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Research Article

Composite Antidisturbance Control for a Class of Nonlinear Stochastic Systems via Disturbance Observer

Yunlong Liu,[1] Yumin Zhang,[1] and Xiuming Yao[2]

[1] *School of Instrumentation Science and Opto-Electronics Engineering, Beihang University, Beijing 100191, China*
[2] *Hebei Engineering Research Center of Simulation and Optimized Control for Power Generation, North China Electric Power University, Baoding 071003, China*

Correspondence should be addressed to Yunlong Liu; liuyunlongbuaa@163.com

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The stabilization problem is investigated in this paper for a class of nonlinear systems with disturbances. The disturbances are supposed to be classified into two types. One type in the input channel is generated by an exogenous system, which can represent the constant or harmonic signals with unknown phase and magnitude. The other type is stochastic disturbance. Two kinds of nonlinear dynamics in the plants are considered, respectively, which correspond to the known and unknown functions. By integrating the disturbance observers with conventional control method, the first type of disturbances can be estimated and rejected. Simultaneously, the desired dynamic performances can be guaranteed. An example is given to show the effectiveness of the proposed scheme.

1. Introduction

Though the stochastic stabilization theory emerged in the 1960s [1], the progress has been slow. This was mainly due to a fundamental theory obstacle in the Lyapunov analysis; the Itô differentiation introduces not only the gradient but also the Hessian term of the Lyapunov function. Along with the advances in differential geometric theory [2] and the discovery of a simple constructive formula for Lyapunov stabilization [3], the stochastic stabilization problem was reexamined and some constructive results have been achieved. The stabilization of nonlinear stochastic systems was considered in the work of Florchinger [4–7], who, among other things, extended the concept of control Lyapunov functions and Sontag's stabilization formula to stochastic setting. Pan and Başer [8] solved the stabilization problem for a class of strict-feedback systems representative of stabilization results for deterministic systems. Deng and Krstić [9] developed a simpler control strategy for strict-feedback systems and then extended the results on inverse optimal stabilization for general systems to the stochastic case. The authors of [10] considered the dissipativity analysis and dissipativity-based sliding mode control for a class of continuous-time switched stochastic systems and [11] designed multistep predictive controller for a class of Markov jump convex polyhedron linear parameter time-varying systems with both constraints on input and output. The adaptive neural tracking control problem was the concern in [12] for a class of strict-feedback stochastic nonlinear systems with unknown dead zone. However, most of these results were focused on systems that only have one kind of disturbance-stochastic disturbance. In [13], Hinrichsen proposed the stochastic robust control method for systems with deterministic and stochastic disturbances, which enabled us to deal with a broader class of systems. However, only stability of the nominal system in the absence of deterministic disturbances was the concern in this approach, which means that the stability cannot be guaranteed in the presence of both deterministic and stochastic disturbances.

Disturbance-observer-based control (DOBC) approach, which is based on the idea of feed-forward compensation, appeared in the late 1980s and has attracted considerable attention in control theory literatures [14–17]. The controller design of this method can be accomplished in two steps. First, a disturbance observer is designed to estimate the deterministic disturbance and then compensate it. Second, feedback controller is designed to stabilize the nominal system without disturbance. DOBC approach has its roots in

many mechanical applications in the last two decades, in particular for linear systems [18–20]. Recently, some attempts have been made to establish theoretic justification of these DOBC applications and extend DOBC from linear systems to nonlinear systems [21, 22]. Besides, [23] designed output feedback controller for a class of Markovian jump repeated scalar nonlinear systems and [24] investigated the problem of composite DOBC and H_∞ control for Markovian jump systems with nonlinearity and multiple disturbances.

This paper considers the application of DOBC approach to a class of nonlinear stochastic systems. The nonlinear dynamics are described by known and unknown nonlinear functions, respectively. And apart from the stochastic noises, the deterministic disturbance is supposed to be generated by an exogenous system as investigated in [18, 19], which is not confined to be bounded in norm [25]. By using the disturbance estimation, the DOBC strategy can be integrated with the conventional stabilization controllers to reject the deterministic disturbance and globally stabilize the closed-loop systems in probability. Finally, simulations on an A4D aircraft model show the effectiveness of the proposed approaches.

