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Research Article

Research on the Thermal Cavitation Problem of a Preexisting Microvoid in a Viscoelastic Sphere

Yajuan Chen[1,2] and Xinchun Shang[1]

[1] *Department of Applied Mechanics, University of Science and Technology Beijing, Beijing 100083, China*
[2] *School of Civil Engineering, Henan Polytechnic University, Jiaozuo 454000, China*

Correspondence should be addressed to Xinchun Shang; sxc1958@126.com

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The cavitation problem of a preexisting microvoid in the incompressible viscoelastic sphere subjected to the uniform temperature field was studied in this paper. Based on the finite logarithmic strain measure for geometrically large deformation, the nonlinear mathematical model of this problem was established by employing the Kelvin-Voigt differential type constitutive equation of thermoviscoelasticity. Adopting the dimensionless transformation of each parameter, growth curves of the microvoid radius increasing with the temperature were given. And the results indicated that the generation of cavity could be regarded as the idealized model of microvoid growth. A parametric study, including the influences of the external temperature, the initial microvoid radius, and the material parameter on the microvoid radius, was also conducted. The sudden growth of infinitely large sphere with a preexisting microvoid could also achieved by the finitely large sphere.

1. Introduction

The cavitation problem caused by the material instability has been attracting attention of many researchers for decades. The cavitation problems of solid materials are divided into two parts, one is the sudden growth of a pre-existing microvoid in the infinitely large solid material, and the other is the sudden generation and growth of a cavity in the finitely large solid material [1, 2]. In 1982, Ball [1] created the nonlinear theory of cavitation and gained the explicit expressions of critical loading in the incompressible hyperelastic material for the first time. Sivaloganathan [3], Chou-Wang and Horgan [4], Horgan and Polignone [5], Shang and Cheng [6], and Ren and Cheng [7–9] have intensively studied the cavitation problems of hyperelastic materials. Similar studies on hyperelastic materials can be found in Lopez-Pamies [10], Cohen and Durban [11], and Ren and Li [12]. For the compressible hyperelastic materials, only several analytical solutions of the cavitation have been found for some specific forms of strain energy function. Horgan and Abeyaratne [2], Sivaloganathan [3] have proved that the growth phenomena of a pre-existing microvoid in hyperelastic materials could be used to explain the cavity formation and growth. Cavitation bifurcation solutions of Hooke elasticity subjected to a radial tensile loading have been discussed by Shang and Cheng [13]. Not considering the external loading, the problem also has come to conclusion with the effect of the temperature on cavity generation of Hooke elastic materials; analytic solutions of parameter form have been derived for thermal dilatation of the composite ball with large elastic deformations by Shang et al. [14]. And other elastic materials have been studied theoretically by many researchers (Murphy [15], Pence and Tsai [16], Rooney and Carroll [17], Henao [18], Negrón-Marrero and Sivaloganathan [19], and Lian and Li [20]).

Metal materials have been the main structure members of the airframe and the aircraft engine, due to the high speed flight. The fatigue life of the aviation material is seriously affected by the transient thermal stress caused by aerodynamic heating. As for the metal materials, their viscoelastic behaviors are close related to vibration or high temperature, and thus, the temperature is a key factor for the cavitation problem in the viscoelastic material. Zhang [21] has

obtained the microscopic damage characteristics inside the material under the thermal shock by means of the experiment method. Collin and Coussios [22] have done the quantitative experiment for a single-bubble cavitation in the viscoelastic media. Zhang and Huang [23] have discussed the growth of a pre-existing void in the nonlinear viscoelastic material subjected to remote hydrostatic tensions with different loading rates. However, few pieces of work have been carried out on the generation and dynamical growth problems of microvoid in the viscoelastic material, only considering the influence of temperature.

The mechanical characteristics are related to time in the viscoelastic material and sensitive to strain rate. Furthermore, the cavitation and bifurcation problems in the viscoelastic material are considered as the instability for nonlinear materials and also the exact solution for such a large deformation problem are very difficult. So, the purpose of this paper is to establish the nonlinear dynamical mathematical mode of the microvoid motion in an incompressible thermoviscoelastic sphere subjected to the uniform temperature field. And by the semianalytical and seminumerical method, variation curves of the microvoid radius with temperature were given. Dynamical variation curves were also obtained to describe the microvoid radius increasing with time. The influences of these parameters on the variation rules of microvoid radius were analyzed.

