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Research Article

Robust Fault Tolerant Control for a Class of Time-Delay Systems with Multiple Disturbances

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A robust fault tolerant control (FTC) approach is addressed for a class of nonlinear systems with time delay, actuator faults, and multiple disturbances. The first part of the multiple disturbances is supposed to be an uncertain modeled disturbance and the second one represents a norm-bounded variable. First, a composite observer is designed to estimate the uncertain modeled disturbance and actuator fault simultaneously. Then, an FTC strategy consisting of disturbance observer based control (DOBC), fault accommodation, and a mixed H_2/H_∞ controller is constructed to reconfigure the considered systems with disturbance rejection and attenuation performance. Finally, simulations for a flight control system are given to show the efficiency of the proposed approach.

1. Introduction

To reduce the influence of model uncertainties and system disturbances, there are several control approaches focusing on nonlinear systems with unknown disturbances (see the survey paper [1] and references therein). The methodologies can be mainly classified into disturbance attenuation methods (such as H_∞ and H_2 control) and disturbance rejection approaches (such as output regulation theory, disturbance observer based control). The disturbance attenuation approaches have conservativeness for bounded stochastic disturbance. The disturbance rejection methods are established based on model-matching conditions. It has been shown that multiple disturbances exist in most practical systems. The idea of disturbance observer based control (DOBC) is to construct an observer to estimate and compensate some external disturbances [2]. A composite control scheme combining DOBC and PD (proportional derivative) control for flexible spacecraft attitude control was proposed in the presence of model uncertainty, elastic vibration, and external disturbances [3]. For nonlinear systems with multiple disturbances, it has been seen that the H_∞ and variable structure control have been integrated with DOBC in [4, 5]. In [6], a composite DOBC and adaptive control approach were proposed for a class of nonlinear systems with multiple disturbances. By constructing a disturbance compensation gain vector in the composite control law, a nonlinear robust DOBC was proposed to attenuate the mismatched disturbances and the influence of parameter variations from output channels [7].

In order to increase the reliability and safety of practical engineering, the issues of fault diagnosis and fault tolerant control (FTC) have become an attractive topic and have been paid much attention in recent years (see [8–13] and references therein). It is difficult to accommodate faults if the disturbances and faults exist simultaneously in the controlled systems. In [14], an optimal fault tolerant control approach was proposed for the nonlinear systems, where generalized H_∞ optimization was applied to estimate the fault and attenuate the disturbances. In [15], a robust observer was proposed to simultaneously estimate system states, faults, and their finite time derivatives and attenuate disturbances; then an FTC approach was designed based on their estimations. For systems with modeled disturbance, a fault diagnosis approach based on disturbance observer was firstly proposed in [16] with disturbance rejection performance. For the nonlinear system with multiple disturbances, [17] addressed a fault tolerant control approach with disturbance rejection and attenuation performances. It is well known that time delay frequently occurs in many practical systems, such as manufacturing systems, telecommunication, and economic systems. Therefore, the problem of fault accommodation for time-delay systems has been a hot topic in the control field. Many important results have been reported in the literature (see [18–21] and references therein). In [18], an adaptive fuzzy fault accommodation control approach was proposed for nonlinear time-delay system. In [19], a fault accommodation was addressed for time-varying delay system using adaptive fault diagnosis observer. [20] dealt with fault tolerant guaranteed cost controller design problem for linear time-delay system against actuator faults.

In this paper, FTC problem is discussed for a class of time-delay systems with actuator fault and multiple disturbances. The first part of multiple disturbances is modeled disturbance formulated by an exogenous system and the second one is norm bounded uncertain variable. A composite observer is designed to estimate the modeled disturbance and time-varying fault. Then, an FTC scheme is addressed with disturbance rejection and attenuation performance by combining fault accommodation and DOBC with a robust H_2/H_∞ controller.

