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Research Article

Analysis of Mode I Periodic Parallel Cracks-Tip Stress Field in an Infinite Orthotropic Plate

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The mechanical behavior near crack tip for periodic parallel cracks in an orthotropic composite plate subjected to the uniformly distributed load within the cracks surface is studied. The mechanical problem is turned into the boundary value problem of partial differential equation. By using the periodicity of the hyperbolic function in the complex domain and constructing proper Westergaard stress function, the periodicity of parallel cracks can be removed. Using the complex variable function method and the undetermined coefficients method, the boundary value problem of partial differential equation can be solved with the help of boundary conditions. The analytic expressions for stress intensity factor, stress, and displacement near the crack tip of periodical parallel cracks are obtained. When the vertical distance of cracks tends to infinity, the stress intensity factor degenerates into a single central crack situation. The stress intensity factor around the crack tip of periodic parallel cracks in an orthotropic composite plate depends on the shape factor. The interaction happens between the cracks. Finally, a numerical analysis of the stress and displacement changed with the polar angle is done.

1. Introduction

Defects in the materials will cause singular stress and cracks. Cracks in the interface, in particular, are the main reason that lessens the structural strength. As we all know, in engineering practice, one certain crack is rare; Crack is always gathered there. It is difficult to deal with a body containing agminate cracks. Therefore, one simple way to model a body containing agminate cracks is to assume that the cracks are arranged in a regular pattern. For simplicity, some of the agminate cracks can be considered ideally as periodic cracks. Periodic crack is the important mechanical model to study the interaction of multiple cracks. Consequently, the research on periodic cracks problem contributes to making an intensive understanding of failure mechanism of materials. Mechanics analysis of crack tip field is very important for engineering practice.

In recent years, the problem of collinear periodic cracks was investigated by many researchers. Hwu, Hu, Guo, et al. [1–4] studied the cracks-tip field problem on collinear periodic cracks in infinite homogeneous materials by means of complex variable function method. The expressions for stress intensity factor, stress, and displacements near crack tip were derived. Erdogan, Ozturk, Chen, and Ding [5–8] studied the antiplane problem of periodical collinear cracks in functionally graded materials by using Fourier transforms method. Sih and Zuo, Hao, Gao et al., Zhao and Meguid [9–13] studied the problem of collinear periodic cracks in piezoelectric material plane. However, to the authors' knowledge, few papers considered the solutions for the problem on periodic parallel cracks. Pak and Goloubeva [14] studied the anti-plane problem of periodic parallel cracks in piezoelectric materials by using distributed dislocation method. The stress and the electric displacement intensity factors were obtained. Using the method of conformal mapping, Hao and Wu [15, 16] considered the anti-plane problem on parallel periodical cracks of finite length starting from the interface of two half planes. The stress intensity factor was obtained. Sanada et al. [17, 18] studied the stress intensity factors for glass-fiber reinforced plastics with an infinite row of parallel cracks at low temperatures under tension in generalized plane strain condition. By using the Fourier transforms to solve a pair of dual integral equations, the expression for the stress intensity factor was obtained. Chen and Liu [19] studied the dynamic

anti-plane problem for a functionally graded piezoelectric strip containing a periodic array of parallel cracks, which were perpendicular to the boundary. By using Laplace transform and Fourier transform, Wang and Mai [20] analyzed the dynamic anti-plane problem in an infinite functionally graded material containing a periodic array of parallel cracks. Using eigenfunction expansion variational method, Chen [21, 22] analyzed the infinite strip problem of periodic parallel cracks subjected to the uniformly distributed load at infinity. The stress intensity factor at the cracks tip and the T-stress were evaluated. Tong, Jiang, Lo and Cheung [23] studied the anti-plane problem of doubly periodic cracks of unequal size in piezoelectric materials. A closed form solution of stress intensity factor was obtained by using complex variable function method. Zhou, Zhang and Li [24] studied the interactions of multiple parallel symmetric and permeable finite length cracks in a piezoelectric material plane subjected to anti-plane shear stress loading by the Schmidt method. Xiao and Jiang [25, 26] used the mapping technique to obtain a closed form solution of stress intensity factor to the problem of periodic open type parallel cracks in an infinite orthotropic elastic body. Bogdanov [27] investigated the axisymmetric problem of fracture of a prestressed composite material with a periodic system of parallel coaxial normal tensile cracks, using the harmonic potential functions and the technique of Hankel integral transformations. Rizk [28, 29] studied two periodic edge cracks in an elastic infinite strip located symmetrically along the free boundaries under thermal shock.

