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Research Article

Strong List Edge Coloring of Subcubic Graphs

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We study strong list edge coloring of subcubic graphs, and we prove that every subcubic graph with maximum average degree less than 15/7, 27/11, 13/5, and 36/13 can be strongly list edge colored with six, seven, eight, and nine colors, respectively.

1. Introduction

All graphs in this paper are finite and simple. For a graph G with vertex set $V(G)$ and edge set $E(G)$, a proper edge coloring of G is an assignment of colors to the edges of G so that no two adjacent edges receive the same color. A strong edge coloring is a proper edge coloring so that two edges adjacent to a common edge receive different colors. The strong chromatic index of G, denoted by $\chi_s'(G)$, is the minimum number of colors needed for a strong edge coloring of G.

Strong edge coloring was introduced by Fouquet and Jolivet [1, 2]. This type of coloring can be used to represent the conflict-free channel assignment in radio networks.

Denote by Δ the maximum degree of the graph. In 1985, Erdös and Nešetřil conjectured that the strong chromatic index of a graph is at most $(5/4)\Delta^2$ when Δ is even and $(1/4)(5\Delta^2 - 2\Delta + 1)$ when Δ is odd. Andersen proved the conjecture for $\Delta = 3$ [3]. Strong edge coloring of cubic Halin graphs has been studied in [4, 5].

Let $\operatorname{mad}(G) = \max_{H\subseteq G, |V(H)|\geq 1}(2|E(H)|/|V(H)|)$ be the maximum average degree of the graph G. Hocquard and Valicov [6] considered the subcubic graphs with bounded maximum average degree, and they proved the following results.

Theorem 1. *Let G be a subcubic graph.*

(i) *If* $\operatorname{mad}(G) < 15/7$, *then* $\chi_s'(G) \leq 6$.

(ii) *If* $\operatorname{mad}(G) < 27/11$, *then* $\chi_s'(G) \leq 7$.

(iii) *If* $\operatorname{mad}(G) < 13/5$, *then* $\chi_s'(G) \leq 8$.

(iv) *If* $\operatorname{mad}(G) < 36/13$, *then* $\chi_s'(G) \leq 9$.

The main purpose of this paper is to generalize the study of list version so that the admissible colors on edges are constrained. An edge list L of a graph G is a mapping that assigns a finite set to each edge of G. Denote $L = \{L(e) : e \in E(G)\}$. We say that L is a k-edge list if $|L(e)| \geq k$ for each edge e in G. The graph G is strongly L-edge colorable if there exists a strong edge coloring c of G such that $c(e) \in L(e)$ for every edge e of G. For a positive integer k, a graph G is strongly k-edge choosable if for every k-edge list L, G is strongly L-edge colorable. The strong list chromatic index $\chi_{ls}'(G)$ is the minimum positive integer k for which G is strongly k-edge choosable.

In this paper, we consider strong list edge coloring of subcubic graphs and extend Theorem 1 to the list version. We prove the following theorem.

Theorem 2. *Let G be a subcubic graph.*

(i) *If* $\operatorname{mad}(G) < 15/7$, *then* $\chi_{ls}'(G) \leq 6$.

(ii) *If* $\operatorname{mad}(G) < 27/11$, *then* $\chi_{ls}'(G) \leq 7$.

(iii) *If* $\operatorname{mad}(G) < 13/5$, *then* $\chi_{ls}'(G) \leq 8$.

(iv) *If* $\operatorname{mad}(G) < 36/13$, *then* $\chi_{ls}'(G) \leq 9$.

The paper is organized as follows. In Section 2, we will prove two lemmas which will be applied a lot in the proof of

Theorem 2. Theorem 2 will be proved in Sections 3, 4, 5, and 6.

Before proceeding we introduce some notations and definitions. The degree of a vertex v in a graph is denoted by $d(v)$. A vertex of degree k is called a k-vertex. A k-neighbor of v is a k-vertex adjacent to v. $N(v)$ is the set of the neighbors of v. A t-thread of G is a path $P_t = x_1x_2 \cdots x_t$ with $d(x_i) = 2$ for $i = 1, 2, \ldots, t$. Two edges are at distance at most 2 if either they are adjacent or they are adjacent to a common edge. Denote by $N_2(e)$ the set of edges at distance at most 2 from the edge e. Define $\mathrm{LSC}(N_2(e))$ as the set of colors used by edges in $N_2(e)$. Denote $L'(e) = L(e) \setminus \mathrm{LSC}(N_2(e))$.

