

Research Article

Dynamics of Artificial Satellites around Europa

Jean Paulo dos Santos Carvalho,¹ Rodolpho Vilhena de Moraes,¹ and Antônio Fernando Bertachini de Almeida Prado²

¹ Instituto de Ciência e Tecnologia, Universidade Federal de São Paulo (UNIFESP), 12.331-280 São José dos Campos, SP, Brazil

² Division of Space Mechanics and Control, INPE, 12.227-010 São José dos Campos, SP, Brazil

Correspondence should be addressed to Jean Paulo dos Santos Carvalho; jeanfeg@gmail.com

Received 22 October 2012; Revised 21 December 2012; Accepted 2 January 2013

Academic Editor: Silvia Maria Giuliatti Winter

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A planetary satellite of interest at the present moment for the scientific community is Europa, one of the four largest moons of Jupiter. There are some missions planned to visit Europa in the next years, for example, Jupiter Europa Orbiter (JEO, NASA) and Jupiter Icy Moon Explorer (JUICE, ESA). In this paper, we search for orbits around Europa with long lifetimes. Here, we develop the disturbing potential in closed form up to the second order to analyze the effects caused on the orbital elements of an artificial satellite around Europa. The equations of motion are developed in closed form to avoid expansions in power series of the eccentricity and inclination. We found polar orbits with long lifetimes. This type of orbits reduces considerably the maintenance cost of the orbit. We show a formula to calculate the critical inclination of orbits around Europa taking into account the disturbing potential due to the nonspherical shape of the central body and the perturbation of the third body.

1. Introduction

A planetary satellite of interest at the present moment for the scientific community is Europa, one of the four largest moons of Jupiter. There are some missions planned to visit Europa in the next years, for example, Jupiter Europa Orbiter (JEO, NASA) and Jupiter Icy Moon Explorer (JUICE, ESA). The dynamics of orbits around a planetary satellite, taking into account the gravitational attraction of a third body and the nonuniform distribution of the mass of the planetary satellite, has been studied by several authors, for instance, [1–5] including inside-cited references. The search for frozen orbits (orbits that keep the periaapsis and the eccentricity almost constant) for planetary satellites is the subject of numerous papers, for instance, [6–8]. A fourth-order analytical theory is presented in [9] for the accurate computation of quasiperiodic frozen orbits.

In this work, we develop the disturbing potential in closed form up to the second order to analyze the effects caused on the orbital elements of the artificial satellite. The equations of motion are developed in closed forms to avoid expansions in power series of the eccentricity and inclination.

The approach for the development of the equations is based on [10–13]. In order to develop the long-period disturbing potential, the averaged method is applied. In this paper, the standard definition for average of periodic functions is applied with respect to eccentric anomaly (E) and true anomaly (f), using known equations from the celestial mechanics [14]. We analyze the disturbing potential effects on some orbital elements, such as eccentricity, inclination, argument of the periaapsis, and longitude of the ascending node.

2. Mathematical Model

In this work, we consider the effects caused by the nonsphericity (J_2, J_3) of the central body (Europa) and the perturbation caused by a third body (R_2 , i.e., the planet Jupiter, assumed to be in circular orbit in the present case) in artificial satellites around Europa. We present an analytical theory using the averaged model, and applications were done by integrating numerically the analytical equations developed. In [15], the disturbing potential is developed including the action of the inclination of the perturbing body. Here, this

inclination is neglected due to the fact that the orbit of Jupiter around Europa has small inclination, and so the disturbing potential is developed taking into account that Jupiter and Europa are on the same plane.

2.1. Disturbing Function. For the model considered in the present paper, it is necessary to calculate the R_2 term of the disturbing function due to the P_2 term. The disturbing potential R_2 can be written in the form [16]:

$$R_2 = \frac{\xi^2 \mu_J}{a_J} \left(\frac{r}{a}\right)^2 \left(\frac{a_J}{r_J}\right)^3 P_2(\cos S), \quad (1)$$

where P_2 is the Legendre polynomial, μ_J is the Jupiter's gravitational parameter, r is the modulus of the radius vector of the artificial satellite, r_J is the modulus of the radius vector of Jupiter, and S is the angle between radius vectors. The artificial satellite is considered as a point mass particle in a three-dimensional orbit with osculating orbital elements: a (semimajor axis), e (eccentricity), i (inclination), g (argument of the periapsis), h (longitude of the ascending node), and n (the mean motion).