The mechanical behavior around the crack tip of periodic parallel cracks in an orthotropic composite plate subjected to the uniformly distributed load within the cracks surface is studied. The mechanical problem is turned into the boundary value problem of partial differential equation. Hyperbolic function is a periodic function in the complex domain. By constructing proper Westergaard stress function and using the periodicity of the hyperbolic function, the problem of periodic parallel cracks subjected to the uniformly distributed load within the cracks surface is ultimately turned into algebra problem. The analytic expressions for stress intensity factor, stress, and displacement near the crack tip of periodical parallel cracks are obtained.

2. Mechanical Model

Consider an infinite linear elastic orthotropic composite plate with periodic parallel cracks of length $2a$ as shown in Figure 1. Suppose that the two coordinate axes are parallel to the principal directions of material elasticity. All cracks are parallel to the x-axis and equally spaced apart from a distance ω along the y-axis. The surfaces of the cracks are subjected to the uniformly distributed stress load σ.

In the plane stress condition, the compatibility equation of the two-dimensional linear elastic body is as follows [30]:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}. \tag{1}$$

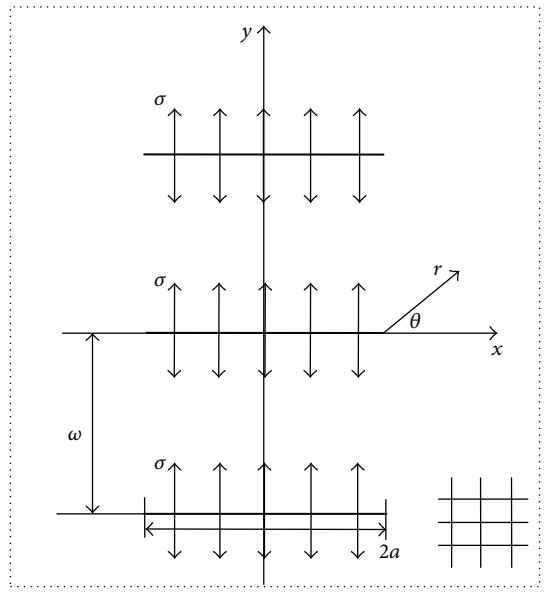

Figure 1: Orthotropic plate with periodic parallel cracks.

Assume that $U = U(x, y)$ is the stress function which is defined as

$$\sigma_x = \frac{\partial^2 U}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 U}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y}. \tag{2}$$

The relations between the strain and the stress are as follows:

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & 0 \\ b_{12} & b_{22} & 0 \\ 0 & 0 & b_{66} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix}, \tag{3}$$

where b_{11}, b_{12}, b_{22}, and b_{66} are the flexibility coefficients in the principal directions of elasticity. From the elasticity theory [31], substituting (2) and (3) into compatibility equation (1), the governing equation of the plane problem in an orthotropic composite plate can be obtained as follows:

$$b_{22} \frac{\partial^4 U}{\partial x^4} + (2b_{12} + b_{66}) \frac{\partial^4 U}{\partial x^2 \partial y^2} + b_{11} \frac{\partial^4 U}{\partial y^4} = 0. \tag{4}$$

As shown in Figure 1, the periodic parallel cracks are subjected to the uniformly distributed stress load within the cracks surface. The boundary conditions are as follows [25, 26]:

$$\begin{aligned} &|x| \longrightarrow \infty, \quad |y| \longrightarrow \infty: \quad \tau_{xy} = \sigma_y = 0, \\ &-a < x < a, \quad y = n\omega \ \ (n = 0, \pm 1, \pm 2 \cdots): \\ &\qquad\qquad \tau_{xy} = 0, \quad \sigma_y = \sigma. \end{aligned} \tag{5}$$

An analysis of fracture problem near cracks tip for periodic parallel cracks subjected to the uniformly distributed load within the cracks surface can be reduced to finding the solution of the boundary value problem of partial differential equations (4) and (5).