2. Lemmas

Lemma 3. *Let G be the graph obtained from a path $x_1x_2x_3x_4x_5$ by adding two vertices u, v so that u is adjacent to x_3 and v is adjacent to x_4. Let L be an edge list of G. If $|L(x_ix_{i+1})| \geq 3$ for $i = 1, 2, 3$, $|L(x_4x_5)| \geq 2$, $|L(x_3u)| \geq 5$, and $|L(x_4v)| \geq 4$, then G has a strong L-edge coloring.*

Proof. If there is a color $a \in L(x_1x_2) \cap L(x_4x_5)$, then we first color the edges x_1x_2 and x_4x_5 with the color a. So we can further color the rest of edges with the available colors in the order of x_2x_3, x_3x_4, x_4v, and x_3u.

Now we assume that $L(x_1x_2) \cap L(x_4x_5) = \emptyset$. Denote $L(x_1x_2) = \{a, b, c\}$ and $L(x_4x_5) = \{d, e\}$. If there is a color $t \in L(x_2x_3) \setminus \{a, b, c\}$, we first color the edge x_2x_3 with the color t. Then after coloring all the other edges, the edge x_1x_2 still has one color available. Therefore, we can color the edges with an available color in the order of x_4x_5, x_3x_4, x_4v, and x_3u. Thus we can further assume $L(x_2x_3) = \{a, b, c\}$. Similarly, we can also assume that $L(x_3x_4) = \{a, b, c\}$. If there is a color $s \in \{a, b, c\} \cap L(x_4v)$, then we first color the edges x_1x_2 and x_4v with the color s. So we can further color the rest of edges with the available colors in the order of x_2x_3, x_3x_4, x_3u, and x_4x_5. Therefore we can assume that $\{a, b, c\} \cap L(x_4v) = \emptyset$. We first color the edges x_1x_2, x_2x_3, x_3x_4, and x_4x_5 with the colors a, b, c, and d, respectively. Then $|L'(x_3u)| \geq 1$, $|L'(x_4v)| \geq 3$, and thus we can further color the edges x_3u and x_4v. This completes the proof of the lemma. □

Lemma 4. *Let $P = xyzuv$ be a path and L an edge list so that $|L(e)| \geq 2$ if $e \in \{xy, zu, uv\}$ and $|L(yz)| \geq 3$. Then P has a strong L-edge coloring.*

Proof. We only need to prove the lemma when each $|L(e)|$ is equal to its lower bound. If there is a color $a \in L(xy) \cap L(uv)$, we color both xy and uv with a. Then $|L'(zu)| \geq 1$ and $|L'(yz)| \geq 2$, so we can further color the edges zu and yz.

Now assume that $L(xy) \cap L(uv) = \emptyset$. Denote $L(xy) = \{a, b\}$ and $L(uv) = \{c, d\}$. If $a \in L(zu)$, we first color zu with a and the edge xy with b and then color the edge yz with an available color. Since $a, b \notin L(uv)$, there is still one color available for the edge uv. Thus P has a strong L-edge coloring. Similarly we can obtain a strong L-edge coloring of P if $L(zu) \cap L(uv) \neq \emptyset$. Now we further assume $L(zu) \cap [L(xy) \cup L(uv)] = \emptyset$. That is, $L(xy)$, $L(zu)$, and $L(uv)$ are mutually disjoint, and it is easy to see that P has a strong L-edge coloring. □

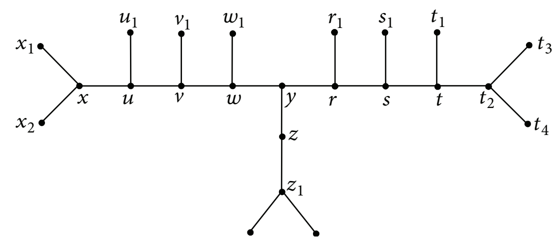

Figure 1: The configuration of Claim 4(1).

3. Proof of (i) of Theorem 2

Let H be a counterexample with $|E(H)|$ as small as possible. Then there exists a 6-edge list L such that H is not strongly L-edge colorable. We can assume that H is connected; otherwise, we can color independently each connected component. A 3-vertex is bad if it is adjacent to a 1-vertex; otherwise it is good.