Here,

$$\xi = \frac{a}{a_J}, \quad (2)$$

where a_J is the semimajor axis of Jupiter.

Using the relation between the angle S and the true anomaly f of the satellite, we get [11]

$$\cos(S) = \alpha \cos(f) + \beta \sin(f), \quad (3)$$

where f is the true anomaly of the artificial satellite.

Considering the disturbing body in a circular orbit, the coefficients α and β may be written in the form [11, 12]:

$$\begin{aligned} \alpha &= \cos(g) \cos(-h + l_J) + \cos(i) \sin(g) \sin(-h + l_J), \\ \beta &= -\sin(g) \cos(-h + l_J) + \cos(i) \cos(g) \sin(-h + l_J), \end{aligned} \quad (4)$$

where l_J (=171.016 degrees) is the mean anomaly of the disturbing body.

The averaged method is applied to eliminate the short-period terms of the artificial satellite. The standard definition for single averaged is applied with respect to the eccentric anomaly E . This is done by using known equations from the celestial mechanics, which are

$$\begin{aligned} \sin(f) &= \frac{\sqrt{1-e^2} \sin(E)}{1-e \cos(E)}, \\ \cos(f) &= \frac{\cos(E) - e}{1-e \cos(E)}, \\ \frac{r}{a} &= 1 - e \cos(E), \\ dl &= (1 - e \cos(E)) dE. \end{aligned} \quad (5)$$

Using the expressions given by (1)–(4), and taking into account the known relationships from the celestial mechanics mentioned above (5), we average the equations according to the eccentric anomalies to eliminate the short-period terms. Thus, we obtain the disturbing potential expanded up to the second order in a small parameter (ξ).

Note that the average was calculated only to eliminate the terms of short period of the artificial satellite. We consider the orbit of Jupiter as circular and fixed in space, so we do not eliminate the mean anomaly of Jupiter, for which we use the numerical values given at JPL (<http://naif.jpl.nasa.gov/naif/index.html>).

The long-period disturbing potential (R_2) can be written as

$$\begin{aligned} R_2 &= \frac{15}{32} n_J^2 a^2 \\ &\times \left(e^2 (\cos(i) + 1)^2 \cos(2g + 2h - 2l_J) + e^2 \right. \\ &\times (\cos(i) - 1)^2 \cos(2g - 2h + 2l_J) - \frac{6}{5} \\ &\times (\cos(i) - 1) \left(e^2 + \frac{2}{3} \right) (\cos(i) + 1) \cos(2h - 2l_J) \\ &+ \left(-2e^2 (\cos(i))^2 + 2e^2 \right) \cos(2g) \\ &\left. + \frac{6}{5} \left(e^2 + \frac{2}{3} \right) \left(-\frac{1}{3} + (\cos(i))^2 \right) \right), \end{aligned} \quad (6)$$

where n_J (=2.0477 $\times 10^{-5}$ rad/s) is the mean motion of the disturbing body.

2.2. Nonsphericity of Europa. To analyze the motion of an artificial satellite around Europa, it is necessary to take into account Europa's nonsphericity [1, 17–20]. Here, we also develop the equations of motion in closed form, but now the average is applied with respect to the true anomaly.

The zonal perturbation due to the oblateness is [19, 21]

$$R_{J2} = \epsilon \frac{\mu}{4r^3} \left(1 - 3 \cos(i)^2 - 3 \sin^2(i) \cos(2f + 2g) \right), \quad (7)$$

where $\epsilon = J_2 R_E^2$, μ (=3202.7 km³/s²) is the gravitational constant of Europa, and R_E is the equatorial radius of Europa ($R_E = 1560.8$ km).

The zonal perturbation due to the pear shaped is defined by [19, 21]

$$\begin{aligned} R_{J3} &= -\frac{\mu \sin(i) \epsilon_1}{8r^4} \\ &\times \left(3 \sin(f + g) - 5 \sin(3f + 3g) + 5 \sin(3f - 3g) \right. \\ &\left. \times \cos^2(i) - 15 \sin(f + g) \cos^2(i) \right), \end{aligned} \quad (8)$$

where $\epsilon_1 = J_3 R_E^3$.