Let

$$U = U\left[x + s\left(y - n\omega\right) + in\omega\right] \quad (n = 0, \pm 1, \pm 2 \cdots). \tag{6}$$

Substituting (6) into (4), we can obtain the characteristic equation as follows:

$$b_{11}s^4 + \left(2b_{12} + b_{66}\right)s^2 + b_{22} = 0. \tag{7}$$

It is a biquadratic equation, and its discriminant is written as

$$\Delta = \left(\frac{2b_{12} + b_{66}}{b_{11}}\right)^2 - 4\frac{b_{22}}{b_{11}}. \tag{8}$$

When $\Delta > 0$, the solutions of the characteristic equation (7) are as follows [32]:

$$s_1 = i\beta_1, \quad s_2 = i\beta_2, \quad s_3 = \overline{s_1}, \quad s_4 = \overline{s_2}, \tag{9}$$

where $\beta_2 > \beta_1 > 0$ and

$$\beta_1^2 = \frac{2b_{12} + b_{66}}{2b_{11}} - \sqrt{\left(\frac{2b_{12} + b_{66}}{2b_{11}}\right)^2 - \frac{b_{22}}{b_{11}}}$$
$$\beta_2^2 = \frac{2b_{12} + b_{66}}{2b_{11}} + \sqrt{\left(\frac{2b_{12} + b_{66}}{2b_{11}}\right)^2 - \frac{b_{22}}{b_{11}}}. \tag{10}$$

When $\Delta < 0$, the solutions of the characteristic equation (7) are as follows [32]:

$$s_1 = \alpha + i\beta, \quad s_2 = -\alpha + i\beta,$$
$$s_3 = \overline{s_1}, \quad s_4 = \overline{s_2}, \tag{11}$$

where $\beta > \alpha > 0$ and

$$2\alpha^2 = \sqrt{\frac{b_{22}}{b_{11}}} - \frac{2b_{12} + b_{66}}{2b_{11}},$$
$$2\beta^2 = \sqrt{\frac{b_{22}}{b_{11}}} + \frac{2b_{12} + b_{66}}{2b_{11}}. \tag{12}$$

Let

$$z_j = x + s_j\left(y - n\omega\right) + in\omega = x_j + iy_j, \quad j = 1, 2. \tag{13}$$

Then, for $\Delta > 0$,

$$x_1 = x, \quad x_2 = x, \quad y_1 = \beta_1\left(y - n\omega\right) + n\omega,$$
$$y_2 = \beta_2\left(y - n\omega\right) + n\omega; \tag{14}$$

for $\Delta < 0$,

$$x_1 = x + \alpha\left(y - n\omega\right), \qquad x_2 = x - \alpha\left(y - n\omega\right),$$
$$y_1 = \beta\left(y - n\omega\right) + n\omega, \tag{15}$$
$$y_2 = \beta\left(y - n\omega\right) + n\omega.$$

Using formulas (14) and (15), the governing equation (4) can be rewritten as a generalized biharmonic equation [32]:

$$\nabla_1^2\nabla_2^2 U = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}\right)\left(\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial y_2^2}\right)U = 0. \tag{16}$$

By the theory of complex variable, the real part and the imaginary part of the analytic function are the solutions of the governing equation (4). The solution for partial differential equation (4) may be chosen as

$$U = \sum_{j=1}^{2}\left[a_j \operatorname{Re}\left(\overline{\overline{U}}_j\right) + b_j \operatorname{Im}\left(\overline{\overline{U}}_j\right)\right], \tag{17}$$

where a_j, b_j $(j = 1, 2)$ are undetermined real parameters, U_j $(j = 1, 2)$ is an analytic function of z_j, and

$$\frac{d\overline{\overline{U}}_j}{dz_j} = \overline{U_j}, \quad \frac{d\overline{U_j}}{dz_j} = U_j, \quad j = 1, 2. \tag{18}$$