2. Model Description

In this paper, we consider the following nonlinear system with time-varying faults, time-delay, and multiple disturbances simultaneously:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\tau) + Gg(x(t)) \\ &\quad + J[u(t) + F(t)] + J_1 d_1(t) + J_2 d_2(t) \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is system state, $u(t)$ is control input, and $y(t) \in R^m$ represents output variable. $F(t)$ is time-varying actuator fault to be diagnosed. A, A_d, C, G, J, J_1, and J_2 represent coefficient matrices of the system with suitable dimensions. τ is a known constant delay. The modeled external disturbance $d_1(t)$ is supposed to be generated by a linear exogenous system described by

$$\begin{aligned} \dot{\omega}(t) &= W\omega(t) + J_3\delta(t), \\ d_1(t) &= V\omega(t), \end{aligned} \tag{2}$$

where $\omega(t)$ is state variable, $W \in R^{p\times p}$, and V and J_3 are known parameter matrices of the exogenous system. $\delta(t)$ is additional disturbance which results from perturbations and uncertainties in the exogenous system. The disturbances $d_2(t)$ and $\delta(t)$ are supposed to have the bounded H_2 norm.

For a known matrix U_1, $g(x(t))$ is a known nonlinear vector function that is supposed to satisfy $g(0) = 0$ and the following norm condition:

$$\left\| g(x_1(t)) - g(x_2(t)) \right\| \le \left\| U_1 (x_1(t) - x_2(t)) \right\| \tag{3}$$

for any $x_1(t)$ and $x_2(t)$.

The following assumptions are required so that the considered problem can be well-posed in this paper.

Assumption 1. (A, J) is controllable; $(W, J_1 V)$ is observable.

Assumption 2. $\operatorname{rank}(J, J_1) = \operatorname{rank}(J)$.

Remark 1. In practical engineering, the exogenous model (2) can represent many kinds of disturbances including harmonic disturbance signal caused by vibration, unknown constant load in the motor, inertial sensor drift represented by first-order Gaussian Markov process, and so on. Compared with the previous works [17, 19, 20], both the time-delay and multiple disturbances are simultaneously considered in this paper. Furthermore, the modeled disturbance $d_1(t)$ and control input are assumed in different channels, while in [17] are in the same channel.

3. Robust Fault Tolerant Controller Design

3.1. Disturbance Observer. In order to reject the modeled external disturbance, disturbance observer should be designed in this subsection. In this paper, we only consider the case of available states. The disturbance observer is formulated as

$$\begin{aligned} \hat{\omega}(t) &= \xi(t) - Lx(t), \\ \hat{d}_1(t) &= V\hat{\omega}(t), \end{aligned} \tag{4}$$

where $\xi(t)$ is auxiliary variable generated by

$$\begin{aligned} \dot{\xi}(t) &= (W + LJ_1V)[\xi(t) - Lx(t)] + L \\ &\quad \times [Ax(t) + A_d x(t-\tau) + Gg(x(t)) \\ &\quad + Ju(t) + Ju_{fc}(t)] \end{aligned} \tag{5}$$

$\hat{\omega}(t)$ is estimation of $\omega(t)$, $\hat{d}_1(t)$ is estimation of modeled disturbance $d_1(t)$, matrix L is the disturbance observer gain to be determined later, and $u_{fc}(t) = \hat{F}(t)$ is compensation term to be designed in fault diagnosis observer, where $\hat{F}(t)$ is denoted as an estimation of fault $F(t)$.

By defining $e_\omega(t) = \omega(t) - \hat{\omega}(t)$ and $e_F(t) = F(t) - \hat{F}(t)$, estimation error system can be obtained from (1), (2), (4), and (5) to show the following:

$$\dot{e}_\omega(t) = (W + LJ_1V)e_\omega(t) + LJe_F(t) + LJ_2d_2(t) + J_3\delta(t). \tag{6}$$

In the following subsection, we will construct a fault diagnosis observer with disturbance estimation so that the modeled disturbance can be rejected and fault can be diagnosed.

3.2. Fault Diagnosis Observer. The following fault diagnosis observer is constructed to diagnose the time-varying actuator fault:

$$\begin{aligned}\hat{F}(t) &= \eta(t) - Kx(t),\\ \dot{\eta}(t) &= KJ[\eta(t) - Kx(t)]\\ &\quad + K[Ax(t) + A_d x(t-\tau)\\ &\quad + Gg(x(t)) + Ju(t) + J_1 u_{dc}(t)],\end{aligned} \tag{7}$$

where $\hat{F}(t)$ is estimation of $F(t)$. The disturbance observer based control term $u_{dc}(t) = \hat{d}_1(t)$ is applied to reject modeled disturbance $d_1(t)$ by its estimation from disturbance observer. K is the fault diagnosis observer gain to be determined later.