2. Mathematic Formulation

Consider a sphere with the pre-existing microvoid composed of incompressible viscoelastic material subjected to a uniform temperature field. Assume that the initial and current radii of the sphere are R_0 and r_0, respectively, and the initial and current microvoid radii are ε and δ, respectively. The profile of the sphere is shown in Figure 1. The initial and the current configurations of the sphere are described by the sphere coordinates (R, Θ, Φ) and (r, θ, φ), respectively. The center of the spherical cavity is the origin of spherical coordinate, and the deformation of sphere is assumed to be completely spherical symmetric. Suppose that the region of initial sphere is

$$D = \{(R, \Theta, \Phi) \mid \varepsilon \le R < R_0, 0 < \Theta \le 2\pi, 0 < \Phi \le \pi\}. \quad (1)$$

The spherically symmetric motion can be expressed as

$$r = r(R, t) > 0, \quad \varepsilon \le R < R_0, \theta = \Theta, \varphi = \Phi. \quad (2)$$

The Kelvin-Voigt differential type constitutive equations for thermo-visco-elasticity [24, 25] are used, and the radial and hoop stresses in the spherical coordinate are expressed as

$$\sigma_r = \lambda\left(1 + t_0\frac{\partial}{\partial t}\right)(\varepsilon_r + 2\varepsilon_\theta) + 2\mu\left(1 + t_0\frac{\partial}{\partial t}\right)\varepsilon_r - \beta T,$$

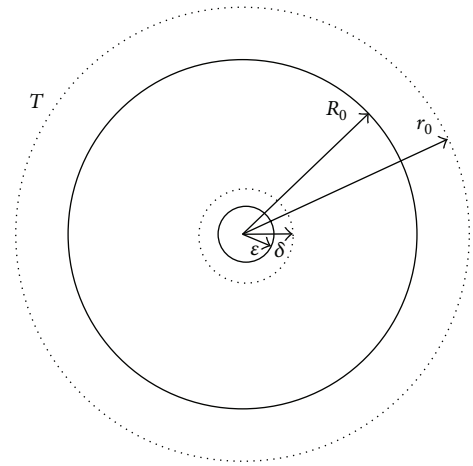

Figure 1: Profile of the sphere.

$$\sigma_\theta = \lambda\left(1 + t_0\frac{\partial}{\partial t}\right)(\varepsilon_r + 2\varepsilon_\theta) + 2\mu\left(1 + t_0\frac{\partial}{\partial t}\right)\varepsilon_\theta - \beta T, \quad (3)$$

where σ_r, σ_θ are the radial and hoop stresses, respectively. ε_r, ε_θ are the radial and hoop strains, respectively. λ, μ are the Láme constants. t_0 is the thermal relaxation time of viscoelasticity. $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion. T is the variable value for the initial temperature.

The differential equation of motion with the absence of body force is

$$\frac{\partial \sigma_r}{\partial r} + \frac{2}{r}(\sigma_r - \sigma_\theta) = \rho\frac{\partial^2 r}{\partial t^2}, \quad (4)$$

where, σ_r, σ_θ are the Cauchy (true) stress. ρ is the material density.

The logarithmic strains are used to describe the finite deformation:

$$\varepsilon_r = \ln\frac{\partial r(R, t)}{\partial R}, \qquad \varepsilon_\theta = \varepsilon_\varphi = \ln\frac{r(R, t)}{R}. \quad (5)$$

In view of the incompressibility condition of the material $\varepsilon_r + 2\varepsilon_\theta = 0$, we have

$$r = r(R, t) = \left(R^3 + \delta^3(t) - \varepsilon^3\right)^{1/3}, \quad t \ge 0, \quad (6)$$

$$\varepsilon_r = \frac{2}{3}\ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{r^3}\right)\right], \qquad \varepsilon_\theta = -\frac{1}{3}\ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{r^3}\right)\right], \quad (7)$$

$$\dot{\varepsilon}_r = -\frac{2\delta^2\dot{\delta}}{r^3}, \qquad \dot{\varepsilon}_\theta = \frac{\delta^2\dot{\delta}}{r^3}, \tag{8}$$

in which $\delta(t) \geq \varepsilon$ is the function to be determined, and it expresses the motion of microvoid with time t in the sphere.

