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Research Article

An Inventory-Theory-Based Inexact Multistage Stochastic Programming Model for Water Resources Management

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An inventory-theory-based inexact multistage stochastic programming (IB-IMSP) method is developed for planning water resources systems under uncertainty. The IB-IMSP is based on inexact multistage stochastic programming and inventory theory. The IB-IMSP cannot only effectively handle system uncertainties represented as probability density functions and discrete intervals but also efficiently reflect dynamic features of system conditions under different flow levels within a multistage context. Moreover, it can provide reasonable transferring schemes (i.e., the amount and batch of transferring as well as the corresponding transferring period) associated with various flow scenarios for solving water shortage problems. The applicability of the proposed IB-IMSP is demonstrated by a case study of planning water resources management. The solutions obtained are helpful for decision makers in not only identifying different transferring schemes when the promised water is not met, but also making decisions of water allocation associated with different economic objectives.

1. Introduction

With speedy population growth, the constantly increasing demand for water in terms of both sufficient quantity and satisfied quality has forced a number of researchers to draw optimal water resources management policies [1–5]. However, water resources systems can be complex with uncertainties which may exist in technical, social, environmental, political, and financial factors. In addition, these complexities and uncertainties could be multiplied by not only the dynamic characteristics of the system, but also the related economic deficits if the targeted demand is not met. Furthermore, because of the temporal and spatial variations of the relationships between water demand and supply, the desired schemes for water allocation may also vary dynamically [6]. Correspondingly, insufficient water may be encountered particularly in the case of continuously low flow levels over a long period. Therefore, it is deemed necessary to develop effective optimization methods for supporting water resources management under such complexities.

Previously, numerous methods were developed for planning water resources management systems under various uncertainties [7–12]. Among them, two-stage stochastic programming (TSP) was an effective technique for problems where an analysis of policy scenarios was desired and the related coefficients are random with known probability distributions. The fundamental idea behind TSP is the concept of recourse, which is the ability to take corrective actions after a random event has taken place [13, 14]. In the past decades, TSP was developed and applied in water resources management under uncertainty [15–21]. However, TSP can only take recourse actions at the second stage to correct any infeasibility, and thus, it can hardly reflect the dynamic variations of system conditions, especially for multistage problems with a sequential structure. To address such a dynamic characteristic, a lot of multistage stochastic programming (MSP) methods were proposed as extensions of dynamic stochastic optimization approaches [13, 22–28]. MSP was improved upon the conventional TSP methods by permitting revised decisions in each time stage based on the information of sequentially realized uncertain events [29]. The uncertain information in an MSP was often modeled through a multilayer scenario tree. The primary advantage of

scenario-based stochastic programming was the flexibility it offered in modeling the decision process and defining the scenarios, particularly if the state dimension was high [22]. A few researchers applied the MSP to water resources management under uncertainty [24, 30–32]. For example, Li and Huang [31] developed a fuzzy-stochastic-based violation analysis approach for the planning of water resources management systems with uncertain information, based on a multistage fuzzy-stochastic integer programming model. Zhou et al. [32] developed a factorial multistage stochastic programming approach to obtain the desired water-allocation schemes and maximize the total net benefit under multiple uncertainties. Although the MSP was useful for dealing with probabilistic uncertainties within the optimization framework, its recourse action was to minimize penalties resulted from water shortages, not to provide a useful alternative to solve this problem positively. Actually, in the case of insufficiency, only penalties analysis is not enough, and more efforts are needed to solve the insufficiency corresponding to various penalties. Undoubtedly, transferring water from abundant regions would be a preferred choice for the water shortage problem. In this case, three questions should be answered by the managers: (a) how much is the amount of transferring water associated with different flow levels? (b) How much is the transferring batch corresponding to varied transferring amount each time? (c) How long is the transferring period between every two transferring actions? Fortunately, all these challenges can be well responded by the inventory theory. The aim of the inventory theory is to design schemes for managers to maximize system benefit/minimize system cost as well as guarantee the users' demand for materials.

