

## Research Article

# **Impulsive Pinning Markovian Switching Stochastic Complex Networks with Time-Varying Delay**

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The synchronization problem of stochastic complex networks with Markovian switching and time-varying delays is investigated by using impulsive pinning control scheme. The complex network possesses noise perturbations, Markovian switching, and internal and outer time-varying delays. Sufficient conditions for synchronization are obtained by employing the Lyapunov-Krasovskii functional method, Itô's formula, and the linear matrix inequality (LMI). Numerical examples are also given to demonstrate the validity of the theoretical results.

#### 1. Introduction and Model Description

Collective behaviors in complex networks and systems have attracted increasing attention in recent years due to their wide applications in physics, mathematics, engineering, biology, and so forth (see [1, 2] and references therein). While complex networks are ubiquitously found in nature and in the modern world, such as neural networks, socially interacting animal species, power networks, wireless sensor networks, Internet, and the World Wide Web.

In the past few decades, the synchronization problems in complex networks have attracted increasing attention. Many synchronization patterns have been studied, including complete synchronization [3], cluster synchronization [4–6], phase synchronization [7], and partial synchronization [8]. There are several control methods to guide the dynamics of a complex network to a desired state, such as adaptive control [9], feedback control [10], intermittent control [11], fuzzy control [12], impulsive control [13, 14], and pinning control [5, 6, 15, 16]. Synchronization of complex networks holds particular promise for applications to many fields [17– 21].

Synchronization in complex dynamical networks is realized via information exchanges among the interconnect nodes [22]. The signal traveling along real physical system is

usually perturbed randomly by the environmental elements, such as noises, the structures of the interconnections, time delays, and the positions of nodes [9]. One popular model is the Markovian switching model driven by continuous-time Markov chains in the sciences and industries (see [23-26] and references therein). In [23, 24], Mao et al. studied stability and controllability of stochastic differential delay equations with Markovian switching, while [25, 26] discussed the exponential stability of stochastic delayed neural networks. Liu et al. [26], on the other hand, investigated the synchronization of discrete-time stochastic complex networks with Markovian jumping and mode-dependent mixed time delays. In [16], Wang et al. investigated the mean-square exponential synchronization of stochastic complex networks with Markovian switching and time-varying delays by using the pinning control method, which is described as

$$dx_{i}(t) = \begin{cases} f(t, x_{i}(t), x_{i}(t - \tau(t))) \\ + \sum_{j=1, i \neq j}^{N} a_{ij}(r(t)) \Sigma(x_{j}(t) - x_{i}(t)) \end{cases}$$

where  $u_i(t)$  (i = 1, 2, ..., N) are the linear state feedback controllers that are defined by

$$u_{i}(t) = \begin{cases} -\varepsilon_{i}(x_{i}(t) - s(t)), & i = 1, 2, \dots, l, \\ 0, & i = l + 1, l + 2, \dots, N, \end{cases}$$
(2)

and  $\varepsilon_i > 0$  (i = 1, 2, ..., l) are the control gains.

Pinning control has been proved to be effective for the synchronization of complex dynamical networks with continuous state coupling [15, 16, 27]. In many systems, the impulsive effect is a common phenomenon due to instantaneous perturbations at certain moments [27, 28]. Impulsive control strategies have been widely used to stabilize and synchronize coupled complex dynamical systems, such as signal processing system, computer networks, automatic control systems, and telecommunications [13]. In [27], pinning impulsive strategy is proposed for the synchronization of stochastic dynamical networks with nonlinear coupling. Zhou et al. studied synchronization in complex delayed dynamical networks with impulsive effects in [28]. And Zhu et al., in [29], investigated the exponential stability of a class of stochastic neural networks with both Markovian jump parameters and mixed fixed time delays. Can the stochastic dynamical network with Markovian switching and timevarying delays be synchronized by impulsive pinning control? This paper is devoted to solving this problem.

In this paper, we study the synchronization of stochastic complex networks with Markovian switching by using the impulse control method. We consider a kind of stochastic complex networks with internal time-varying delayed couplings, Markovian switching, and Wiener processes. By applying the Lyapunov-Krasovskii functional method, Itö's formula and the linear matrix inequality (LMI), some sufficient conditions for synchronization of these networks are derived. Numerical examples are finally given to demonstrate the effectiveness of the proposed impulsive pinning strategy.

Notations. Throughout this paper,  $\mathbb{R}^n$  will denote the *n*-dimensional Euclidean space and  $\mathbb{R}^{n \times n}$  the set of all  $n \times n$  real matrices. The superscript *T* will denote the transpose of a matrix or a vector. And  $\text{Tr}(\cdot)$  stands for the trace of the corresponding matrix.  $\mathbf{1}_n = (1, 1, \dots, 1)^T \in \mathbb{R}^n$ , and  $I_n$  is the *n*-dimensional identity matrix. For square matrices *M*, the notation M > 0 (<0) will mean that *M* is a positive-definite (negative-definite) matrix.  $\lambda_{\max}(A)$  and  $\lambda_{\min}(A)$  will denote the greatest and least eigenvalues of a symmetric matrix, respectively.  $\check{p} = \max\{p_1, p_2, \dots, p_n\}$ , and  $\widehat{p} = \min\{p_1, p_2, \dots, p_n\}$ .

#### 2. Preliminaries

Let  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$  be a complete probability space with a filtration  $\{\mathcal{F}_t\}_{t\geq 0}$  that is right continuous with  $\mathcal{F}_0$  containing all the  $\mathcal{P}$ -null sets.  $C([-\tau, 0]; \mathbb{R}^n)$  will denote the family of continuous functions  $\phi$  from  $[-\tau, 0]$  to  $\mathbb{R}^n$  with the uniform norm  $\|\phi\|^2 = \sup_{-\tau \leq s \leq 0} \phi(s)^T \phi(s)$ . And  $C^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$  denotes the family of all  $\mathcal{F}_0$  measurable,  $C([-\tau, 0]; \mathbb{R}^n)$ -valued stochastic variables  $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$  such that  $\int_{-\tau}^0 \mathbb{E} \|\xi(s)\|^2 ds \leq \infty$ , where  $\mathbb{E}$  stands for the correspondent expectation operator with respect to the given probability measure  $\mathcal{P}$ .