The outer surface of the microvoid is traction free for the radial stress:

$$\sigma_r(\delta, t) = 0, \quad t \geq 0. \tag{9}$$

The free boundary condition of the outermost layer in the sphere is

$$\sigma_r(r_0, t) = 0, \qquad r_0 = r(R_0, t) = \left(R_0{}^3 + \delta^3(t) - \varepsilon^3\right)^{1/3}. \tag{10}$$

Supposing the sphere is in the undeformed state at $t \leq 0$, the initial condition is

$$\delta(0) = \varepsilon, \qquad \dot{\delta}(0) = 0. \tag{11}$$

3. Analytic Solution

Differentiating twice the incompressible condition (6) with respect to t and substituting the obtained result and (3) into the motion equation (4), we have

$$\frac{\partial\sigma_r}{\partial r} = \rho\frac{\delta^2}{r^2}\ddot{\delta} + 2\rho\left(\frac{\delta}{r^2} - \frac{\delta^4}{r^5}\right)\dot{\delta}^2 + \frac{12}{r}\mu t_0\frac{\delta^2\dot{\delta}}{r^3} - \frac{4}{r}\mu\ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{r^3}\right)\right]. \tag{12}$$

Integrating (12) with respect to r from δ to r, then the radial stress σ_r of thermo-visco-elastic region is obtained:

$$\begin{aligned}\sigma_r = {} & \rho\left(\delta - \frac{\delta^2}{r}\right)\ddot{\delta} + \rho\left(\frac{\delta^4}{2r^4} - \frac{2\delta}{r} + \frac{3}{2}\right)\dot{\delta}^2 \\ & - 4\mu t_0\delta^2\left(\frac{1}{r^3} - \frac{1}{\delta^3}\right)\dot{\delta} \\ & - 4\mu\int_\delta^r \ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{s^3}\right)\right]\frac{ds}{s} + p(\delta, t).\end{aligned} \tag{13}$$

Combining the boundary condition (9), we have

$$p(\delta, t) = 0. \tag{14}$$

Then, the expressions for the radial and hoop Euler stresses are obtained:

$$\begin{aligned}\sigma_r = {} & \rho\left(\delta - \frac{\delta^2}{r}\right)\ddot{\delta} + \rho\left(\frac{\delta^4}{2r^4} - \frac{2\delta}{r} + \frac{3}{2}\right)\dot{\delta}^2 \\ & - 4\mu t_0\delta^2\left(\frac{1}{r^3} - \frac{1}{\delta^3}\right)\dot{\delta} \\ & - 4\mu\int_\delta^r \ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{s^3}\right)\right]\frac{ds}{s},\end{aligned} \tag{15}$$

$$\begin{aligned}\sigma_\theta = {} & \rho\left(\delta - \frac{\delta^2}{r}\right)\ddot{\delta} + \rho\left(\frac{\delta^4}{2r^4} - \frac{2\delta}{r} + \frac{3}{2}\right)\dot{\delta}^2 \\ & + 2\mu t_0\delta^2\left(\frac{1}{r^3} + \frac{2}{\delta^3}\right)\dot{\delta} \\ & - 2\mu\ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{r^3}\right)\right] \\ & - 4\mu\int_\delta^r \ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{s^3}\right)\right]\frac{ds}{s}.\end{aligned} \tag{16}$$

Combining the boundary condition (10) and (3), (15), then (17) is obtained:

$$\begin{aligned}& \rho\left(\delta - \frac{\delta^2}{r_0}\right)\ddot{\delta} + \rho\left(\frac{\delta^4}{2r_0^4} - \frac{2\delta}{r_0} + \frac{3}{2}\right)\dot{\delta}^2 \\ & + \frac{4\mu t_0}{\delta}\dot{\delta} - 4\mu\int_\delta^{r_0} \ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{s^3}\right)\right]\frac{ds}{s} \\ & - \frac{4}{3}\mu\ln\left[1 - \left(\frac{\delta^3 - \varepsilon^3}{r_0^3}\right)\right] + \beta T = 0.\end{aligned} \tag{17}$$

Equation (17) is a nonlinear second-order ordinary differential equation. For a given temperature T, it provides an exact relationship between the microvoid radius $\delta(t) \geq \varepsilon$ and time t. So, (17) is called the motion equation of the microvoid.