Claim 1. A 1-vertex is adjacent to a 3-vertex in H and each bad 3-vertex is adjacent to two 3-vertices.

Proof. Let u be a 1-vertex and v its neighbor. Since H is a minimum counterexample, $H \setminus \{uv\}$ has a strong L-edge coloring. If $d(v) = 2$ or v is adjacent to only one 3-vertex, we have $|L'(uv)| \geq 1$, and thus we can easily extend the coloring to H, a contradiction. □

Claim 2. H does not contain a t-thread with $t \geq 3$.

Proof. Suppose that H contains a t-thread $x_1x_2 \cdots x_t$ with $t \geq 3$. Then $H' = H \setminus \{x_1x_2, x_2x_3\}$ has a strong L-edge coloring by the minimality of H. It is easy to see that $|L'(x_1x_2)| \geq 2$ and $|L'(x_2x_3)| \geq 2$. Hence we can extend the coloring of H' to H, a contradiction. □

Claim 3. H does not contain a path $x_2x_3x_4x_5$ such that x_2, x_3, x_4, x_5 are all bad 3-vertices.

Proof. Suppose that H contains such a path $x_2x_3x_4x_5$. Let x_1, u, v be the 1-neighbors of x_2, x_3, x_4, respectively. By the minimality of H, $H \setminus [\{x_ix_{i+1} \mid i = 1, \ldots, 4\} \cup \{x_3u, x_4v\}]$ has a strong L-edge coloring f. Since $|L(e)| \geq 6$, we have $|L'(x_ix_{i+1})| \geq 3$ for each $i = 1, 2, 3$, $|L'(x_4x_5)| \geq 2$, $|L'(x_3u)| \geq 5$, and $|L'(x_4v)| \geq 4$. By Lemma 3, we can further extend the coloring to the rest of the edges of H to obtain a strong L-edge coloring of H, a contradiction. □

Claim 4. H does not contain the following three configurations:

(1) a path $xuvwyrstt_2$ such that u, v, w, r, s, and t are bad 3-vertices, y is a good vertex, and another neighbor z of y is a 2-vertex (see Figure 1),

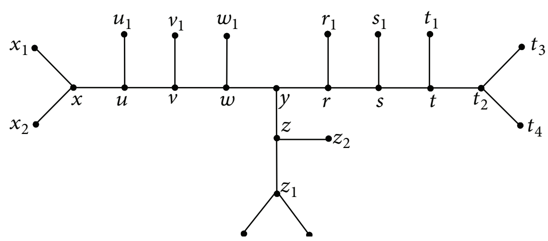

Figure 2: The configuration of Claim 4(2).

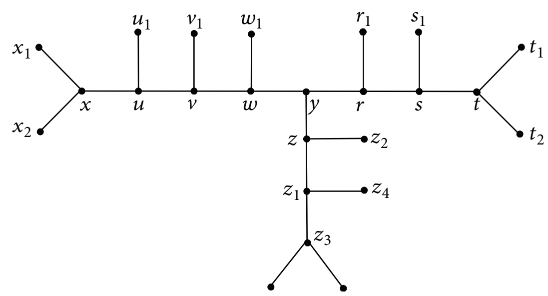

Figure 3: The configuration of Claim 4(3).

(2) a path $xuvwyrstt_2$ such that u, v, w, r, s, and t are bad 3-vertices, y is a good vertex, and another neighbor z of y is a bad 3-vertex (see Figure 2),

(3) a path $xuvwyrs$ such that u, v, w, r, and s are bad 3-vertices, y is a good vertex, and another neighbor z of y is a bad 3-vertex which is also adjacent to a bad 3-vertex (see Figure 3).