In the past decades, a number of methods based on inventory theory were developed for solving the problems of materials' supply and demand [33–38]. For instance, Axsäter [39] considered a two-echelon distribution inventory system with a central warehouse and a number of retailers, where the system is controlled by continuous review installation stock policies with given batch quantities; Gupta et al. [40] proposed a discrete-time model for setting clearance prices for clearing retail inventories of fashion goods, where a heuristic procedure was developed to find near-optimal prices; Yadavalli et al. [41] considered a continuous review inventory system at a service facility, wherein an item demanded by a customer was issued to him/her only after performing service of random duration on the item; Arnold et al. [42] developed a deterministic optimal control approach optimizing the procurement and inventory policy of an enterprise that is processing a raw material when the purchasing price, holding cost, and the demand rate are fluctuating over time; Schmitt et al. [43] modeled a retailer whose supplier was subject to complete supply disruptions, where discrete event uncertainty and continuous sources of uncertainty were combined. Tsao and Lu [44] addressed an integrated facility location and inventory allocation problem through considering two types of transportation cost discounts: quantity discounts for inbound transportation cost and distance discounts for outbound transportation cost. However, most of the past inventory models were rarely developed and applied in water resources management. Although Suo et al. [21] proposed an inventory-theory-based two-stage stochastic programming model for solving water shortage problem, this model did not consider the dynamic variations of system conditions, particularly for sequential influences of different flow levels among multiple stages. In addition, multiple uncertainties existed in water resources management systems, such as the continuously changed water availabilities, various targeted water demand associated with timely policy scenarios, and fluctuant water benefit as well as related transferring cost. The conventional inexact optimization methods had difficulties in tackling such complexities.

Therefore, as an extension of the previous efforts, an inventory-theory-based inexact multistage stochastic programming model (IB-IMSP) will be developed for supporting water resources management planning. The IB-IMSP is an integrated method of inventory theory, inexact optimization, and multistage stochastic programming. It can tackle uncertainties represented as not only probability density functions but also discrete intervals as well as identify the system dynamics and decision processes under a series of scenarios. In addition, water transferring is exercised with recourse against any infeasibility, which allows exhaustive analyses of different policy scenarios with respect to varied levels of economic consequences if the targeted water allocations are infringed. Correspondingly, reasonable transferring schemes (the transferring amount, batch, and the corresponding transferring period) associated with various flow scenarios would be provided for decision makers for solving water shortage problem. A hypothetical case study of water resources management will then be provided to validate the applicability of the developed approach. The results obtained can help the managers gain insight into the water resources management with maximizing economic objectives and satisfying targeted water demands from users.

2. Methodology

2.1. Inexact Multistage Linear Programming. In many real-world problems, uncertainties can be denoted as random variables, and the related study systems are of dynamic feature. Thus, the relevant decisions must be made at each time stage under varying probability levels. Such a problem can be formulated as a scenario-based multistage stochastic programming (MSP) model with recourse as follows:

$$\max \quad f = \sum_{t=1}^{T} U_t X_t - \sum_{t=1}^{T} \sum_{h=1}^{h_t} p_{th} K_{th} Y_{th} \tag{1a}$$

$$\text{s.t.} \quad A_{rt} X_t \le B_{rt}, \quad r = 1, 2, \ldots, m_1;\ t = 1, 2, \ldots, T, \tag{1b}$$

$$A_{it} X_t + A'_{ith} Y_{th} \ge w_{ith}, \quad i = 1, 2, \ldots, m_2;\quad t = 1, 2, \ldots, T;\ h = 1, 2, \ldots, h_t, \tag{1c}$$

$$x_{jt} \ge 0, x_{jt} \in X_t, \quad j = 1, 2, \ldots, n_1;\ t = 1, 2, \ldots, T, \tag{1d}$$

$$y_{jth} \ge 0, y_{jth} \in Y_{th}, \quad j = 1, 2, \ldots, n_1;\quad t = 1, 2, \ldots, T;\ h = 1, 2, \ldots, h_t, \tag{1e}$$

