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Research Article

Robust Fuzzy H_∞ Output Feedback Control for a Class of Nonlinear Uncertain Systems with Mixed Time Delays

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This paper studies the problem of delay-dependent robust H_∞ output feedback control for a class of uncertain fuzzy neutral systems with both discrete and distributed delays. The system is described by a state-space Takagi-Sugeno fuzzy model with distributed delays and norm-bounded parameter uncertainties. The purpose is to design a fuzzy dynamic output feedback controller which ensures the robust asymptotic stability of the closed-loop fuzzy neutral system and satisfies an H_∞ norm bound constraint for all admissible uncertainties. In terms of linear matrix inequalities, sufficient conditions for the solvability of this problem are presented. Finally, a numerical example is included to demonstrate the effectiveness of the proposed method.

1. Introduction

Fuzzy control, as a promising way to approach nonlinear control problems, has had an impact on the control community [1–4]. Furthermore, the Takagi-Sugeno (T-S) fuzzy dynamic model [5–9] is nonlinear system described by fuzzy IF-THEN rules which give local linear representations of the underlying systems [10, 11]. It has been shown that such models can describe a wide class of nonlinear systems. Hence it is important to investigate their stability analysis and controller design problems, and in the past two decades many stability and control issues related to the T-S fuzzy systems have been studied; see, for example, [12–15] and the references cited therein.

On the other hand, time delays exist commonly in dynamic systems due to measurement, transmission, transport lags, and so forth [16], which have been generally regarded as a main source of instability and poor performance. Thus the analysis of time delay systems and controller design for them is very important [17–20]. Recently, T-S fuzzy systems with time delays have attracted a great deal of interests. For example, in [21], the stability analysis and stabilization problems for T-S fuzzy delay systems were considered, and state feedback fuzzy controllers and fuzzy observers were designed. The robust H_∞ control problem for T-S fuzzy systems with time delays was investigated in [22, 23], and state feedback fuzzy controllers were designed; the corresponding results for the discrete case can be found in [24, 25], while in [26, 27], the robust H_∞ output feedback controllers were designed for the continuous and discrete fuzzy time-delay systems, respectively.

Quite recently, T-S fuzzy time-delay systems of neutral type were introduced in [28], where both the stabilization and H_∞ control problems were studied. It should be point out that distributed delays were not taken into account. While in [29], authors considered the problems of robust stabilization and robust H_∞ control for uncertain T-S fuzzy neutral systems with both discrete and distributed time delays. However, the results in [28, 29] were all delay-independent. It is known that delay-dependent results are less conservative than delay-independent ones, especially in the case when the size of the delay is small. On the other hand, these obtained results, however, are mainly dealt with through a state feedback controller design method that requires all state variables to be available. In many cases, this condition is too restrictive. So it is meaningful to investigate the output feedback control method. To the best of our knowledge, so far, there are no results of delay-dependent robust H_∞ output feedback control for uncertain fuzzy neutral systems with both discrete and distributed delays. This motives the present studies.

In this paper, we consider the delay-dependent robust H_∞ output feedback control problem for fuzzy neutral systems with both discrete and distributed delays. The system to be considered is described by a state-space T-S fuzzy model with mixed delays and norm-bounded parameter uncertainties. The distributed delays are assumed to appear in the state equation, and the uncertainties are allowed to be time varying but norm bounded. The aim of this paper is to design a full-order fuzzy dynamic output feedback controller such that the resulting closed-loop system is robustly asymptotically stable while satisfying an H_∞ norm condition with a prescribed level irrespective of the parameter uncertainties. A sufficient condition for the solvability of this problem is proposed in terms of linear matrix inequalities (LMIs). When these LMIs are feasible, an explicit expression of a desired output feedback controller is also given.

Notation. Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite). I is an identity matrix with appropriate dimension. $\mathbb{N}$ is the set of natural numbers. $\mathscr{L}_2[0,\infty)$ refers to the space of square summable infinite vector sequences. $\|\cdot\|_2$ stands for the usual $\mathscr{L}_2[0,\infty)$ norm. The notation M^T represents the transpose of the matrix M. Matrices, if not explicitly stated, are assumed to have compatible dimensions. "$*$" is used as an ellipsis for terms induced by symmetry.

2. Problem Formulation

A continuous T-S fuzzy neutral model with distributed delays and parameter uncertainties can be described by the following.

Plant Rule i: if $s_1(t)$ is μ_{i1} and $\cdots$ and $s_p(t)$ is μ_{ip}, then

$$\begin{aligned}
\dot{x}(t) = {} & [A_i + \Delta A_i(t)]\, x(t) + [A_{1i} + \Delta A_{1i}(t)]\, x(t-\tau_1) \\
& + [A_{2i} + \Delta A_{2i}(t)]\, \dot{x}(t-\tau_2) \\
& + [A_{3i} + \Delta A_{3i}(t)] \int_{t-\tau_3}^{t} x(s)\, ds \\
& + [B_i + \Delta B_i(t)]\, u(t) + D_{1i} w(t), \\
y(t) = {} & [C_i + \Delta C_i(t)]\, x(t) + C_{1i} x(t-\tau_1) + D_{2i} w(t), \\
z(t) = {} & E_i x(t) + E_{1i} x(t-\tau_1) + G_i u(t), \\
x(t) = {} & \phi(t) \quad \forall t \in [-\overline{\tau}, 0],\ i = 1, 2, \ldots, r,
\end{aligned} \tag{1}$$

where μ_{ij} is the fuzzy set, r is the number of IF-THEN rules, and $s_1(t), \ldots, s_p(t)$ are the premise variables. Throughout this paper, it is assumed that the premise variables do not depend on control variables; $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^s$ is the measured output; $z(t) \in \mathbb{R}^q$ is the controlled output; $w(t) \in \mathbb{R}^p$ is the noise signal; $\tau_i > 0$ $(i = 1, 2, 3)$ are integers representing the time delay of the fuzzy systems; $\overline{\tau} = \max\{\tau_1, \tau_2, \tau_3\}$; A_i, A_{1i}, A_{2i}, A_{3i}, B_i, C_i, C_{1i}, D_{1i}, D_{2i}, E_i, E_{2i}, and G_i are known real constant matrices; $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, $\Delta A_{3i}(t)$, $\Delta B_i(t)$, and $\Delta C_i(t)$ are real-valued unknown matrices representing time-varying parameter uncertainties and are assumed to be of the form

$$\begin{aligned}
& [\Delta A_i(t) \ \ \Delta A_{1i}(t) \ \ \Delta A_{2i}(t) \ \ \Delta A_{3i}(t) \ \ \Delta B_i(t) \ \ \Delta C_i(t)] \\
& \quad = M_i F_i(t) [N_{0i} \ \ N_{1i} \ \ N_{2i} \ \ N_{3i} \ \ N_{4i} \ \ N_{5i}] \quad i = 1, 2, \ldots, r,
\end{aligned} \tag{2}$$

where M_i, N_{0i}, N_{1i}, N_{2i}, N_{3i}, N_{4i}, and N_{5i} are known real constant matrices and $F_i(\cdot) : \mathbb{N} \to \mathbb{R}^{l_1 \times l_2}$ are unknown time-varying matrix function satisfying

$$F_i(t)^T F_i(t) \leq I, \quad \forall t. \tag{3}$$

The parameter uncertainties $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, $\Delta A_{3i}(t)$, $\Delta B_i(t)$, and $\Delta C_i(t)$ are said to be admissible if both (2) and (3) hold.

Then the final output of the fuzzy neutral system is inferred as follows:

$$\begin{aligned}
\dot{x}(t) = {} & \sum_{i=1}^{r} h_i(s(t)) \Big\{ [A_i + \Delta A_i(t)]\, x(t) \\
& + [A_{1i} + \Delta A_{1i}(t)]\, x(t-\tau_1) \\
& + [A_{2i} + \Delta A_{2i}(t)]\, \dot{x}(t-\tau_2) \\
& + \Big[A_{3i} + \Delta A_{3i}(t) \int_{t-\tau_3}^{t} x(s)\, ds \Big] \\
& + [B_i + \Delta B_i(t)]\, u(t) + D_{1i} w(t) \Big\}, \\
y(t) = {} & \sum_{i=1}^{r} h_i(s(t)) \{ [C_i + \Delta C_i(t)]\, x(t) \\
& + C_{1i} x(t-\tau_1) + D_{2i} w(t) \}, \\
z(t) = {} & \sum_{i=1}^{r} h_i(s(t)) [E_i x(t) + E_{1i} x(t-\tau_1) + G_i u(t)],
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
h_i(s(t)) &= \frac{\varpi_i(s(t))}{\sum_{i=1}^{r} \varpi_i(s(t))}, \\
\varpi_i(s(t)) &= \prod_{j=1}^{p} \mu_{ij}(s_j(t)), \\
s(t) &= [s_1(t) \ \ s_2(t) \ \ \cdots \ \ s_p(t)],
\end{aligned} \tag{5}$$

in which $\mu_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in μ_{ij}. Then, it can be seen that

$$\begin{aligned}
& \varpi_i(s(t)) \geq 0, \quad i = 1, \ldots, r, \\
& \sum_{i=1}^{r} \varpi_i(s(t)) > 0,
\end{aligned} \tag{6}$$

for all t. Therefore, for all t,

$$h_i(s(t)) \geq 0, \quad i = 1, \ldots, r,$$
$$\sum_{i=1}^{r} h_i(s(t)) = 1, \quad \forall t. \tag{7}$$

Now, by the parallel distributed compensation (PDC), the following full-order fuzzy dynamic output feedback controller for the fuzzy neutral system in (4) is considered.