Substituting (17) and (18) into (2), the stress expressions can be written as

$$\sigma_x = \frac{\partial^2 U}{\partial y^2} = \sum_{j=1}^{2}\left[a_j \operatorname{Re}\left(s_j^2 U_j\right) + b_j \operatorname{Im}\left(s_j^2 U_j\right)\right],$$
$$\sigma_y = \frac{\partial^2 U}{\partial x^2} = \sum_{j=1}^{2}\left[a_j \operatorname{Re}\left(U_j\right) + b_j \operatorname{Im}\left(U_j\right)\right], \tag{19}$$
$$\tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y} = -\sum_{j=1}^{2}\left[a_j \operatorname{Re}\left(s_j U_j\right) + b_j \operatorname{Im}\left(s_j U_j\right)\right].$$

3. Westergaard Stress Function

In order to solve the boundary value problem of partial differential equations (4) and (5), the Westergaard stress function is selected as follows [25]:

$$U_j = \sigma\left[\frac{\sinh\left(2\pi z_j/\omega\right)}{\sqrt{\sinh^2\left(2\pi z_j/\omega\right) - \sinh^2\left(2\pi\left(a + n\omega i\right)/\omega\right)}} - 1\right] \quad j = 1, 2, \tag{20}$$

when $|x| \to \infty$, $|y| \to \infty : U_j = 0$.

When $-a < x < a$, $y = n\omega$ $(n = 0, \pm 1, \pm 2 \cdots)$:

$$U_j = \sigma\left[\frac{-\sinh\left(2\pi x/\omega\right)i}{\sqrt{\sinh^2\left(2\pi a/\omega\right) - \sinh^2\left(2\pi x/\omega\right)}} - 1\right]. \tag{21}$$

When $\Delta > 0$, substituting (9), (19), and (21) into boundary conditions (5), we can obtain a system of linear equations in 4 unknowns about coefficients a_j, b_j $(j = 1, 2)$:

$$a_1\beta_1 + a_2\beta_2 = 0, \qquad a_1 + a_2 = -1$$
$$b_1 + b_2 = 0, \qquad b_1\beta_1 + b_2\beta_2 = 0. \tag{22}$$

Solving nonhomogeneous linear equations, the unique solution of equations can be derived as follows:

$$a_1 = \frac{-\beta_2}{\beta_2 - \beta_1}, \quad a_2 = \frac{\beta_1}{\beta_2 - \beta_1}, \qquad b_1 = 0, \quad b_2 = 0. \tag{23}$$

When $\Delta < 0$, substituting (11), (19), and (21) into boundary conditions (5), we can obtain a system of linear equations in 4 unknowns about coefficients a_j, b_j $(j = 1, 2)$:

$$\begin{aligned} &a_1 + a_2 = -1, \quad b_1 + b_2 = 0, \\ &a_1\beta + a_2\beta - b_1\alpha + b_2\alpha = 0, \\ &-a_1\alpha + a_2\alpha - b_1\beta - b_2\beta = 0. \end{aligned} \tag{24}$$

Solving the non-homogeneous linear equations, the unique solution of equations can be derived as follows:

$$a_1 = -\frac{1}{2}, \quad a_2 = -\frac{1}{2}, \quad b_1 = -\frac{\beta}{2\alpha}, \quad b_2 = \frac{\beta}{2\alpha}. \tag{25}$$

Substituting (23) and (25) into (17), the real analytic solution U is obtained, which meets the governing equation (4) and the boundary conditions (5).

4. Stress Intensity Factor

Fully considering the loadings and geometry size of orthotropic composite plate, we introduce the following stress intensity factor:

$$K_I = \lim_{z_j \to a+n\omega i} \left[2\pi\left(z_j - (a + n\omega i)\right)\right]^{1/2} U_j\left(z_j\right). \tag{26}$$

Substituting (20) into (26), we obtain

$$K_I = \sigma\sqrt{\frac{\omega \tanh(2\pi a/\omega)}{2}} = Y\sigma\sqrt{\pi a}, \tag{27}$$

where $Y = \sqrt{(\omega/2\pi a)\cdot\tanh(2\pi a/\omega)}$ is called the shape factor.