The fault estimation error system yields

$$\begin{aligned}\dot{e}_F(t) &= \dot{F}(t) - \dot{\hat{F}}(t)\\ &= \dot{F}(t) + KJe_F(t) + KJ_1Ve_\omega(t) + KJ_2d_2(t).\end{aligned} \tag{8}$$

In the next subsection, a composite fault tolerant controller should be determined for reconfiguring the systems with disturbance rejection and attenuation performance.

3.3. Composite Fault Tolerant Controller. In this section, the object is to construct a control approach to guarantee that the system (1) is stable in the presence (or absence) of faults and multiple disturbances simultaneously for the considered time-delay system. The structure of composite fault tolerant controller is formulated as

$$u(t) = u_{sc}(t) - u_{fc}(t) - J^* J_1 u_{dc}(t), \tag{9}$$

where $u_{fc}(t) = \hat{F}(t)$, $u_{dc}(t) = \hat{d}_1(t)$, and $u_{sc}(t) = Sx(t)$, S is the state feedback controller gain to be determined later. Substituting (9) into (1), it can be seen that

$$\begin{aligned}\dot{x}(t) &= (A + JS)x(t) + A_d x(t-\tau) + Gg(x(t))\\ &\quad + Je_F(t) + J_1d_1(t) - JJ^*J_1\hat{d}_1(t) + J_2d_2(t).\end{aligned} \tag{10}$$

From Assumption 2, it can be seen that the vector space spanned by the columns of J_1 is a subset of the space spanned by the column vectors of J [19]; that is, span $(J_1) \subset$ span (J), which is equivalent to the existence of J, such that

$$J_1 - JJ^*J_1 = 0. \tag{11}$$

Then, it can be concluded that

$$\begin{aligned}\dot{x}(t) &= (A + JS)x(t) + A_d x(t-\tau) + Gg(x(t))\\ &\quad + Je_F(t) + J_1Ve_\omega(t) + J_2d_2(t).\end{aligned} \tag{12}$$

Combing estimation error equations (6) and (8) with (12) yields

$$\begin{aligned}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{A}_d\bar{x}(t-\tau) + \bar{G}g(\bar{x}(t)) + \bar{J}\bar{d}(t),\\ z_2(t) &= \bar{C}_2\bar{x}(t) + \bar{C}_{d2}\bar{x}(t-\tau),\\ z_\infty(t) &= \bar{C}_\infty\bar{x}(t) + \bar{C}_{d\infty}\bar{x}(t-\tau) + D\bar{d}(t),\end{aligned} \tag{13}$$

where

$$\begin{aligned}\bar{x}(t) &= \begin{bmatrix} x(t) \\ e_\omega(t) \\ e_F(t) \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} A + JS & J_1V & J \\ 0 & W + LJ_1V & LJ \\ 0 & KJ_1V & KJ \end{bmatrix},\\ \bar{G} &= \begin{bmatrix} G \\ 0 \\ 0 \end{bmatrix},\\ \bar{A}_d &= \begin{bmatrix} A_d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \bar{J} = \begin{bmatrix} J_2 & 0 & 0 \\ LJ_2 & J_3 & 0 \\ KJ_2 & 0 & I \end{bmatrix},\\ \bar{d}(t) &= \begin{bmatrix} d_2(t) \\ \delta(t) \\ \dot{F}(t) \end{bmatrix},\\ g(\bar{x}(t)) &= g(x(t))\end{aligned} \tag{14}$$

$z_2(t)$ is H_2 reference output,

$$z_2(t) = C_{21}x(t) + C_{22}\left[e_\omega^T(t) \;\; e_F^T(t)\right]^T + C_{d2}x(t-\tau) \tag{15}$$

$z_\infty(t)$ is H_∞ reference output,

$$\begin{aligned}z_\infty(t) &= C_{\infty1}x(t) + C_{\infty2}\left[e_\omega^T(t) \;\; e_F^T(t)\right]^T\\ &\quad + C_{d\infty}x(t-\tau) + D\bar{d}(t),\end{aligned} \tag{16}$$

where C_{21}, C_{22}, C_{d2}, $C_{\infty1}$, $C_{\infty2}$, $C_{d\infty}$, and D are the selected weighting matrices.