2. Problem Statement

The following MIMO stochastic system with nonlinearity is described as

$$\begin{aligned} dx(t) = {} & \{Ax(t) + F_0 f(x(t), t) + B[u(t) + v(t)]\} dt \\ & + A_0 x(t) d\omega, \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $v(t) \in R^m$, are the state, control input, and disturbance, respectively. $f(x(t), t)$ is a nonlinear vector function satisfying bounded condition as described in Assumption 1. ω is an r-dimensional standard Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, \mathscr{P})$ with Ω being a sample space, $\mathscr{F}$ being a σ-field, and $\mathscr{P}$ being the probability measure. A, B, F_0, and A_0 are given system matrices with corresponding dimensions.

Assumption 1. For any $x_j(t) \in R^n$, $j = 1, 2$, nonlinear function $f(x(t), t)$ satisfy

$$\begin{gathered} f(0, t) = 0, \\ \|f(x_1(t), t) - f(x_2(t), t)\| \le \|U(x_1(t) - x_2(t))\|, \end{gathered} \tag{2}$$

where U is a given constant weighting matrix.

Assumption 2. The disturbance $v(t)$ in the control input path is supposed to be generated by the following exogenous systems:

$$\dot{\varpi}(t) = W\varpi(t), \qquad v(t) = V\varpi(t), \tag{3}$$

where $W \in R^{r\times r}$, $V \in R^{m\times r}$ are known constant weighting matrices.

Remark 3. In fact, many kinds of disturbances in engineering can be described by Assumption 2, for example, unknown constant and harmonics with unknown phase and magnitude.

The following assumption is the necessary condition for the DOBC problem.

Assumption 4. (A, B) is controllable and (W, BV) is observable.

In this paper, we suppose system states are available, which means that only the estimation of disturbance needs to be focused on. In this situation, the objective of DOBC is to design an observer for system (1) to estimate the unknown disturbance $v(t)$, and then construct a composite controller with the disturbance estimation and a conventional controller so that the disturbance can be rejected and the stability in probability of the resulting composite system can be guaranteed.

3. DOBC for the Case with Known Nonlinearity

In this section, the nonlinearity function $f(x(t), t)$ is supposed to be given and Assumptions 1, 2, and 4 hold. Due to the fact that the states of system are available, the disturbance observer is designed as

$$\hat{v}(t) = V\hat{\varpi}(t), \qquad \hat{\varpi}(t) = s(t) - Lx(t), \tag{4}$$

where $\hat{\varpi}(t)$ is the estimation of $\varpi(t)$ and $s(t)$ is the auxiliary variable generated by

$$\begin{aligned} ds(t) = {} & [(W + LBV)(s(t) - Lx(t)) \\ & + L(Ax(t) + F_0 f(x, t) + Bu(t))] dt. \end{aligned} \tag{5}$$

Denote the estimation error as $e_\varpi(t) = \varpi(t) - \hat{\varpi}(t)$. Based on (1), (3), (4), and (5), the error dynamics satisfy

$$de_\varpi(t) = (W + LBV) e_\varpi dt + LA_0 x(t) d\omega. \tag{6}$$

The objective of disturbance rejection can be achieved by designing the observer gain L such that (6) satisfies the desired stability in probability.

For DOBC strategy, the controller is usually selected as [22, 26–28]

$$u(t) = -\hat{v}(t) + Kx(t), \tag{7}$$

where $\hat{v}(t)$ is to compensate the disturbance in control input path and K is the conventional feedback gain needed to be determined later.

