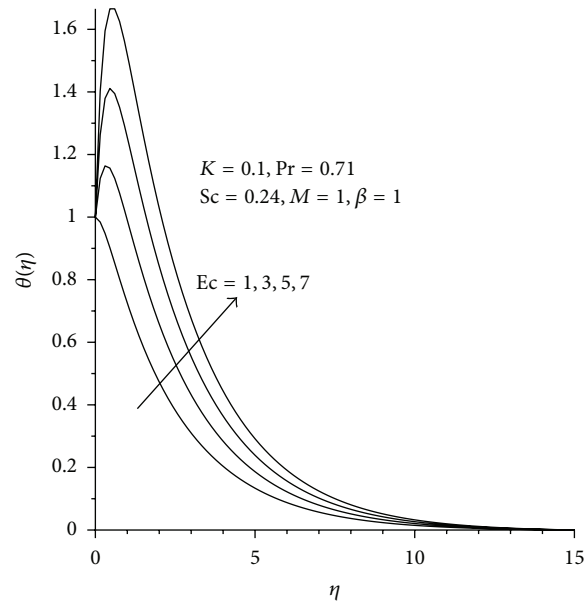

Figure 6: Temperature profiles for varying values of Eckert number (Ec).

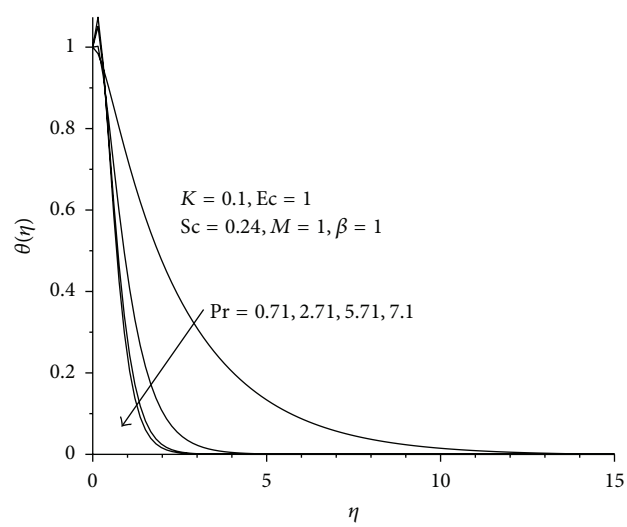

Figure 7: Temperature profiles for varying values of Prandtl number (Pr).

showed an increase with increasing magnetic parameter due to Ohmic heating, the concentration profiles indicate a slight increase. It is further noted that Prandtl number (Pr) and the radiation parameter (K) have no effects on the velocity and chemical concentration profiles which is clearly obvious from (12) and (14). Figures 5, 6, and 7 illustrate the effects of K, Ec, and Pr, respectively, on the temperature profiles. It is observed that increasing the radiation parameter (K) and the Eckert number (Ec) increases the thermal boundary layer thickness whilst the reverse is observed for increasing values of the Prandtl number (Pr). This is due to the fact that Pr decreases the thermal diffusivity resulting in the heat being diffused away from the surface more slowly and in consequence increases the temperature gradient at the surface. The influences of the Schmidt number (Sc) and the reaction rate parameter (β) on the concentration profiles are, respectively, illustrated in Figures 8 and 9. It can be observed that increases in both Sc and β reduce the concentration boundary layer. In all the illustrations (Figures 2–9), it is observed that the far field boundary conditions are satisfied asymptotically, supporting the accuracy of the numerical procedure.

5. Conclusions

The effect of radiation and chemical reaction on the steady MHD boundary layer flow over an exponentially stretching

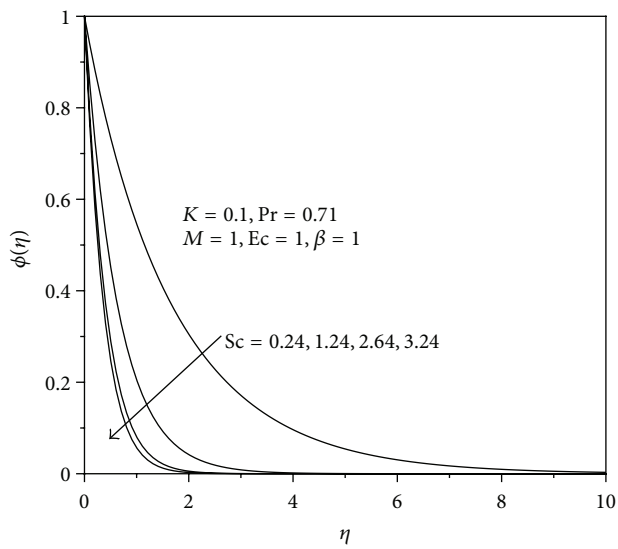

Figure 8: Concentration profiles for varying values of Schmidt number (Sc).

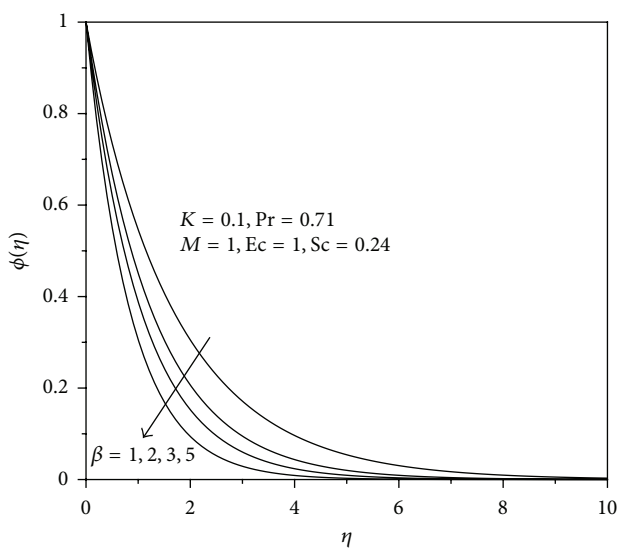

Figure 9: Concentration profiles for varying reaction rate parameter (β).

sheet was investigated. The numerical results showed good agreement with previously reported cases available in the literature. It was found that the surface shear stress increases with increasing magnetic parameter (M) whilst the heat transfer rate increases with Prandtl number Pr. It however decreases with both magnetic (M) and radiation (K) parameters. Furthermore, the chemical concentration boundary layer was found to decrease near the boundary with increasing reaction rate parameter and the Schmidt numbers.

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