Definition 2. For constants $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$, the H_∞ performance is denoted as follows:

$$\begin{aligned}J_\infty &= \|z_\infty(t)\|^2 - \gamma_1^2\|d_2(t)\|^2 - \gamma_2^2\|\delta(t)\|^2\\ &\quad - \gamma_3^2\|\dot{F}(t)\|^2 - \delta(P_1, Q),\end{aligned} \tag{17}$$

where

$$\delta(P_1, Q) = \phi^T(0)P_1\phi(0) + \int_{-d}^{0}\phi^T(\tau)Q\phi(\tau)\,d\tau. \tag{18}$$

Definition 3. The H_2 performance measure for (13) is defined as $J_2 = \|z_2(t)\|^2$.

Remark 4. Compared with [17], H_2/H_∞ mixed multiobjective optimization technique is used for the composite system (13). In the proposed approach, the modeled disturbance and fault are rejected by their estimations, while H_∞ performance is adopted to attenuate norm bounded uncertain disturbances and H_2 performance index is applied to optimize estimation error.

At this stage, the objective is to find K, L, and S such that system (13) is stable. The following result provides a design method based on convex optimization technology [22].

Theorem 5. *If for the parameter $\lambda > 0$, r_i $(i = 1, \ldots, 4)$, matrices C_{21}, C_{22}, C_{d2}, $C_{\infty 1}$, $C_{\infty 2}$, $C_{d\infty}$, and D, there exist matrices $P_0 > 0$, $P_2 > 0$, $Q_0 > 0$, R_0, and R_2 and constants $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$, if the following LMI-based optimization problem holds:*

$$\min\left\{r_1\gamma_1 + r_2\gamma_2 + r_3\gamma_3 + r_4\phi^T(0)P_0^{-1}\phi(0)\right\} \tag{19}$$

subject to

$$\begin{bmatrix} \Xi_1 & A_dQ_0 & G & \overline{V} & \Xi_{15} & P_0U_1^T & P_0C_{\infty 1}^T & P_0C_{21}^T & P_0 \\ * & -Q_0 & 0 & 0 & 0 & 0 & C_{d\infty}^T & C_{d2}^T & 0 \\ * & * & -\dfrac{1}{\lambda}I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_4 & \Xi_{45} & 0 & C_{\infty 2}^T & C_{22}^T & 0 \\ * & * & * & * & \Xi_5 & 0 & D^T & 0 & 0 \\ * & * & * & * & * & -\lambda I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -Q_0 \end{bmatrix} < 0, \tag{20}$$

where

$$\Xi_1 = \operatorname{sym}(AP_0 + JR_0), \qquad \Xi_{15} = [J_2 \;\; 0 \;\; 0],$$
$$\Xi_4 = \operatorname{sym}\left(P_2\overline{W} + R_2\overline{V}\right),$$
$$\Xi_{45} = \left[R_2J_2 \;\; P_2\overline{J}_3 \;\; P_2\overline{J}_1\right],$$
$$\Xi_5 = \begin{bmatrix} -\gamma_1^2 I & 0 & 0 \\ 0 & -\gamma_2^2 I & 0 \\ 0 & 0 & -\gamma_3^2 I \end{bmatrix},$$
$$\overline{W} = \begin{bmatrix} W & 0 \\ 0 & 0 \end{bmatrix}, \qquad \overline{V} = [J_1V \;\; J],$$
$$\overline{J}_1 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \qquad \overline{J}_3 = \begin{bmatrix} J_3 \\ 0 \end{bmatrix}, \qquad \overline{C} = [C_1 \;\; C_2], \tag{21}$$

then with gains $S = R_0P_0^{-1}$ and $\overline{L} = \left[\begin{smallmatrix} L \\ K \end{smallmatrix}\right] = P_2^{-1}R_2$, error system (13) *is stable and satisfies $J_\infty < 0$ and $J_2 \le \delta(P_1, Q)$. The symmetric terms in a symmetric matrix are denoted by $*$. The symbol* sym() *represents* $\operatorname{sym}(\Theta) := \Theta + \Theta^T$.