By substituting (7) into (1), the closed loop system can be written in the following form:

$$\begin{aligned} dx(t) = {} & \{(A + BK)x(t) + F_0 f(x(t), t) + BVe_\varpi(t)\} dt \\ & + A_0 x(t) d\omega. \end{aligned} \tag{8}$$

Combining (8) with (6), the composite system is described as

$$d\overline{x}(t) = \left\{\overline{A}\overline{x}(t) + Ff(x, t)\right\} dt + \overline{A}_0 \overline{x}(t) d\omega, \tag{9}$$

where

$$\overline{x}(t) = \begin{bmatrix} x(t) \\ e_\omega(t) \end{bmatrix}, \qquad \overline{A} = \begin{bmatrix} A + BK & BV \\ 0 & W + LBV \end{bmatrix},$$
$$F = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}, \qquad \overline{A}_0 = \begin{bmatrix} A_0 & 0 \\ LA_0 & 0 \end{bmatrix}. \tag{10}$$

In the following, the objective is to design gain L and K such that system (9) is asymptotical stabilization in probability. For the convenience of research, the following lemma is presented.

Lemma 5 (see [29, 30]). *For a given stochastic system*

$$dx(t) = g(x,t)\,dt + h(x,t)\,d\omega, \quad x(t_0) = x_0, \tag{11}$$

if there exists function $V(x,t) \in C^{1,2}$, $\mu_1(\cdot), \mu_2(\cdot) \in \mathscr{K}_\infty$, constants $c_1 > 0$, and a nonnegative $M(x,t)$, such that

$$\mu_1(x) \le V(x,t) \le \mu_2(x), \qquad \mathscr{L}V \le -c_1 M(x,t), \tag{12}$$

then, one has the following.

(i) *The equilibrium $x = 0$ is globally stable in probability and the solution $x(t)$ satisfied $P\{\lim_{t\to\infty} M(x(t)) = 0\} = 1$, when $g(0,t) = 0$, $h(0,t) = 0$, and $M(x,t) = M(x)$ is continuous.*

(ii) *The equilibrium $x = 0$ is globally asymptotically stable in probability, when $g(0,t) = 0$, $h(0,t) = 0$, and $M(x,t)$ is positive definite.*

Here, the differential operator $\mathscr{L}$ for differentiable function $V(x,t)$ is defined as

$$\mathscr{L}V = \frac{\partial V}{\partial x} g(x,t) + \frac{1}{2} Tr\left\{h(x,t)^T \frac{\partial^2 V}{\partial x^2} h(x,t)\right\}. \tag{13}$$

Theorem 6. *Consider system (1) with disturbance (3) under Assumptions 1, 2, and 4. For some $\lambda > 0$, if there exist $P_1 = P_1^T > 0$, R_1 and constant $\beta > 0$ satisfying*

$$\begin{bmatrix} sym(P_1 W + R_1 BV) & V^T B^T \\ BV & -\beta^2 I \end{bmatrix} < 0 \tag{14}$$

and $Q_2 = Q_2^T > 0$, R_2 satisfying

$$\begin{bmatrix} \Xi & Q_2 A_0^T & Q_2 A_0^T R_1^T & Q_2 U^T \\ A_0 Q_2 & -Q_2 & 0 & 0 \\ R_1 A_0 Q_2 & 0 & -P_1 & 0 \\ U Q_2 & 0 & 0 & -\lambda^2 I \end{bmatrix} < 0, \tag{15}$$

where $\Xi = sym(AQ_2 + BR_2) + \lambda^2 F_0 F_0^T + \beta^2 I$, then the closed loop system (8) under DOBC law (7) with gain $K = R_2 Q_2^{-1}$ and observer (4) with gain $L = P_1^{-1} R_1$ are global asymptotical stabilization in probability.

Proof. Define

$$\Sigma_1(e_\omega, t) = e_\omega^T P_1 e_\omega,$$
$$\Sigma_2(x,t) = x^T P_2 x + \frac{1}{\lambda^2}\int_0^t \left[\|Ux(t)\|^2 - \|f(x,t)\|^2\right] d\tau, \tag{16}$$

where $P_2^{-1} = Q_2$.