4. Numerical Results and Discussion

Using the dimensionless transformation $w = \rho R_0^2/\mu t_0^2$, $\tau = t/t_0$, $T_0 = (\beta/\mu)T$, $x = \delta/R_0$, $x_0 = \varepsilon/R_0$, introducing $\xi = (1 - (\delta^3 - \varepsilon^3)/s^3)^{-1/3}$, and utilizing the conversion relationship $ds/s = (\xi^{-1}/(1 - \xi^3))d\xi$ between ξ and s, (17) is turned into

$$\begin{aligned}& wx^2\left(\frac{1}{x} - \frac{1}{(1 + x^3 - x_0^3)^{1/3}}\right)\frac{d^2x}{d\tau^2} \\ & + 2wx\left(\frac{x^3}{4(1 + x^3 - x_0^3)^{4/3}} - \frac{1}{(1 + x^3 - x_0^3)^{1/3}} + \frac{3}{4x}\right)\left(\frac{dx}{d\tau}\right)^2 + \frac{4}{x}\frac{dx}{d\tau} \\ & - 12\int_{(x/x_0)}^{(1+x^3-x_0^3)^{1/3}} \frac{(\ln\xi)}{\xi^4 - \xi}d\beta \\ & + \frac{4}{3}\ln\left(1 + x^3 - x_0^3\right) + T_0 = 0.\end{aligned} \tag{18}$$

Letting $dx/d\tau = 0$ in (18), then (19) is obtained:

$$\frac{4}{3}\ln\left(1+x^3-x_0^3\right)-12\int_{(x/x_0)}^{(1+x^3-x_0^3)^{1/3}}\frac{(\ln\xi)}{\xi^4-\xi}d\beta+T_0=0. \tag{19}$$

The quasistatic solution of the thermo-viscoelastic sphere can be obtained from (19). Figure 2 shows the growth curves of microvoid radius x with temperature T_0 under the different initial microvoid radius x_0. When x_0 is infinitely close to 0, the cavity generation in a solid sphere can be regarded as the ideal model of microvoid growth, and the dimensionless critical temperature of cavity generation can be obtained from (19); it is 2.193. If the outside temperature is lower than the critical temperature, there will be no cavity in the sphere. If the outside temperature exceeds the critical temperature, the cavity will appear suddenly, and the cavity radius increases very rapidly with the rising of temperature. When x_0 is not equal to 0, the microvoid increases very slowly. However, if the temperature is close to the critical temperature, the microvoid will increase very rapidly, and the growth curve of a pre-existing microvoid is more close to the bifurcation curve of cavity along with the continuous decrease of x_0. It proves that the generation of cavity can be regarded as the idealized model of microvoid growth.

Letting $\tau' = d\tau/dx$, then (18) becomes

$$\begin{aligned}&-wx^2\left(\frac{1}{x}-\frac{1}{(1+x^3-x_0^3)^{1/3}}\right)\tau''\\&+\left[-12\int_{(x/x_0)}^{(1+x^3-x_0^3)^{1/3}}\frac{(\ln\xi)}{\xi^4-\xi}d\beta\right.\\&\left.+\frac{4}{3}\ln\left(1+x^3-x_0^3\right)+T_0\right]\tau'^3\\&+\frac{4}{x}\tau'^2+2wx\left(\frac{x^3}{4(1+x^3-x_0^3)^{4/3}}\right.\\&\left.-\frac{1}{(1+x^3-x_0^3)^{1/3}}+\frac{3}{4x}\right)\tau'=0.\end{aligned} \tag{20}$$

Let

$$\begin{aligned}a(x)&=-wx^2\left(\frac{1}{x}-\frac{1}{(1+x^3-x_0^3)^{1/3}}\right),\\b_1(x)&=-12\int_{(x/x_0)}^{(1+x^3-x_0^3)^{1/3}}\frac{(\ln\xi)}{\xi^4-\xi}d\beta\\&\quad+\frac{4}{3}\ln\left(1+x^3-x_0^3\right)+T_0,\\b_2(x)&=\frac{4}{x},\end{aligned}$$

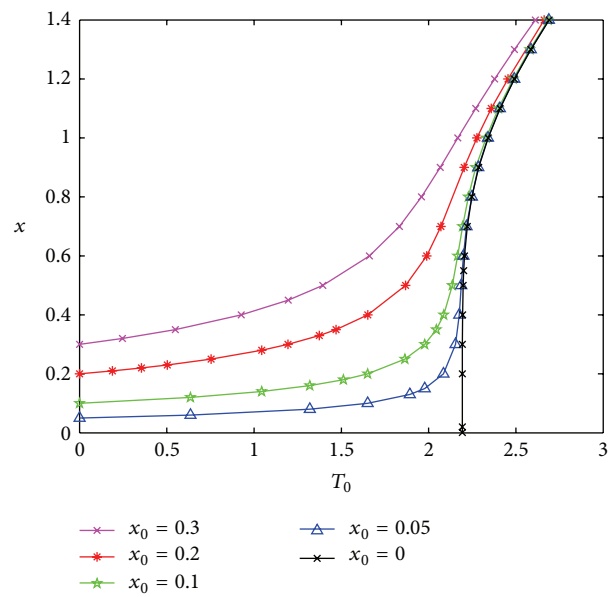

Figure 2: Curves of microvoid radius x with temperature T_0 under the different initial microvoid radius x_0.