Consider a complex network consisting of N identical nodes with Markovian switching

$$dx_{i}(t) = \begin{cases} f(x_{i}(t), x_{i}(t - \tau(t))) \\ + \sum_{j=1}^{N} a_{ij}^{[r(t)]} \Sigma x_{j}(t) \\ + \sum_{j=1}^{N} b_{ij}^{[r(t)]} \Sigma x_{j}(t - \tau_{c}(t)) \end{cases} dt \\ + \sigma_{i}^{[r(t)]} (x(t), x(t - \tau(t)), x(t - \tau_{c}(t))) dw(t), \\ i = 1, 2, \dots, N, \end{cases}$$
(3)

where  $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$  is the state vector of the *i*th node of the network,  $f(x_i(t), x_i(t - \tau(t))) = [f_1(x_i(t), x_i(t - \tau(t))), f_2(x_i(t), x_i(t - \tau(t))), \dots, f_n(x_i(t), x_i(t - \tau(t)))]^T$  is a continuous vector-valued function,  $\Sigma = \text{diag}\{\varrho_1, \varrho_2, \dots, \varrho_n\}$  is an inner coupling matrix of the networks that satisfies  $\varrho_j > 0$ ,  $j = 1, 2, \dots, n$ , and r(t) are the continuous-time Markov processes that describe the evolution of the nodes at time *t*. Here,  $A^{[r(t)]} = [a_{ij}^{[r(t)]}] \in \mathbb{R}^{n \times n}$  and  $B^{[r(t)]} = [b_{ij}^{[r(t)]}] \in \mathbb{R}^{n \times n}$  are the outer coupling matrices of the network at time *t* at nodes  $r(t), t - \tau_c(t)$ , respectively, such that  $a_{ij}^{[r(t)]} \ge 0$  for  $i \neq j$ ,  $a_{ii}^{[r(t)]} = -\sum_{j=i,j \neq i}^{N} a_{ij}^{[r(t)]}, b_{ij}^{[r(t)]} \ge 0$  for  $i \neq j$  and  $b_{ii}^{[r(t)]} = -\sum_{j=i,j \neq i}^{N} b_{ij}^{[r(t)]}$ . Figure 1 shows the topology structures of the switching networks for 5 nodes.  $\tau(t)$  is the inner time-varying delay satisfying  $\tau \ge \tau(t) \ge 0$  and  $\tau_c(t)$  is the coupling time-varying delay satisfying  $\tau_c \ge \tau_c(t) \ge 0$ . Finally,  $\sigma_i^{[r(t)]}(x(t), x(t - \tau(t)), x(t - \tau_c(t))) = \sigma_i^{[r(t)]}(x_1(t), \dots, x_n(t), x_1(t - \tau(t)), \dots, x_n(t - \tau_c(t))) = \sigma_i^{[r(t)]}(x_1(t), \dots, x_n(t), x_1(t - \tau(t)), \dots, x_n(t - \tau_c(t))) = \sigma_i^{[r(t)]}(x_1(t), \dots, x_n(t), x_1(t - \tau(t)), \dots, x_n(t - \tau_c(t)))$ .

$$\mathbb{E}w_{j}(t) = 0, \qquad \mathbb{E}w_{j}^{2}(t) = 1,$$

$$\mathbb{E}w_{i}(t)w_{i}(s) = 0 \qquad (s \neq t).$$
(4)



FIGURE 1: The topology structures of the switching networks for 5 nodes; (A1) and (A2) the topology structures of the coupling matrix  $A^1$  and  $A^2$ , respectively; (B1) and (B2) the topology structures of the coupling matrix  $B^1$  and  $B^2$ , respectively.

Let r(t), t > 0 be a right-continuous Markov chain on a probability space that takes values in a finite state space S = 1, 2, ..., M with a generator  $\Gamma = [\gamma_{ij}] \in \mathbb{R}^{M \times M}$  given by

$$P\left\{r\left(t+\Delta\right)=j\mid r\left(t\right)=i\right\}=\begin{cases}\gamma_{ij}\Delta+o\left(\Delta\right)&\text{if }i\neq j,\\1+\gamma_{ii}\Delta+o\left(\Delta\right)&\text{if }i=j,\end{cases}$$
(5)

for some  $\Delta > 0$ . Here  $\gamma_{ij} \ge 0$  is the transition rate from *i* to *j* if  $i \ne j$  and  $\gamma_{ii} = -\sum_{i \ne j} \gamma_{ij}$ . In this paper,  $A^{[r]}$  is assumed to be symmetric and irreducible, and  $B^{[r]}$  is assumed to be symmetric, for r = 1, 2, ..., M.

The initial conditions associated with (3) are

$$x_i(s) = \xi_i(s), \quad -\check{\tau} \le s \le 0, \quad i = 1, 2, \dots, N,$$
 (6)

where  $\check{\tau} = \max\{\tau(t), \tau_c(t)\}, \xi_i \in C^b_{\mathcal{F}_0}([-\check{\tau}, 0], \mathbb{R}^n)$  with the norm  $\|\xi_i\|^2 = \sup_{-\check{\tau} \le s \le 0} \xi_i(s)^T \xi_i(s)$ .

The impulse controllers are defined by

$$\Delta x_{i}(t_{k}) = x_{i}(t_{k}^{+}) - x_{i}(t_{k}^{-}) = \epsilon_{ik}x_{i}(t_{k}^{-}),$$
  

$$t = t_{k}, \quad k \in Z^{+}, \quad i = 1, 2, \dots, N,$$
(7)

where  $\epsilon_{ik}$  are constants, and  $\{t_1, t_2, t_3, ...\}$  is the impulsive sequence of strictly increasing impulsive instants satisfying  $\lim_{k \to \infty} t_k = +\infty$ , and  $t_k - t_{k-1} = T$  for k > 1.