It is noted that for all x and e_ω, $\Sigma_1 \ge 0, \Sigma_2 \ge 0$. In addition, along with (6), (8), the Itó differential of Σ_1, Σ_2 satisfies

$$\begin{aligned} \mathscr{L}\Sigma_1 &= e_\omega^T \left[P_1(W + LBV) + (W + LBV)^T P_1\right] e_\omega \\ &\quad + x^T A_0^T L^T P_1 L A_0 x, \\ \mathscr{L}\Sigma_2 &= x^T \left[P_2(A + BK) + (A + BK)^T P_2\right] x \\ &\quad + 2x^T P_2 F_0 f + 2x^T P_2 BV e_\omega + \frac{1}{\lambda^2} x^T U^T U x \\ &\quad + x^T A_0^T P_2 A_0 x - \frac{1}{\lambda^2} f^T f. \end{aligned} \tag{17}$$

Via Young inequality, we get

$$\begin{aligned} 2x^T P_2 F_0 f &\le \lambda^2 x^T P_2 F_0 F_0^T P_2 x + \frac{1}{\lambda^2} f^T f, \\ 2x^T P_2 BV e_\omega &\le \beta^2 x^T P_2 P_2 x + \frac{1}{\beta^2} e_\omega^T V^T B^T BV e_\omega. \end{aligned} \tag{18}$$

Then we have

$$\begin{aligned} \mathscr{L}\Sigma_2 \le x^T \Big\{ &\mathrm{sym}\left[P_2(A + BK)\right] + A_0^T P_2 A_0 + \frac{1}{\lambda^2} U^T U \\ &+ \lambda^2 P_2 F_0 F_0^T P_2 + \beta^2 P_2 P_2 \Big\} x + \frac{1}{\beta^2} e_\omega^T V^T B^T BV e_\omega. \end{aligned} \tag{19}$$

A Lyapunov function candidate for (9) is chosen as $\Sigma(x, e_\omega, t) = \Sigma_1(e_\omega, t) + \Sigma_2(x,t)$; hence, it is easy to get

$$\begin{aligned} \mathscr{L}\Sigma &\le e_\omega^T \left\{\mathrm{sym}\left[P_1(W + LBV)\right] + \frac{1}{\beta^2} V^T B^T BV\right\} e_\omega \\ &\quad + x^T \Big\{\mathrm{sym}\left[P_2(A + BK)\right] + A_0^T P_2 A_0 + \frac{1}{\lambda^2} U^T U \\ &\quad + \lambda^2 P_2 F_0 F_0^T P_2 + \beta^2 P_2 P_2 + A_0^T L^T P_1 L A_0 \Big\} x \\ &= e_\omega^T \Pi_1 e_\omega + x^T \Pi_2 x, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \Pi_1 &= \mathrm{sym}\left[P_1(W + LBV)\right] + \frac{1}{\beta^2} V^T B^T BV, \\ \Pi_2 &= \mathrm{sym}\left[P_2(A + BK)\right] + A_0^T P_2 A_0 + \frac{1}{\lambda^2} U^T U \\ &\quad + \lambda^2 P_2 F_0 F_0^T P_2 + \beta^2 P_2 P_2 + A_0^T L^T P_1 L A_0. \end{aligned} \tag{21}$$

Using Lemma 5, it can be verified that system (9) is global asymptotical stabilization in probability if $\Pi_1 < 0$ and $\Pi_2 < 0$ hold.

Based on Schur complement, $\Pi_1 < 0$ is equivalent to $\Pi_{10} < 0$, where

$$\Pi_{10} = \begin{bmatrix} \mathrm{sym}\left[P_1 W + R_1 BV\right] & V^T B^T \\ BV & -\beta^2 I \end{bmatrix}, \tag{22}$$

and $\Pi_2 < 0$ is equivalent to $\Pi_{21} < 0$, where

$$\Pi_{21} = \begin{bmatrix} \Xi_0 & A_0^T & A_0^T L^T P_1 & U^T \\ * & -P_2^{-1} & 0 & 0 \\ * & * & -P_1 & 0 \\ * & * & * & -\lambda^2 I \end{bmatrix} \tag{23}$$

and $\Xi_0 = \mathrm{sym}[P_2(A+BK)]+\lambda^2 P_2 F_0 F_0^T P_2+\beta^2 P_2 P_2$. In addition, $*$ represents the corresponding elements in the symmetric matrix.