Proof. Consider the following Lyapunov function:

$$\Pi(t) = x^T(t)P_1x(t) + \int_{t-\tau}^{t} x^T(\tau)Qx(\tau)\,d\tau + \overline{e}^T(t)P_2\overline{e}(t) + \lambda\int_0^t \left[\|Ux(\tau)\|^2 - \|g(\tau)\|^2\right]d\tau. \tag{22}$$

It is verified that $\Pi(t) \ge 0$ holds for all arguments. Along with the trajectories of (13), it can be shown that

$$\begin{aligned} \dot{\Pi}(t) &= 2x^T(t)P_1\dot{x}(t) + x^T(t)Qx(t) - x(t-\tau)^TQx(t-\tau) \\ &\quad + 2\overline{e}^T(t)P_2\dot{\overline{e}}(t) + \lambda\left[\|Ux(t)\|^2 - \|g(x(t))\|^2\right] \\ &= 2x^T(t)P_1\left[(A + JS)x(t) + A_dx(t-\tau) + Gg(x(t))\right. \\ &\qquad \left. + J_1Ve_\omega(t) + Je_F(t) + J_2d_2(t)\right] \\ &\quad + x^T(t)Qx(t) - x(t-\tau)^TQx(t-\tau) \\ &\quad + \lambda x^T(t)U^TUx(t) - \lambda\|g(x(t))\|^2 + 2\overline{e}^T(t) \\ &\quad \times P_2\left[\left(\overline{W} + \overline{L}\,\overline{V}\right)\overline{e}(t) + \overline{L}J_2d_2(t) + \overline{J}_3\delta(t) + \overline{J}_1\dot{F}(t)\right]. \end{aligned} \tag{23}$$

In the absence of $\overline{d}(t)$ (i.e., $\overline{d}(t) = 0$), it can be seen that

$$\dot{V}(t) = s^T(t)\left[\Phi - \zeta\zeta^T\right]s(t), \tag{24}$$

where

$$s^T(t) = \left[x^T(t) \;\; x^T(t-\tau) \;\; g^T(x(t)) \;\; \overline{e}^T(t)\right],$$
$$\zeta^T = [C_{21} \;\; C_{d2} \;\; 0 \;\; C_{22}],$$
$$\Phi = \begin{bmatrix} \Phi_{11} & P_1A_d + C_{21}^TC_{d2} & P_1G & P_1\overline{V} + C_{21}^TC_{22} \\ * & -Q + C_{d2}^TC_{d2} & 0 & 0 \\ * & * & -\lambda I & 0 \\ * & * & * & \Phi_{44} \end{bmatrix},$$

$$\Phi_{11} = \operatorname{sym}\left(P_1\left(A + JS\right)\right) + \lambda U^T U + Q + C_{21}^T C_{21}, \quad \Phi_{44} = \operatorname{sym}\left(P_2\left(W + \overline{L}\,\overline{V}\right)\right) + C_{22}^T C_{22}. \tag{25}$$

From (20), it can be seen that

$$\begin{bmatrix} \Xi_1 + P_0 Q_0^{-1} P_0 + \frac{1}{\lambda} P_0 U_0^T U_0 P_0 & Q_0 A_d & G & \overline{V} & \Xi_{15} & P_0 C_{\infty 1}^T & P_0 C_{21}^T \\ * & -Q_0 & 0 & 0 & 0 & C_{d\infty}^T & C_{d2}^T \\ * & * & -\frac{1}{\lambda} I & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_4 & \Xi_{45} & C_{\infty 2}^T & C_{22}^T \\ * & * & * & * & \Xi_5 & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{26}$$

From the first, second, third, fourth, and seventh columns and rows of the left matrix in inequality (26) it can be verified that

$$\Phi_0 = \begin{bmatrix} \Xi_1 + P_0 Q_0^{-1} P_0 + \frac{1}{\lambda} P_0 U_0^T U_0 P_0 & Q_0 A_d & G & \overline{V} & P_0 C_{21}^T \\ * & -Q_0 & 0 & 0 & C_{d2}^T \\ * & * & -\frac{1}{\lambda} I & 0 & 0 \\ * & * & * & \Xi_4 & C_{22}^T \\ * & * & * & * & -I \end{bmatrix} < 0. \tag{27}$$