Proof. (1) Suppose that there exists a path $xuvwyrstt_2$ such that u, v, w, r, s, and t are bad 3-vertices, y is a good vertex, and y is adjacent to a 2-vertex (see Figure 1). Denote $N(u) = \{x, u_1, v\}$, $N(v) = \{u, v_1, w\}$, $N(w) = \{v, w_1, y\}$, $N(r) = \{y, r_1, s\}$, $N(s) = \{r, s_1, t\}$, and $N(t) = \{s, t_1, t_2\}$, where u_1, v_1, w_1, r_1, s_1, t_1 are 1-vertices, and $N(x) = \{u, x_1, x_2\}$, $N(y) = \{w, r, z\}$, $N(z) = \{y, z_1\}$, and $N(t_2) = \{t, t_3, t_4\}$. Then $H \setminus \{uu_1, uv, vv_1, vw, ww_1, wy, yz, yr, rr_1, rs, ss_1, st, tt_1\}$ has a strong L-edge coloring C by the minimality of H. We are going to extend the coloring C to H. For each uncolored edge e in H, we use $L'(e)$ to denote the set of colors available for e. Then $L'(uu_1) = L(uu_1) \setminus \{C(xu), C(xx_1), C(xx_2)\}$ has at least three colors because $|L(e)| \geq 6$ for each edge e in H. Similarly we have the following:

(1) $|L'(e)| \geq 3$ if $e \in \{uu_1, uv, st, tt_1, yz\}$,

(2) $|L'(e)| \geq 5$ if $e \in \{vv_1, vw, wy, yr, rs, ss_1\}$,

(3) $|L'(e) \geq 6$ if $e \in \{ww_1, rr_1\}$.

By Lemma 3, we only need to

(i) either extend the coloring C to $yz, yr, rs, ss_1, st, tt_1$ so that there are still two colors available for rr_1,

(ii) or extend the coloring C to $yz, yr, rr_1, rs, ss_1, st, tt_1$ so that $|\{C(yz), C(yr), C(rr_1), C(rs)\} \cap [L(wy) \setminus \{C(zz_1)\}]| \leq 3$.

If there is a color $a \in L'(yz) \cap L'(st)$, we color the edges yz and st with the color a. Then we may further color the edges tt_1, ss_1, rs, and yr. This gives a coloring in (i). Thus $L'(yz) \cap L'(st) = \emptyset$. Similarly we can show $L'(yz) \cap L'(ss_1) = \emptyset$. Denote $L'(yz) = \{a, b, c\}$.

If $a \notin L'(yw)$, we first color the edge yz with the color a, and then by Lemma 3, we can further extend C to the edges yz, yr, rr_1, rs, ss_1, st, and tt_1. Since the edge yz is colored with a not in $L'(wy)$, such extension satisfies (ii). Therefore $L'(yz) \subseteq L'(wy)$. Similarly we can show that $L'(wy) = L'(yr) = L'(rs)$. Denote $L'(wy) = \{a, b, c, d, e\}$. We first color the edges yz, yr, and rs with a, b, and c, respectively. Then $L'(wy) \setminus \{a, b, c\}$ has two colors. By Lemma 3, we can first extend C to the edges uu_1, uv, vv_1, vw, ww_1, and wy. Since $\{a, b, c\} \cap [L'(st) \cup L'(ss_1)] = \emptyset$, we can further color the edges tt_1, st, rr_1, and ss_1 in order.

In each case, we can extend the coloring C to a strong L-edge coloring of H, a contradiction. Therefore, the configuration in Figure 1 does not exist.

Similarly we can also show that the configurations in Figures 2 and 3 do not exist either. □

Let $M(x) = d(x) - (15/7)$ be the initial charge of x for each vertex x. Then $\sum_{x \in V(H)} M(x) < 0$. We assign a new charge to each vertex according to the following rules.

R1. For each good 3-vertex x, if $xx_1x_2 \cdots x_t x_{t+1}$ is a path in which $x_1 x_2 \cdots x_t$ is a maximal t-thread, then x sends $1/14$ to each x_i for $i = 1, \ldots, t$, and if $xx_1x_2 \cdots x_t x_{t+1}$ is a path in which x_i is a bad 3-vertex for each $i = 1, \ldots, t$, x sends $1/7$ to each x_i for $i = 1, \ldots, t$.

R2. Each bad 3-vertex sends $8/7$ to its 1-neighbor.

Now we consider the new charge $M'(x)$ for each vertex x.

(1) If $d(x) = 1$, then by Claim 1, x is adjacent to a bad 3-vertex. Thus $M'(x) = 1 - (15/7) + (8/7) = 0$.

(2) If $d(x) = 2$, then by R1, x receives $2 \times (1/14) = (1/7)$ in total from some 3-vertices. Thus $M'(x) = 2 - (15/7) + (1/7) = 0$.

(3) If x is a bad 3-vertex, then $M'(x) = (8/7) - (8/7) = 0$.