$$b_3(x)=2wx\left(\frac{x^3}{4(1+x^3-x_0^3)^{4/3}}-\frac{1}{(1+x^3-x_0^3)^{1/3}}+\frac{3}{4x}\right). \tag{21}$$

Letting $y_1 = d\tau/dx$, $y_2 = \tau$. Equations (20), (11) can be turned into (22), (23), respectively:

$$\frac{dy_1}{dx}=-\frac{b_1(x)}{a(x)}y_1^3-\frac{b_2(x)}{a(x)}y_1^2-\frac{b_3(x)}{a(x)}y_1, \tag{22}$$

$$\begin{gathered}y_1(0)=\infty,\\y_2(0)=0,\qquad y_2(x)=\int_0^x y_1(s)\,ds.\end{gathered} \tag{23}$$

Combining the initial condition (23) and using the Runge-Kutta method, numerical computation for (22) is done, then the numerical solution of $x = x(\tau)$ by solving the inverse function can be obtained.

Observing (20), it is easy to know that the variation rules of dimensionless microvoid radius x increasing with dimensionless time t are mainly dependent on the three parameters: the dimensionless initial microvoid radius x_0, the dimensionless temperature T_0, and the parameter w. Figures 3–6 give the results of numerical computation.

Figure 3 shows the variation curves of microvoid radius x with time τ under the different initial microvoid radius x_0, when the outside temperature T_0 and the parameter w keep constant. Figure 4 shows the variation ratio of microvoid radius $dx/d\tau$ with time τ. It is seen that the larger the initial microvoid radius is, the more rapidly the void increases. If they achieve the same cavity radius, it will take shorter time

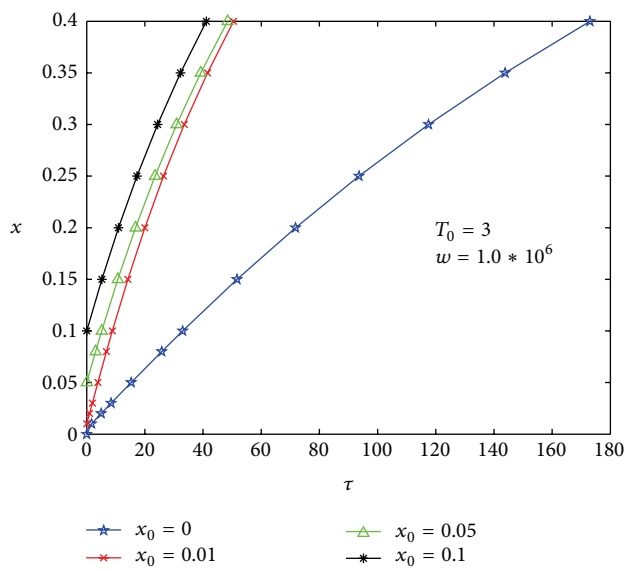

FIGURE 3: Variation curves of microvoid radius x with time τ under the different initial microvoid radius x_0.

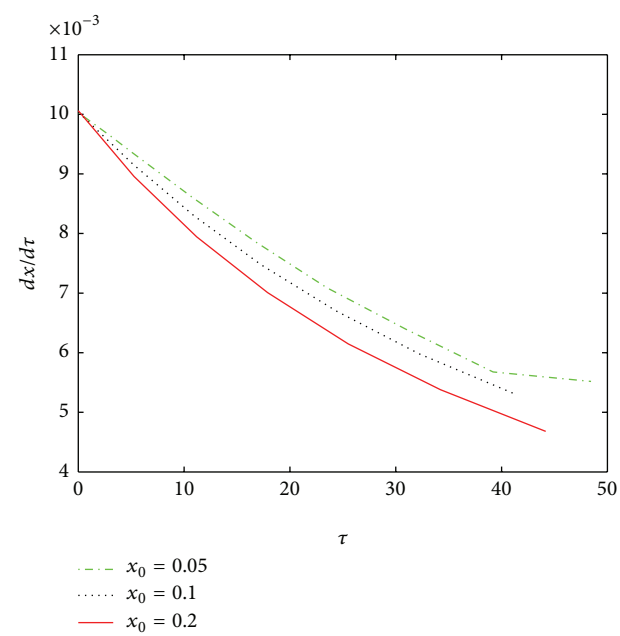

FIGURE 4: Variation curves of $dx/d\tau$ with time τ under the different initial microvoid radius x_0.