TABLE 1: Numerical values for J_2 and J_3 .

J_2	4.355×10^{-4}
J_3	1.3784×10^{-4}

To eliminate the short-period terms of the potential given by (7) and (8), the averaged method is applied with respect to the true anomaly using known equations from the celestial mechanics, which are

$$\frac{a}{r} = \frac{1 + e \cos(f)}{1 - e^2}, \quad (9)$$

and we also used the area integral in the form [10]:

$$dl = \frac{1}{\sqrt{1 - e^2}} \frac{r^2}{a^2} df. \quad (10)$$

Using the expressions given by (7)–(10), we average the equations with respect to the true anomaly to eliminate short-period terms. Thus, considering the equatorial plane of Europa as the reference plane, we obtain the disturbing potential (R_{J_2}, R_{J_3}):

$$R_{J_2} = -\frac{1}{4} \frac{e n^2 (-2 + 3 \sin^2(i))}{(1 - e^2)^{3/2}}, \quad (11)$$

$$R_{J_3} = -\frac{3}{8} \frac{e e_1 n^2 \sin(i) (-4 + 5 \sin^2(i)) \sin(g)}{(1 - e^2)^{5/2} a}. \quad (12)$$

As mentioned above, only the harmonic coefficients J_2 and J_3 are considered. Their numerical values [22] are given in Table 1. In the case of Europa, there is no high-order model for the spherical harmonics, only J_2 is measured with some accuracy. The J_2 term was calculated during the passage of the Galileo spacecraft. The J_3 term is used by some authors with an approximate value.

3. Results

The disturbing potential of the orbital motion of artificial satellites orbiting Europa taking into account the gravitational attraction of a third body (R_2) and the nonuniform distribution of mass (J_2, J_3) of the planetary satellite can be written in the form:

$$R = R_2 + R_{J_2} + R_{J_3}. \quad (13)$$

To analyze the effects of the potential on the orbital elements of the orbit of the spacecraft, (13) is replaced by the Lagrange planetary equations [23], and a set of four nonlinear differential equations is numerically integrated using the software MapleSoft. Polar orbits are very important because they allow a better coverage of the surface of Europa when compared to other types of orbits. Orbits around Europa are generally of short duration, around 160 days [2, 3], so a study with a duration of around 300 days covers a time long enough for several types of missions. Figures 1 to 8 show the behavior

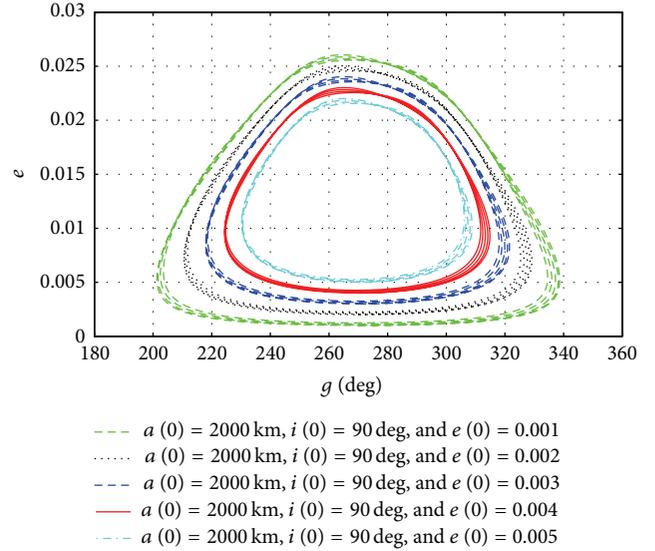


FIGURE 1: Time evolution (300 days) of the eccentricity e and the argument of the periaapsis g . Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