Labeling $K_I' = \sigma\sqrt{\pi a}$, K_I' is the stress intensity factor of a single central crack in an infinite linear elastic orthotropic composite plate subjected to the uniformly distributed load within the crack surface. It can be concluded from (27) that stress intensity factor around the tip of periodic parallel cracks depends on the shape factor Y. When $\omega \to \infty$, $Y = \sqrt{(\omega/2\pi a)\cdot\tanh(2\pi a/\omega)} \to 1$ then $K_I \to K_I'$; namely, the stress intensity factor degenerates into a single central crack situation when the vertical distance of periodic parallel cracks tends to infinity. The variation curves of K_I', Y, and K_I with the cracks spacing ω are given as shown in Figure 2. It can be seen from Figure 2 that the stress intensity factor and the shape factor increase rapidly with the increase of the distance between cracks and then reach a steady state, that is, when $\omega \to \infty$, $Y \to 1$, and $K_I \to K_I'$. In other words, the periodic parallel cracks problem degenerates into a single central crack situation when the vertical distance of periodic parallel cracks tends to infinity, and it is entirely consistent with the previous results.

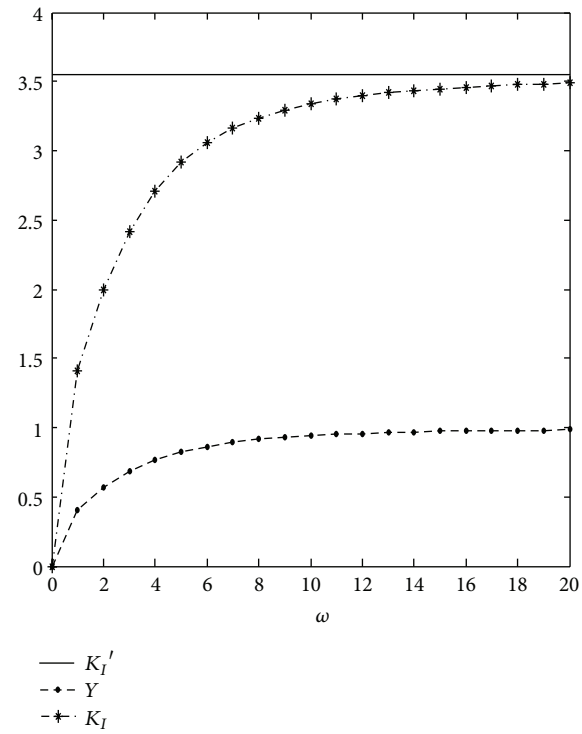

Figure 2: Variation curves of K_I', Y, and K_I with crack spacing ω.

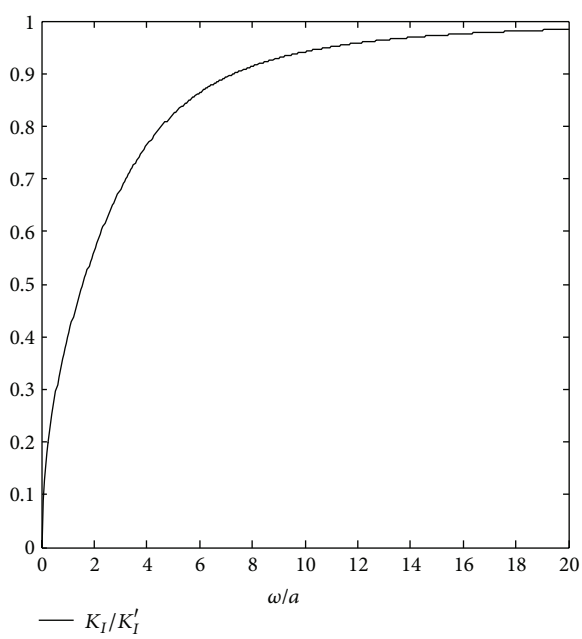

Figure 3: Interaction of periodic parallel cracks.

In order to research the interaction between cracks, the variation curve of K_I/K_I' with ω/a is given as shown in Figure 3. It can be seen from Figure 3, when $\omega/a < 10$, K_I/K_I' increases quickly with the increase of ω/a, and the distance

between cracks is the main reason that influenced the interaction between cracks; when $\omega/a > 10$, K_I/K_I' increases slowly with the increase of ω/a, and $K_I/K_I' \rightarrow 1$; the reason is that the interaction between the cracks is small with the increase of the distance between cracks, and so, it is considered that the periodic parallel cracks problem degenerates into a single crack problem.