Π_{21} is premultiplied and postmultiplied simultaneously by $\mathrm{diag}\{Q_2, I, I, I\}$; then it is equivalent to Π_{20}, where

$$\Pi_{20} = \begin{bmatrix} \Xi & Q_2 A_0^T & Q_2 A_0^T R_1^T & Q_2 U^T \\ * & -Q_2 & 0 & 0 \\ * & * & -P_1 & 0 \\ * & * & * & -\lambda^2 I \end{bmatrix} \tag{24}$$

and $\Xi = \mathrm{sym}[(A + BK)Q_2] + \lambda^2 F_0 F_0^T + \beta^2 I$.

Thus, (14), (15) can be obtained.

On the other hand, (14), (15) hold, meaning that there exist $\alpha_1 > 0$, $\alpha_2 > 2$ such that $\Pi_{10} < -\alpha_1 I$, $\Pi_{20} < -\alpha_2 I$; that is, $\Pi_1 < -\alpha_1 I$, $\Pi_2 < -\alpha_2 I$. Hence, we have

$$\begin{aligned} \mathscr{L}\Sigma &\le e_\omega^T \Pi_1 e_\omega + x^T \Pi_2 x \\ &\le -\alpha_1 \|e_\omega\|^2 - \alpha_2 \|x\|^2 \\ &\le -\min\{\alpha_1, \alpha_2\} \left(\|x\|^2 + \|e_\omega\|^2\right) \\ &= -\min\{\alpha_1, \alpha_2\} \|\overline{x}\|^2. \end{aligned} \tag{25}$$

Therefore, the closed-loop system (9) is global asymptotical stabilization in probability when the control gain is selected as $K = R_2 Q_2^{-1}$ and observer gain is selected as $L = P_1^{-1} R_1$. The proof is completed. □

4. DOBC for the Case with Unknown Nonlinearity

In this section, the nonlinear function $f(x(t), t)$ is supposed to be unknown, which means disturbance observer should be designed different from Section 3. In this case, the disturbance observer can be constructed as

$$\begin{gathered} \widehat{v}(t) = V\widehat{\omega}(t), \quad \widehat{\omega}(t) = s(t) - Lx(t), \\ ds(t) = (W + LBV)(s(t) - Lx(t)) + L(Ax(t) + Bu(t))\,dt. \end{gathered} \tag{26}$$

Compared with (6), the estimation error $e_\omega(t) = \omega(t) - \widehat{\omega}(t)$ satisfies

$$de_\omega(t) = \left[(W + LBV)e_\omega + LF_0 f(x,t)\right]dt + LA_0 x(t)\,d\omega. \tag{27}$$

Thus, the composite system combined (8) with (27) is given by

$$d\overline{x}(t) = \left\{\overline{A}\overline{x}(t) + Ff(x,t)\right\}dt + \overline{A}_0 \overline{x}(t)\,d\omega, \tag{28}$$

where

$$\begin{gathered} \overline{x}(t) = \begin{bmatrix} x(t) \\ e_\omega(t) \end{bmatrix}, \qquad \overline{A} = \begin{bmatrix} A + BK & BV \\ 0 & W + LBV \end{bmatrix}, \\ F = \begin{bmatrix} F_0 \\ LF_0 \end{bmatrix}, \qquad \overline{A}_0 = \begin{bmatrix} A_0 & 0 \\ LA_0 & 0 \end{bmatrix}. \end{gathered} \tag{29}$$

The objective of this section is similar to Section 3, that is, to design gain L and K such that system (28) is asymptotically stable in probability.