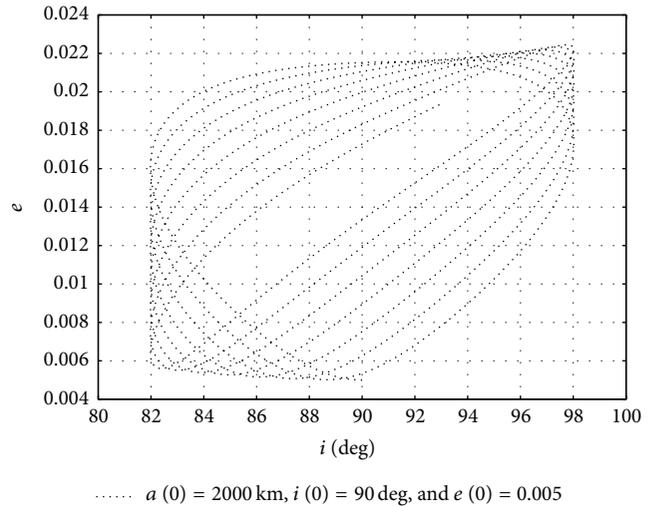


FIGURE 2: Diagram e versus i . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

of a polar orbit using the analytical equations developed. The diagram e versus g (Figures 1 and 5) shows the regions of libration for several values of the eccentricity.

Figures 1 to 4 show the results for orbits with a semimajor axis equal to 2000 km. Figures 5 to 8 show the results for orbits with a semimajor axis equal to 2341 km. The main difference among these figures is that the magnitude of the orbit with a semimajor axis of 2341 km has smaller libration amplitude than the orbit with a semimajor axis of 2000 km as shown in Figures 1 and 5. Performing this search, we found orbits with long lifetimes (frozen orbits, see [6, 8, 19]) that librate around an equilibrium point of $g = 270^\circ$, for example, with inclination $i = 90^\circ$ and a semimajor axis in the interval $2000 \leq a \leq 3000 \text{ km}$.

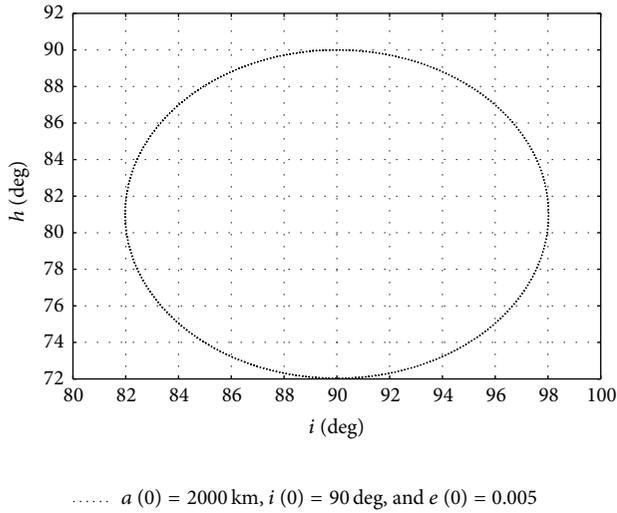


FIGURE 3: Diagram h versus i . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

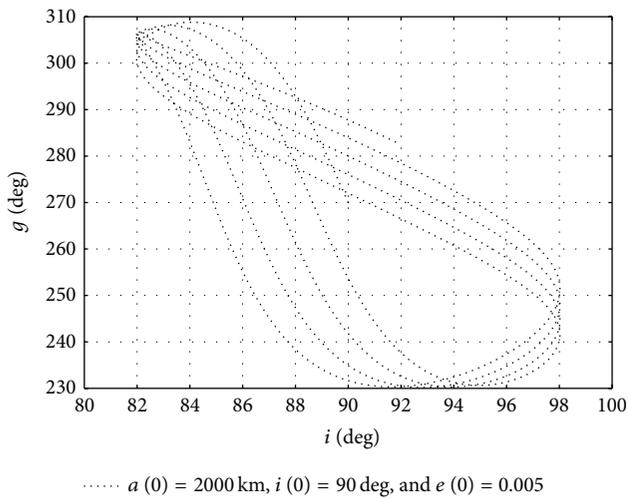


FIGURE 4: Diagram g versus i . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

As a result, the behavior of e versus g librates around an equilibrium point of $g = 270^\circ$ with small amplitudes. The set of initial conditions for orbits with long lifetimes is given by $a = 2341 \text{ km}$, $e = 0.005$, $i = 90^\circ$, $g = 270^\circ$, and $h = 90^\circ$. These initial conditions can be used by an Europa's orbiter to study the planetary satellite with lower cost of station keeping.