5. Stress Field and Displacement Field

Substituting (13) and (26) into (20), in the vicinity of the cracks tip, we can obtain

$$U_j\left(z_j\right) = \frac{K_I}{\left[2\pi\left(z_j - (a + n\omega i)\right)\right]^{1/2}}, \quad (n = 0, \pm 1, \pm 2 \cdots), \text{ as } z_j \longrightarrow a. \tag{28}$$

Let

$$z_j - (a + n\omega i) = r\left(\cos\theta + s_j \sin\theta\right) \tag{29}$$

so,

$$U_j\left(z_j\right) = \frac{K_I}{\left[2\pi r\left(\cos\theta + s_j \sin\theta\right)\right]^{1/2}}, \quad \text{as } r \longrightarrow 0, \tag{30}$$

where polar radius r is the distance to the crack tip which is the shortest distance away from the point.

Substituting (9), (23), (30), (11), (25), and (30) into (19), respectively, and according to the relationships of the stress-strain and strain-displacement, the unified analytic expressions for stress and displacement of the periodical parallel cracks tip are achieved as follows. Consider

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \operatorname{Re}\left\{\frac{s_1 s_2}{s_1 - s_2}\left[\frac{s_1}{\left(\cos\theta + s_1 \sin\theta\right)^{1/2}} - \frac{s_2}{\left(\cos\theta + s_2 \sin\theta\right)^{1/2}}\right]\right\}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \operatorname{Re}\left\{\frac{1}{s_1 - s_2}\left[\frac{s_2}{\left(\cos\theta + s_1 \sin\theta\right)^{1/2}} - \frac{s_1}{\left(\cos\theta + s_2 \sin\theta\right)^{1/2}}\right]\right\}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \operatorname{Re}\left\{\frac{s_1 s_2}{s_1 - s_2}\left[\frac{1}{\left(\cos\theta + s_2 \sin\theta\right)^{1/2}} - \frac{1}{\left(\cos\theta + s_1 \sin\theta\right)^{1/2}}\right]\right\}$$

$$u = K_I\sqrt{\frac{r}{2\pi}} \operatorname{Re}\left\{\frac{1}{s_1 - s_2}\left[s_2\left(s_1^2 b_{11} + b_{12}\right) \times \left(\cos\theta + s_1 \sin\theta\right)^{1/2} - s_1\left(s_2^2 b_{11} + b_{12}\right) \times \left(\cos\theta + s_2 \sin\theta\right)^{1/2}\right]\right\}$$

$$v = K_I\sqrt{\frac{r}{2\pi}} \operatorname{Re}\left\{\frac{1}{s_1 - s_2}\left[\frac{s_2}{s_1}\left(s_1^2 b_{12} + b_{22}\right) \times \left(\cos\theta + s_1 \sin\theta\right)^{1/2} - \frac{s_1}{s_2}\left(s_2^2 b_{12} + b_{22}\right) \times \left(\cos\theta + s_2 \sin\theta\right)^{1/2}\right]\right\}. \tag{31}$$

For an orthotropic composite plate, the material parameters are as follows:

$$\begin{aligned} &E_1 = 70\,\text{GPa}, \quad E_2 = 11.4\,\text{GPa} \\ &\nu_{12} = 0.23, \quad G_{12} = 5.3\,\text{GPa} \\ &a = 0.01\,\text{m}, \quad r = 0.001\,\text{m}. \end{aligned} \tag{32}$$

For the different a/ω, the variations curves of the stress and displacement with the polar angle are given as shown in Figures 4 and 5. It can be seen from Figure 4 that stress can reach maximum value and minimum value in the range of $[-90^\circ$ to $90^\circ]$. For the different a/ω, the angles in which stress reaches the maximum (or minimum) value are the same, and this property is very important for S-fracture criterion and Z-fracture criterion. As seen from Figure 5, the displacement increases with the increase of a/ω. For the different a/ω, the angles in which displacement reach maximum (or minimum) value are the same, and this theory is useful to study the fracture criterion.