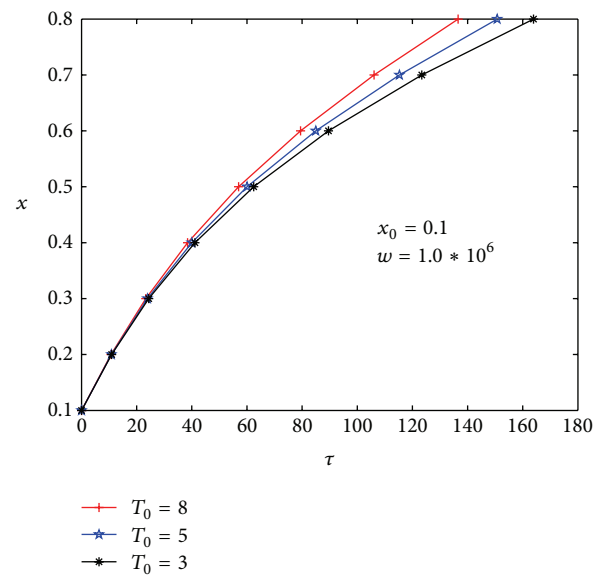

FIGURE 5: Variation curves of microvoid radius x with time τ under the different temperature T_0.

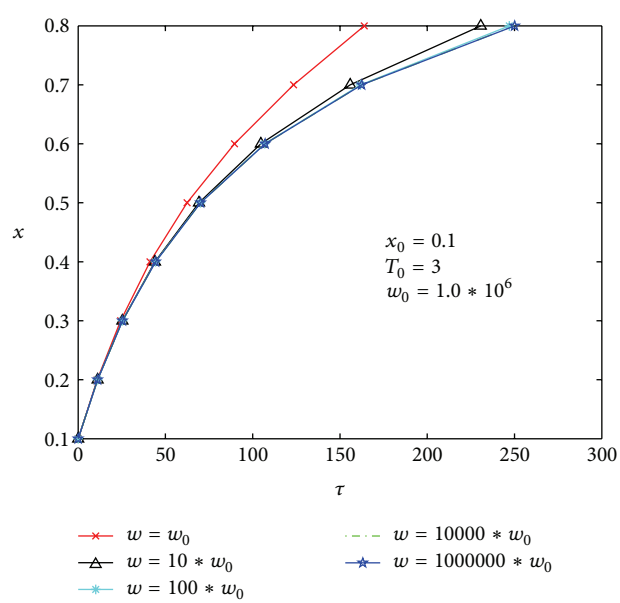

FIGURE 6: Variation curves of microvoid radius x with time τ under the different material parameter w.

for the larger initial microvoid radius. When the microvoid radius x_0 is infinitely close to 0, it can be used to describe the dynamically increasing rules of cavity generation in a solid sphere.

Figure 5 shows the variation curves of microvoid radius x with time τ under the different external temperature T_0, when the initial microvoid radius x_0 and the parameter w keep constant. It displays that the microvoid radius increases quickly with the growth of temperature, and the higher the temperature is, the more rapidly the microvoid radius increases.

Figure 6 shows the variation curves of microvoid radius x with time τ under the different parameter w, when the initial microvoid radius x_0 and the external temperature T_0 keep constant. It displays that the smaller the parameter w is, the more rapidly the microvoid radius increases. From the parameter $w = \rho R_0^2/\mu t_0^2$, we can know that when the values of material parameter ρ, μ, t_0 are certain, the radius is 10, 100, and 1000 times the initial radius R_0; that is, w is $100w_0$, $10000w_0$, and $1000000w_0$, and three curves almost keep

coincidence together. We can simulate the sudden growth of an infinitely large sphere with a pre-existing microvoid by use of the sphere, in which the parameter w is $100w_0$.

5. Conclusions

In this paper, the microvoid dynamical growth problem in an incompressible thermo-viscoelastic sphere subjected to a uniform temperature field is researched. An exactly differential relation between the microvoid radius and the outside temperature field is obtained. It is concluded that it will spend shorter time for the larger initial microvoid radius with the higher temperature and the smaller parameter w to reach a certain higher value of cavity radius. Since the generation and growth of cavity are the important factors to material damage, this paper provides a valuable method for the generation and expansion of crack in the viscoelastic materials.

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