Figures 2 and 6 show the diagrams e versus i . Comparing these figures, we see that for the orbit where $a = 2341 \text{ km}$, the eccentricity ranged from 0.005 to 0.0064, while when considering $a = 2000 \text{ km}$, the eccentricity varies from 0.005 to 0.024. Figures 3 and 7 display the diagrams h versus i . In this case, the two figures show practically the same behavior. Figures 4 and 8 represent the diagrams g versus i . Note that for the orbit where $a = 2341 \text{ km}$, the g term ranges from 264° to 276° , while for the situation where $a = 2000 \text{ km}$, the

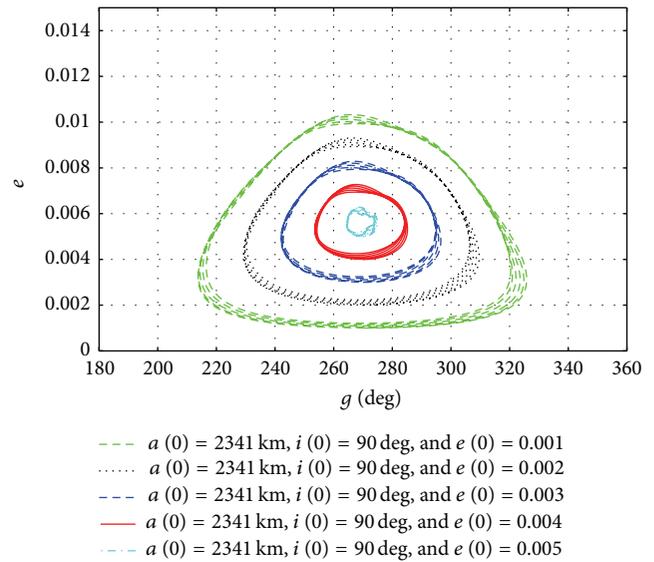


FIGURE 5: Time evolution (300 days) of the eccentricity e and periapsis g . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

g term varies from 230° to 310° . Note that the inclination was well behaved in all cases analyzed, ranging between 82° and 98° . Therefore, the orbit with $a = 2341 \text{ km}$ shows an important characteristic, where the orbital elements have small variations compared with the orbit where $a = 2000 \text{ km}$, which may be considered for future missions for artificial satellites around Europa.

Frozen orbits are very important for real missions because they require small amounts of fuel for orbital maintenance. In this situation, the cost is low to keep orbits that pass at the same altitude for a given latitude, benefiting the users with this regularity. In other words, this type of orbit maintains an almost constant altitude over any point on the surface of the central body. Since a satellite in polar orbit with a low altitude (about 100 km) around Europa collides in a short time period [2, 3], it is necessary to look for polar orbits in which the satellite presents a longer lifetime. Indeed, we found a region of initial semimajor axis where the polar orbits survive longer than that found in the literature [2, 3]. A semimajor axis of 2341 km is a good location for a spacecraft because it implies in an altitude around 768 km from the surface of Europa, and the orbit remains for a period of time longer than 300 days, as shown in Figure 5. We found near circular frozen polar orbits whose semimajor axis is in the interval between 2000 and 3000 km. This is high enough to avoid a strong effect from the non-Keplerian terms of the gravitational potential of Europa, but still not too high to cause problems in the observation due to the large distance between the spacecraft and Europa. The region found for the semimajor axis is due to the coupling of the perturbations of Jupiter and the nonspherical shape of the central body. It is worth noting that the orbits found survive for more than 300 days, since they are frozen (see Figure 5).

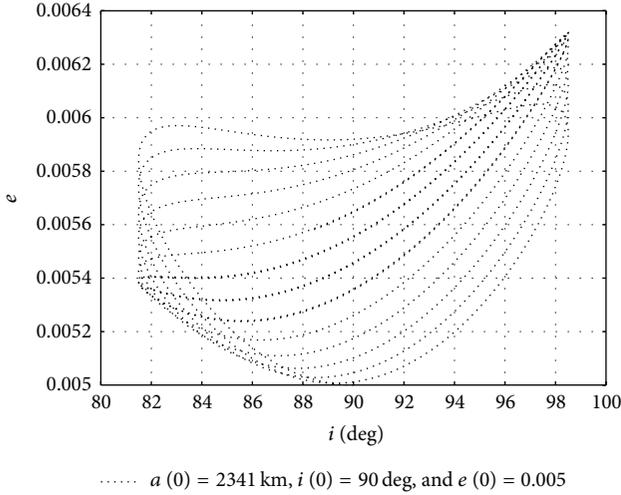


FIGURE 6: Diagram e versus i . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