In the case that system (3) reaches synchronization, that is,  $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$ , we have the following synchronized state equation:

$$ds(t) = f(s(t), s(t - \tau(t))) dt + \sigma(s(t), s(t - \tau(t))) dw(t).$$
(8)

In the paper, we would control the system (3) to the desired trajectory s(t). Define  $e_i(t) = x_i(t) - s(t)$  (i = 1, 2, ..., N) as the synchronization error. Then, according to the controller (7), the error system is

$$de_{i}(t) = \left\{ f\left(x_{i}(t), x_{i}(t-\tau(t))\right) - f\left(s(t), s(t-\tau(t))\right) + \sum_{j=1}^{N} a_{ij}^{[r(t)]} \Sigma e_{j}(t) + \sum_{j=1}^{N} b_{ij}^{[r(t)]} \Sigma e_{j}(t-\tau_{c}(t)) \right\} dt + \sigma_{i}^{[r(t)]} \left(e(t), e(t-\tau(t)), e(t-\tau_{c}(t))\right) dw(t), t \neq t_{k}, k \in \mathbb{Z}^{+}, i = 1, 2, ..., N,$$

$$\Delta e_{i}(t_{k}) = \epsilon_{ik} e_{i}(t_{k}^{-}), t = t_{k}, k \in \mathbb{Z}^{+}, i = 1, 2, ..., N,$$
(9)

where 
$$\sigma_i^{[r(t)]}(e(t), e(t - \tau(t)), e(t - \tau_c(t))) = \sigma_i^{[r(t)]}(x(t), x(t - \tau(t)), x(t - \tau_c(t))) - \sigma(s(t), s(t - \tau(t))).$$

*Definition 1* (see [16, 27]). The complex network (3) is said to be exponentially synchronized in mean square if the trivial solution of system (9) is such that

$$\sum_{i=1}^{N} \mathbb{E} \left\| e_i(t, t_0, \xi_i) \right\|^2 \le K e^{-\kappa t},$$
(10)

for some K > 0 and  $\kappa > 0$  under any initial data  $\xi_i \in \mathscr{C}^b_{\mathscr{F}_0}([-\check{\tau}, 0]; \mathbb{R}^n)$ .

*Definition 2* (see [9, 11, 16]). A continuous function f(x, y) :  $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$  is said to belong to the function class QUAD,

denoted by  $f \in \text{QUAD}(P, \Delta, \eta, \theta)$  for some given matrix  $\Sigma = \text{diag}\{\varrho_1, \varrho_2, \dots, \varrho_n\}$  if there exists a positive definite diagonal matrix  $P = \text{diag}\{p_1, p_2, \dots, p_n\}$ , a diagonal matrix  $\Delta = \text{diag}\{\delta_1, \delta_2, \dots, \delta_n\}$  and constants  $\eta > 0, \theta > 0$  such that  $f(\cdot)$  satisfies the condition

$$(x-y)^{T}P\left(\left(f\left(x,z\right)-f\left(y,w\right)\right)-\Delta\Sigma\left(x-y\right)\right)$$
  
$$\leq -\eta(x-y)^{T}\left(x-y\right)+\theta(z-w)^{T}\left(z-w\right)$$
(11)

for all  $x, y, z, w \in \mathbb{R}^n$ .

The following assumptions are usual and will be used throughout this paper for establishing the synchronization conditions [9, 11, 16].

- (H1)  $\tau(t)$  and  $\tau_c(t)$  are bounded and continuously differentiable functions such that  $0 < \tau(t) \le \tau$ ,  $\dot{\tau}(t) < \overline{\tau} < 1$ ,  $0 < \tau_c(t) \le \tau_c$  and  $\dot{\tau}_c(t) < \overline{\tau}_c < 1$ . Let  $\dot{\overline{\tau}} = \max{\{\overline{\tau}, \overline{\tau}_c\}}$ .
- (H2) There exist positive definite constant matrices  $\Upsilon_{i1}^{[r]}$ ,  $\Upsilon_{i2}^{[r]}$  and  $\Upsilon_{i3}^{[r]}$  for i = 1, 2, ..., N and r = 1, 2, ..., M such that

$$\operatorname{Tr} \left[ \sigma_{i}^{[r]}(e(t), e(t - \tau(t)), e(t - \tau_{c}(t)))^{T} \times \sigma_{i}^{[r]}(e(t), e(t - \tau(t)), e(t - \tau_{c}(t))) \right] \\ \leq \sum_{j=1}^{N} e_{j}(t)^{T} \Upsilon_{i1}^{[r]} e_{j}(t) \\ + \sum_{j=1}^{N} e_{j}(t - \tau(t))^{T} \Upsilon_{i2}^{[r]} e_{j}(t - \tau(t)) \\ + \sum_{j=1}^{N} e_{j}(t - \tau_{c}(t))^{T} \Upsilon_{i3}^{[r]} e_{j}(t - \tau_{c}(t)).$$

$$(12)$$

*Remark 3.* Considering Definition 2 and assumption (H2), there exists a unique solution of (9) under the initial data  $\xi_i \in \mathscr{C}^b_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$  (see [23, 24]).