(4) Assume that x is a good 3-vertex. Denote by t and s the numbers of bad 3-vertices and 2-vertices receiving charges from x, respectively. By Claim 2 and Claim 4, we have $0 \leq t \leq 6$ and $0 \leq s \leq 6$. Hence $M'(x) = 3 - (15/7) - (t/7) - (s/14) = (6/7) - (t/7) - (s/14)$.

If $t = 0$, then $s \leq 6$. Hence $M'(x) \geq (6/7) - (6/14) > 0$.

If $0 < t \leq 3$, then $s \leq 4$. Hence $M'(x) \geq (6/7) - (3/7) - (4/14) > 0$.

If $t = 4$, then $s \leq 2$. Hence $M'(x) \geq (6/7) - (4/7) - (2/14) > 0$.

If $t = 5$, then $s \leq 2$. Hence $M'(x) \geq (6/7) - (5/7) - (2/14) \geq 0$.

If $t = 6$, then $s = 0$. Hence $M'(x) \geq (6/7) - (6/7) \geq 0$.

Therefore we have $0 \leq \sum_{x \in V(H)} M'(x) = \sum_{x \in V(H)} M(x) < 0$.

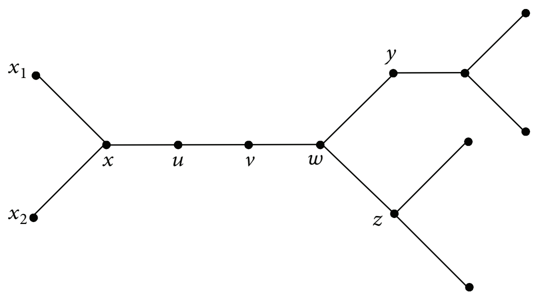

Figure 4: The configuration of Claim 7(1).

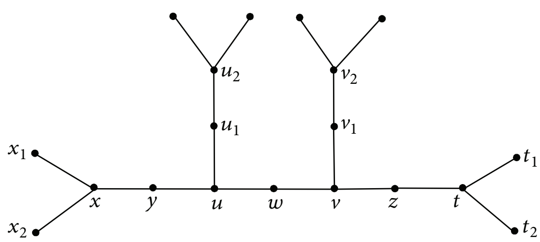

Figure 5: The configuration of Claim 7(2).

4. Proof of (ii) of Theorem 2

Let H be a counterexample with $|E(H)|$ as small as possible. Then there exists a 7-edge list L such that H is not strongly L-edge colorable.

Claim 5. There is no 1-vertex in H.

Proof. Suppose to the contrary that H contains a 1-vertex u, such that v is its neighbor. Since H is a minimum counterexample, $H' = H \setminus \{uv\}$ has a strong L-edge coloring. Hence $|L'(uv)| \geq 1$, we can easily extend this coloring to H, a contradiction. □

Claim 6. H does not contain a t-thread with $t \geq 3$.

Proof. The proof is similar to that of Claim 2 and thus omitted. □

Claim 7. H does not contain the following two configurations:

(1) A path $xuvw$ such that u and v are 2-vertices, x is a 3-vertex, and w is a 3-vertex which has two 2-neighbors and one 3-neighbor (see Figure 4).

(2) A path uwv where $d(u) = d(v) = 3$ and $d(w) = 2$ and both u and v have three 2-neighbors (see Figure 5).

Proof. (1) Suppose that H contains a path $xuvw$ in Figure 4. Let $H' = H \setminus \{uv, vw, wy\}$. Since H is a minimal counterexample, H' has a strong L-edge coloring. Then $|L'(wy)| \geq 1$, $|L'(vw)| \geq 2$, and $|L'(uv)| \geq 3$, and we can extend the coloring to H, a contradiction.

(2) Suppose that H contains the configuration in Figure 5, where y, u_1, w, v_1, and z are 2-vertices and u and v are 3-vertices. Since H is a minimum counterexample, $H' = H \setminus \{yu, uu_1, uw, wv, vv_1, vz\}$ has a strong L-edge coloring. Now $|L'(uu_1)| \geq 3$ and $|L'(vv_1)| \geq 3$. We first color the two edges uu_1 and vv_1 to obtain a strong L-edge coloring of $H \setminus \{yu, uw, wv, vz\}$. It is easy to check that $|L'(yu)| \geq 2$, $|L'(uw)| \geq 3$, $|L'(wv)| \geq 3$, and $|L'(vz)| \geq 2$. By Lemma 4, we can further extend the coloring to H, a contradiction. □

Let $M(x) = d(x) - (27/11)$ be the initial charge of x for each vertex x. Then $\sum_{x\in V(H)} M(x) < 0$. A 2_k-vertex is a 2-vertex with k 3-neighbors. We assign a new charge to each vertex according to the following rules.