The dynamics of orbits around planetary satellites, taking into account the gravitational attraction of a third body and the nonuniform distribution of mass of the planetary satellite, was studied in [19]. Lifetimes for these orbits are computed through the single- and double-averaged method. Comparison between the results obtained by the single- and double-averaged method was presented. Reference [19] also shows that the single-averaged model is more realistic than the double-averaged model. Considering the single-averaged method, [19] found unstable polar orbits where the satellite does not impact the surface of Europa for at least 200 days. Here we find, using the single-averaged model, stable polar orbits (frozen) around Europa where the satellite has longer lifetimes. It is important to emphasize that the value of the semimajor axis found here is different from the one presented in [19].

4. Critical Inclination

In [24], a formula to calculate the critical inclination of the case of a lunar orbit is presented taking into account the principal perturbations suffered by a satellite in a low orbit around Europa, which are the terms due to J_2 and C_{22} . In the case of a lunar orbit, the perturbation of the third body could be neglected for some analyses (see [6]). In the case of Europa, the disturbing body cannot be neglected in the dynamics because of the strong perturbation of Jupiter, the opposite of a lunar orbit where it is possible to neglect the perturbation of the Earth for some analysis. Then, we consider the two major perturbations for an orbit around Europa taking into account the influence of the third body and the nonuniform distribution of mass of the central body to develop a formula to calculate the critical inclination of these orbits.

The value of the inclination where $dg/dt = 0$ is called critical inclination. Now, applying the double averaged in (6) to eliminate the mean anomaly of Jupiter to simplify the

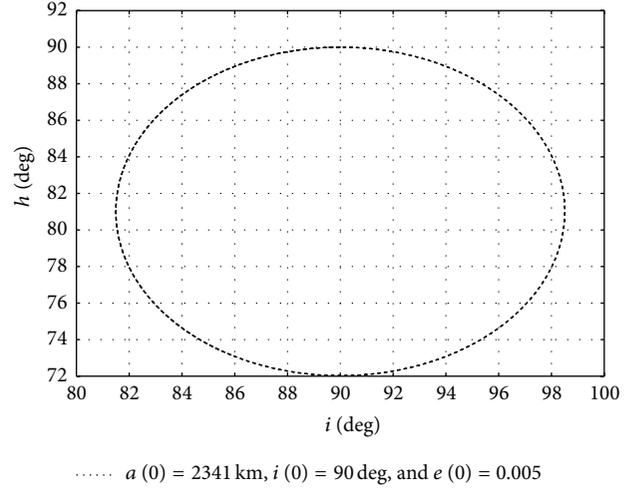


FIGURE 7: Diagram h versus i . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

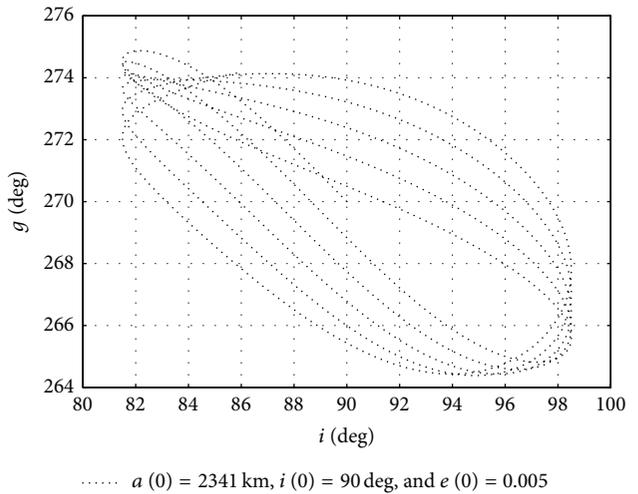


FIGURE 8: Diagram g versus i . Time evolution 300 days. Initial conditions: $h = 90^\circ$ and $g = 270^\circ$.

equations, we find a simplified formula to calculate the critical inclination of orbits around Europa. We get

$$R_{2DA} = -\frac{1}{16}a^2n_J^2 \times \left(2 + 3e^2 - 9e^2\cos^2(i) - 6\cos^2(i) + 15e^2\cos^2(i)\cos(2g) - 15e^2\cos(2g) \right). \quad (14)$$