Control Rule i: if $s_1(t)$ is μ_{i1} and $\cdots$ and $s_p(t)$ is μ_{ip}, then

$$\dot{\hat{x}}(t) = A_{ki}\hat{x}(t) + B_{ki}y(t), \tag{8}$$

$$u(t) = C_{ki}\hat{x}(t), \quad i = 1, 2, \ldots, r, \tag{9}$$

$$\hat{x}(t) = 0, \quad t \leq 0, \tag{10}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the controller state and A_{ki}, B_{ki}, and C_{ki} are matrices to be determined later. Then, the overall fuzzy output feedback controller is given by

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(s(t)) \left[A_{ki}\hat{x}(t) + B_{ki}y(t)\right], \tag{11}$$

$$u(t) = \sum_{i=1}^{r} h_i(s(t)) C_{ki}\hat{x}(t), \quad i = 1, 2, \ldots, r. \tag{12}$$

From (4) and (11), one can obtain the closed-loop system as

$$\begin{aligned}\dot{e}(t) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t)) h_j(s(t)) \\ & \times \Big\{\left[A_{ij} + \Delta A_{ij}(t)\right] e(t) \\ & + \left[A_{1ij} + \Delta\widetilde{A}_{1i}(t)\right] e(t-\tau_1) \\ & + \left[\widetilde{A}_{2i} + \Delta\widetilde{A}_{2i}(t)\right] \dot{e}(t-\tau_2) \\ & + \left[\widetilde{A}_{3i} + \Delta\widetilde{A}_{3i}(t)\right] \\ & \times \int_{t-\tau_3}^{t} e(s)\,ds + D_{ij}w(t)\Big\},\end{aligned} \tag{13}$$

$$z(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t)) h_j(s(t)) \left[E_{ij}e(t) + \widetilde{E}_{1i}e(t-\tau_1)\right], \tag{14}$$

where

$$\begin{gathered}\dot{e}(t) = \begin{bmatrix}\dot{x}(t) \\ \dot{\hat{x}}(t)\end{bmatrix}, \\ A_{ij} = \begin{bmatrix}A_i & B_iC_{kj} \\ B_{kj}C_i & A_{kj}\end{bmatrix}, \\ \Delta A_{ij}(t) = \begin{bmatrix}\Delta A_i(t) & \Delta B_i(t)C_{kj} \\ B_{kj}\Delta C_i(t) & 0\end{bmatrix}, \\ A_{1ij} = \begin{bmatrix}A_{1i} & 0 \\ B_{kj}C_{1i} & 0\end{bmatrix}, \qquad \Delta\widetilde{A}_{1i}(t) = \begin{bmatrix}\Delta A_{1i}(t) & 0 \\ 0 & 0\end{bmatrix}, \\ \widetilde{A}_{2i} = \begin{bmatrix}A_{2i} & 0 \\ 0 & 0\end{bmatrix}, \qquad \Delta\widetilde{A}_{2i}(t) = \begin{bmatrix}\Delta A_{2i}(t) & 0 \\ 0 & 0\end{bmatrix}, \\ \widetilde{A}_{3i} = \begin{bmatrix}A_{3i} & 0 \\ 0 & 0\end{bmatrix}, \qquad \Delta\widetilde{A}_{3i}(t) = \begin{bmatrix}\Delta A_{3i}(t) & 0 \\ 0 & 0\end{bmatrix}, \\ D_{ij} = \begin{bmatrix}D_{1i} \\ B_{kj}D_{2i}\end{bmatrix}, \qquad E_{ij} = \left[E_i \;\; G_iC_{kj}\right], \\ \widetilde{E}_{1i} = \left[E_{1i} \;\; 0\right].\end{gathered} \tag{15}$$

Then, the problem of robust fuzzy H_∞ control problem to be addressed in the previous is formulated as follows: given an uncertain distributed delay fuzzy neutral system in (4) and a scalar $\gamma > 0$, determine a dynamic output feedback fuzzy controller in the form of (8) and (9) such that the closed-loop system in (13) and (14) is robustly asymptotically stable when $w(t) = 0$ and

$$\|z\|_2 < \gamma\|w\|_2, \tag{16}$$

under zero-initial conditions for any nonzero $w(t) \in \mathscr{L}_2[0, \infty)$ and all admissible uncertainties.

3. Main Results

In this section, an LMI approach will be developed to solve the problem of robust output feedback H_∞ control of uncertain distributed delay fuzzy neutral systems formulated in the previous section. We first give the following results which will be used in the proof of our main results.

Lemma 1 (see [30]). *Let $\mathscr{A}$, $\mathscr{D}$, $\mathscr{S}$, $\mathscr{W}$, and F be real matrices of appropriate dimensions with $\mathscr{W} > 0$ and F satisfying $F^TF \leq I$. Then one has the following.*

(1) *For any scalar $\epsilon > 0$ and vectors x, $y \in \mathbb{R}^n$,*

$$2x^T\mathscr{D}\mathscr{F}\mathscr{S}y \leq \epsilon^{-1}x^T\mathscr{D}\mathscr{D}^Tx + \epsilon y^T\mathscr{S}^T\mathscr{S}y. \tag{17}$$

(2) *For any scalar $\epsilon > 0$ such that $\mathscr{W} - \epsilon\mathscr{D}\mathscr{D}^T > 0$,*

$$\begin{aligned}&(\mathscr{A} + \mathscr{D}\mathscr{F}\mathscr{S})^T\mathscr{W}^{-1}(\mathscr{A} + \mathscr{D}\mathscr{F}\mathscr{S}) \\ &\quad \leq \mathscr{A}^T\left(\mathscr{W} - \epsilon\mathscr{D}\mathscr{D}^{\mathscr{T}}\right)^{-1}\mathscr{A} + \epsilon^{-1}\mathscr{S}^T\mathscr{S}.\end{aligned} \tag{18}$$

Lemma 2 (see [31]). *Given any matrices $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{Z}$ with appropriate dimensions such that $\mathcal{Y} > 0$, then, one has*

$$\mathcal{X}^T\mathcal{Z} + \mathcal{Z}^T\mathcal{X} \le \mathcal{X}^T\mathcal{Y}\mathcal{X} + \mathcal{Z}^T\mathcal{Y}^{-1}\mathcal{Z}. \tag{19}$$

Theorem 3. *The uncertain fuzzy neutral delay system in* (13) *and* (14) *is asymptotically stable, and* (16) *is satisfied if there exist matrices $P > 0$, $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, R_1, and R_2 and scalars $\epsilon_{ij} > 0$, $1 \le i \le j \le r$, such that the following LMIs hold:*

$$\begin{bmatrix} \Pi_{1ii} & \Pi_{2ii} & P\widetilde{A}_{2i} & P\widetilde{A}_{3i} & PD_{ii} & \tau_1 R_1 & E_{ii}^T & \Gamma_{ii} & PM_{ii} & N_{ii}^T \\ * & J_1 & 0 & 0 & 0 & \tau_1 R_2 & E_{1i}^T & \Gamma_{1ii} & 0 & 0 \\ * & * & -P_2 & 0 & 0 & 0 & 0 & \Gamma_{2i} & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & \Gamma_{3i} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & \Gamma_{4ii} & 0 & 0 \\ * & * & * & * & * & -\tau_1 Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -J_2^T & J_2^T M_{ii} & 0 \\ * & * & * & * & * & * & * & * & -\epsilon_{ii} I & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_{ii}^{-1} I \end{bmatrix} < 0, \quad 1 \le i \le r,$$

$$\begin{bmatrix} \Lambda_{1ij} & \Lambda_{2ij} & \Lambda_{3ij} \\ * & \Lambda_{4ij} & \Lambda_{5ij} \\ * & * & \Lambda_{6ij} \end{bmatrix} < 0, \quad 1 \le i < j \le r, \tag{20}$$

where

$$\Lambda_{1ij} = \begin{bmatrix} \Pi_{1ij} + \Pi_{1ji} & \Pi_{2ij} + \Pi_{2ji} & P\widetilde{A}_{2i} + P\widetilde{A}_{2j} & P\widetilde{A}_{3i} + P\widetilde{A}_{3j} \\ * & 2J_1 & 0 & 0 \\ * & * & -2P_2 & 0 \\ * & * & * & -2Q_2 \end{bmatrix},$$

$$\Lambda_{2ij} = \begin{bmatrix} PD_{ij} + PD_{ji} & 2\tau_1 R_1 & E_{ij}^T + E_{ji}^T & \Gamma_{ij} + \Gamma_{ji} \\ 0 & 2\tau_1 R_2 & \widetilde{E}_{1i}^T + \widetilde{E}_{1j}^T & \Gamma_{1ij} + \Gamma_{1ji} \\ 0 & 0 & 0 & \Gamma_{2i} + \Gamma_{2j} \\ 0 & 0 & 0 & \Gamma_{3i} + \Gamma_{3j} \end{bmatrix},$$

