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Research Article

He's Max-Min Approach for Coupled Cubic Nonlinear Equations Arising in Packaging System

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He's inequalities and the Max-Min approach are briefly introduced, and their application to a coupled cubic nonlinear packaging system is elucidated. The approximate solution is obtained and compared with the numerical solution solved by the Runge-Kutta algorithm yielded by computer simulation. The result shows a great high accuracy of this method. The research extends the application of He's Max-Min approach for coupled nonlinear equations and provides a novel method to solve some essential problems in packaging engineering.

1. Introduction

Various kinds of nonlinear oscillation problems exist in the engineering field, which are usually difficult to be solved analytically. However, the analytical solution is significant for the further intensive study. Among the methods for analytical solution, the Perturbation method [1] is one of the most well-known approaches and is based on the existence of small or large parameters which is not commonly contained in many nonlinear problems. Besides, in order to avoid some restrictions of Perturbation Method, some other methods are developed, including the homotopy perturbation method (HPM), the variational interation method (VIM), many well-established asymptotic methods [2], a novel Max-Min method [3]. The Max-Min approach is developed from the idea of ancient Chinese math and owns the property of convenient application, less calculation and high accuracy, and so forth. Among current researches about He's Max-Min approach and its applications [4–10], few involve coupled nonlinear problems such in packaging engineering, especially the higher-dimensional coupled nonlinear problems.

In this paper, He's Max-Min approach is applied to the second order coupled cubic nonlinear packaging system to get its frequencies and periods under different situations. What's more, the obtained analytical solution is compared with the solution of computer simulation by Matlab. Consequently, the comparison shows the efficiency of this method.

2. He's Inequalities and the Max-Min Approach [3]

According to He's Max-Min approach, in order to obtain the exact solution of certain variable x, its minimum of Max values and maximum of Min values should firstly gained as follows:

$$\frac{a}{b} < x < \frac{d}{c}, \tag{1}$$

where a, b, c, and d are real numbers, and then

$$\frac{a}{b} < \frac{ma+nd}{mb+nc} < \frac{d}{c}, \tag{2}$$

and x is approximated by

$$x = \frac{ma+nd}{mb+nc} = \frac{ka+d}{kb+c}, \tag{3}$$

where m and n are weighing factors and $k = m/n$.

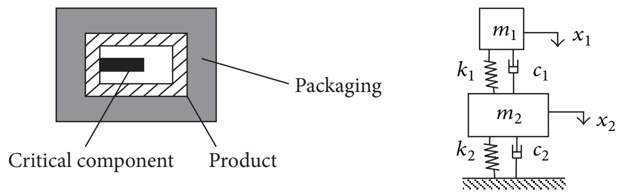

FIGURE 1: The model of a packaging system with a critical component.

The changing progress of k from zero to infinite is just that of x from d/c to a/b. Thus there must exist a certain value of k while the corresponding value of x locates at its exact solution.

However, the method to determine the value of k is varied. In this paper, the method in [3] is used to determine the value of k.

3. Modelling and Equations

Packaged products can be potentially damaged by dropping [11, 12], and it is very important to investigate the oscillation process of the packaging system. Most products, especially mechanical and electronic products, are composed of large numbers of elements, and the damage generally occurs at the so-called critical component [13]. In order to prevent any damage, a critical component and a cushioning packaging are always included in a packaging system [14], as shown in Figure 1. Here the coefficients m_1 and m_2 denote, respectively, the mass of the critical component and main part of product, while k_1 and k_2 are, respectively, the coupling stiffness of the critical component and that of cushioning pad.

The oscillation in the packaging system is of inherent nonlinearity. The governing equations of cubic nonlinear cushioning packaging system with the critical component can be expressed as [14]

$$\begin{gathered} m_1 x'' + k_1 (x - y) = 0, \\ m_2 y'' + k_2 y + r_2 y^3 - k_1 (x - y) = 0, \end{gathered} \tag{4}$$

where

$$\begin{gathered} x(0) = 0, \qquad y(0) = 0, \\ x'(0) = \sqrt{2gh}, \qquad y'(0) = \sqrt{2gh}. \end{gathered} \tag{5}$$