Considering the effect of the third body (R_{2DA}) given by (14) and the oblateness of Europa (J_2) given by (11), replacing it in the Lagrange planetary equations and solving

the equation $dg/dt = 0$, we found a formula for the critical inclination. We get

$$\begin{aligned} \cos(i_c) = & \pm \frac{1}{5} \times \sqrt{5} \left((-2\epsilon n^2 - a^2 n_J^2 + a^2 \cos(2g) n_J^2) \right. \\ & \times \left. (-a^2 n_J^2 + 5a^2 \cos(2g) n_J^2 - 2\epsilon n^2) \right)^{1/2} \\ & \times (-2\epsilon n^2 - a^2 n_J^2 + a^2 \cos(2g) n_J^2)^{-1}, \end{aligned} \quad (15)$$

and replacing the initial conditions, we obtain

(1) $a = 1685$ km:

(a) in the case of a prograde orbit, the solution found is $i_c = 47.8^\circ$,

(b) in the case of a retrograde orbit, the solution found is $i_c = 132.2^\circ$,

(2) $a = 2000$ km:

(a) in the case of a prograde orbit, the solution found is $i_c = 43.9^\circ$,

(b) in the case of a retrograde orbit, the solution found is $i_c = 136.1^\circ$,

(3) $a = 2341$ km:

(a) in the case of a prograde orbit, the solution found is $i_c = 41.6^\circ$,

(b) in the case of a retrograde orbit, the solution found is $i_c = 138.4^\circ$.

Considering a low-altitude orbit, for example, $a = 1685$ km and neglecting the perturbation of the third body, we obtain the classical result for the critical inclination. Making $n_J = 0$ in (15), we get $i_c = 63.4^\circ$ and $i_c = 116.6^\circ$. Now neglecting the perturbation due to the nonspherical shape of Europa, we get the classical result for the perturbation of the third body ($\epsilon = 0$ in (15)) $i_c = 39.2^\circ$ and $i_c = 140.8^\circ$.

Our results are restricted to the influence of the gravitational attraction of Jupiter and Europa and of the nonuniform distribution of mass of Europa on the motion of artificial satellites with low and medium orbits. The solar gravitational attraction was not considered, and for orbits with large semimajor axis, the influence of the gravitational attraction of the Io and Ganymede satellites must be analyzed.

5. Conclusions

An analytical theory has been developed where the disturbing potential was obtained in closed form to avoid expansions in power series of the eccentricity and inclination. We have considered in the dynamics the influence of the third body (R_2) in a circular orbit and the nonuniform distribution of mass (J_2, J_3) of the planetary satellite. Numerical simulations were performed to analyze several orbits around Europa with the equations developed.

We applied the single-averaged method to eliminate the mean anomaly of the artificial satellite and keep the mean anomaly of Jupiter in the dynamics, considering its orbit fixed in the space. We found a region of a semimajor axis between $2000 < a$ (km) < 3000 , where polar orbits around Europa have longer lifetimes. Polar orbits that librate around an equilibrium point for different amplitudes which depends on the semimajor axis were found. As a result, it is shown that the behavior of e versus g librates around an equilibrium point for $g = 270^\circ$ with small amplitudes. We show two cases. The first one has an orbit with a semimajor axis equal to 2341 km, and it presents less libration amplitudes than the second case, that is, an orbit with $a = 2000$ km. Integrations were performed by a time period of 300 days. It was found that the coupled perturbations help to control the increase of the eccentricity and to keep the orbits that librate around the equilibrium point.

We show a formula to calculate the critical inclination of orbits around Europa considering the terms R_2 (third body) and J_2 (nonsphericity of the central body). The critical inclination for orbit with a semimajor axis equal to 2341 km is $i_c = 46.1^\circ$ for the prograde case and $i_c = 138.4^\circ$ for the retrograde case.

Acknowledgments

The authors are grateful to São Paulo Research Foundation (FAPESP) under the Contract nos. 2011/05671-5, 2011/09310-7, and 2011/08171-3, SP, Brazil, National Council for Scientific and Technological Development (CNPq), Brazil, for Contracts nos. 304700/2009-6 and 3003070/2011-0, and CAPES.

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