Defining $P_0 = P_1^{-1}$, $Q^{-1} = Q_0$, premultiplied and postmultiplied simultaneously by $\operatorname{diag}\{P_0^{-1}, Q_0^{-1}, I, I, I\}$, it can be seen by using Schur complement formula that $\Phi_0 < 0$ leads to $\Phi < 0$. When $\Phi < 0$ holds, we have

$$\dot{V}(t) < -s^T(t)\,\zeta\zeta^T s(t) = -z_2^T z_2 \le 0. \tag{28}$$

It follows that (13) is asymptotically stable in the absence of the exogenous input $\overline{d}(t)$. Next, we consider the performances to be optimized for (13). Consider two auxiliary functions as follows:

$$\begin{aligned} J_0 &= z_2^T(t)\, z_2(t) + \dot{V}(t), \\ J_1 &= z_\infty^T(t)\, z_\infty(t) - \gamma_1^2 d_2(t)^T d_2(t) - \gamma_2^2 \delta(t)^T \delta(t) \\ &\quad - \gamma_3^2 \dot{F}(t)^T \dot{F}(t) + \dot{V}(t). \end{aligned} \tag{29}$$

Following the definition of the H_2 performance, we only consider the case in the absence of $\overline{d}(t)$. It can be verified that $J_0 \le s^T(t)\Phi s(t)$.

In the presence of $\overline{d}(t)$, it can be seen that $J_1 = q^T(t)\Psi q(t)$, where

$$q^T(t) = \left[x^T(t) \;\; x^T(t-\tau) \;\; g^T(x(t)) \;\; \overline{e}^T(t) \;\; \overline{d}^T(t)\right],$$

$$\Psi = \begin{bmatrix} \Psi_{11} & P_1 A_d & P_1 G & P_1 \overline{V} & \Psi_{15} \\ * & -Q & 0 & 0 & 0 \\ * & * & -\frac{1}{\lambda} I & 0 & 0 \\ * & * & * & \Psi_{44} & \Psi_{45} \\ * & * & * & * & \Xi_5 \end{bmatrix} + \begin{bmatrix} C_{\infty 1}^T \\ C_{d\infty}^T \\ 0 \\ C_{\infty 2}^T \\ 0 \end{bmatrix} \begin{bmatrix} C_{\infty 1}^T \\ C_{d\infty}^T \\ 0 \\ C_{\infty 2}^T \\ 0 \end{bmatrix}^T, \tag{30}$$

where

$$\begin{aligned} \Psi_{11} &= \operatorname{sym}\left(P_1 A + P_1 JS\right) + \frac{1}{\lambda} U^T U + Q, \\ \Psi_{15} &= \left[P_1 J_2 \;\; 0 \;\; 0\right], \\ \Psi_{44} &= \operatorname{sym}\left(P_2 \overline{W} + P_2 \overline{L}\,\overline{V}\right), \\ \Psi_{45} &= \left[P_2 \overline{L} J_2 \;\; P_2 \overline{J}_3 \;\; P_2 \overline{J}_1\right]. \end{aligned} \tag{31}$$

Denote

$$\Psi_0 = \Psi + \left[C_{21} \;\; C_{d2} \;\; 0 \;\; C_{22} \;\; 0\right]^T \left[C_{21} \;\; C_{d2} \;\; 0 \;\; C_{22} \;\; 0\right]. \tag{32}$$

It can be seen by using Schur complement formula that (20) leads to $\Psi_0 < 0$, and then $\Psi < 0$ holds. It can be verified that both $J_0 < 0$ and $J_1 < 0$ hold. This completes the proof. □

4. Simulation Examples

In this section, we consider the longitudinal dynamics of A4D aircraft at a flight condition of 15000 ft altitude and 0.9 Mach given in [2]. The longitudinal dynamics can be denoted as

$$\begin{aligned}\dot{x}(t) = {} & Ax(t) + Gg(x(t)) + J(u(t) + F(t)) \\ & + J_1 d_1(t) + J_2 d_2(t),\end{aligned} \tag{33}$$

where $x(t)$ are measurable by using sensors technique. The coefficient matrices of aircraft model are given by

$$A = \begin{bmatrix} -0.0605 & 32.37 & 0 & 32.2 \\ -0.00014 & -1.475 & 1 & 0 \\ -0.0111 & -34.72 & -2.793 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$J = \begin{bmatrix} 0 \\ -0.1064 \\ -33.8 \\ 0 \end{bmatrix}, \qquad J_1 = \begin{bmatrix} 0 \\ -0.0532 \\ -16.9000 \\ 0 \end{bmatrix}, \qquad J_2 = \begin{bmatrix} 0.1 \\ 0 \\ -3 \\ 0.1 \end{bmatrix}. \tag{34}$$