R1. Let x be a 2_2-vertex and u a 3-neighbor of x. If u is adjacent to three 2-vertices, then u sends $2/11$ to x; otherwise u sends $3/11$ to x.

R2. Let x be a 2_1-vertex and u be its 3-neighbor. u sends $5/11$ to x.

(1) If x is a 2_1-vertex, then by R2, x receives $5/11$ from its 3-neighbor. Thus $M'(x) = 2 - (27/11) + (5/11) = 0$.

(2) If x is a 2_2-vertex with two neighbors u and v and if one of u, v has three 2-neighbors, then by Claim 7 the other one has at most two 2-neighbors. Hence x receives $(2/11) + (3/11) = 5/11$ from its neighbors. If neither u nor v has three 2-neighbors, then x receives $(3/11) + (3/11) = 6/11$ from its neighbors. Therefore $M'(x) \geq 2 - (27/11) + (5/11) \geq 0$.

(3) Assume $d(x) = 3$. By Claim 7, we only consider the following three cases: (a) if x is adjacent to three 2_2-vertices, then x sends out $3 \times (2/11) = 6/11$ to its neighbors; (b) if x is adjacent to at most two 2_2-vertices, then it sends out at most $2 \times (3/11) = 6/11$ to its neighbors; (c) if x is adjacent to one 2_1-vertex, then it sends out $5/11$ to its neighbors. In each case, we have $M'(x) \geq 3 - (27/11) - (6/11) = 0$.

Therefore we have $0 \leq \sum_{x\in V(H)} M'(x) = \sum_{x\in V(H)} M(x) < 0$.

5. Proof of (iii) of Theorem 2

Let H be a counterexample with $|E(H)|$ as small as possible. Then there exists an 8-edge list L such that H is not strongly L-edge colorable.

Claim 8. There is no 1-vertex in H.

Proof. The proof is similar to that of Claim 5 and thus omitted. □

Claim 9. There are no two adjacent 2-vertices in H.

Proof. Suppose that there are two adjacent 2-vertices u and v. Let w be the other neighbor of v. Since H is a minimum counterexample, $H \setminus \{uv, vw\}$ has a strong L-edge coloring. Then $|L'(uv)| \geq 3$ and $|L'(vw)| \geq 1$. Hence, we can extend the coloring to H easily, a contradiction. □

Claim 10. A 3-vertex is not adjacent to three 2-vertices in H.

Proof. Suppose to the contrary that a 3-vertex u is adjacent to three 2-vertices x, v, and w. Since H is a minimum counterexample, $H \setminus \{xu, uv, uw\}$ has a strong L-edge coloring. Thus $|L'(xu)| \geq 3$, $|L'(uv)| \geq 3$, and $|L'(uw)| \geq 3$. Therefore, we can extend the coloring to H, a contradiction. □

Claim 11. H does not contain a path $P = uvwxy$ where u, w, and y are 2-vertices and v and x are 3-vertices.

Proof. Suppose to the contrary that H contains a path $uvwxy$, where u, w, and y are 2-vertices and v and x are 3-vertices. By the minimality of H, $H \setminus \{vw, wx\}$ has a strong L-edge coloring. Uncolor uv and xy. It is easy to check that $|L'(uv)| \geq 2$, $|L'(xy)| \geq 2$, $|L'(vw)| \geq 3$, and $|L'(wx)| \geq 3$. By Lemma 4, we can extend the coloring to the path $uvwxy$ to obtain a strong L-edge coloring of H, a contradiction. □

Let $M(x) = d(x) - (13/5)$ be the initial charge of x for each vertex x. Then $\sum_{x \in V(H)} M(x) < 0$. We assign a new charge to each vertex according to the following rule.

R. Let x be a 3-vertex and t the number of 2-neighbors of x. x sends $2/5t$ to each adjacent 2-vertices if $t \neq 0$.

Obviously $M'(x) \geq 0$ if $d(x) = 3$.

