

# Research Article Performance of Networked Control Systems

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Data packet dropout is a special kind of time delay problem. In this paper, predictive controllers for networked control systems (NCSs) with dual-network are designed by model predictive control method. The contributions are as follows. (1) The predictive control problem of the dual-network is considered. (2) The predictive performance of the dual-network is evaluated. (3) Compared to the popular networked control systems, the optimal controller of the new NCSs with data packets dropout is designed, which can minimize infinite performance index at each sampling time and guarantee the closed-loop system stability. Finally, the simulation results show the feasibility and effectiveness of the controllers designed.

### 1. Introduction

With the rapid development of computer networks technology, the control system based on NCSs has become one of the hot research tasks in the current international control field. As stability analysis of NCSs subjected to packet dropping has received much attention in [1-3], various approaches for the delay issue in NCSs have been presented in [4–8], and so on. Compared to the traditional control systems, the main advantages of NCSs are lower cost, simpler installation, and higher reliability [9-11]. Because of these attractive advantages, typical application of these systems ranges from a wide field, such as automotive [12], mobile [13], and advanced aircraft [14]. However, the introduction of communication networks in the control loops makes the analysis and design of NCSs complex. For example, network-induced delays and data packet dropout problem may be inevitable during transmitting communication.

Data packet dropout is a special kind of time delay problems. Data packet dropout which is a kind of uncertainty that may happen due to node failures or network congestion is a common problem in networked systems. This loss will deteriorate the performance and may even cause the system to be unstable. Recently, the effect of data packet dropout on the stability and performance of NCSs has received great

attention. In [15], the stability of a linear networked control system in the presence of dropped packets was studied. A stability analysis of model-based NCSs can be found in [2, 16-19], where an additional model was used for estimating the plant state between transmission times and generating a control signal. In [20], though turning the model of NCSs into an asynchronous dynamic system, hybrid system technology has been used to handle the system with time delay and data dropouts. A stability condition was obtained for the Try-Once-Discard networked protocol in [3]. In [21], Hadjicostis and Touri analyzed the performance when lost data were replaced by zeros. In [22, 23], Ling and Lemmon posed the problem of optimal compensator design for the case when data loss was independent and identically distributed. Reference [24] addresses the random time delays and packet losses issues of NCSs within the framework of jump linear systems with mode-dependent time delays. Jump linear systems with Markov chains [25, 26] also were used to analyze the effect of dropouts on system stability and performance. In [27], a delay-dependent stability condition was presented for discrete-time jump time delay system where the time delay was dependent on the system mode. In [28], the problem of stability analysis and controller design has been proposed based on a new model with packet dropouts.

Model predictive control (MPC) can now be found in a wide variety of application areas including chemicals, food

processing, and so on. MPC is also an effective method to incorporate the input and output constraints into online optimization, which increases the possibility of its application in the synthesis and analysis of NCSs [29]. In [30], the MPC strategy for multivariable plants was presented. Wu et al. [31] introduced MPC into NCSs with time delay and designed an optimal control rule. In [4], the networked predictive control with modified MPC was proposed.

In this paper, the contributions are as follows. (1) The predictive control problem of the dual-network is considered. (2) The predictive performance of the dual-network is evaluated. (3) Compared to the popular networked control systems, the optimal controller of the new NCSs with data packets dropout is designed, which can minimize infinite performance index at each sampling time and guarantee the closed-loop system stability.

The remainder of this paper is organized as follows. The problem descriptions and model of NCSs with packet dropouts are given in Section 2. The optimization method of the new closed-loop systems is proposed in Section 3. The stability analysis is given in Section 4. A numerical example and an industrial example show the effectiveness of the proposed method in Section 5, and some conclusions are given in Section 6.

## 2. Problem Descriptions and Modeling of NCSs with Packet Dropouts

Consider that NCSs model which is shown in Figure 1, sensors, controllers, and actuators are connected by networks, and we suppose that the communication link between primary sensor and controller  $(S_1/C_1)$  is ideal. Based on such a structure, the problem of packet dropouts in network transmission mainly exists in the actuator and secondary sensor and controller  $(S_2/C_2 \text{ and } C_2/A)$ .

In Figure 1,  $S_i$  (i = 1, 2),  $C_i$  (i = 1, 2), and  $P_i$  (i = 1, 2) denote the sensor, controller, and plant of primary and secondary, respectively, and A is actuator.  $u_1(k)$  and  $\overline{u}_2(k)$  are outputs of primary and secondary controllers at sampling time k.  $u_2(k)$  is input of actuator at sampling time k,  $d_k^{C_2A}$  is the quantity of packet dropouts between sampling current time k and the latest communicate time successfully  $(k-d_k^{C_2A})$  on  $C_2/A$  side, and  $d_k^{S_2C_2}$  is the quantity of packet dropouts at current time k and the latest communicate successfully  $(k-d_k^{S_2C_2})$  on  $S_2/C_2$  side. The quantity of packet dropouts on  $S_2/C_2$  and  $C_2/A$  sides is assumed to satisfy

$$d_m \le d_k^{S_2 C_2} \le d_M, \qquad \tau_m \le d_k^{C_2 A} \le \tau_M, \tag{1}$$

where  $d_m$ ,  $d_M$ ,  $\tau_m$ , and  $\tau_M$  are constant positive scalars representing the minimum and maximum quantities of packet dropouts on  $S_2/C_2$  and  $C_2/A$  sides, respectively, where  $d_k^{S_2C_2}$  and  $d_k^{C_2A}$  are two independent Markov chains. Without loss of generality, define

$$0 \le d_k^{S_2 C_2} \le d_M, \qquad 0 \le d_k^{C_2 A} \le \tau_M. \tag{2}$$

Suppose that under the condition  $d_k^{S_2C_2} = i$ , the probability that the state of packet dropouts at time k + 1 is  $j (d_{k+1}^{S_2C_2} = j)$  is