$$\Lambda_{3ij} = \begin{bmatrix} PM_{ij} & PM_{ji} & N_{ij}^T & N_{ji}^T \\ 0 & 0 & \widetilde{N}_{1i}^T & \widetilde{N}_{1j}^T \\ 0 & 0 & \widetilde{N}_{2i}^T & \widetilde{N}_{2j}^T \\ 0 & 0 & \widetilde{N}_{3i}^T & \widetilde{N}_{3j}^T \end{bmatrix},$$

$$\Lambda_{4ij} = \begin{bmatrix} -2\gamma^2 I & 0 & 0 & \Gamma_{4ij} + \Gamma_{4ji} \\ * & -2\tau_1 Q_1 & 0 & 0 \\ * & * & -2I & 0 \\ * & * & * & -2J_2^T \end{bmatrix},$$

$$\Lambda_{5ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ J_2^T M_{ij} & J_2^T M_{ji} & 0 & 0 \end{bmatrix},$$

$$\Lambda_{6ij} = \begin{bmatrix} -\epsilon_{ij} I & 0 & 0 & 0 \\ * & -\epsilon_{ji} I & 0 & 0 \\ * & * & -\epsilon_{ij}^{-1} I & 0 \\ * & * & * & -\epsilon_{ji}^{-1} I \end{bmatrix},$$

$$\begin{gathered} \Upsilon_1 = H^T\left(P_1 + \tau_3^2 Q_2 - R_1 - R_1^T\right)H, \\ \Pi_{1ij} = \Upsilon_1 + PA_{ij} + A_{ij}^T P, \\ \Upsilon_2 = H^T\left(R_1 - R_2^T\right), \qquad \Pi_{2ij} = \Upsilon_2 + PA_{1ij}, \\ \Gamma_{ij} = A_{ij}^T\left(P_2 + \tau_1 Q_1\right), \\ \Gamma_{1ij} = A_{1ij}^T\left(P_2 + \tau_1 Q_1\right), \qquad \Gamma_{2i} = \widetilde{A}_{2i}^T\left(P_2 + \tau_1 Q_1\right), \\ \Gamma_{3i} = \widetilde{A}_{3i}^T\left(P_2 + \tau_1 Q_1\right), \qquad \Gamma_{4ij} = D_{ij}^T\left(P_2 + \tau_1 Q_1\right), \\ J_1 = -P_1 + R_2 + R_2^T, \qquad J_2 = P_2 + \tau_1 Q_1, \\ M_{ij} = \begin{bmatrix} M_i & 0 \\ 0 & B_{kj} M_i \end{bmatrix}, \qquad \widetilde{F}_i(t) = \begin{bmatrix} F_i(t) & 0 \\ 0 & F_i(t) \end{bmatrix}, \\ N_{ij} = \begin{bmatrix} N_{0i} & N_{3i} C_{kj} \\ N_{4i} & 0 \end{bmatrix}, \qquad \widetilde{N}_{1i} = \begin{bmatrix} N_{1i} & 0 \\ 0 & 0 \end{bmatrix}, \\ \widetilde{N}_{2i} = \begin{bmatrix} N_{2i} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \widetilde{N}_{3i} = \begin{bmatrix} N_{3i} & 0 \\ 0 & 0 \end{bmatrix}. \end{gathered} \tag{21}$$

Proof. To establish the robust stability of the system in (13), we consider (13) with $w(t) \equiv 0$; that is,

$$\begin{aligned} \dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} & h_i(s(t))\, h_j(s(t)) \\ & \times \Big\{ \left[A_{ij} + \Delta A_{ij}(t)\right] e(t) \\ & + \left[A_{1ij} + \Delta\widetilde{A}_{1i}(t)\right] e(t - \tau_1) \\ & + \left[\widetilde{A}_{2i} + \Delta\widetilde{A}_{2i}(t)\right] \dot{e}(t - \tau_2) \\ & + \left[\widetilde{A}_{3i} + \Delta\widetilde{A}_{3i}(t)\right] \int_{t-\tau_3}^{t} e(s)\, ds \Big\}. \end{aligned} \tag{22}$$

For this system, we define the following Lyapunov function candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t), \tag{23}$$

where

$$\begin{aligned} V_1(t) &= e(t)^T P e(t), \\ V_2(t) &= \int_{t-\tau_1}^{t} e(s)^T P_1 e(s)\, ds, \\ V_3(t) &= \int_{t-\tau_2}^{t} \dot{e}(s)^T P_2 \dot{e}(s)\, ds, \\ V_4(t) &= \int_{-\tau_1}^{0} \int_{t+\beta}^{t} \dot{e}(\alpha)^T Q_1 \dot{e}(\alpha)\, d\alpha\, d\beta, \\ V_5(t) &= \int_{t-\tau_3}^{t} \left[\int_{s}^{t} e(\theta)^T d\theta \right] Q_2 \left[\int_{s}^{t} e(\theta)\, d\theta \right] ds, \\ V_6(t) &= \int_{0}^{\tau_3} ds \int_{t-\beta}^{t} (\theta - t + \beta)\, e(\alpha)^T Q_2 e(\alpha)\, d\alpha\, d\beta. \end{aligned} \tag{24}$$

The time derivative of $V(t)$ is given by

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t), \tag{25}$$

where

$$\begin{aligned} \dot{V}_1(t) &= 2e(t)^T P \dot{e}(t), \\ \dot{V}_2(t) &= e(t)^T P_1 e(t) - e(t-\tau_1)^T P_1 e(t-\tau_1), \\ \dot{V}_3(t) &= \dot{e}(t)^T P_2 \dot{e}(t) - \dot{e}(t-\tau_2)^T P_2 \dot{e}(t-\tau_2), \\ \dot{V}_4(t) &= \tau_1 \dot{e}(t)^T Q_1 \dot{e}(t) - \int_{t-\tau_1}^{t} \dot{e}(\alpha)^T Q_1 \dot{e}(\alpha)\, d\alpha, \\ \dot{V}_5(t) &= 2\int_{t-\tau_3}^{t} (\theta - t + \tau_3)\, e(t)^T Q_2 e(\theta)\, d\theta \\ &\quad - \left[\int_{t-\tau_3}^{t} e(\theta)^T d\theta \right] Q_2 \left[\int_{t-\tau_3}^{t} e(\theta)\, d\theta \right], \\ \dot{V}_6(t) &= \frac{1}{2}\tau_3^2 e(t)^T Q_2 e(t) - \int_{t-\tau_3}^{t} (\theta - t + \tau_3)\, e(\theta)^T Q_2 e(\theta)\, d\theta. \end{aligned} \tag{26}$$

Now, by Lemma 1, it can be shown that

$$2e(t)^T Q_2 e(\theta) \le e(t)^T Q_2 e(t) + e(\theta)^T Q_2 e(\theta). \tag{27}$$

Therefore

$$\begin{aligned} \dot{V}_5(t) \le{}& \frac{1}{2}\tau_3^2 e(t)^T Q_2 e(t) \\ &+ \int_{t-\tau_3}^{t} (\theta - t + \tau_3)\, e(\theta)^T Q_2 e(\theta)\, d\theta \\ &- \left[\int_{t-\tau_3}^{t} e(\theta)^T d\theta \right] Q_2 \left[\int_{t-\tau_3}^{t} e(\theta)\, d\theta \right]. \end{aligned} \tag{28}$$

This together with (26) implies

$$\dot{V}_5(t) + \dot{V}_6(t) \le \tau_3^2 e(t)^T Q_2 e(t) - \alpha(t)^T Q_2 \alpha(t), \tag{29}$$

where

$$\alpha(t) = \int_{t-\tau_3}^{t} e(\theta)\, d\theta. \tag{30}$$

Then, there holds

$$\begin{aligned} \dot{V}(t) \le{}& 2e(t)^T P \dot{e}(t) \\ &+ e(t)^T P_1 e(t) - e(t-\tau_1)^T P_1 e(t-\tau_1) \\ &+ \dot{e}(t)^T P_2 \dot{e}(t) - \dot{e}(t-\tau_2)^T P_2 \dot{e}(t-\tau_2) \\ &+ \tau_1 \dot{e}(t)^T Q_1 \dot{e}(t) - \int_{t-\tau_1}^{t} \dot{e}(\alpha)^T Q_1 \dot{e}(\alpha)\, d\alpha \\ &+ \tau_3^2 e(t)^T Q_2 e(t) - \alpha(t)^T Q_2 \alpha(t) \\ &+ 2e(t)^T R_1 \int_{t-\tau_1}^{t} \dot{e}(\alpha)\, d\alpha \\ &- 2e(t)^T R_1 [e(t) - e(t-\tau_1)] \\ &+ 2e(t-\tau_1)^T R_2 \int_{t-\tau_1}^{t} \dot{e}(\alpha)\, d\alpha \\ &- 2e(t-\tau_1)^T R_2 [e(t) - e(t-\tau_1)]. \end{aligned} \tag{31}$$