Let x be a 2-vertex and u, v its neighbors. By Claims 9 and 10, both u and v are 3-vertices and have at most two 2-neighbors. If one 3-neighbor of x is adjacent to two 2-vertices, then by Claim 11, the other neighbor of x is adjacent to only one 2-vertex. Hence x receives $(1/5) + (2/5) = 3/5$ from its neighbors. If each neighbor of x is adjacent to only one 2-vertex, then x receives $(2/5) + (2/5) = 4/5$ from its neighbors. Hence $M'(x) \geq 2 - (13/5) + (3/5) = 0$.

Therefore we have $0 \leq \sum_{x \in V(H)} M'(x) = \sum_{x \in V(H)} M(x) < 0$. This contradiction completes the proof.

6. Proof of (iv) of Theorem 2

Let H be a counterexample with $|E(H)|$ as small as possible. Then there exists a 9-edge list L such that H is not strongly L-edge colorable.

Claim 12. There is no 1-vertex in H.

Proof. The proof is similar to that of Claim 5 and thus omitted. □

Claim 13. There are no two adjacent 2-vertices in H.

Proof. The proof is similar to that of Claim 9 and thus omitted. □

Claim 14. No 3-vertex is adjacent to two 2-vertices in H.

Proof. Suppose to the contrary that a 3-vertex u is adjacent to two 2-vertices v, w. Let x be the third neighbor of u. Since H is a minimum counterexample, $H \setminus \{uv, uw, ux\}$ has a strong L-edge coloring. Then $|L'(ux)| \geq 1$, $|L'(uv)| \geq 3$, and $|L'(uw)| \geq 3$. Hence we can extend the coloring to H, a contradiction. □

Claim 15. If a 3-vertex x has a 2-neighbor, then each 3-neighbor of x is not adjacent to a 2-vertex.

Proof. Suppose to the contrary that x has a 3-neighbor y such that y is also adjacent to a 2-vertex u. Let z be the 2-neighbor of x. We first assume that $u = z$. By the choice of H, $H \setminus \{xz, yz\}$ has a strong L-edge coloring with the edges xz and yz uncolored. Then it is easy to see that $|L'(xz)| \geq 3$ and $|L'(yz)| \geq 3$, and thus we may further color the edges xz and yz to get a strong L-edge coloring of H, a contradiction.

Now we assume that $u \neq z$. Let v be the other neighbor of z. Let C be a strong L-edge coloring of $H \setminus \{vz, zx, xy, yu\}$ obtained from a strong L-edge coloring of $H - \{z\}$ by uncoloring the edges xy and yu. It is easy to see that $|L'(vz)| \geq 2$, $|L'(zx)| \geq 3$, $|L'(xy)| \geq 2$, and $|L'(yu)| \geq 2$. By Lemma 4, C can be extended to those four uncolored edges, and thus H has a strong L-edge coloring, a contradiction. □

Let $M(x) = d(x) - (36/13)$ be the initial charge of x for each vertex x. Then $\sum_{x \in V(H)} M(x) < 0$. We assign a new charge to each vertex according to the following rules.

R1. Each 2-vertex receives 5/13 from each adjacent vertex.

R2. If a 3-vertex x is adjacent to a 2-vertex then x receives 1/13 from each 3-neighbor.

Obviously if $d(x) = 2$, then $M'(x) = 2 - (36/13) + (10/13) = 0$.

If $d(x) = 3$ and x is not adjacent to a 2-vertex, then $M'(x) \geq 3 - (36/13) - 3 \times (1/13) = 0$.

If $d(x) = 3$ and x is adjacent to a 2-vertex, then by Claims 14 and 15, $M'(x) = 3 - (36/13) - (5/13) + 2 \times (1/13) = 0$.

Therefore we have $0 \leq \sum_{x \in V(H)} M'(x) = \sum_{x \in V(H)} M(x) < 0$. This contradiction completes the proof.

7. Conclusion

This paper studies strong list edge coloring of subcubic graphs. The result can be used to deal with the conflict-free channel assignment problem in wireless radio networks when the admissible channels on the links between transceivers are constrained. We believe that the upper bounds on the maximum average degree in Theorem 2 are not sharp. It would be interesting to find sharp upper bounds for the maximum average degree.

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