$$\Pr\left\{d_{k+1}^{S_2C_2} = j \mid d_k^{S_2C_2} = i\right\} = \pi_{ij} \quad \forall i, j \in \chi_1,$$
(3)

where  $\chi_1 = \{0, 1, 2, ..., d_M\}$ . That is also to say that the transition probability of  $d_k^{S_2C_2}$  jumping from mode *i* to *j* is (3), and the transition probability of  $d_k^{C_2A}$  jumping from mode *r* to *s* is

$$\Pr\left\{d_{k+1}^{C_2A} = s \mid d_k^{C_2A} = r\right\} = \lambda_{rs} \quad \forall r, s \in \boldsymbol{\chi}_2, \tag{4}$$

where  $\chi_2 = \{0, 1, 2, \dots, \tau_M\}, \ \pi_{ij} \ge 0, \ \sum_{j=0}^{d_M} \pi_{ij} = 1, \ \sum_{j=0, j \neq i}^{d_M} \pi_{ij} = 1 - \pi_{ii}, \ \text{and} \ \lambda_{rs} \ge 0, \ \sum_{s=0}^{\tau_M} \lambda_{rs} = 1.$ 

For analysis convenience and without loss of generality, we suppose that Markov chains jump no more than one step. Thus, Markov chain transferring probability matrix is satisfied [28]. Consider

$$\pi_{ij} = 0, \quad \text{if } j \neq i+1, \ j \neq 0,$$

$$\lambda_{rs} = 0, \quad \text{if } s \neq r+1, \ s \neq 0.$$
(5)

And the two equations are used to produce the definition of data packet dropout. *i* denotes the quantity of the packet dropouts at time k - 1. *j* denotes the quantity of packet dropouts at time *k*. The system's conditional probability of data packets dropout is equal to zero when the difference of the quantity of packet dropouts between time *k* and k - 1 is not equal to 1 or there is no dropout at time *k*.

Some assumptions of NCSs are introduced as follows.

- (1) The sensors, controllers, and actuators are clockdriven.
- (2) The buffers are big enough to hold all the data arrived, and rule of the buffer is last-in-first-out.
- (3) Transmission link of primary control loop is ideal and data packet dropouts only happen in secondary control loop.
- (4) The transition of NCSs is single-packet transmission and no timing sequence disordered.

The linear time-invariant discrete-time models of primary and secondary plants in Figure 1 are described as follows:

$$P_{1}: \begin{cases} x_{1}(k+1) = \Phi_{1}x_{1}(k) + \Gamma_{1}y_{2}(k), \\ y_{1}(k) = C_{1}x_{1}(k), \end{cases}$$

$$P_{2}: \begin{cases} x_{2}(k+1) = \Phi_{2}x_{2}(k) + \Gamma_{2}u_{2}(k), \\ y_{2}(k) = C_{2}x_{2}(k), \end{cases}$$
(6)

where  $x_1(k)$  and  $x_2(k)$  are the states of primary and secondary plants, respectively, at sampling time k,  $u_2(k)$  is the input of secondary plant  $P_2$  at sampling time k, and  $\Phi_1$ ,  $\Gamma_1$ ,  $C_1$ ,  $\Phi_2$ ,  $\Gamma_2$ ,  $C_2$  are known constant matrixes with appropriate dimensions.



FIGURE 1: Networked control system model.

Due to the existence of data packet dropouts, transmission link in network cannot normally communicate. The controller and actuator can use the buffer rule, named firstin-last-out, to pick up signal [28]. Take the controller, for example; when a secondary sensor data  $x_2(k)$  is false to transmit, the secondary controller gets the latest data  $\overline{x}_2(k-1)$ from buffer and uses it as  $\overline{x}_2(k)$  to calculate new control input. Otherwise, the new sensor data  $x_2(k)$  will be saved to buffer and used by the secondary controller as  $\overline{x}_2(k)$ . Thus,

$$\overline{x}_{2}(k) = \begin{cases} x_{2}(k), & d_{k}^{S_{2}C_{2}} = 0, \\ \\ \overline{x}_{2}(k-1), & d_{k}^{S_{2}C_{2}} > 0. \end{cases}$$
(7)

Similarly

$$u_{2}(k) = \begin{cases} \overline{u}_{2}(k), & d_{k}^{C_{2}A} = 0, \\ u_{2}(k-1), & d_{k}^{C_{2}A} > 0. \end{cases}$$
(8)

It can easily derive

$$\overline{x}_{2}\left(k\right) = x_{2}\left(k - d_{k}^{S_{2}C_{2}}\right).$$
(9)

From the model of NCSs, as shown in Figure 1,  $u_1(k)$  is the output of primary controller at sampling time *k* as

$$u_1(k) = F_1 x_1(k), (10)$$

where  $F_1$  is to be designed by MPC method.

 $\overline{u}_2(k)$  is the output of secondary controller as

$$\overline{u}_{2}(k) = u_{1}(k) + F_{2}\left(d_{k}^{S_{2}C_{2}}\right)\overline{x}_{2}(k), \qquad (11)$$

where  $F_2(d_k^{S_2C_2})$  is to be designed by MPC method. Substituting (9) and (10) into (11), we have

$$\overline{u}_{2}(k) = F_{1}x_{1}(k) + F_{2}\left(d_{k}^{S_{2}C_{2}}\right)x_{2}\left(k - d_{k}^{S_{2}C_{2}}\right).$$
(12)

Therefore, (8) can be rewritten as

$$u_{2}(k) = \begin{cases} F_{1}x_{1}(k) + F_{2}\left(d_{k}^{S_{2}C_{2}}\right)x_{2}\left(k - d_{k}^{S_{2}C_{2}}\right), & d_{k}^{C_{2}A} = 0, \\ \\ u_{2}(k-1), & d_{k}^{C_{2}A} > 0. \end{cases}$$
(13)