It is supposed that

$$G = \begin{bmatrix} 0 & 0 & 50 & 0 \end{bmatrix}^T, \qquad g(x(t)) = \sin(2\pi 5t)\, x_2(t). \tag{35}$$

then, the matrix U can be selected as $U = \operatorname{diag}\{0,1,0,0\}$ and the norm condition (3) can be satisfied. $A_d = 0.5 \times A$, $\tau = 2$. Periodic disturbance $d_1(t)$ caused by rotating aerial propeller is assumed to be an unknown harmonic disturbance described by (2) with

$$W = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}, \qquad E = \begin{bmatrix} 25 & 0 \end{bmatrix}, \qquad H_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \tag{36}$$

$\delta(t)$ is the additional disturbance signal resulting from the perturbations and uncertainties in the exogenous system (2) and satisfies 2-norm boundedness. In simulation, we select $\delta(t)$ as the random signal with upper 2-norm bound 1. Wind gust and system noises $d_2(t)$ can also be considered as the random signal with upper 2-norm bounded.

The initial values of the states are supposed to be $x^T(0) = [2 \quad -2 \quad 3 \quad 2]$. For the reference output, it is denoted that

$$\begin{gathered} C_{\infty 1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \qquad C_{\infty 2} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \\ C_{d\infty} = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}, \\ C_{21} = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}, \qquad C_{22} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}, \\ C_{d2} = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}. \end{gathered} \tag{37}$$

For $\lambda = 1$, $\gamma_1 = 1$, $\gamma_2 = 1$, and $\gamma_3 = 1$, it can be solved via LMI related to (20) that the gain of fault diagnosis observer (7) is

$$K = \begin{bmatrix} 20.9299 & 10.0231 & 1.3553 & 20.9682 \end{bmatrix}, \tag{38}$$

the gain of disturbance observer (4) is

$$L = \begin{bmatrix} -0.2584 & -0.5402 & -0.0142 & -0.2610 \\ 2.3246 & 0.7523 & 0.1526 & 2.3269 \end{bmatrix}, \tag{39}$$

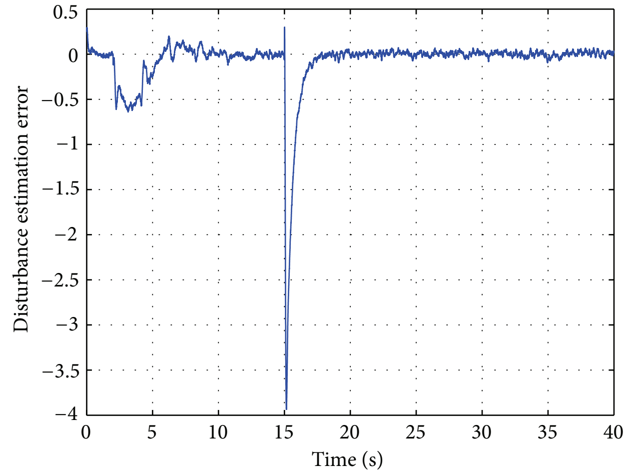

Figure 1: Disturbances estimation error in disturbance observer.

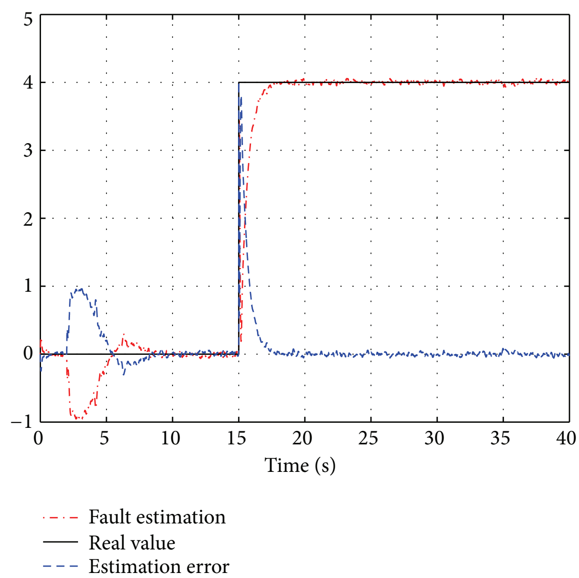

Figure 2: Bias fault, its estimation, and error.