It follows from (22) and Lemma 2 that

$$\begin{aligned} &2e(t)^T P \dot{e}(t) \\ &\quad = 2e(t)^T P \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(s(t))\, h_j(s(t)) \\ &\qquad \times \Big\{ \left[A_{ij} + \Delta A_{ij}(t) \right] e(t) \\ &\qquad\quad + \left[A_{1ij} + \Delta \widetilde{A}_{1i}(t) \right] e(t-\tau_1) \\ &\qquad\quad + \left[\widetilde{A}_{2i} + \Delta \widetilde{A}_{2i}(t) \right] \dot{e}(t-\tau_2) \\ &\qquad\quad + \left[\widetilde{A}_{3i} + \Delta \widetilde{A}_{3i}(t) \right] \int_{t-\tau_3}^{t} e(s)\, ds \Big\} \\ &\quad \le 2\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(s(t))\, h_j(s(t))\, e(t)^T P \overline{A}_{ij} \beta(t) \\ &\qquad + \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(s(t))\, h_j(s(t)) \\ &\qquad\quad \times \left[\varepsilon_{1ij}^{-1} e(t)^T P M_{ij} M_{ij}^T P e(t) \right. \\ &\qquad\quad\quad \left. + \varepsilon_{1ij} \beta(t)^T \widetilde{N}_{ij}^T \widetilde{N}_{ij} \beta(t) \right], \end{aligned} \tag{32}$$

where

$$\begin{aligned} \overline{A}_{ij} &= \left[A_{ij} \;\; A_{1ij} \;\; \widetilde{A}_{2i} \;\; \widetilde{A}_{3i} \right], \\ \beta(t) &= \left[e(t)^T \;\; e(t-\tau_1)^T \;\; \dot{e}(t-\tau_2)^T \;\; \alpha(t)^T \;\; \dot{e}(\alpha)^T \right], \\ \widetilde{N}_{ij} &= \left[N_{ij} \;\; \widetilde{N}_{1i} \;\; \widetilde{N}_{2i} \;\; \widetilde{N}_{3i} \right]. \end{aligned} \tag{33}$$

It also can be verified that

$$\begin{aligned}
&\dot{e}(t)^T P_2 \dot{e}(t) \\
&= \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_i(s(t))\, h_j(s(t))\, h_u(s(t))\, h_v(s(t))\, \beta(t)^T \\
&\qquad \times \Big\{\big[\overline{A}_{ij} + M_{ij}\widetilde{F}_i(t)\widetilde{N}_{ij}\big]^T \\
&\qquad\quad \times P_2\big[\overline{A}_{uv} + M_{uv}\widetilde{F}_u(t)\widetilde{N}_{uv}\big] \\
&\qquad\quad + \big[\overline{A}_{uv} + M_{uv}\widetilde{F}_u(t)\widetilde{N}_{uv}\big]^T \\
&\qquad\quad \times P_2\big[\overline{A}_{ij} + M_{ij}\widetilde{F}_i(t)\widetilde{N}_{ij}\big]\Big\}\beta(t) \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))\, h_j(s(t))\, \beta(t)^T \\
&\qquad \times \big[\overline{A}_{ij} + M_{ij}\widetilde{F}_i(t)\widetilde{N}_{ij}\big]^T P_2 \big[\overline{A}_{ij} + M_{ij}\widetilde{F}_i(t)\widetilde{N}_{ij}\big] \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))\, h_j(s(t))\, \beta(t)^T \\
&\qquad \times \Big[\overline{A}_{ij}^T\big(P_2^{-1} - \varepsilon_{2ij}^{-1}M_{ij}M_{ij}^T\big)^{-1}\overline{A}_{ij} + \varepsilon_{2ij}\widetilde{N}_{ij}^T\widetilde{N}_{ij}\Big]\beta(t), \\
&\tau_1\dot{e}(t)^T Q_1 \dot{e}(t) \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))\, h_j(s(t))\, \beta(t)^T \\
&\qquad \times \Big\{\overline{A}_{ij}^T\big[(\tau_1 Q_1)^{-1} - \varepsilon_{3ij}^{-1}M_{ij}M_{ij}^T\big]^{-1}\overline{A}_{ij} \\
&\qquad\quad + \varepsilon_{3ij}\widetilde{N}_{ij}^T\widetilde{N}_{ij}\Big\}\beta(t).
\end{aligned} \tag{34}$$

Hence, with the support of the above conditions, we have

$$\begin{aligned}
\dot{V}(t) &\le 2e(t)^T P\dot{e}(t) + \dot{e}(t)^T P_2\dot{e}(t) \\
&\quad + \tau_1\dot{e}(t)^T Q_1\dot{e}(t) + e(t)^T\big(P_1 + \tau_3^2 Q_2\big)e(t) \\
&\quad - e(t-\tau_1)^T P_1 e(t-\tau_1) \\
&\quad - \dot{e}(t-\tau_2)^T P_2\dot{e}(t-\tau_2) - \int_{t-\tau_1}^{t}\dot{e}(\alpha)^T Q_1\dot{e}(\alpha)\,d\alpha \\
&\quad - \alpha(t)^T Q_2\alpha(t) + 2e(t)^T R_1\int_{t-\tau_1}^{t}\dot{e}(\alpha)\,d\alpha \\
&\quad - 2e(t)^T R_1\left[e(t) - e(t-\tau_1)\right] \\
&\quad + 2e(t-\tau_1)^T R_2\int_{t-\tau_1}^{t}\dot{e}(\alpha)\,d\alpha - 2e(t-\tau_1)^T \\
&\quad \times R_2\left[e(t) - e(t-\tau_1)\right] \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))\, h_j(s(t))\, \beta(t)^T \\
&\quad \times \Big[\Xi_{ij} + \overline{A}_{ij}\big(P_2^{-1} - \varepsilon_{2ij}^{-1}M_{ij}M_{ij}^T\big)^{-1}\overline{A}_{ij} \\
&\quad + \overline{A}_{ij}\big(\tau_1^{-1}Q_1^{-1} - \varepsilon_{3ij}^{-1}M_{ij}M_{ij}^T\big)^{-1}\overline{A}_{ij} + \delta_{ij}\widetilde{N}_{ij}^T\widetilde{N}_{ij}\Big]\beta(t),
\end{aligned} \tag{35}$$

where

$$\Xi_{ij} = \begin{bmatrix} \Sigma_{ij} & PA_{1ij} + R_1 - R_2^T & P\widetilde{A}_{2i} & P\widetilde{A}_{3i} & \tau_1 R_1 \\ * & -P_1 + R_2 + R_2^T & 0 & 0 & \tau_1 R_2 \\ * & * & -P_2 & 0 & 0 \\ * & * & * & -Q_2 & 0 \\ * & * & * & * & -\tau_1 Q_1 \end{bmatrix},$$

$$\Sigma_{ij} = PA_{ij} + A_{ij}^T P + P_1 + \tau_3^2 Q_2 - R_1 - R_1^T + \varepsilon_{1ij}^{-1}PM_{ij}M_{ij}^T P, \tag{36}$$

for $1 \le i \le j \le r$. On the other hand, by applying the Schur complement formula to (20), we have that for $1 \le i < j \le r$,

$$\Gamma_{ij} < 0, \tag{37}$$

and from this and (7), we have that $\dot{V}(t) < 0$ for all $\beta(t) \neq 0$. Therefore, the system in (13) is robustly asymptotically stable.

This completes the proof. □

Next, we will establish the robust H_∞ performance of the system in (13) and (14) under the zero initial condition. To this end, we introduce

$$J(t) = \int_0^t \left[z(s)^T z(s) - \gamma^2 w(s)^T w(s)\right] ds, \tag{38}$$

where $t > 0$. Noting the zero initial condition, it can be shown that for any nonzero $w(t) \in \mathscr{L}_2[0,\infty)$ and $t > 0$,

$$\begin{aligned}
J(t) &= \int_0^t \left[z(s)^T z(s) - \gamma^2 w(s)^T w(s) + \dot{V}(s)\right] ds - V(t) \\
&\le \int_0^t \left[z(s)^T z(s) - \gamma^2 w(s)^T w(s) + \dot{V}(s)\right] ds,
\end{aligned} \tag{39}$$

where $V(s)$ is defined in (23), and then we have

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(t))\, h_j(s(t)) \\
&\quad \times \Big\{2e(t)^T P\big[A_{ij}(t)e(t) + A_{1ij}(t)e(t-\tau_1) \\
&\quad + A_{2i}(t)\dot{e}(t-\tau_2) \\
&\quad + A_{3i}(t)\alpha(t) + D_{ij}w(t)\big]\Big\}
\end{aligned}$$

$$
\begin{aligned}
&+ e(t)^T P_1 e(t) - e(t-\tau_1)^T P_1 e(t-\tau_1) \\
&+ \dot{e}(t)^T P_2 \dot{e}(t) - \dot{e}(t-\tau_2)^T P_2 \dot{e}(t-\tau_2) \\
&+ \tau_1 \dot{e}(t)^T Q_1 \dot{e}(t) \\
&- \int_{t-\tau_1}^{t} \dot{e}(t)^T Q_1 \dot{e}(\alpha)\, d\alpha + \tau_3^2 e(t)^T Q_2 e(t) \\
&- \alpha(t)^T Q_2 \alpha(t) \\
&+ 2e(t)^T R_1 \int_{t-\tau_1}^{t} \dot{e}(\alpha)\, d\alpha - 2e(t)^T \\
&\times R_1 \left[e(t) - e(t-\tau_1)\right] \\
&+ 2e(t-\tau_1)^T R_2 \int_{t-\tau_1}^{t} \dot{e}(\alpha)\, d\alpha - 2e(t-\tau_1)^T \\
&\times R_2 \left[e(t) - e(t-\tau_1)\right].
\end{aligned} \tag{40}
$$