In order to simplify the expression of the closed-loop control systems, a function  $\alpha = \begin{cases} 0, d_k^{C_2A} = 0\\ 1, d_k^{C_2A} > 0 \end{cases}$  is introduced, which is dependent on whether packet dropped or not, instead of the quantity of packet dropouts. Combining (6) and (13), it can obtain that

$$u_{2}(k) = \alpha u_{2}(k-1) + [1-\alpha] F_{1}x_{1}(k) + [1-\alpha] F_{2}(d_{k}^{S_{2}C_{2}}) x_{2}(k-d_{k}^{S_{2}C_{2}}),$$
(14)

$$x_{1}(k+1) = \Phi_{1}x_{1}(k) + \Gamma_{1}C_{2}x_{2}(k), \qquad (15)$$

$$x_{2} (k + 1) = \Phi_{2} x_{2} (k) + \Gamma_{2} \alpha u_{2} (k - 1) + \Gamma_{2} [1 - \alpha] F_{1} x_{1} (k) + \Gamma_{2} [1 - \alpha]$$
(16)  
$$\times F_{2} (d_{k}^{S_{2}C_{2}}) x_{2} (k - d_{k}^{S_{2}C_{2}}).$$

Combining (10), (14), (15), and (16) and augmenting the state vectors, the new resulting closed-loop control systems and the augmenting vector (18) formed by predictive controller (10) and (14) designed by MPC method are as follows:

$$\begin{aligned} x(k+1) &= A\left(d_{k}^{C_{2}A}\right) x(k) + B\left(d_{k}^{S_{2}C_{2}}, d_{k}^{C_{2}A}\right) x\left(k - d_{k}^{S_{2}C_{2}}\right), \end{aligned}$$
(17)  
$$u(k) &= \begin{pmatrix} u_{1}(k) \\ u_{2}(k) \end{pmatrix} \\ &= C\left(d_{k}^{C_{2}A}\right) x(k) + D\left(d_{k}^{S_{2}C_{2}}, d_{k}^{C_{2}A}\right) x\left(k - d_{k}^{S_{2}C_{2}}\right), \end{aligned}$$
(18)

where 
$$x(k) = \left(x_{1}^{T}(k) \ x_{2}^{T}(k) \ u_{2}^{T}(k-1)\right)^{T}$$
,  
 $x\left(k - d_{k}^{S_{2}C_{2}}\right)$   
 $= \left(x_{1}^{T}\left(k - d_{k}^{S_{2}C_{2}}\right) \ x_{2}^{T}\left(k - d_{k}^{S_{2}C_{2}}\right) \ u_{2}^{T}\left(k - d_{k}^{S_{2}C_{2}} - 1\right)\right)^{T}$ ,  
 $A\left(d_{k}^{C_{2}A}\right) = \left(\begin{array}{ccc} \Phi_{1} & \Gamma_{1}C_{2} & 0\\ \Gamma_{2} \left[1 - \alpha\right]F_{1} & \Phi_{2} & \Gamma_{2}\alpha\\ \left[1 - \alpha\right]F_{1} & 0 & \alpha\end{array}\right)$ ,  
 $B\left(d_{k}^{S_{2}C_{2}}, d_{k}^{C_{2}A}\right) = \left(\begin{array}{ccc} 0 & 0 & 0\\ 0 & \Gamma_{2} \left[1 - \alpha\right]F_{2} & \left(d_{k}^{S_{2}C_{2}}\right) & 0\\ 0 & \left[1 - \alpha\right]F_{2} & \left(d_{k}^{S_{2}C_{2}}\right) & 0\end{array}\right)$ ,  
 $C\left(d_{k}^{C_{2}A}\right) = \left(\begin{array}{ccc} F_{1} & 0 & 0\\ \left[1 - \alpha\right]F_{1} & 0 & \alpha\end{array}\right)$ ,  
 $D\left(d_{k}^{S_{2}C_{2}}, d_{k}^{C_{2}A}\right) = \left(\begin{array}{ccc} 0 & 0 & 0\\ 0 & \left[1 - \alpha\right]F_{2} & \left(d_{k}^{S_{2}C_{2}}\right) & 0\end{array}\right)$ .  
(19)

*Remark 1.* The new closed-loop control systems (17) and the augmenting vector (18) formed by predictive controller (10) and (14) are linear jumping systems, where their communications are described by Markov chain which is the description of the quantity of packet dropouts at sampling time k on  $C_2/A$  and  $S_2/C_2$  sides. The value of  $\alpha$  depends on whether the designed control signal is successfully transmitted or not. Therefore we can use the results of linear jumping system with delay to analyze this class of NCSs with packets dropped.

#### 3. Optimum Analysis Based on MPC

Assume that predictive horizon  $p = \infty$ , control horizon  $q = \infty$ , and the *m* steps control sequences  $u(k + m \mid k)$ ,  $m = 0, 1, 2, ..., \infty$ , are computed by minimizing the following performance function:

$$J_{\infty} = \sum_{m=0}^{\infty} \left[ \left\| x \left( k + m \mid k \right) \right\|_{Q}^{2} + \left\| u \left( k + m \mid k \right) \right\|_{R}^{2} \right].$$
(20)

The norm terms in the performance function are defined as

$$\|x\|_{O}^{2} = x^{\mathrm{T}}Qx.$$
 (21)

The exact measurement state of NCSs at each sampling time k is

$$x(k \mid k) = x(k).$$
 (22)

By using the control input given in (18), the first control move is

$$u(k) = u(k | k)$$
  
=  $C(d_{k|k}^{C_2A}) x(k | k)$   
+  $D(d_{k|k}^{S_2C_2}, d_{k|k}^{C_2A}) x(k - d_{k|k}^{S_2C_2} | k).$  (23)

According to (17) and (22), the predicted state at time k + m is obtained as (24) which is predicted based on the exact measurement state x(k | k). Consider

$$x (k + m + 1 | k)$$
  
=  $A \left( d_{k+m|k}^{C_2 A} \right) x (k + m | k) + B \left( d_{k+m|k}^{S_2 C_2}, d_{k+m|k}^{C_2 A} \right)$  (24)  
 $\times x \left( k + m - d_{k+m|k}^{S_2 C_2} | k \right),$ 

where  $d_{k+m|k}^{S_2C_2}$  is model-dependent time invariable delay on  $S_2/C_2$  side, which is *m*-step ahead prediction based on the measurement time *k*.  $d_{k+m|k}^{C_2A}$  is the predicted quantity of packet dropouts at time k + m.