Here r_2 is the incremental rate of linear elastic coefficient for cushioning pad, and h is the dropping height. Equation (4) can be equivalently written in the following forms:

$$\begin{gathered} \ddot{X} + \omega_1^2 (X - Y) = 0, \\ \ddot{Y} + Y + Y^3 + \left(1 - \omega_2^2\right)(X - Y) = 0, \end{gathered} \tag{6}$$

where

$$X = \frac{x}{\sqrt{k_2/r_2}}, \tag{7}$$

$$Y = \frac{y}{\sqrt{k_2/r_2}}, \tag{8}$$

$$\tau = \frac{t}{\sqrt{m_2/k_2}}, \tag{9}$$

$$\omega_1 = \sqrt{\frac{m_2 k_1}{m_1 k_2}}, \tag{10}$$

$$\omega_2 = \sqrt{\frac{1 + m_1 \omega_1^2}{m_2}}, \tag{11}$$

$$X(0) = 0, \tag{12}$$

$$Y(0) = 0, \tag{13}$$

$$\dot{X}(0) = \frac{\sqrt{m_2 r_2}}{k_2} \sqrt{2gh}, \tag{14}$$

$$\dot{Y}(0) = \frac{\sqrt{m_2 r_2}}{k_2} \sqrt{2gh}. \tag{15}$$

4. Application of He's Max-Min Approach

From (6), we can easily obtain

$$Y^{(4)} + \left(\omega_1^2 + \omega_2^2 + 3Y^2\right)\ddot{Y} + \omega_1^2 \left(Y + Y^3\right) = 0. \tag{16}$$

Rewrite (16) in the following form:

$$Y^{(4)} = -\left[\left(\frac{\omega_1^2 + \omega_2^2}{Y} + 3Y\right)\ddot{Y} + \omega_1^2 \left(1 + Y^2\right)\right] Y. \tag{17}$$

According to the Max-Min method, we choose a trial function in the following form:

$$Y = A \sin(\Omega \tau) \tag{18}$$

which meets the initial conditions as described in (13) and (15).

By simple analysis, from (17)-(18) we know that

$$\Omega^4 = \left(\omega_1^2 + \omega_2^2\right)\Omega^2 - \omega_1^2 + \left(3\Omega^2 - \omega_1^2\right) A^2 \sin^2 \Omega \tau. \tag{19}$$

The maximal and minimal value of $\sin^2 \Omega \tau$ are, respectively, 1 and 0. So we can immediately obtain

$$\begin{gathered} f_{\min} = \left(3\Omega^2 - \omega_1^2\right) A^2 + \left(\omega_1^2 + \omega_2^2\right)\Omega^2 - \omega_1^2 \\ < \Omega^4 < \left(\omega_1^2 + \omega_2^2\right)\Omega^2 - \omega_1^2 = f_{\max}. \end{gathered} \tag{20}$$

According to He Chengtian's interpolation [6], we obtain

$$\Omega^4 = \frac{m f_{\min} + n f_{\max}}{m + n} = \left(\omega_1^2 + \omega_2^2\right)\Omega^2 - \omega_1^2 + kM, \tag{21}$$