**Lemma 4** (see [23, 24]). *Consider a stochastic delayed differential equation with Markovian switching of the form* 

$$dx(t) = f(x(t), x(t-\tau), r(t)) dt$$
  
+  $\sigma(x(t), x(t-\tau), r(t)) d\omega(t)$  (13)

on  $t \ge 0$  with initial value  $x_0 = \xi \in C^b_{F_0}([-\tau, 0]; \mathbb{R}^n)$ , where

$$f: \mathbb{R}^n \times \mathbb{R}^n \times S \to \mathbb{R}^n, \qquad \sigma: \mathbb{R}^n \times \mathbb{R}^n \times S \to \mathbb{R}^{n \times m}.$$
(14)

Let  $C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n; \mathbb{R}_+)$  be the family of all the nonnegative functions V(t, x, r) on  $\mathbb{R}_+ \times \mathbb{R}^n \times S$  that are twice continuously differentiable in x and once differentiable in t. Let  $V \in$ 

 $C^{2,1}(\mathbb{R}_+\times\mathbb{R}^n\times S;\mathbb{R}_+)$ . Define an operator  $\mathscr{D}V$  from  $\mathbb{R}^n\times\mathbb{R}_+\times S$  to  $\mathbb{R}^n$  by

$$\mathcal{L}V(t, x, r) = V_t(t, x, r) + V_x(t, x, r) f(x, r)$$
$$+ \frac{1}{2} \operatorname{Tr} \left[ \sigma(x, r)^T V_{xx} \sigma(x, r) \right]$$
$$+ \sum_{i=1}^M \gamma_{ij} V(t, x, j),$$
(15)

where  $V_t(t, x, r) = \partial V(t, x, r)/\partial t$ ,  $V_x(t, x, r) = (\partial V(t, x, r)/\partial x_1, \dots, \partial V(t, x, r)/\partial x_n)$ , and  $V_{xx}(t, x, r) = (\partial^2 V(t, x, r)/\partial x_i x_j)_{n \times n}$ . If  $V \in C^{2,1}(\mathbb{R}_+ \times \mathbb{R}^n \times S; \mathbb{R}_+)$ , and then

$$\mathbb{E}V(t, x(t), r) = \mathbb{E}V(t_0, x(t_0), r) + \mathbb{E}\int_{t_0}^t \mathscr{L}V(s, x(s), r) ds$$
(16)

for all  $\infty > t > t_0 \ge 0$ , as long as the expectations of the integrals exist.

#### 3. Main Result

In this section, we will deduce our main results.

**Theorem 5.** Let assumptions (H1) and (H2) be true and let  $f \in \text{QUAD}(P, \Delta, \eta, \theta)$ . If there exist positive constants  $\alpha_r$  and  $\beta_r$  such that

$$\begin{bmatrix} A^{[r]} + \check{\delta}I_N - \alpha_r I_N & \frac{B^{[r]}}{2} \\ \frac{B^{[r]}}{2} & -\beta_r I_N \end{bmatrix} \leq 0, \quad \text{for } r = 1, 2, \dots, M,$$

$$\check{\tau} \leq \theta T, \quad \check{\tau} \leq (1 - \theta) T, \quad 0 \leq \check{\overline{\tau}} \leq 1 - \frac{\check{q}\left(\check{b} + \check{c}\right)}{1 + \theta},$$

$$\varphi\left(\check{\tau} + T\right) + 2\ln\frac{\check{q}}{\hat{q}}\left|1 + \epsilon_v\right| - \gamma T < 0,$$

$$\left(\frac{1}{b_1 + c_1}, \frac{1}{b_2 + c_2}, \dots, \frac{1}{b_M + c_M}\right)^T > \widetilde{\Gamma}^{-1} \mathbf{1}_M,$$
(17)

where

à

$$\varphi = 1 + \theta + \gamma \check{q} + \frac{\check{b}\check{q}}{1 - \bar{\tau}} e^{\gamma \tau} + \frac{\check{c}\check{q}}{1 - \bar{\tau}_c} e^{\gamma \tau_c},$$

$$\widetilde{\Gamma} = \operatorname{diag} \{a_1, a_2, \dots, a_M\} + \Gamma,$$

$$= \max_{r \in S} a_r, \qquad a_r = \frac{\lambda_{\max} \left(-2\eta I_n + \check{p} \sum_{i=1}^N \Upsilon_{i1}^{[r]} + 2\alpha_r P\Sigma\right)}{\check{p}},$$

$$\check{b} = \max_{r \in S} b_r, \qquad b_r = \frac{\lambda_{\max} \left(\sum_{i=1}^N P\Upsilon_{i2}^{[r]} + 2\theta I_N\right)}{\widehat{p}},$$

$$\check{c} = \max_{r \in S} c_r, \qquad c_r = \frac{\lambda_{\max} \left(\sum_{i=1}^N P\Upsilon_{i3}^{[r]} + 2\beta_r P\Sigma\right)}{\widehat{p}}, \quad (18)$$

then the solutions  $e_1(t)$ ,  $e_2(t)$ ,..., and  $e_N(t)$  of system (9) are exponentially stable in mean square. It means that the complex dynamical network (3) can be exponentially controlled to the objective trajectory s(t) under the controllers (7).

*Proof.* By (44), there exists a sufficiently small constant  $\theta > 0$  such that

$$\left(\frac{1}{b_1+c_1}, \frac{1}{b_2+c_2}, \dots, \frac{1}{b_M+c_M}\right)^T \ge (1+\theta) \,\tilde{\Gamma}^{-1} \mathbf{1}.$$
(19)

Set  $(1 + \theta)\tilde{\Gamma}^{-1}\mathbf{1} = q = (q_1, q_2, \dots, q_M)^T$ . Then  $\tilde{\Gamma}q = (1 + \theta)\mathbf{1}_M$ , that is,

$$(b_r + c_r) q_r \le 1, \qquad a_r q_r + \sum_{s=1}^M \gamma_{rs} q_s = 1 + \theta.$$
 (20)