6. Conclusions

In this paper the mechanical behaviour around the periodic parallel cracks in orthotropic composite plate is studied. The mechanical problem is turned into the boundary value problem of partial differential equation. By constructing proper Westergaard stress function and using the complex variable function method, the analytic expressions for stress intensity factor, stress, and displacement are obtained with the help of boundary conditions.

(1) The stress intensity factor around the crack tip of periodic parallel cracks depends on the shape factor Y. The interaction happens between the cracks. When ω/a is small, strong interaction between the cracks can be found. With the increase of ω/a, the mutual

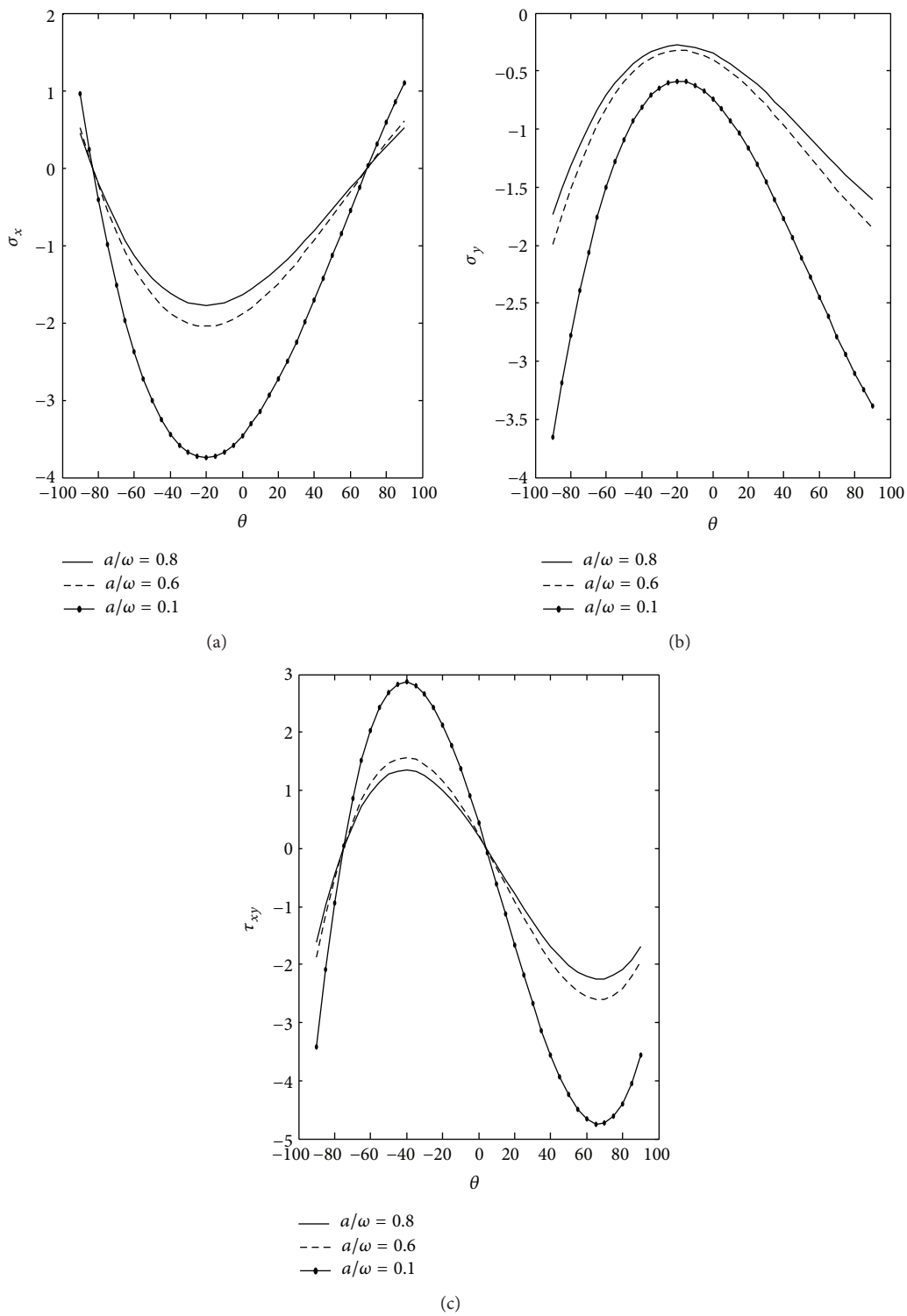

Figure 4: Variation curve of the stress with the polar angle.

influence between the cracks decreases; that is, the interaction between the cracks decreases.