And the key of predictive controller rule is to compute the matrixes  $F_1$  and  $F_2(d_k^{S_2C_2})$  in (14). The predicted controller at time k + m based on the first control move  $u(k \mid k)$  is

$$u(k+m \mid k) = C\left(d_{k+m|k}^{C_2A}\right) x(k+m \mid k) + D\left(d_{k+m|k}^{S_2C_2}, d_{k+m|k}^{C_2A}\right) x\left(k+m - d_{k+m|k}^{S_2C_2} \mid k\right),$$
(25)

where  $C(d_k^{C_2A})$  and  $D(d_k^{S_2C_2}, d_k^{C_2A})$  are matrixes which contain the control gain matrixes  $F_1$  and  $F_2(d_k^{S_2C_2})$ , respectively.

Next we will introduce a method to solve the optimal control sequence which can minimize the following performance index at each sampling time:

$$\min_{u(k+m|k),m=0,1,2,...,\infty} \int_{\infty}^{\infty} = \sum_{m=0}^{\infty} \left[ x^{\mathrm{T}} \left( k+m \mid k \right) Qx \left( k+m \mid k \right) + u^{\mathrm{T}} \left( k+m \mid k \right) Ru \left( k+m \mid k \right) \right].$$
(26)

Q and R are symmetric positive definite weight matrixes.

In order to simplify, let  $d_{k+m|k}^{S_2C_2} = v$ ,  $d_{k+m|k}^{C_2A} = \varsigma$ ,  $d_{k+m+1|k}^{S_2C_2} = v_1$ .

First, consider a Lyapunov function candidate  $V(X(k + m \mid k), v)$  with  $X(k + m \mid k) = [x^{T}(k + m \mid k), x^{T}(k + m - 1 \mid k), \dots, x^{T}(k + m - v \mid k)]^{T}$  as follows:

$$V(X(k+m \mid k), v) = V_1 + V_2 + V_3,$$
(27)

where

$$V_{1}(X(k+m \mid k), v) = x^{T}(k+m \mid k) P(v) x(k+m \mid k),$$

$$V_{2} (X (k + m | k), v) = \sum_{l=k+m-v}^{N m - 1} x^{T} (l | k) Sx (l | k),$$
  

$$V_{3} (X (k + m | k), v)$$
  

$$= (1 - \pi_{m}) \sum_{\theta = -d_{M}+1}^{0} \sum_{l=k+m+\theta}^{k+m-1} x^{T} (l | k) Sx (l | k).$$
(28)

Matrixes P(v) and S are positive definite with appropriate dimensions.

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Second, suppose that at sampling time k, V(X(k + m | k), v) satisfies (29) for any x(k + m | k) and u(k + m | k). Consider

$$V(X(k+m+1 | k), v_1) - V(X(k+m | k), v)$$
  

$$\leq -[x^{T}(k+m | k)Qx(k+m | k) + u^{T}(k+m | k)Ru(k+m | k)].$$
(29)

For the infinite performance index  $J_{\infty}(k)$  to be finite, we must set  $x(\infty \mid k) = 0$ ; hence,  $V(x(\infty \mid k), d_{\infty \mid k}^{S_2 C_2}) = 0$ . Summing (29) from m = 0 to  $m = \infty$ , it can be obtained that

$$J_{\infty}(k) \le V\left(X(k \mid k), d_{k|k}^{S_2 C_2}\right).$$
(30)

Therefore, the infinite optimization problem at time k has transformed into optimizing upper function  $V(X(k | k), d_{k|k}^{S_2C_2})$  at each sampling time k. As a standard in MPC, only the first control move u(k|k) is implemented, and at the next sampling time k + 1, the state x(k + 1) is measured and the optimization is repeated to compute matrixes  $F_1$  and  $F_2(d_k^{S_2C_2})$ .

Third, a sufficient condition for existing of a set of control sequence is

$$J_{\infty} < \Upsilon,$$
 (31)

where Y is a suitable nonnegative coefficient to be minimized. Before proceeding, the following lemma needs to be

introduced.

**Lemma 2** (Schur Complement Lemma). Given constant matrixes  $Z_1$ ,  $Z_2$ ,  $Z_3$ , where  $Z_1 = Z_1^T$ ,  $Z_2 = Z_2^T$ , then  $Z_1 + Z_3^T Z_2^{-1} Z_3 < 0$  holds if  $\begin{pmatrix} Z_1 & Z_3^T \\ Z_3 & -Z_2 \end{pmatrix} < 0$  or  $\begin{pmatrix} -Z_2 & Z_3 \\ Z_3^T & Z_1 \end{pmatrix} < 0$ .

Assumption 3. The values of  $d_k^{S_2C_2}$  and  $\alpha$  are known to the controller.