Theorem 7. *Consider system* (1) *with disturbance* (3) *under Assumptions 1, 2, and 4. For some $\lambda_1 > 0$, $\lambda_2 > 0$, if there exist $P_1 = P_1^T > 0$, R_1 and constant $\beta > 0$ satisfying*

$$\begin{bmatrix} sym(P_1 W + R_1 BV) & R_1 F_0 & V^T B^T \\ * & -\lambda_1^{-2} I & 0 \\ * & * & -\beta^2 I \end{bmatrix} < 0 \tag{30}$$

and $Q_2 = Q_2^T > 0$, R_2 satisfying

$$\begin{bmatrix} \overline{\Xi} & Q_2 A_0^T & Q_2 A_0^T R_1^T & Q_2 U^T & Q_2 U^T \\ * & -Q_2 & 0 & 0 & 0 \\ * & * & -P_1 & 0 & 0 \\ * & * & * & -\lambda_1^2 I & 0 \\ * & * & * & * & -\lambda_2^2 I \end{bmatrix} < 0, \tag{31}$$

where $\overline{\Xi} = sym(AQ_2 + BR_2) + \lambda_2^2 F_0 F_0^T + \beta^2 I$, then the closed loop system (28) *under DOBC law* (7) *with gain $K = R_2 Q_2^{-1}$ and observer* (26) *with gain $L = P_1^{-1} R_1$ is global asymptotical stabilization in probability.*

Proof. Let

$$\begin{gathered} \overline{\Sigma}_1(e_\omega, t) = e_\omega^T P_1 e_\omega + \frac{1}{\lambda_1^2} \int_0^t \left[\|Ux(t)\|^2 - \|f(x,t)\|^2\right] d\tau, \\ \overline{\Sigma}_2(x,t) = x^T P_2 x + \frac{1}{\lambda_2^2} \int_0^t \left[\|Ux(t)\|^2 - \|f(x,t)\|^2\right] d\tau, \end{gathered} \tag{32}$$

where $P_2^{-1} = Q_2$. And a Lyapunov function candidate for (28) is chosen as $\overline{\Sigma}(x, e_\omega, t) = \overline{\Sigma}_1(e_\omega, t) + \overline{\Sigma}_2(x, t)$. The following proof procedure can be given similarly to that of the proof for Theorem 6. □

5. Simulation Example

In [21], DOBC strategy was employed to a deterministic system of A4D aircraft with disturbance, and better system performance was obtained than some previous results [31]. However, stochastic noise should be considered when higher precision of system performance was required. In this section, the stochastic system that represents the longitudinal dynamics of A4D aircraft is considered, which is described as follows:

$$\begin{aligned} dx(t) = &\left\{Ax(t) + F_0 f(x,t) + B[u(t) + v(t)]\right\}dt \\ &+ A_0 x(t)\,d\omega \end{aligned} \tag{33}$$

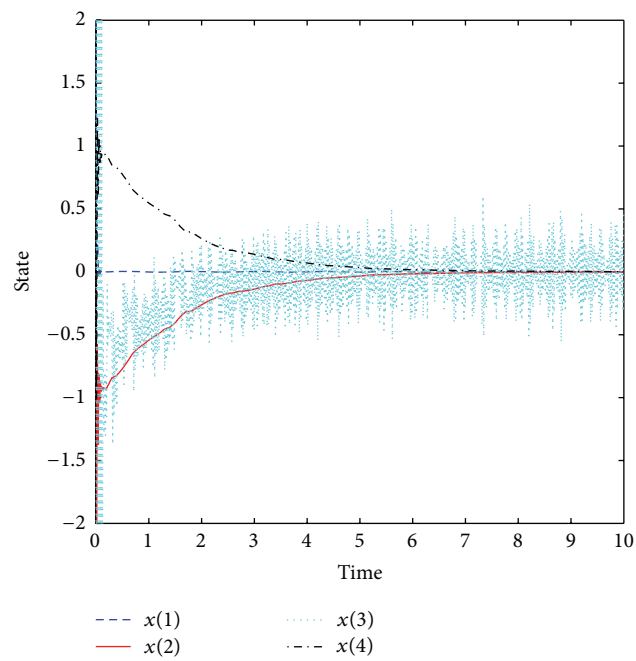

Figure 1: System performance using stochastic robust control strategy.