(2) The analytic expressions for stress and displacement of the periodical parallel cracks tip are obtained. The variations curves of the stress and displacement with the polar angles are given. For the different a/ω, stress and displacement can reach maximum value and minimum value, and the angles in which stress and displacement reach maximum (or minimum) value are the same. This property is very important for S-fracture criterion and Z-fracture criterion.

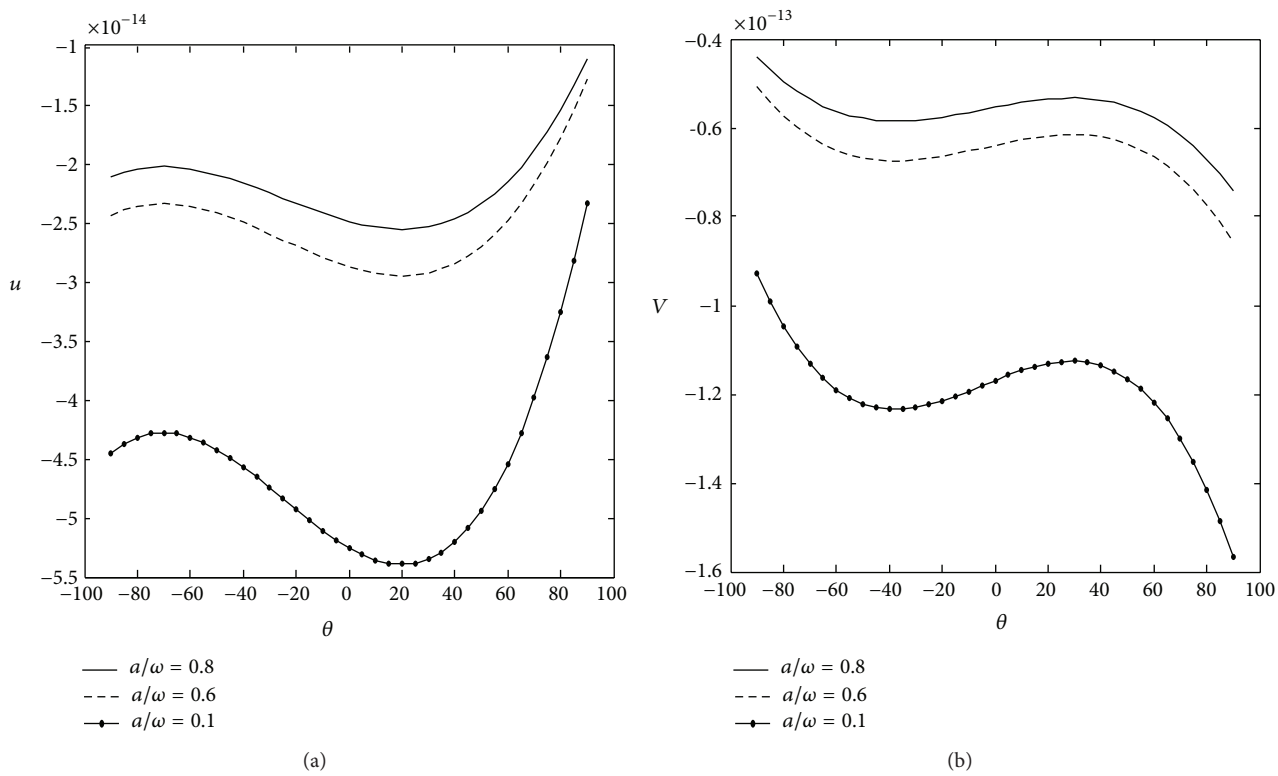

Figure 5: Variation curve of the displacement with the polar angle.

Conflict of Interests

The authors declare no direct financial relation with any commercial entities mentioned in the paper that might lead to a conflict of interests.

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