For  $1 \le r \le M$ , define the Lyapunov-Krasovskii function

$$V(e(t), r) = q_r \frac{1}{2} \sum_{i=1}^{N} e_i(t)^T P e_i(t), \qquad (21)$$

and let  $\tilde{e}^{k}(t) = (e_{1k}(t), e_{2k}(t), \dots, e_{Nk}(t))^{T}, k = 1, 2, \dots, n$ . For any  $t \in (t_{k-1}, t_{k}], k = 1, 2, \dots$ , by Lemma 4, we have  $\mathscr{L}V(e(t), r)$ 

$$\begin{split} &= q_{r} \sum_{i=1}^{N} e_{i}(t)^{T} P\left\{f\left(x_{i}\left(t\right), x_{i}\left(t-\tau\left(t\right)\right)\right)\right.\\ &\quad -f\left(s\left(t\right), s\left(t-\tau\left(t\right)\right)\right) + \sum_{j=1}^{N} a_{ij}^{[r]} \Sigma e_{j}\left(t\right)\\ &\quad +\sum_{j=1}^{N} b_{ij}^{[r]} \Sigma e_{j}\left(t-\tau_{c}\left(t\right)\right)\right\}\\ &\quad + \frac{1}{2} q_{r} \sum_{i=1}^{N} \operatorname{Tr}\left\{\sigma_{i}(x(t), x(t-\tau(t))), x(t-\tau_{c}(t)), r\right)^{T}\\ &\quad \times P \sigma_{i}\left(x\left(t\right), x\left(t-\tau\left(t\right)\right)\right), x\left(t-\tau_{c}\left(t\right)\right), r\right)\right\}\\ &\quad +\sum_{s=1}^{M} \gamma_{rs} q_{s} \frac{1}{2} \sum_{i=1}^{N} e_{i}(t)^{T} P e_{i}\left(t\right)\\ &\leq q_{r}\left\{-\eta \sum_{i=1}^{N} e_{i}(t)^{T} e_{i}\left(t\right) + \theta \sum_{i=1}^{N} e_{i}(t-\tau(t))^{T} e_{i}\left(t-\tau\left(t\right)\right)\\ &\quad +\sum_{k=1}^{n} p_{k} \varrho_{k} \overline{e}^{k}(t)^{T} \overline{e}^{k}\left(t\right) + \sum_{k=1}^{n} p_{k} \varrho_{k} \overline{e}^{k}(t)^{T} A^{[r]} \overline{e}^{k}\left(t\right)\\ &\quad + \frac{1}{2} \underline{p} \sum_{k=1}^{N} \left[\sum_{i=1}^{N} e_{i}(t)^{T} Y_{j1}^{[r]} e_{i}\left(t\right)\right] \end{split}$$

$$+ \sum_{i=1}^{N} e_i (t - \tau(t))^T \Upsilon_{j2}^{[r]} e_i (t - \tau(t))$$

$$+ \sum_{i=1}^{N} e_i (t - \tau_c (t))^T \Upsilon_{j3}^{[r]} e_i (t - \tau_c (t))$$

$$\begin{split} &+ \sum_{s=1}^{M} \gamma_{rs} q_{s} \frac{1}{2} \sum_{i=1}^{N} e_{i}(t)^{T} P e_{i}(t) \\ &= q_{r} \left\{ \sum_{i=1}^{N} e_{i}(t)^{T} \left( -\eta I_{N} + \frac{1}{2} \check{p} \sum_{j=1}^{N} \Upsilon_{j1}^{[r]} + \alpha_{r} P \Sigma \right) e_{i}(t) \\ &+ \sum_{i=1}^{N} e_{i}(t - \tau(t))^{T} \left( \theta + \frac{1}{2} \check{p} \sum_{j=1}^{N} \Upsilon_{j2}^{[r]} \right) e_{i}(t - \tau(t)) \\ &+ \sum_{i=1}^{N} e_{i}(t - \tau_{c}(t))^{T} \left( \frac{1}{2} \check{p} \sum_{j=1}^{N} \Upsilon_{j3}^{[r]} + \beta_{r} P \Sigma \right) e_{i}(t - \tau_{c}(t)) \right\} \\ &+ \sum_{s=1}^{M} \gamma_{rs} q_{s} \frac{1}{2} \sum_{i=1}^{N} e_{i}(t)^{T} P e_{i}(t) \\ &+ q_{r} \left\{ \sum_{k=1}^{n} p_{k} \varrho_{k} \bar{e}^{k}(t)^{T} \left[ A^{[r]} + \left( \check{\delta} - \alpha_{r} \right) I_{N} \right] \bar{e}^{k}(t) \\ &+ \sum_{k=1}^{n} p_{k} \varrho_{k} \bar{e}^{k}(t)^{T} B^{[r]} \bar{e}^{k}(t - \tau_{c}(t)) \\ &- \sum_{k=1}^{n} p_{k} \varrho_{k} \bar{e}^{k}(t - \tau_{c}(t))^{T} \beta_{r} \bar{e}^{k}(t - \tau_{c}(t)) \right\} \\ &\leq q_{r} \left\{ \sum_{i=1}^{N} e_{i}(t)^{T} \left( -\eta I_{N} + \frac{1}{2} \check{p} \sum_{j=1}^{N} \Upsilon_{j1}^{[r]} + \alpha_{r} P \Sigma \right) e_{i}(t) \\ &+ \sum_{i=1}^{N} e_{i}(t - \tau(t))^{T} \left( \theta I_{N} + \frac{1}{2} \check{p} \sum_{j=1}^{N} \Upsilon_{j2}^{[r]} \right) e_{i}(t - \tau(t)) \\ &+ \sum_{i=1}^{N} e_{i}(t - \tau_{c}(t))^{T} \left( \frac{1}{2} \check{p} \sum_{j=1}^{N} \Upsilon_{j3}^{[r]} + \beta_{r} P \Sigma \right) e_{i}(t - \tau_{c}(t)) \right\} \\ &+ \sum_{s=1}^{M} \gamma_{rs} q_{s} \frac{1}{2} \sum_{i=1}^{N} e_{i}(t)^{T} P e_{i}(t) . \end{split}$$