**Theorem 4.** Suppose the states  $x(k | k), x(k-1 | k), ..., x(k-d_M | k)$  of system (17) are measured; then, the control rule in (18) makeing (29) and  $J_{\infty} < \Upsilon$  hold, if existing matrixes P(v) > 0, S > 0,  $C(\varsigma), X(v) > 0$ ,  $D(v, \varsigma)$ , F, and scalar  $\Upsilon > 0$  make the following optimization feasible:

$$\min \Upsilon$$
 (32)

subject to

$$x_{k+m|k}^{\mathrm{T}}P(v) x_{k+m|k} + \sum_{l=k+m-d_{k}^{SC}}^{\kappa+m-1} x_{l|k}^{\mathrm{T}}Sx_{l|k}$$
(33)

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$$+ (1 - \pi_m) \sum_{\theta = -d_M + 1}^{0} \sum_{l=k+m+\theta}^{k+m-1} x_{l|k}^{\mathrm{T}} S x_{l|k} \leq \Upsilon,$$

$$\begin{pmatrix} -P(v) + (1 + \mu)S + Q & 0 & A^{T}(\varsigma) & C^{T}(\varsigma) \\ 0 & -S & B^{T}(v,\varsigma) & D^{T}(v,\varsigma) \\ A(\varsigma) & B(v,\varsigma) & -\hat{P}^{-1}(v) & 0 \\ C(\varsigma) & D(v,\varsigma) & 0 & -R^{-1} \end{pmatrix} < 0,$$
(34)

or

$$\begin{pmatrix} -P(v) + (1+\mu)S + Q & * & * & * \\ 0 & -S & * & * \\ \widehat{A}(\varsigma) & \widehat{B}(\varsigma) & -\Lambda & * \\ \widetilde{I}(\varsigma) + \widetilde{J}(\varsigma)F\widetilde{H}_{1} & \widetilde{K}(\varsigma)F\widetilde{H} & 0 & -R^{-1} \end{pmatrix} < 0,$$
$$\begin{pmatrix} P & I \\ I & X \end{pmatrix} \ge 0, \qquad PX = I,$$
(35)

where  $\pi_m = \min\{\pi_{ii}, i \in \chi_1\}, \hat{P}(v) = \sum_{j=d_m, j \neq i}^{d_M} \pi_{ij}P(j), i$ denotes the quantity of packet dropouts,  $i \in \chi_1, \chi_1 = \{0, 1, 2, ..., d_M\}, \mu = d_M(1 - \pi_m), \hat{A}(\varsigma) = [\sqrt{\pi_{i0}}A(\varsigma), \sqrt{\pi_{i1}}A(\varsigma), ..., \sqrt{\pi_{id_M}}A(\varsigma)]^{\mathrm{T}}, \hat{B}(\varsigma) = [\sqrt{\pi_{i0}}B(\varsigma), \sqrt{\pi_{i1}}B(\varsigma), ..., \sqrt{\pi_{id_M}}B(\varsigma)]^{\mathrm{T}}, \Lambda = \operatorname{diag}(X_0, X_1, ..., X_i), \quad i \in (0, 1, 2, ..., d_M), X(v) = P^{-1}(v), \Theta = \operatorname{diag}[-[1 + (1 - \pi_m)(d_M - 1)S]^{-1}, -[1 + (1 - \pi_m)(d_M - 2)S]^{-1}, ..., -[(1 - \pi_m)S]^{-1}], x_p = [x(k - 1 \mid k), x(k - 2 \mid k), ..., x(k - d_M + 1 \mid k)]^{\mathrm{T}}, F = [F_1, F_2].$ 

In order to express conveniently, let  $x_{k+m|k} = x(k+m \mid k)$ ,  $x_{k+m-v|k} = x(k+m-v \mid k)$ ,  $u_{k+m|k} = u(k+m \mid k)$ ,  $x_{l|k} = x(l \mid k)$ . Thus, (24) becomes  $x_{k+m+1|k} = A(\varsigma)x_{k+m|k} + B(v,\varsigma)x_{k+m-v|k}$ .

The proof of the previous theorem is divided into the following two steps: Step 1, we design a Lyapunov function V(X(k + m | k), v) for the systems; Step 2, an optimal controller is designed such that the closed-loop systems are stable and the optimal characteristics are satisfied.

#### Proof

 $V_3$ 

 $\Delta V_1$ 

Step 1. Consider a Lyapunov function candidate  $V(X(k + m | k), v) = V_1 + V_2 + V_3$  with  $\mathbf{x} \neq 0$ , where

$$V_{1} (X (k + m | k), v) = x_{k+m|k}^{T} P(v) x_{k+m|k},$$
(36)  

$$V_{2} (X (k + m | k), v) = \sum_{l=k+m-v}^{T} x_{l|k}^{T} S x_{l|k},$$
(36)  

$$(X (k + m | k), v) = (1 - \pi_{m}) \sum_{\theta=-d_{M}+1}^{0} \sum_{l=k+m+\theta}^{k+m-1} x_{l|k}^{T} S x_{l|k},$$

$$= E [V_{1} (X_{k+m+1|k}, v)] - V_{1} (X_{k+m|k}, v)$$

$$= E [x_{k+m+1|k}^{T} P(v) x_{k+m+1|k}] - x_{k+m|k}^{T} P(v) x_{k+m|k}$$

$$= x_{k+m+1|k}^{T} \widehat{P}(v) x_{k+m+1|k} - x_{k+m|k}^{T} P(v) x_{k+m|k}$$

$$= \eta_{k+m|k}^{T} \left( \begin{array}{c} A^{T} (\varsigma) \widehat{P}(v) \\ * & B^{T} (v, \varsigma) \widehat{P}(v) B(v, \varsigma) \end{array} \right) \eta_{k+m|k},$$