Then, by noting (14) and using Lemma 2, we have

$$
\begin{aligned}
z(s)^T z(s) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_i(s(s))\, h_j(s(s))\, h_u(s(s)) \\
&\qquad \times h_v(s(s))\, \zeta(s)^T \widetilde{E}_{ij}^T \widetilde{E}_{uv} \zeta(s) \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(s))\, h_j(s(s))\, \zeta(s)^T \widetilde{E}_{ij}^T \widetilde{E}_{ij} \zeta(s).
\end{aligned} \tag{41}
$$

It can be deduced that

$$
\begin{aligned}
&z(s)^T z(s) - \gamma^2 w(s)^T w(s) + \dot{V}(s) \\
&\quad \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(s))\, h_j(s(s))\, \zeta(s)^T \Pi_{1ij} \zeta(s) \\
&\quad = \sum_{i=1}^{r} h_i(s(s))^2 \zeta(s)^T \Xi_{1ii} \zeta(s) \\
&\qquad + 2\sum_{i,j=1, i<j}^{r} h_i(s(s))\, h_j(s(s))\, \zeta(s)^T \frac{\Xi_{1ij} + \Xi_{1ji}}{2} \zeta(s),
\end{aligned} \tag{42}
$$

where

$$
\Xi_{1ij} = \begin{bmatrix}
\Sigma_{ij} & PA_{1ij}(t) + R_1 - R_2^T & PA_{2i}(t) & PA_{3i}(t) & PD_{ij} & \tau_1 R_1 & E_{ij}^T & A_{ij}^T(t)(P_2 + \tau_1 Q_1) \\
* & J_1 & 0 & 0 & 0 & \tau_1 R_2 & \widetilde{E}_{1i}^T & A_{1ij}^T(t)(P_2 + \tau_1 Q_1) \\
* & * & -P_2 & 0 & 0 & 0 & 0 & A_{2i}^T(t)(P_2 + \tau_1 Q_1) \\
* & * & * & -Q_2 & 0 & 0 & 0 & A_{3i}^T(t)(P_2 + \tau_1 Q_1) \\
* & * & * & * & -\gamma^2 I & 0 & 0 & D_{ij}^T(P_2 + \tau_1 Q_1) \\
* & * & * & * & * & -\tau_1 Q_2 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & -(P_2 + \tau_1 Q_1)^T
\end{bmatrix},
$$

$$
\zeta(s) = \left[e(t)^T \;\; e(t-\tau_1)^T \;\; \dot{e}(t-\tau_2)^T \;\; \alpha(t)^T \;\; w(t)^T \;\; \dot{e}(\alpha)^T\right]^T,
$$

$$
\widetilde{E}_{ij} = \left[E_{ij} \;\; \widetilde{E}_{1i} \;\; 0 \;\; 0 \;\; 0 \;\; 0\right], \tag{43}
$$

for $1 \le i < j \le r$. Similar to the previous section, applying the Schur complement formula to the LMI in (20) results in $\Xi_{1ij} < 0, 1 \le i < j \le r$, which together with (39) gives $J(t) < 0$ for any nonzero $w(t) \in \mathscr{L}_2[0,\infty)$ and $t > 0$. Therefore, we have $\|z\|_2 < \gamma \|w\|_2$.

Now, we are in a position to present a solution to the robust output feedback H_∞ control problem.

Theorem 4. *Consider the uncertain fuzzy neutral delay system in* (1), *and let $\gamma > 0$ be a prescribed constant scalar. The robust H_∞ problem is solvable if there exist matrices $X > 0$, $Y > 0$, Q_1, Q_2, Φ_i, Ψ_i, and Ω_i and scalars ϵ_{ij}, $1 \le i \le j \le r$, such that the following LMIs hold:*

$$
\begin{bmatrix}
\Sigma_{ii} & \Sigma_{1ii} & \Sigma_{2ii} & \Sigma_{3ii} & R_5 \\
* & -R_{11} & R_{2ii} & 0 & 0 \\
* & * & R_{3ii} & 0 & R_{6ii} \\
* & * & * & -R_4 & 0 \\
* & * & * & * & -R_7
\end{bmatrix} < 0, \quad 1 \le i \le r, \tag{44}
$$

$$\begin{bmatrix} \Sigma_{ij}+\Sigma_{ji} & \Sigma_{1ij}+\Sigma_{1ji} & \widetilde{\Sigma}_{2ij} & \Sigma_{4ij} & \Sigma_{5ij} & \Sigma_{6ij} & 0 \\ * & -2R_{11} & \widetilde{R}_{2ij} & 0 & 0 & 0 & 0 \\ * & * & \widetilde{R}_{3ij} & 0 & 0 & 0 & \widetilde{R}_{6ij} \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -R_8 & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad 1 \le i < j \le r, \tag{45}$$

$$\begin{bmatrix} -Y & -I \\ -I & -X \end{bmatrix} < 0, \tag{46}$$

where

$$\Theta_{ij} = \begin{bmatrix} A_iX + XA_i^T + B_i\Psi_j + \Psi_j^TB_i^T & A_i + \Omega_i^T \\ A_i^T + \Omega_i & YA_i + A_i^TY + \Phi_jC_i + C_i^T\Phi_j^T \end{bmatrix} - R_1 - R_1^T,$$

$$\Theta_{1ij} = \begin{bmatrix} A_{1i}X & A_{1i} \\ 0 & YA_{1i} + \Phi_jC_{1i} \end{bmatrix} + R_1 - R_2^T,$$

$$\Theta_{2i} = \begin{bmatrix} A_{2i} & 0 \\ YA_{2i} & 0 \end{bmatrix}, \qquad \Theta_{3i} = \begin{bmatrix} A_{3i} & 0 \\ YA_{3i} & 0 \end{bmatrix},$$

$$\Theta_{4ij} = \begin{bmatrix} D_{1i} \\ YD_{1i} + \Phi_jD_{2i} \end{bmatrix}, \qquad \Theta_{5ij} = \begin{bmatrix} XE_i^T + \Psi_j^TG_i^T \\ E_i^T \end{bmatrix},$$

$$\Theta_{6ij} = \begin{bmatrix} XA_i^T + \Psi_j^TB_i^T & \Omega_j^T \\ A_i^T & A_i^TY + C_i^T\Phi_j^T \end{bmatrix},$$

$$\Theta_{7ij} = \begin{bmatrix} M_i & 0 \\ YM_i & \Phi_jM_i \end{bmatrix},$$

$$\Theta_{8ij} = \begin{bmatrix} XN_{0i}^T + \Psi_j^TN_{4i}^T & XN_{5i}^T \\ N_{0i}^T & N_{5i}^T \end{bmatrix}, \qquad J_{3i} = \begin{bmatrix} XE_{1i}^T \\ E_{1i}^T \end{bmatrix},$$

$$J_{4ij} = \begin{bmatrix} XA_{1i} & 0 \\ A_{1i} & A_{1i}Y + C_{1i}^T\Phi_j^T \end{bmatrix},$$

$$J_{5i} = \begin{bmatrix} XN_{1i}^T & 0 \\ N_{1i}^T & 0 \end{bmatrix}, \qquad J_{6i} = \begin{bmatrix} A_{2i}^T & A_{2i}^TY \\ 0 & 0 \end{bmatrix},$$

$$J_{7i} = \begin{bmatrix} A_{3i}^T & A_{3i}^TY \\ 0 & 0 \end{bmatrix},$$

$$J_{8ij} = \begin{bmatrix} D_{1i}^T & D_{1i}^TY + D_{2i}^T\Phi_j^T \end{bmatrix},$$

$$J_9 = \Pi_2^T + \Pi_2 - P_2 - \tau_1Q_1,$$

$$J_{10ij} = \begin{bmatrix} M_i & 0 \\ YM_i & \Phi_jM_i \end{bmatrix},$$

$$\Sigma_{ij} = \begin{bmatrix} \Theta_{ij} & \Theta_{1ij} & \Theta_{2i} \\ * & -U + R_2 + R_2^T & 0 \\ * & * & -P_2 \end{bmatrix},$$

$$\Sigma_{1ij} = \begin{bmatrix} \Theta_{3i} & \Theta_{4ij} & \tau_1R_1 \\ 0 & 0 & \tau_1R_2 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_{2ij} = \begin{bmatrix} \Theta_{5ij} & \Theta_{6ij} & \Theta_{7ij} & \Theta_{8ij} \\ J_{3i} & J_{4ij} & 0 & J_{5i} \\ 0 & J_{6i} & 0 & \widetilde{N}_{2i}^T \end{bmatrix},$$