Let

$$E(t) = \frac{1}{2} \sum_{i=1}^{N} e_i(t)^T P e_i(t); \qquad (23)$$

then we have

$$\mathcal{L}V(t) \leq a_{r}q_{r}E(t) + b_{r}q_{r}E(t - \tau(t)) + c_{r}q_{r}E(t - \tau_{c}(t)) + \sum_{s=1}^{M}\gamma_{rs}q_{s}E(t), \qquad (24)$$

and by (20), we have

$$\mathscr{L}V(t) \le (1+\theta) E(t) + \check{b}\check{q}E(t-\tau(t)) + \check{c}\check{q}E(t-\tau_{c}(t)).$$
(25)

Define

$$W(t) = e^{\gamma t} V(t) . \qquad (26)$$

Use (25) to compute the operator

$$\mathscr{L}W(t) = e^{\gamma t} \left[ \gamma V(t) + \mathscr{L}V(t) \right]$$
  
$$\leq e^{\gamma t} \left[ \gamma \check{q}E(t) + (1+\theta)E(t) + \check{b}\check{q}E(t-\tau(t)) + \check{c}\check{q}E(t-\tau_c(t)) \right].$$
(27)

The generalized Itô's formula gives

$$e^{\gamma t} \mathbb{E}V(t) = e^{\gamma t_0} \mathbb{E}V(t_0) + \mathbb{E}\int_{t_0}^t \mathscr{D}W(s) \, ds, \qquad (28)$$

for any  $t_k > t > t_0 > t_{k-1} \ge 0$ . Hence we have

$$e^{\gamma t} \mathbb{E} V(t) \leq e^{\gamma t_{0}} \mathbb{E} V(t_{0})$$

$$+ \mathbb{E} \int_{t_{0}}^{t} e^{\gamma s} \left[ \gamma \check{q} E(s) + (1 + \theta) E(s) + \check{b} \check{q} E(s - \tau(s)) + \check{c} \check{q} E(s - \tau_{c}(s)) \right] ds$$

$$\leq \check{q} e^{\gamma t_{0}} \mathbb{E} E(t_{0}) + (\gamma \check{q} + 1 + \theta) \qquad (29)$$

$$\times \int_{t_{0}}^{t} e^{\gamma s} \mathbb{E} E(s) ds$$

$$+ \check{b} \check{q} e^{\gamma \tau} \int_{t_{0}}^{t} e^{\gamma (s - \tau(s))} \mathbb{E} E(s - \tau(s)) ds$$

$$+ \check{c} \check{q} e^{\gamma \tau_{c}} \int_{t_{0}}^{t} e^{\gamma (s - \tau_{c}(s))} \mathbb{E} E(s - \tau_{c}(s)) ds.$$

By changing variable  $s - \tau(s) = u$ , we have

$$\int_{t_0}^{t} e^{\gamma(s-\tau(s))} \mathbb{E}E\left(s-\tau\left(s\right)\right) ds$$
$$= \int_{t_0-\tau(t_0)}^{t-\tau(t)} e^{\gamma u} \mathbb{E}E\left(u\right) \frac{du}{1-\dot{\tau}\left(t\right)}$$
(30)
$$\leq \frac{1}{1-\bar{\tau}} \int_{t_0-\tau}^{t} e^{\gamma u} \mathbb{E}E\left(u\right) du.$$

Similarly, we have

$$\int_{t_0}^t e^{\gamma(s-\tau_c(s))} \mathbb{E}E\left(s-\tau\left(s\right)\right) ds \le \frac{1}{1-\overline{\tau}_c} \int_{t_0-\tau_c}^t e^{\gamma u} \mathbb{E}E\left(u\right) du.$$
(31)

Substituting (30) and (31) into (29), we get

$$e^{\gamma t} \mathbb{E}V(t) \le q_r e^{\gamma t_0} \mathbb{E}V(t_0) + \varphi \int_{t_0 - \check{\tau}}^t e^{\gamma u} \mathbb{E}E(u) \, du.$$
(32)

By using Gronwall inequality, we have

$$\mathbb{E}E\left(t\right) \leq \frac{\check{q}}{\widehat{q}} \mathbb{E}E\left(t_{0}\right) e^{\varphi\left(t-t_{0}+\check{\tau}\right)+\gamma\left(t_{0}-t\right)}.$$
(33)

On the other hand, from the construction of E(t), we have

$$E(t_k) \le (1 + \epsilon_k)^2 E(t_k^-), \qquad (34)$$

where  $|1 + \epsilon_k| = \max_{i=1,2,\dots,N} |1 + \epsilon_{ik}|$ . According to (33)-(34), let  $k = \lfloor (t - t_0)/T \rfloor$ , for any  $t \in [t_{k-1}, t_k)$ , and we get

$$\mathbb{E}V(t) \leq \frac{\check{q}}{\widehat{q}} \mathbb{E}E(t_k) e^{\varphi(t-t_k+\check{\tau})+\gamma(t_k-t)}$$

$$\leq \frac{\check{q}}{\widehat{q}} \mathbb{E}V(t_{k-1}^-) e^{\varphi(t-t_{k-1}+\check{\tau})+\gamma(t_k-t)+2\ln|1+\epsilon_{k-1}|} \qquad (35)$$

$$\leq \cdots \leq \left(\frac{\check{q}}{\widehat{q}}\right)^{k-1} \mathbb{E}E(0) e^{\varphi(t+k\check{\tau})-\gamma t+\sum_{\nu=1}^{k-1}2\ln|1+\epsilon_{\nu}|}.$$

Let  $|1 + \epsilon| = \max_{\nu \in Z^+} |1 + \epsilon_{\nu}|$ , and we have

$$\mathbb{E}E(t) \le \mathbb{E}E(0) e^{\varphi(t+k\check{\tau})-\gamma t+2(k-1)\ln(\check{q}/\hat{q})|1+\epsilon|}.$$
 (36)

Using condition (43) of Theorem 5, there exist a number  $\eta$  such that  $\mathbb{E}E(t) \leq \mathbb{E}E(t_0)e^{-\eta t}$ . Hence,  $\mathbb{E}||e_i(t)|| \leq (E(t_0)/\check{p})^{1/2}e^{-(\eta/2)(t-t_0)}$ . The proof of Theorem 5 is completed.