(37)

where  $\eta_{k+m|k} = (x_{k+m|k} \ x_{k+m-v})^{T}$ , and E is the mathematical expectation. Consider

$$\Delta V_{2} = \mathbb{E} \left[ V_{2} \left( X_{k+m+1|k}, v_{1} \right) \right] - V_{2} \left( X_{k+m|k}, v \right)$$

$$= \mathbb{E} \left[ \sum_{l=k+m+1-d_{k}^{SC}}^{k+m} x_{l|k}^{T} S x_{l|k} \right] - \sum_{l=k+m-d_{k}^{SC}}^{k+m-1} x_{l|k}^{T} S x_{l|k}$$

$$= x_{k+m|k}^{T} S x_{k+m|k} - x_{k+m-d_{k}^{SC}|k}^{T} S x_{k+m-d_{k}^{SC}|k}$$

$$+ \sum_{j=0,i\neq j}^{d_{M}} \pi_{ij} \left( \sum_{l=k+m+1-d_{k+1}^{SC}}^{k+m-1} - \sum_{l=k+m-d_{k}^{SC}}^{k+m-1} \right) x_{l|k}^{T} S x_{l|k},$$
(38)

and  $\sum_{j=0, j \neq i}^{d_M} \pi_{ij} = 1 - \pi_{ii} \le 1 - \pi_m, 0 \le v \le d_M$ ; thus,

$$\Delta V_{2} = \mathbb{E} \left[ V_{2} \left( X_{k+m+1|k}, v_{1} \right) \right] - V_{2} \left( X_{k+m|k}, v \right)$$

$$\leq x_{k+m|k}^{\mathrm{T}} S x_{k+m|k} - x_{k+m-v|k}^{\mathrm{T}} S x_{k+m-v|k}$$

$$+ \left( 1 - \pi_{m} \right) \sum_{l=k+m-d_{M}+1}^{k+m} x_{l|k}^{\mathrm{T}} S x_{l|k}.$$
(39)

Similarly,

$$\Delta V_{3} = \mathbb{E} \left[ V_{3} \left( X_{k+m+1|k}, v_{1} \right) \right] - V_{3} \left( X_{k+m|k}, v \right)$$

$$\leq d_{M} \left( 1 - \pi_{m} \right) x_{l|k}^{\mathrm{T}} S x_{l|k} - \left( 1 - \pi_{m} \right) \sum_{l=k+m-d_{M}+1}^{k+m} x_{l|k}^{\mathrm{T}} S x_{l|k}.$$
(40)

Therefore,  $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$  and

$$\leq \eta_{k+m|k}^{\mathrm{T}} \begin{pmatrix} A^{\mathrm{T}}(\varsigma) \, \hat{P}(\upsilon) \, A(\varsigma) & A^{\mathrm{T}}(\varsigma) \, \hat{P}(\upsilon) \, B(\upsilon,\varsigma) \\ -P(\upsilon) + (1+\mu) \, S & & \\ * & B^{\mathrm{T}}(\upsilon,\varsigma) \, \hat{P}(\upsilon) \, B(\upsilon,\varsigma) - S \end{pmatrix} \eta_{k+m|k},$$
(A1)

where \* denotes the block determined by symmetry. Substituting (25) and (A1) into (29), we have

$$V\left(X_{k+m+1|k}, v_{1}\right) - V\left(X_{k+m|k}, v\right) + x_{k+m|k}^{\mathrm{T}}Qx_{k+m|k} + u_{k+m|k}^{\mathrm{T}}Ru_{k+m|k} \leq \eta_{k+m|k}^{\mathrm{T}}\Xi\eta_{k+m|k},$$

$$\Xi = \begin{pmatrix} \Phi & A^{\mathrm{T}}\left(\varsigma\right)\widehat{P}\left(v\right)B\left(v,\varsigma\right) + C^{\mathrm{T}}\left(\varsigma\right)RD\left(v,\varsigma\right) \\ * & B^{\mathrm{T}}\left(v,\varsigma\right)\widehat{P}\left(v\right)B\left(v,\varsigma\right) - S + D^{\mathrm{T}}\left(v,\varsigma\right)RD\left(v,\varsigma\right) \end{pmatrix},$$
(41)

where  $\Phi = A^{\mathrm{T}}(\varsigma)\widehat{P}(\upsilon)A(\varsigma) - P(\upsilon) + (1+\mu)S + Q + C^{\mathrm{T}}(\varsigma)RC(\varsigma)$ . According to Lemma 2, we obtain

$$\begin{pmatrix} -P(v) + (1+\mu)S + Q & 0 & A^{T}(\varsigma) & C^{T}(\varsigma) \\ 0 & -S & B^{T}(v,\varsigma) & D^{T}(v,\varsigma) \\ A(\varsigma) & B(v,\varsigma) & -\hat{P}^{-1}(v) & 0 \\ C(\varsigma) & D(v,\varsigma) & 0 & -R^{-1} \end{pmatrix}.$$
(42)

According to (34), then

$$\eta_{k+m|k}^{\mathrm{T}} \Xi \eta_{k+m|k} < 0. \tag{43}$$

Hence, (29) holds.

And

$$x_{k+m|k}^{\mathrm{T}} P(v) x_{k+m|k} + \sum_{l=k+m-v}^{k+m-1} x_{l|k}^{\mathrm{T}} S x_{l|k} + (1-\pi_m) \sum_{\theta=-d_M+1}^{0} \sum_{l=k+m+\theta}^{k+m-1} x_{l|k}^{\mathrm{T}} S x_{l|k} \leq \Upsilon.$$
(44)

That is,

$$V\left(X_{k+m|k}, \nu\right) \le \Upsilon. \tag{45}$$

From (30), we have

$$J_{\infty}(k) < \Upsilon. \tag{46}$$

Step 2. A congruence transformation to (17) and (18) leads to

$$A(\varsigma) = \widetilde{D}(\varsigma) + \widetilde{E}(\varsigma) F\widetilde{H}_{1}, \qquad B(v,\varsigma) = \widetilde{E}(\varsigma) F\widetilde{H},$$
  

$$C(\varsigma) = \widetilde{I}(\varsigma) + \widetilde{J}(\varsigma) F\widetilde{H}_{1}, \qquad D(v,\varsigma) = \widetilde{K}(\varsigma) F\widetilde{H},$$
(A2)

where

$$\widetilde{D}(\varsigma) = \begin{pmatrix} \Phi_1 & \Gamma_1 C_2 & 0\\ 0 & \Phi_2 & \Gamma_2 \alpha\\ 0 & 0 & \alpha \end{pmatrix},$$