$$\widetilde{\Sigma}_{2ij} = \begin{bmatrix} \Theta_{5ij}+\Theta_{5ji} & \Theta_{6ij}+\Theta_{6ji} & \Theta_{7ij} & \Theta_{7ji} & \Theta_{8ij} & \Theta_{8ji} \\ J_{3i}+J_{3j} & J_{4ij}+J_{4ji} & 0 & 0 & J_{5i} & J_{5j} \\ 0 & J_{6i}+J_{6j} & 0 & 0 & \widetilde{N}_{2i}^T & \widetilde{N}_{2j}^T \end{bmatrix},$$

$$\Sigma_{3ij} = \begin{bmatrix} \Pi_1^T & \tau_3\Pi_1^T & T_{ij} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad T_{ij} = \begin{bmatrix} 0 \\ YA_{1i} + \Phi_jC_{1i} \end{bmatrix},$$

$$R_5 = \begin{bmatrix} 0 & 0 & 0 \\ T_1 & T_1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} x \\ 0 \end{bmatrix}, \qquad \Sigma_{4ij} = \begin{bmatrix} T_{2ij} & T_{3ij} & T_{4ij} & T_{5ij} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$T_{2ij} = \begin{bmatrix} \left(C_ix - C_jx\right)^T \\ 0 \end{bmatrix},$$

$$T_{3ij} = \begin{bmatrix} 0 \\ \Phi_j - \Phi_i \end{bmatrix}, \qquad T_{4ij} = \begin{bmatrix} \left(\Psi_j - \Psi_i\right)^T \\ 0 \end{bmatrix},$$

$$T_{5ij} = \begin{bmatrix} 0 \\ YB_i - YB_j \end{bmatrix},$$

$$\Sigma_{5ij} = \begin{bmatrix} \Pi_1^T & \tau_3\Pi_1^T & T_{ij} & 0 \\ 0 & 0 & 0 & T_1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_{6ij} = \begin{bmatrix} T_{ji} & 0 & T_4 & T_2 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R_{11} = \operatorname{diag}\left(Q_2 \quad \gamma^2 I \quad V\right), \qquad R_{2ij} = \begin{bmatrix} 0 & J_{7i} & 0 & \widetilde{N}_{3i}^T \\ 0 & J_{8ij} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{R}_{2ij} = \begin{bmatrix} 0 & J_{7i} + J_{7j} & 0 & 0 & \widetilde{N}_{3i}^T & \widetilde{N}_{3j}^T \\ 0 & J_{8ij} + J_{8ji} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R_{3ij} = \begin{bmatrix} -I & 0 & 0 & 0 \\ * & -J_9 & J_{10ij} & 0 \\ * & * & -\epsilon_{ij} I & 0 \\ * & * & * & -\epsilon_{ij}^{-1} I \end{bmatrix},$$

$$\widetilde{R}_{3ij} = \begin{bmatrix} -2I & 0 & 0 & 0 & 0 & 0 \\ * & -2J_9 & J_{10ij} & J_{10ji} & 0 & 0 \\ * & * & -\epsilon_{ij} I & 0 & 0 & 0 \\ * & * & * & -\epsilon_{ji} I & 0 & 0 \\ * & * & * & * & -\epsilon_{ij}^{-1} I & 0 \\ * & * & * & * & * & -\epsilon_{ji}^{-1} I \end{bmatrix},$$

$$R_4 = \operatorname{diag}\left(P_1^{-1} \quad Q_2^{-1} \quad I\right),$$

$$R_{6ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & T_{6ij} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \widetilde{R}_{6ij} = \begin{bmatrix} 0 & 0 \\ T_{5ij} & T_{3ij} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$T_{6ij} = \begin{bmatrix} 0 \\ YA_{1i}^T + \Phi_j C_{1i} \end{bmatrix},$$

$$R_7 = \operatorname{diag}\left(I \quad I \quad I\right), \qquad R_8 = \operatorname{diag}\left(\frac{1}{2}P_1^{-1} \quad \frac{1}{2}Q_2^{-1} \quad I \quad I\right),$$

$$\Pi_1 = \begin{bmatrix} X & I \\ X & 0 \end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} X & I \\ X & 0 \end{bmatrix}. \tag{47}$$

Furthermore a desired robust dynamic output feedback controller is given in the form of (11) *with parameters as follows:*

$$A_{Ki} = \left(X^{-1} - Y\right)^{-1}\left(\Omega_i - YA_iX - YB_i\Psi_i - \Phi_iC_iX\right)X^{-1},$$

$$B_{Ki} = \left(X^{-1} - Y\right)^{-1}\Phi_i, \quad C_{Ki} = \Psi_iX^{-1}, \quad 1 \le i \le r. \tag{48}$$

Proof. Applying the Schur complements formula to (44) and (45) and using Lemma 2 result in

$$\begin{bmatrix} \widetilde{\Sigma}_{ii} & \Sigma_{1ii} & \widehat{\Sigma}_{2ii} \\ * & -R_{11} & R_{2ii} \\ * & * & R_{3ii} \end{bmatrix} < 0, \quad 1 \le i \le r,$$

$$\begin{bmatrix} \widehat{\Sigma}_{ij} & \Sigma_{1ij} + \Sigma_{1ji} & \overline{\Sigma}_{2ij} \\ * & -2R_{11} & \widetilde{R}_{2ij} \\ * & * & \widetilde{R}_{3ij} \end{bmatrix} < 0, \quad 1 \le i < j \le r, \tag{49}$$

where

$$\widetilde{\Sigma}_{ij} = \begin{bmatrix} \Theta_{ij} + \Pi_1^T P_1 \Pi_1 + \left(\tau_3\Pi_1\right)^T Q_2 \left(\tau_3\Pi_1\right) & \Theta_{1ij} + \begin{bmatrix} 0 & 0 \\ \left(YA_{1i} + \Phi_jC_{1i}\right)X & 0 \end{bmatrix} & \Theta_{2i} \\ * & -U + R_2 + R_2^T & 0 \\ * & * & -P_2 \end{bmatrix},$$

$$\widehat{\Sigma}_{2ij} = \begin{bmatrix} \Theta_{5ij} & \Theta_{6ij} & \Theta_{7ij} & \Theta_{8ij} \\ J_{3i} & J_{4ij} + \begin{bmatrix} 0 & X\left(A_{1i}Y + C_{1i}^T\Phi_j^T\right) \\ 0 & 0 \end{bmatrix} & 0 & J_{5i} \\ 0 & J_{6i} & 0 & \widetilde{N}_{2i}^T \end{bmatrix},$$

$$\widehat{\Sigma}_{ij} = \widetilde{\Sigma}_{ij} + \widetilde{\Sigma}_{ji} + \begin{bmatrix} 0 & \left(C_iX - C_jX\right)^T\left(\Phi_j - \Phi_i\right)^T + \left(\Psi_j - \Psi_i\right)^T\left(YB_i - YB_j\right)^T & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}, \tag{50}$$

$$\overline{\Sigma}_{2ij} = \widetilde{\Sigma}_{2ij} + \begin{bmatrix} 0 & \left(\Psi_j^T - \Psi_i^T\right)\left(B_i^TY - B_j^TY\right) + \left(XC_i^T - XC_j^T\right)\left(\Phi_j^T - \Phi_i^T\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Note that

$$\Pi_2^T + \Pi_2 - P_2 - \tau_1 Q_1 \le \Pi_2^T (P_2 + \tau_1 Q_1)^{-1} \Pi_2. \quad (51)$$

Then, by this inequality, it follows from (49) that

$$\begin{bmatrix} \widetilde{\Sigma}_{ii} & \Sigma_{1ii} & \widehat{\Sigma}_{2ii} \\ * & -R_{11} & R_{2ii} \\ * & * & \widehat{R}_{3ii} \end{bmatrix} < 0, \quad 1 \le i \le r,$$

$$\begin{bmatrix} \widehat{\Sigma}_{ij} & \Sigma_{1ij} + \Sigma_{1ji} & \overline{\Sigma}_{2ij} \\ * & -2R_{11} & \widetilde{R}_{2ij} \\ * & * & \overline{R}_{3ij} \end{bmatrix} < 0, \quad 1 \le i < j \le r, \quad (52)$$

where

$$\widehat{R}_{3ij} = \begin{bmatrix} -I & 0 & 0 & 0 \\ * & -\Pi_2^T (P_2 + \tau_1 Q_1)^{-1} \Pi_2 & J_{10ij} & 0 \\ * & * & -\epsilon_{ij} I & 0 \\ * & * & * & -\epsilon_{ij}^{-1} I \end{bmatrix},$$

$$\overline{R}_{3ij} = \begin{bmatrix} -2I & 0 & 0 & 0 & 0 & 0 \\ * & -2\Pi_2^T (P_2 + \tau_1 Q_1)^{-1} \Pi_2 & J_{10ij} & J_{10ji} & 0 & 0 \\ * & * & -\epsilon_{ij} I & 0 & 0 & 0 \\ * & * & * & -\epsilon_{ji} I & 0 & 0 \\ * & * & * & * & -\epsilon_{ij}^{-1} I & 0 \\ * & * & * & * & * & -\epsilon_{ji}^{-1} I \end{bmatrix}. \quad (53)$$