with the following coefficient:

$$A = \begin{bmatrix} -0.0605 & 32.37 & 0 & 32.2 \\ -0.00014 & -1.475 & 1 & 0 \\ -0.0111 & -34.72 & -2.793 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$F_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 60 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ -0.1064 \\ -33.8 \\ 0 \end{bmatrix}, \tag{34}$$

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0.2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}.$$

The parameter matrices for disturbance $v(t)$ that are described by (3) are given by

$$W = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} 25 & 10 \end{bmatrix}. \tag{35}$$

Case 1 (with known nonlinearity). In this case, the nonlinear dynamic is supposed to be denoted by $f(x,t) = \sin(10\pi t)x_2(t)$ in the simulation. In order to satisfy Assumption 1, U is selected as diag$\{0, 1, 0, 0\}$. The initial value of state is taken to be $x(0) = [2, -2, 3, 0]$ and λ is selected as 20. Based on Theorem 6, it can be solved that

$$L = \begin{bmatrix} 0 & -0.0001 & 0.0019 & 0 \\ 0 & -0.0001 & 0.0012 & 0 \end{bmatrix},$$
$$K = \begin{bmatrix} 0.2952 & 2.4053 & 0.3805 & 4.8333 \end{bmatrix}. \tag{36}$$

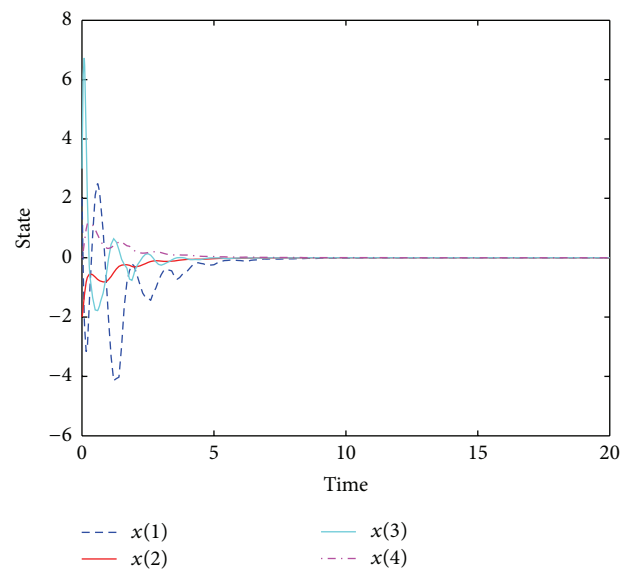

Figure 2: System performance using DOBC strategy for known nonlinearity.

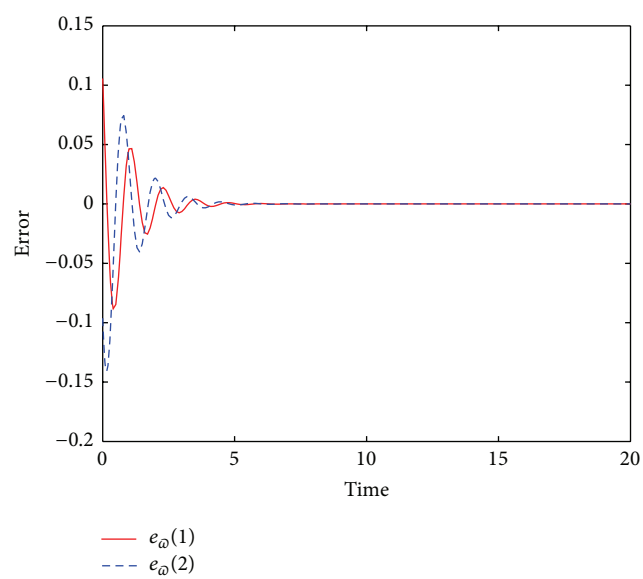

Figure 3: Estimation error of disturbance for known nonlinearity.

When the stochastic robust control strategy is applied to (33), which was first given in [13], it can be shown from Figure 1 that asymptotical stabilization in probability cannot be guaranteed in the presence of disturbance $v(t)$. Figures 2 and 3 show the system response and estimation error of system disturbance for the case with known nonlinearity, respectively. The simulation results show that asymptotical stabilization can be achieved using the method proposed in this paper and that the proposed disturbance observer is fine and effective.