*Remark 6.* The stochastic networks studied before are without topological switch, and the time delays are always assumed to be fixed. However, for the sake of applications in the real work, these two points above should be taken into consideration. Of course, it will enhance the difficulties of the investigations on this network. For example, if the network has Markovian switching topology, the structure of the network is fast varying and the Lyapunov function is hard to be determined. By using the Lyapunov-Krasovskii functional, Itö's formula, and LMI, the exponential stability criterion of the pinning impulsive controlled Markovian switching stochastic dynamical network with time-varying delays was obtained. This also showed that the impulsive pinning control is a kind of cheap control strategy for guiding complex dynamical networks to the objective trajectory.

To make Theorem 5 more applicative, we give the following corollaries.

When complex dynamic networks (3) are considered without coupled delay time ( $B^{[r(t)]} = B$ ), we can get the following corollary.

**Corollary 7.** Let assumptions (H1) and (H2) be true and let  $f \in \text{QUAD}(P, \Delta, \eta, \theta)$ . If there exist positive constants  $\alpha_r$  and  $\beta_r$  such that

$$A^{[r]} + \check{\delta}I_N - \alpha_r I_N \leq 0, \quad \text{for } r = 1, 2, \dots, M,$$
  

$$\tau \leq \theta T, \qquad \tau \leq (1 - \theta) T, \qquad 0 \leq \overline{\tau} \leq 1 - \frac{\check{q}\check{b}}{1 + \theta},$$
  

$$\varphi \left(\check{\tau} + T\right) + 2\ln\frac{\check{q}}{\widehat{q}} |1 + \epsilon| - \gamma T < 0,$$
  

$$\left(\frac{1}{b_1}, \frac{1}{b_2}, \dots, \frac{1}{b_M}\right)^T > \widetilde{\Gamma}^{-1} \mathbf{1}_M,$$
(37)

where

$$\varphi = 1 + \theta + \gamma \check{q} + \frac{\check{b}\check{q}}{1 - \overline{\tau}} e^{\gamma\tau},$$

$$\widetilde{\Gamma} = \operatorname{diag} \left\{ a_1, a_2, \dots, a_M \right\} + \Gamma,$$

$$\check{a} = \max_{r \in S} a_r, \qquad a_r = \frac{\lambda_{\max} \left( -2\eta I_n + \check{p} \sum_{i=1}^N \Upsilon_{i1}^{[r]} + 2\alpha_r P \Sigma \right)}{\check{p}},$$

$$\check{b} = \max_{r \in S} b_r, \qquad b_r = \frac{\lambda_{\max} \left( \sum_{i=1}^N P \Upsilon_{i2}^{[r]} + 2\theta I_N \right)}{\widehat{p}},$$
(38)

then the solutions  $e_1(t), e_2(t), \ldots$ , and  $e_N(t)$  of system (9) are exponentially stable in mean square.

In another case, when we consider the system (3) without Markov switching, that is,  $A^{[r(t)]} = A$ ,  $B^{[r(t)]} = B$ , and  $\sigma_i^{[r(t)]}(e(t), e(t - \tau(t)), e(t - \tau_c(t))) = \sigma_i(e(t), e(t - \tau(t)), e(t - \tau_c(t)))$ , we can get another corollary.

**Corollary 8.** Let assumptions (H1) and (H2) be true and let  $f \in \text{QUAD}((P, \Delta, \eta, \theta))$ . If there exist positive constants  $\alpha$  and  $\beta$  such that

$$\begin{bmatrix} A + \check{\delta}I_N - \alpha I_N & \frac{B}{2} \\ \frac{B}{2} & -\beta I_N \end{bmatrix} \le 0,$$
  

$$\check{\tau} \le \theta T, \quad \check{\tau} \le (1 - \theta) T, \qquad 0 \le \check{\tau} \le 1 - \frac{b + c}{a}, \quad (39)$$
  

$$\varphi \left(\check{\tau} + T\right) + 2\ln\left|1 + \epsilon_v\right| - \gamma T < 0,$$
  

$$h + c \le a.$$

where

$$\varphi = a + \gamma + \frac{b}{1 - \overline{\tau}} e^{\gamma \tau} + \frac{c}{1 - \overline{\tau}_c} e^{\gamma \tau_c},$$

$$a = \frac{\lambda_{\max} \left(-2\eta I_n + \check{p} \sum_{i=1}^N \Upsilon_{i1} + 2\alpha P\Sigma\right)}{\check{p}},$$

$$b = \frac{\lambda_{\max} \left(\sum_{i=1}^N P\Upsilon_{i2} + 2\theta I_N\right)}{\widehat{p}},$$

$$c = \frac{\lambda_{\max} \left(\sum_{i=1}^N P\Upsilon_{i3} + 2\beta_r P\Sigma\right)}{\widehat{p}},$$
(40)

then the solutions  $e_1(t), e_2(t), \ldots$ , and  $e_N(t)$  of system (9) are exponentially stable in mean square.

Furthermore, when we address the system (3) with fixed time delays, that is,  $\tau(t) = \tau$ ,  $\tau_c(t) = \tau_c$ , the following corollary also can be obtained.