$$\widetilde{E}(\varsigma) = \begin{pmatrix} 0\\ \Gamma_2 & [1-\alpha]\\ [1-\alpha] \end{pmatrix}, \quad F = \begin{bmatrix} F_1, F_2 \end{bmatrix},$$

$$\widetilde{H} = \begin{pmatrix} 0 & 0 & 0\\ 0 & I & 0 \end{pmatrix}, \quad \widetilde{I}(\varsigma) = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & \alpha \end{pmatrix},$$

$$\widetilde{J}(\varsigma) = \begin{pmatrix} I\\ [1-\alpha] \end{pmatrix}, \quad \widetilde{H}_1 = \begin{pmatrix} I & 0 & 0\\ 0 & 0 & 0 \end{pmatrix},$$

$$\widetilde{K}(\varsigma) = \begin{pmatrix} 0\\ [1-\alpha] \end{pmatrix}.$$
(47)

Substituting (A2) into (34), (35) can be obtained. The proof of Theorem 4 is completed.

*Remark 5.* According to the proposed method, the optimal input is unique. Y is a scalar and Y > 0. Unique resolution is given finally by using Toolbox of Matlab. Its convergence can be found in Figure 3.

# 4. Stability Analysis of NCSs with Packet Dropouts

**Theorem 6.** Suppose that the optimization problem in Theorem 4 is feasible at time k; then, the new resulting closed-loop systems in (17) are asymptotically stable by the optimal control law in (18) which is obtained from Theorem 4.



FIGURE 2: The number of data packet dropouts of *S* to *C* and  $C_2$  to *A*.



FIGURE 3: The upper of performance index.

*Proof.* First, the Lyapunov function,  $V(X(k + m | k), v) = V_1 + V_2 + V_3$  is discussed and  $V(X(k + m | k), d_{k+1|k}^{S_2C_2}) - V(X(k | k), d_{k|k}^{S_2C_2}) < 0$  holds only when  $x \neq 0$ .

Next, in the following we will consider the situation at time k (i.e., m = 0):

$$V\left(X\left(k \mid k\right), d_{k|k}^{SC}\right) = x_{k|k}^{T} P_{k} x_{k|k} + \sum_{l=k-d_{k|k}^{SC}}^{k-1} x_{l|k}^{T} S_{k} x_{l|k} + \left(1 - \pi_{m}\right) \sum_{\theta = -d_{M}+1}^{0} \sum_{l=k+\theta}^{k-1} x_{l|k}^{T} S_{k} x_{l|k},$$
(48)

where  $P_k > 0$ ,  $S_k > 0$  are obtained from the optimal solution at time k. Suppose that the optimization problem in Theorem 4 is feasible at time k, and according to the introduction in [8], these optimization problems are also

feasible for all k + 1. Let us note values of  $P_k > 0$ ,  $S_k > 0$ ,  $F_{1k}$ ,  $F_{2k}$ ,  $Y_k$ , and  $P_{k+1} > 0$ ,  $S_{k+1} > 0$ ,  $F_{1k+1}$ ,  $F_{2k+1}$  obtained from the optimal solution at time k and k + 1, respectively. Therefore, we have

$$\begin{aligned} x_{k+1|k+1}^{\mathrm{T}} P_{k+1} x_{k+1|k+1} + \sum_{l=k+1-d_{k+1|k+1}}^{k} x_{l|k+1}^{\mathrm{T}} S_{k+1} x_{l|k+1} \\ &+ (1 - \pi_m) \sum_{\theta = -d_M + 1}^{0} \sum_{l=k+1+\theta}^{k} x_{l|k+1}^{\mathrm{T}} S_{k+1} x_{l|k+1} \\ &\leq x_{k+1|k+1}^{\mathrm{T}} P_k x_{k+1|k+1} + \sum_{l=k+1-d_{k|k}}^{k} x_{l|k+1}^{\mathrm{T}} S_k x_{l|k+1} \\ &+ (1 - \pi_m) \sum_{\theta = -d_M + 1}^{0} \sum_{l=k+1+\theta}^{k} x_{l|k+1}^{\mathrm{T}} S_k x_{l|k+1}. \end{aligned}$$
(49)

That is because  $P_{k+1} > 0$ ,  $S_{k+1} > 0$  are optimal, whereas  $P_k > 0$ ,  $S_k > 0$  are only feasible at time k.

From (29), it can be obtained that

$$V\left(X\left(k+1 \mid k\right), d_{k+1|k}^{S_{2}C_{2}}\right) - V\left(X\left(k \mid k\right), d_{k|k}^{S_{2}C_{2}}\right)$$
  

$$\leq -\left[x^{T}\left(k \mid k\right) Qx\left(k \mid k\right) + u^{T}\left(k \mid k\right) Ru\left(k \mid k\right)\right]$$
  

$$\leq -x^{T}\left(k \mid k\right) Qx\left(k \mid k\right) \leq -\lambda_{\min}\left(Q\right) \left\{ \|x\left(k \mid k\right)\|^{2} \right\}.$$
(50)

Because *Q* is symmetric positive definite, we have  $\lambda_{\min}(Q) > 0$  (the minimal eigenvalue of matrix *Q*), and then

$$V\left(X\left(k+1\mid k\right), d_{k+1\mid k}^{S_{2}C_{2}}\right) - V\left(X\left(k\mid k\right), d_{k\mid k}^{S_{2}C_{2}}\right) < 0.$$
(51)

When m = 0, (24) is becoming

$$x (k + 1 | k) = A \left( d_{k|k}^{C_2 A} \right) x (k | k) + B \left( d_{k|k}^{S_2 C_2}, d_{k|k}^{C_2 A} \right) x \left( k - d_{k|k}^{S_2 C_2} | k \right).$$
(52)