Case 2 (with unknown nonlinearity). When nonlinear term $f(x,t)$ is unknown, we assume $f(x,t) = r(t)x_2(t)$ in

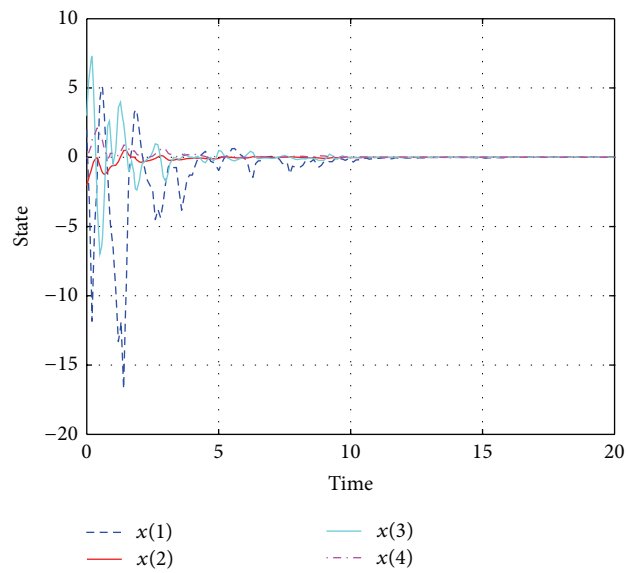

Figure 4: System performance using DOBC strategy for unknown nonlinearity.

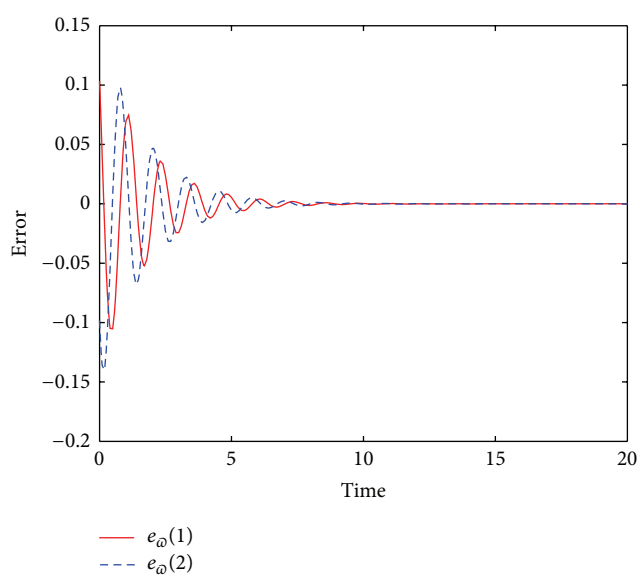

Figure 5: Estimation error of disturbance for unknown nonlinearity.

simulation, where $r(t)$ is supposed to be stochastic input that obeys uniform distribution. Based on Theorem 7, it can be solved that

$$L = \begin{bmatrix} 0 & 0.0001 & 0.0011 & 0 \\ 0 & 0.0001 & 0.0006 & 0 \end{bmatrix}, \qquad K = \begin{bmatrix} 0.2209 & 0.9695 & 0.2536 & 3.1946 \end{bmatrix}. \tag{37}$$

It is clear from Figure 4 that all states of system converge to zero and estimation errors of disturbance also converge to zero as shown in Figure 5. As has been shown above, we can see that asymptotical stabilization in probability is guaranteed and satisfied performance of the closed-loop control systems is achieved.

6. Conclusion

In this paper, the DOBC approach is investigated for a class of nonlinear systems with deterministic and stochastic disturbances. Feasible design procedures are proposed under different conditions to estimate and reject deterministic disturbance for the plants with known and unknown nonlinearity. Based on the estimation of disturbances, the composite control laws can guarantee the composite closed-loop systems to be global asymptotical stabilization in probability in the presence of disturbances. Simulation for an aircraft model shows the efficiency of the proposed algorithms.

Acknowledgments

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