Now, set

$$\widetilde{P} = \Pi_2 \Pi_1^{-1}. \quad (54)$$

Then, by (46), it can be verified that $\widetilde{P} > 0$. Noting the parameters in (48) and pre- and postmultiplying (3) by

$$\operatorname{diag}\left(\Pi_1^{-T}, \Pi_1^{-T}, I, I, I, \Pi_1^{-T}, I, \Pi_2^{-T}, I, \ldots, I\right) \quad (55)$$

and its transpose, respectively, we have

$$\begin{bmatrix} \widetilde{\Pi}_{1ii} & \widetilde{\Pi}_{2ii} & P\widetilde{A}_{2i} & P\widetilde{A}_{3i} & PD_{ii} & \tau_1 \widetilde{R}_1 & E_{ii}^T & \Gamma_{ii} & PM_{ii} & N_{ii}^T \\ * & J_1 & 0 & 0 & 0 & \tau_1 \widetilde{R}_2 & E_{1i}^T & \Gamma_{1ii} & 0 & 0 \\ * & * & -P_2 & 0 & 0 & 0 & 0 & \Gamma_{2i} & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & \Gamma_{3i} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & \Gamma_{4ii} & 0 & 0 \\ * & * & * & * & * & -\tau_1 Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -J_2^T & J_2^T M_{ii} & 0 \\ * & * & * & * & * & * & * & * & -\epsilon_{ii} I & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_{ii}^{-1} I \end{bmatrix} < 0, \quad 1 \le i \le r,$$

$$\begin{bmatrix} \Lambda_{1ij} & \Lambda_{2ij} & \Lambda_{3ij} \\ * & \Lambda_{4ij} & \Lambda_{5ij} \\ * & * & \Lambda_{6ij} \end{bmatrix} < 0, \quad 1 \le i < j \le r, \quad (56)$$

where

$$\widetilde{\Lambda}_1 = H^T \left(P_1 + \tau_3^2 Q_2 - \widetilde{R}_1 - \widetilde{R}_1^T\right) H,$$

$$\widetilde{\Pi}_{1ij} = \widetilde{\Lambda}_1 + PA_{ij} + A_{ij}^T P,$$

$$\widetilde{\Lambda}_2 = H^T \left(\widetilde{R}_1 - \widetilde{R}_2^T\right),$$

$$\widetilde{\Pi}_{2ij} = \widetilde{\Lambda}_2 + PA_{1ij},$$

$$\widetilde{R}_1 = \Pi_1^T R_1 \Pi_1. \quad (57)$$

Finally, by Theorem 3, the desired result follows immediately. □

Remark 5. Theorem 4 provides a sufficient condition for the solvability of the robust H_∞ output feedback control problem for uncertain fuzzy neutral systems with both discrete and distributed delays. It is worth pointing out that the result in Theorem 4 can be readily extended to the case with multiple delays.

Remark 6. In Theorem 4, if we set $A_{21} = 0$, $M_i = 0$, and $N_{2i} = 0$ $(i = 1, 2, \ldots r)$, then the results in [32] are included in our paper. Also, the controller design method in our paper can be the reference for designing the observer-based output feedback controllers and so forth.

4. Simulation Example

In this section, we provide one example to illustrate the output feedback H_∞ controller design approach developed in this paper.

The uncertain fuzzy system with distributed delays considered in this example is with two rules.

Plant Rule 1: if $x_1(t)$ is μ_1, then

$$\begin{aligned}\dot{x}(t) = {} & [A_1 + \Delta A_1(t)]\, x(t) + [A_{11} + \Delta A_{11}(t)]\, x(t-\tau_1) \\ & + [A_{21} + \Delta A_{21}(t)]\, \dot{x}(t-\tau_2) \\ & + [A_{31} + \Delta A_{31}(t)] \int_{t-\tau_3}^{t} x(s)\, ds \\ & + [B_1 + \Delta B_1(t)]\, u(t) + D_{1i} w(t), \\ y(t) = {} & [C_1 + \Delta C_1(t)]\, x(t) + C_{11} x(t-\tau_1) + D_{21} w(t), \\ z(t) = {} & E_1 x(t) + E_{11} x(t-\tau_1) + G_1 u(t).\end{aligned} \tag{58}$$

Plant Rule 2: if $x_1(t)$ is μ_2, then

$$\begin{aligned}\dot{x}(t) = {} & [A_2 + \Delta A_2(t)]\, x(t) + [A_{12} + \Delta A_{12}(t)]\, x(t-\tau_1) \\ & + [A_{22} + \Delta A_{22}(t)]\, \dot{x}(t-\tau_2) \\ & + [A_{32} + \Delta A_{32}(t)] \int_{t-\tau_3}^{t} x(s)\, ds \\ & + [B_2 + \Delta B_2(t)]\, u(t) + D_{12} w(t), \\ y(t) = {} & [C_2 + \Delta C_2(t)]\, x(t) + C_{12} x(t-\tau_1) + D_{22} w(t), \\ z(t) = {} & E_2 x(t) + E_{12} x(t-\tau_1) + G_2 u(t),\end{aligned} \tag{59}$$

where

$$\begin{gathered}A_1 = \begin{bmatrix} 0.05 & 0 \\ -0.2 & -0.06 \end{bmatrix}, \qquad A_{11} = \begin{bmatrix} 0.05 & 0.02 \\ -0.05 & 0.01 \end{bmatrix}, \\ A_{21} = \begin{bmatrix} -0.002 & 0.008 \\ 0.006 & 0 \end{bmatrix}, \\ A_{31} = \begin{bmatrix} -0.05 & 0.08 \\ 0.06 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.01 & 0 \\ 0.03 & 0.06 \end{bmatrix}, \\ C_1 = \begin{bmatrix} -0.03 & -0.1 \\ 0.2 & 0.02 \end{bmatrix}, \qquad C_{11} = \begin{bmatrix} -0.03 & -0.01 \\ 0.02 & 0.02 \end{bmatrix}, \\ D_{11} = \begin{bmatrix} 0.03 \\ 0.06 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}, \\ E_1 = [-0.2 \;\; 0], \qquad E_{11} = [-0.02 \;\; 0], \\ G_1 = [0 \;\; -0.03], \\ A_2 = \begin{bmatrix} -0.05 & 0 \\ -0.2 & -0.06 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} -0.05 & 0.2 \\ -0.5 & 0.01 \end{bmatrix}, \\ A_{22} = \begin{bmatrix} 0.008 & -0.002 \\ -0.003 & 0.02 \end{bmatrix}, \\ A_{32} = \begin{bmatrix} 0.05 & -0.2 \\ -0.3 & 0.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.01 & -0.08 \\ 0.02 & 0.05 \end{bmatrix}, \\ C_2 = \begin{bmatrix} 0.03 & -0.1 \\ 0.2 & 0.02 \end{bmatrix}, \\ C_{12} = \begin{bmatrix} -0.03 & -0.01 \\ 0.02 & 0.2 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} -0.01 \\ 0.02 \end{bmatrix}, \\ D_{22} = \begin{bmatrix} -0.3 \\ 0 \end{bmatrix}, \\ E_2 = [-0.01 \;\; 0], \qquad E_{12} = [-0.01 \;\; 0], \\ G_2 = [0 \;\; 0.02],\end{gathered} \tag{60}$$

and $\Delta A_i(t)$, $\Delta A_{1i}(t)$, $\Delta A_{2i}(t)$, $\Delta A_{3i}(t)$, $\Delta B_i(t)$, $\Delta C_i(t)$ $(i = 1, 2)$ can be represented in the form of (2) and (3) with

$$\begin{gathered}M_1 = \begin{bmatrix} -0.02 \\ 0 \end{bmatrix}, \qquad N_{01} = [-0.2 \;\; 0.1], \\ N_{11} = [-0.2 \;\; 0.2], \\ N_{21} = [-0.02 \;\; 0.01], \qquad N_{31} = [-0.02 \;\; 0.01], \\ N_{41} = [-0.02 \;\; 0.01], \\ N_{51} = [-0.02 \;\; 0.01], \qquad M_2 = \begin{bmatrix} 0.03 \\ 0 \end{bmatrix}, \\ N_{02} = N_{01}, \qquad N_{12} = N_{11}, \\ N_{22} = N_{21}, \qquad N_{32} = N_{31}, \\ N_{42} = N_{41}, \qquad N_{52} = N_{51}.\end{gathered} \tag{61}$$

Then, the final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{2} h_i(s(t)) &\times \{[A_i + \Delta A_i(t)]x(t) \\ &+ [A_{1i} + \Delta A_{1i}(t)]x(t-\tau_1) \\ &+ [A_{2i} + \Delta A_{2i}(t)]\dot{x}(t-\tau_2) \\ &+ [A_{3i} + \Delta A_{3i}(t)]\int_{t-\tau_3}^{t} x(s)\,ds \\ &+ [B_i + \Delta B_i(t)]u(t) + D_{1i}w(t)\}, \\ y(t) = \sum_{i=1}^{2} h_i(s(t)) &\times \{[C_i + \Delta C_i(t)]x(t) \\ &+ C_{1i}x(t-\tau_1) + D_{2i}w(t)\}, \\ z(t) = \sum_{i=1}^{2} h_i(s(t))&\{E_i x(t) + E_{1i}x(t-\tau_1) + G_i u(t)\},\end{aligned} \tag{62}$$

where

$$\begin{aligned} h_1(x_1(t)) &= \begin{cases} \frac{1}{3}, & \text{for } x_1 < -1, \\ \frac{2}{3} + \frac{1}{3}x_1, & \text{for } |x_1| \le 1, \\ 1, & \text{for } x_1 > 1, \end{cases} \\ h_2(x_1(t)) &= \begin{cases} \frac{2}{3}, & \text{for } x_1 < -1, \\ \frac{1}{3} - \frac{1}{3}x_1, & \text{for } |x_1| \le 1, \\ 0, & \text{for } x_1 > 1. \end{cases}\end{aligned} \tag{63}$$

In this example, we choose the H_∞ performance level $\gamma = 3$.