and the gain of state feedback controller is

$$S = \begin{bmatrix} 89.6396 & 471.0670 & 74.2419 & 465.7941 \end{bmatrix}. \tag{40}$$

When the disturbance observer is constructed based on (4) and (5), the estimation error of exogenous disturbances is shown in Figure 1. The actuator bias fault is supposed to occur at 15th second as $F = 4$. The estimation and its error of fault with system disturbances are demonstrated in Figure 2, where the solid line represents the real fault signal, the dash-dotted line is its estimation, and the dash line denotes its estimation error. In Figure 3, the state response signals of the control system are illustrated. It can be seen that the proposed fault tolerant controller has a good control ability for configuring fault and rejecting and attenuating disturbances simultaneously. In Figure 4, the ramp fault is assumed to occur at 15th second with slope 0.1. From

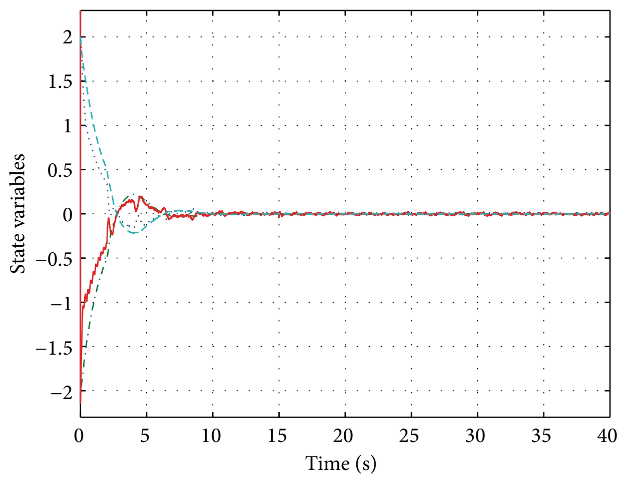

Figure 3: The responses of state variable.

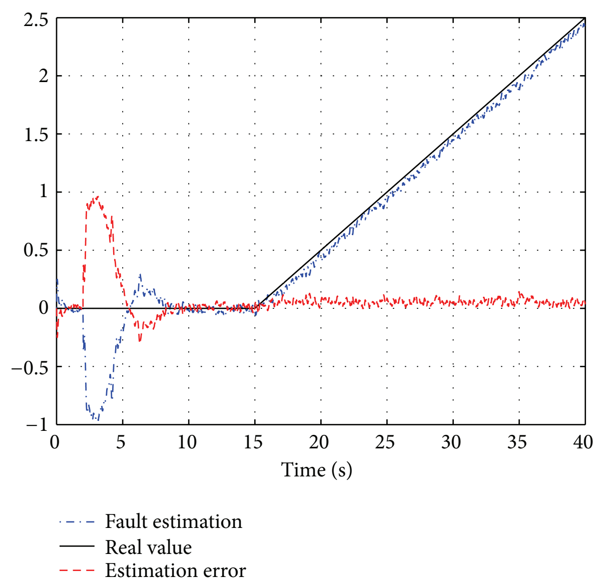

Figure 4: Ramp fault, its estimation, and error.

Figure 4, it is shown that the time-varying fault can also be well estimated.

5. Conclusion

In this paper, a robust antidisturbance fault tolerant control problem is investigated for nonlinear time delay systems with faults and multiple disturbances. There are the following features of the proposed algorithm compared with the previous results. First, the multiple disturbances and state time-delay are considered simultaneously in this paper. Second, an FTC scheme is addressed with disturbance rejection and attenuation performance by combining fault accommodation and DOBC with a mixed H_2/H_∞ controller, with which the fault can be accommodated and the disturbances can be rejected and attenuated simultaneously. Finally, simulation for a flight control system is given to show the efficiency of the proposed approach.

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