Because of the measured state,

$$x (k + 1 | k + 1)$$
  
= x (k + 1)  
= A  $(d_k^{C_2 A}) x (k) + B (d_k^{S_2 C_2}, d_k^{C_2 A}) x (k - d_k^{S_2 C_2}).$   
(53)

So we have

$$x(k+1 \mid k+1) = x(k+1 \mid k).$$
(54)

Comparing with (A1), it can be obtained that

$$V\left(X\left(k+1 \mid k+1\right), d_{k+1|k+1}^{S_{2}C_{2}}\right) - V\left(X\left(k \mid k\right), d_{k|k}^{S_{2}C_{2}}\right) < 0.$$
(55)



FIGURE 4: (a) State response of state  $x_2$  with designed optimal controller, (b) state response of state  $x_1$  with designed optimal controller, (c) state trajectories of state  $x_2$ , (d) state trajectories of state  $x_1$ , (e) state response of state  $x_2$ , and (f) state trajectories of state  $x_1$ .



FIGURE 5: The number of data packet dropouts of *S* to *C* side.

Therefore, the Lyapunov function  $V(X(k \mid k), d_{k|k}^{S_2C_2})$  is decreasing for the new closed loop, and  $\lim_{k\to\infty} x(k \mid k) = 0$  is concluded. The stochastic stability is obtained.

#### 5. Simulation Examples

In order to show the effectiveness of the proposed method, we will give some cases and simulations.

5.1. A Simple Example. Considering the coefficient matrixes of  $P_1$  and  $P_2$ 

$$\Phi_1 = \begin{pmatrix} 0.9512 & 1.051 \\ -0.6065 & -1.1618 \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} 8.5530 \\ 29.4735 \end{pmatrix}, \quad (56)$$

 $C_1 = (-0.0045 \ 0.1004),$ 

$$\Phi_2 = \begin{pmatrix} -0.3667 & -0.0111 \\ 1 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{57}$$

 $C_2 = \begin{pmatrix} 0 & 0.0111 \end{pmatrix}.$  (58)

Markov models of the whole closed control systems are consisted. Suppose that  $d_k^{S_2C_2} = i \in (0, 1)$  and  $d_k^{C_2A} = r \in$ (0, 1), as shown in Figures 2 and 5, which means that the number of data packet dropouts is *i* at sensor to controller of secondary side and, similarly, is *r* at secondary controller to actuator side at time *k*. In the method, Y is a scalar and Y > 0. Its convergence is shown in Figure 3. Unique resolution is given finally by using Toolbox of Matlab. The simulation results under the optimal state feedback controller are shown in Figure 4.

From the simulation (1) Figure 3 is the upper of performance index. (2) In Figures 4(a) and 4(b), system state trajectories will eventually be more stable. It is shown that NCSs with data packet dropouts are stable with the optimal controller which is designed by Theorem 4. (3) Through comparing to LQR controller, which is showed in Figures 4(c), 4(d), 4(e), and 4(f), the system overshoots are shortened, and state trajectory paths are more superior. Thus, the optimal controller designed in our paper is effective.

5.2. Industrial Systems. In order to verify the method proposed earlier, a networked control system for main furnace temperature in industrial systems is taken for an example. It is assumed that the transfer functions of the inertial and leading sections are  $G_{p1}(s) = (1/[(30s + 1)(3s + 1)])$  and  $G_{p2}(s) = (1/[(s + 1)^2(10s + 1)])$ , respectively. After discretization, the following state space models are available:

$$P_{1}:\begin{cases} x_{1}(k+1) = \begin{pmatrix} 0.6887 & -0.0093\\ 0.8356 & 0.9951 \end{pmatrix} x_{1}(k) \\ + \begin{pmatrix} 0.8356\\ 0.4437 \end{pmatrix} y_{2}(k), \\ y_{1}(k) = \begin{pmatrix} 0 & 0.0111 \end{pmatrix} x_{1}(k), \end{cases}$$

$$P_{2}:\begin{cases} x_{2}(k+1) = \begin{pmatrix} -0.0342 & -0.4364 & -0.0342\\ 0.3425 & 0.6849 & -0.0254\\ 0.2542 & 0.8762 & 0.9899 \end{pmatrix} x_{2}(k) \\ + \begin{pmatrix} 0.3425\\ 0.2542 & 0.8762 & 0.9899 \end{pmatrix} x_{2}(k), \\ y_{2}(k) = \begin{pmatrix} 0 & 0 & 0.1 \end{pmatrix} x_{2}(k). \end{cases}$$
(59)

Figures 6(a) and 6(b) can be obtained, from which the systems in (17) are stabilized by our designed controller. Through comparing to LQR controller, as shown in Figures 6(c), 6(d), 6(e), and 6(f), and the system overshoots shortened, state trajectories are more superior and performance of the control systems is further improved. Thus, the proposed optimal controller designed is effective and feasible.

#### 6. Conclusions

In this paper, the modeling, optimal, and control problems for a class of NCSs under data packet dropout effect have been studied. The data packet dropouts are described by Markov chains. The Markov chains describe that the quantity of packet dropouts between current time k and its latest communicates successfully instead of whether the data packets dropped or not. Though augmenting the state vectors, the resulting closed-loop control system is transformed into a jump system with time delays. Model predictive control method is applied to study optimization and stability problems of the resulting closed-loop NCSs. Sufficient conditions are proposed for the optimization and stability of the resulting closed-loop NCSs. Some simulations are given in the last section and it can be seen that the designed controllers are feasible and effective. In this work, the full state feedback controllers are designed. However, it is difficult to obtain



FIGURE 6: (a) State response of state  $x_2$  with designed optimal controller, (b) state response of state  $x_1$  with designed optimal controller, (c) state trajectories of state  $x_2$ , (d) state trajectories of state  $x_1$ , (e) state response of state  $x_2$ , and (f) state trajectories of state  $x_1$ .

the full state. Output feedback controllers should be considered. Dynamic output feedback can be investigated in the future work.

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