In order to design a fuzzy H_∞ output feedback controller for the T-S model, we first choose the initial condition that is $x(0) = [0.3 \ \ -0.1]^T$, and the disturbance input $w(t)$ is assumed to be

$$w(t) = \frac{1}{t+0.1}, \quad t \ge 0. \tag{64}$$

Then, solving the LMIs in (44), (45), and (46), we obtain the solution as follows:

$$\begin{aligned} X &= \begin{bmatrix} 0.9868 & 0.6071 \\ 0.6071 & 1.3948 \end{bmatrix}, \\ Y &= \begin{bmatrix} 12.9348 & 0.1814 \\ 0.1814 & 6.3384 \end{bmatrix}, \\ \Omega_1 &= \begin{bmatrix} -1.6069 & -1.6112 \\ -0.0417 & -3.2117 \end{bmatrix}, \\ \Omega_2 &= \begin{bmatrix} 8.2076 & -3.7031 \\ -5.8461 & 5.3292 \end{bmatrix}, \\ \Phi_1 &= \begin{bmatrix} 3.1848 & 0.0758 \\ -0.1899 & -2.5282 \end{bmatrix}, \\ \Phi_2 &= \begin{bmatrix} 2.0547 & 0.1643 \\ -0.2930 & -0.7113 \end{bmatrix}, \\ \psi_1 &= \begin{bmatrix} -140.1869 & -27.8001 \\ 5.7867 & -77.8562 \end{bmatrix}, \\ \psi_2 &= \begin{bmatrix} -79.1853 & -88.2601 \\ -88.1012 & -95.7242 \end{bmatrix}. \end{aligned} \tag{65}$$

Now, by Theorem 4, a desired fuzzy output feedback controller can be constructed as in (11) and (12) with

$$\begin{aligned} A_{k1} &= \begin{bmatrix} 87.3277 & -68.0397 \\ 139.9266 & -107.2394 \end{bmatrix}, \\ A_{k2} &= \begin{bmatrix} -24.9856 & 15.8880 \\ 11.1983 & -12.2840 \end{bmatrix}, \\ B_{k1} &= \begin{bmatrix} 10.9115 & -0.3725 \\ 16.5830 & 1.9936 \end{bmatrix}, \\ B_{k2} &= \begin{bmatrix} 1.4642 & -0.0212 \\ 3.1940 & 0.5922 \end{bmatrix}, \\ C_{k1} &= \begin{bmatrix} -16.0423 & 12.7577 \\ 20.2647 & -14.8918 \end{bmatrix}, \\ C_{k2} &= \begin{bmatrix} 14.2596 & -10.4929 \\ 3.2304 & -1.1934 \end{bmatrix}. \end{aligned} \tag{66}$$

With the output feedback fuzzy controller, the simulation results of the state response of the nonlinear system are given in Figure 1. Figure 2 shows the control input, while Figures 3 and 4 present the corresponding measured output and the controlled output, respectively.

From these simulation results, it can be seen that the designed fuzzy output feedback controller ensures the robust asymptotic stability of the uncertain fuzzy neutral system and guarantees a prescribed H_∞ performance level.

5. Conclusion and Future Work

The problem of delay-dependent robust output feedback H_∞ control for uncertain T-S fuzzy neutral systems with parameter uncertainties and distributed delays has been studied. In

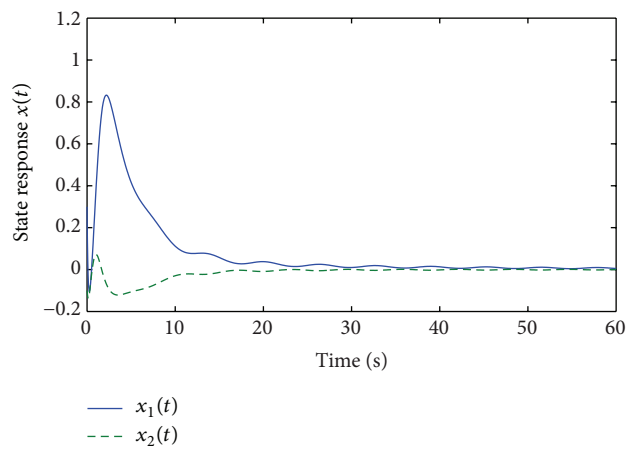

FIGURE 1: State response $x_1(t)$ (continuous line) and $x_2(t)$ (dashed line).

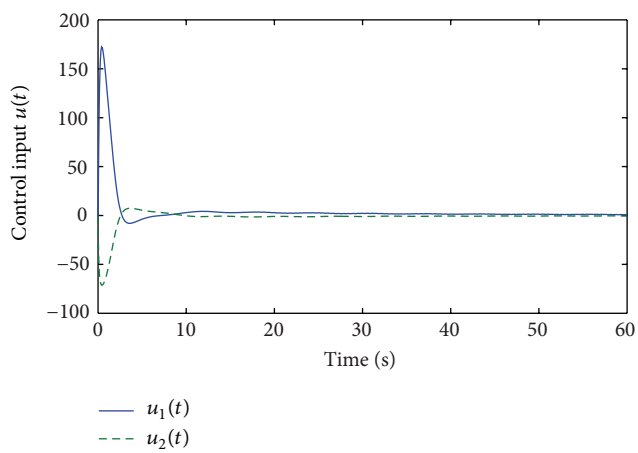

FIGURE 2: Control input $u_1(t)$ (continuous line) and $u_2(t)$ (dashed line).

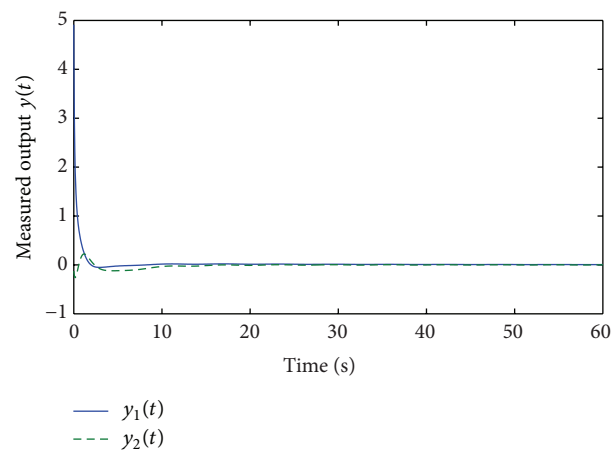

FIGURE 3: Measured output $y_1(t)$ (continuous line) and $y_2(t)$ (dashed line).

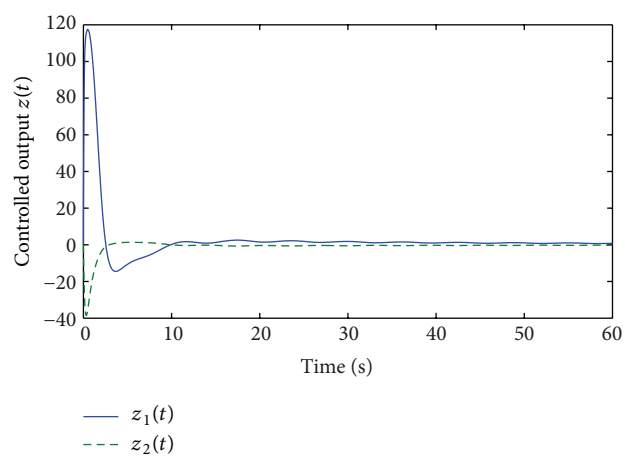

FIGURE 4: Controlled output $z_1(t)$ (continuous line) and $z_2(t)$ (dashed line).

terms of LMIs, a sufficient condition for the existence of a full-order fuzzy dynamic output feedback controller, which robustly stabilizes the uncertain fuzzy neutral delay systems and guarantees a prescribed level on disturbance attenuation, has been obtained.

Future work will mainly cover the problem of robust output feedback H_∞ control for uncertain fractional-order (FO) T-S fuzzy neutral systems with state and distributed delays. Firstly, investigate the robust output feedback H_∞ control for FO T-S fuzzy systems with state delays; then, study the robust output feedback H_∞ control for FO T-S fuzzy systems with both state and distributed delays; finally, obtain the results about the robust output feedback H_∞ control for uncertain FO T-S fuzzy neutral systems with state and distributed delays.

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