**Corollary 9.** Let assumptions (H1) and (H2) be true and let  $f \in \text{QUAD}(P, \Delta, \eta, \theta)$ . If there exist positive constants  $\alpha_r$  and  $\beta_r$  such that

$$\begin{bmatrix} A^{[r]} + \check{\delta}I_N - \alpha_r I_N & \frac{B^{[r]}}{2} \\ \frac{B^{[r]}}{2} & -\beta_r I_N \end{bmatrix} \le 0, \quad \text{for } r = 1, 2, \dots, M,$$
(41)

$$\check{\tau} \le \theta T, \qquad \check{\tau} \le (1-\theta) T,$$
 (42)

$$\varphi\left(\check{\tau}+T\right)+2\ln\frac{\check{q}}{\hat{q}}\left|1+\epsilon_{\nu}\right|-\gamma T<0, \tag{43}$$

$$\left(\frac{1}{b_1+c_1}, \frac{1}{b_2+c_2}, \dots, \frac{1}{b_M+c_M}\right)^T > \tilde{\Gamma}^{-1} \mathbf{1}_M, \quad (44)$$

where

ă

$$\varphi = 1 + \theta + \gamma \check{q} + \check{b} \check{q} e^{\gamma \tau} + \check{c} \check{q} e^{\gamma \tau_c},$$

$$\widetilde{\Gamma} = \operatorname{diag} \{a_1, a_2, \dots, a_M\} + \Gamma,$$

$$= \max_{r \in S} a_r, \qquad a_r = \frac{\lambda_{\max} \left(-2\eta I_n + \check{p} \sum_{i=1}^N \Upsilon_{i1}^{[r]} + 2\alpha_r P \Sigma\right)}{\check{p}},$$

$$\check{b} = \max_{r \in S} b_r, \qquad b_r = \frac{\lambda_{\max} \left(\sum_{i=1}^N P \Upsilon_{i2}^{[r]} + 2\theta I_N\right)}{\widehat{p}},$$

$$\check{c} = \max_{r \in S} c_r, \qquad c_r = \frac{\lambda_{\max} \left(\sum_{i=1}^N P \Upsilon_{i3}^{[r]} + 2\beta_r P \Sigma\right)}{\widehat{p}},$$
(45)

then the solutions  $e_1(t), e_2(t), \ldots$ , and  $e_N(t)$  of system (9) are exponentially stable in mean square.

*Remark 10.* In [29], the exponential stability of a class of stochastic neural networks with both Markovian jump parameters and mixed fixed time delays were investigated. Therefore, we could see our results as a further research about the stochastic dynamic network of [29].

#### 4. Numerical Simulation

In this section, we present some numerical simulation results that validate the theorem in the previous section.

Consider the chaotic delayed neural network

$$ds(t) = \{-Cs(t) + Af(s(t)) + Bg(s(t - \tau(t)))\} dt + \sigma(s(t), s(t - \tau(t))) dw(t),$$
(46)

where  $f(s) = g(s) = \tanh(s), \tau(t) = 1, \sigma(s(t), s(t - \tau(t))) = \text{diag}\{s_1(t), s_2(t)\},\$ 

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad A = \begin{bmatrix} 2 & -0.1 \\ -5 & 4.5 \end{bmatrix},$$
$$B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{bmatrix}.$$
(47)



FIGURE 2: The trajectories of the state variables of  $x_{i1}$  and  $x_{i2}$  (i = 1, 2, ..., 5) in system (48) by impulse control.



FIGURE 3: The time evolution of  $e_{i1}$  and  $e_{i2}$  (i = 1, 2, ..., 5) in system (48) by impulse control.

(49)

Taking  $P = \text{diag}\{1, 2\}$  and  $\Delta = \text{diag}\{5, 11, 5\}$ , we have  $\eta = 0.15$  and  $\theta = 3.25$  so that condition (11) is satisfied. Thus

$$dx_{i}(t) = \left\{ f\left(x_{i}(t), x_{i}(t-\tau(t))\right) + \sum_{j=1}^{5} a_{ij}^{[r]} \Sigma x_{j}(t) + \sum_{j=1}^{5} b_{ij}^{[r]} \Sigma x_{j}(t-\tau_{c}(t)) \right\} dt + \sigma_{i}^{[r]} \left(x(t), x(t-\tau(t)), x(t-\tau_{c}(t))\right) dw_{i}(t),$$

$$i = 1, 2, \dots, 5, \quad r = 1, 2,$$
(48)

and 
$$\Gamma = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix}, \tau_c(t) = 0.1(e^t/(1+e^t)),$$
  
 $\sigma_i^{[1]}(x(t), x(t-\tau(t)), x(t-\tau_c(t)))$   
 $= 0.1 \operatorname{diag} \{x_{i1}(t), x_{i2}(t)\},$   
 $\sigma_i^{[2]}(x(t), x(t-\tau(t)), x(t-\tau_c(t)))$   
 $= 0.1 \operatorname{diag} \{x_{i1}(t-\tau(t)), x_{i2}(t-\tau(t))\}.$ 

Computations then yield  $\tau = 1$ ,  $\overline{\tau} = 0$ ,  $\tau_c = 0.1$ ,  $\overline{\tau}_c = 0.1$ and  $\Upsilon_{ij} = 0.1I_2$  for i = 1, 2, ..., N. Then the solutions of inequalities (41)–(44) are (by using the Matlab LMI toolbox)  $\alpha_1 = 2.500$ ,  $\beta_1 = 0.001$ ,  $a_1 = 4.305$ ,  $b_1 = 5.055$ ,  $c_1 = 0.109$ ;  $\alpha_2 = 4.006$ ,  $\beta_2 = 0.009$ ,  $a_2 = 5.105$ ,  $b_2 = 5.030$ , and  $c_2 = 0.065$ . The initial conditions for this simulation are  $x_{ij}(t_0)$  which are constants, for i = 1, 2, ..., 5, j = 1, 2 and the trajectories of the impulse control gains are shown in Figure 2. Figure 3 shows the time evolution of the synchronization errors with impulse control.

#### 5. Conclusion

In this paper, we investigated the synchronization problem for stochastic complex networks with Markovian switching and nondelayed and time-varying delayed hybrid couplings. We achieved synchronization by applying an impulse control scheme to a small fraction of the nodes and derived sufficient conditions for stability of synchronization. Finally, we considered some numerical examples that illustrate the theoretical analysis.

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