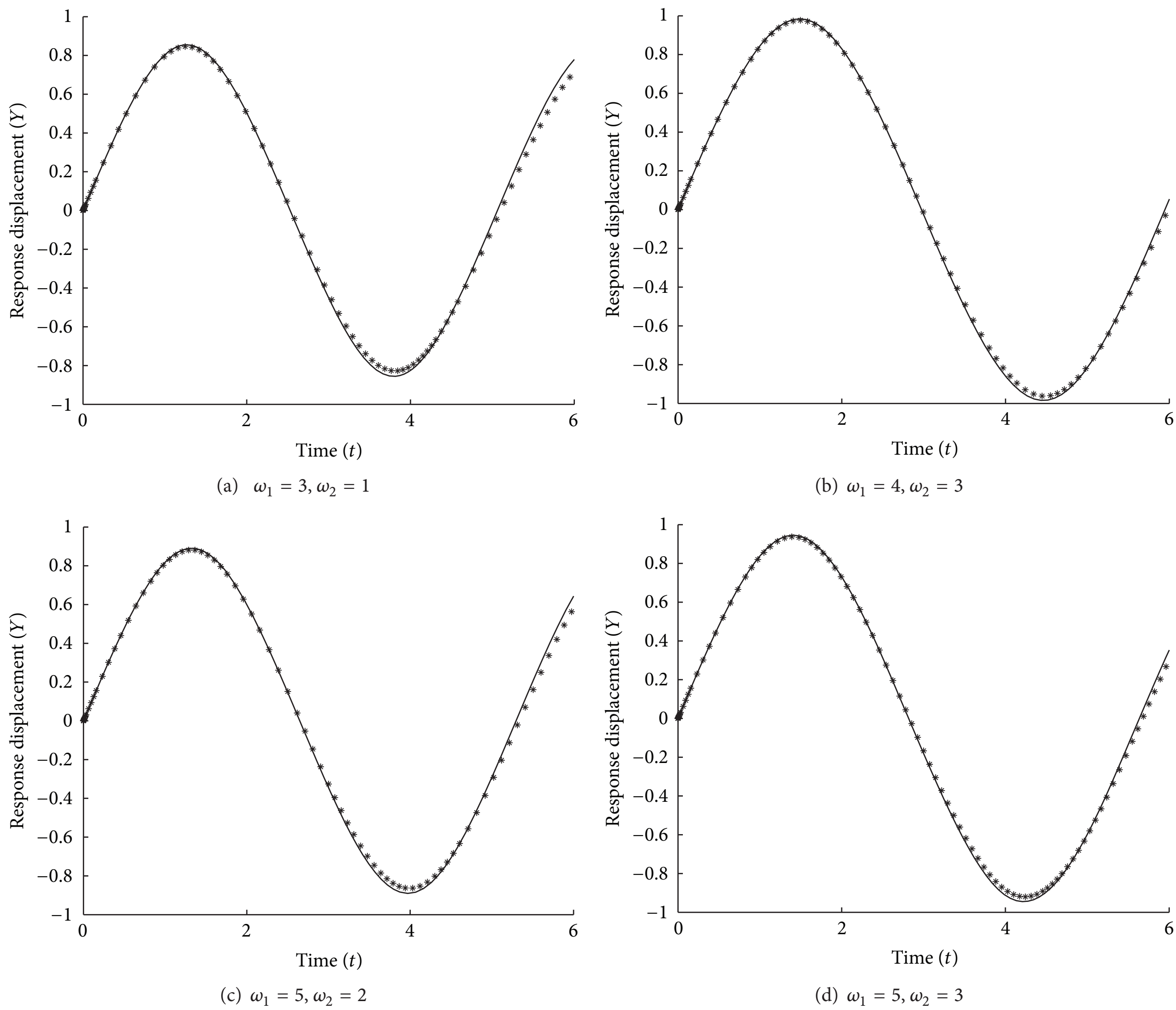

Figure 2: Comparison of the approximate solution by the Max-Min approach with the numerical simulation solution solved by the Runge-Kutta algorithm. (Asterisk: solution by the Max-Min approach; continuous line: solution by the Runge-Kutta method).

where m and n are weighting factors, $k = m/(m+n)$, $M = (3\Omega^2 - \omega_1^2)A^2$.

Then, the approximate solution of (16) can be written as

$$Y = A \sin\left[\left(\omega_1^2 + \omega_2^2\right)\Omega^2 - \omega_1^2 + kM\right]^{1/4}\tau. \qquad (22)$$

To determine the value of k, substituting (22) into (16) results in the following residual [4]:

$$R(\tau, k) = \left(3\Omega^2 - \omega_1^2\right)Y^3 - kMY. \qquad (23)$$

And by setting

$$\int_0^{T/4} R(\tau, k) \sin\Omega\tau \, d\tau = 0, \qquad (24)$$

where $T = 2\pi/\Omega$, we obtain the k value as

$$k = \frac{3}{4}. \qquad (25)$$

Substituting (25) into (21) yields

$$\Omega^4 = \left(\omega_1^2 + \omega_2^2\right)\Omega^2 - \omega_1^2 + \frac{3}{4}A^2\left(3\Omega^2 - \omega_1^2\right). \qquad (26)$$

From (26), we can easily obtain the frequency value Ω, which can be used to obtain the approximate solution of (16). Figure 2 shows the approximate solution, (22), agrees well with the numerical solution by the Runge-Kutta method for various different values of ω_1 and ω_2, where the initial velocity is assumed as $X(0) = Y(0) = 0$, and $\dot{X}(0) = \dot{Y}(0) = A\Omega = 1$, as illustrated in (14) and (15). The parameter ω_1 for typical packaging system ranges from 3 to 5, and ω_2 from 1 to 3. As shown in Figure 2, the deviation of the solution by the Max-Min approach from that by the Runge-Kutta method is very small, taking Figure 2(a), for example, the whole deviation $\sum_{i=1}^{120}(|\Delta Y_i/Y_i|) = 1.72\%$, where $\Delta Y_i/Y_i$ represents the relating error of the solution by the Max-Min approach from that by the Runge-Kutta method.

5. Conclusion

The Max-Min method, which has been widely applied to many kinds of strong nonlinear equations such as pendulum and Duffing equations, is applied to study the nonlinear response of coupled cubic nonlinear packaging system in this study for the first time. The method is a well-established method for analyzing nonlinear systems and can be easily extended to many kinds of nonlinear equation. We demonstrated the accuracy and efficiency of the method in solving the coupled equations, showing that this method can be easily used in engineering application with high accuracy without cumbersome calculation.

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