where p_{th} is the probability of occurrence for scenario h in period t, with $p_{th} \geq 0$ and $\sum_{h=1}^{h_t} p_{th} = 1$. K_{th} are coefficients of recourse variables (Y_{th}) in the objective function; A'_{ith} are coefficients of Y_{th} in constraint i; w_{ith} is the random variable of constraint i, which is associated with probability levels q_{th}; h_t is the number of scenarios in period t, with the total being $H = \sum_{t=1}^{T} h_t$. In model (1a)–(1e), the decision variables are divided into two subsets: those that must be determined before the realizations of random variables are known (i.e., x_{jt}), and those (recourse variables) that can be determined after the realized random-variable values are available (i.e., y_{jth}).

Obviously, model (1a)–(1e) can only deal with uncertainties in the right-hand sides expressed as PDFs (probability density functions) when coefficients in A and U are deterministic. However, in real-world problems, the quality of information that can be obtained is often not good enough to be expressed as probabilistic distributions; in addition, even though these distributions are available, reflection of them in large-scale MSP models could be extremely challenging [45]. Correspondingly, interval parameters can be introduced into the multistage programming framework to identify uncertainties in parameters. This leads to an integrated inexact MSP (IMSP) model as follows:

$$\text{Max} \quad f^{\pm} = \sum_{t=1}^{T} U_t^{\pm} X_t^{\pm} - \sum_{t=1}^{T} \sum_{h=1}^{h_t} p_{th} K_{th}^{\pm} Y_{th}^{\pm} \tag{2a}$$

$$\text{s.t.} \quad A_{rt}^{\pm} X_t^{\pm} \leq B_{rt}^{\pm}, \quad r = 1, 2, \ldots, m_1; \quad t = 1, 2, \ldots, T, \tag{2b}$$

$$A_{it}^{\pm} X_t^{\pm} + A_{ith}^{\pm\prime} Y_{th}^{\pm} \geq w_{ith}^{\pm}, \quad i = 1, 2, \ldots, m_2; \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{2c}$$

$$x_{jt}^{\pm} \geq 0, \; x_{jt}^{\pm} \in X_t^{\pm}, \quad j = 1, 2, \ldots, n_1; \; t = 1, 2, \ldots, T, \tag{2d}$$

$$y_{jth}^{\pm} \geq 0, y_{jth}^{\pm} \in Y_{th}^{\pm}, \quad j = 1, 2, \ldots, n_1; \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{2e}$$

where $U_t^{\pm}$, $X_t^{\pm}$, $K_{th}^{\pm}$, $Y_{th}^{\pm}$, $A_{rt}^{\pm}$, $B_{rt}^{\pm}$, $A_{it}^{\pm}$, $A_{ith}^{\pm\prime}$, and $w_{ith}^{\pm}$ are interval parameters/variables. An interval is defined as a number with known upper and lower bounds but unknown distribution information [46]. Let U_t^{-} and U_t^{+} be the lower and upper bounds of $U_t^{\pm}$, respectively. When $U_t^{-} = U_t^{+}$, $U_t^{\pm}$ becomes a deterministic number.

However, in water resources management system, with the shortage of water availability, water transferring from other abundant regions is considered as an adaptive measure to meet the water demands in arid regions. In this case, it is noticed that the reservoir storage capacity, available water transferring, and the related costs happened in the transferring process (e.g., communication cost, unit cost, and reservoir's protection cost) should be considered. In addition, transferring too much water cannot only make the cost of reservoir operation, insurance, and protection increase, but also can bring on a high risk for the reservoir's storage capacity; transferring too little water is not enough to satisfy the water demand, and can increase the transferring times. Hence, reasonable transferring batch size and period are needed to optimize the transferring process, which is actually an inventory problem. Therefore, it is necessary to introduce inventory theory into the water resources management system [21].

2.2. Inventory-Theory-Based Inexact Multistage Stochastic Programming. The aim of inventory theory is to ascertain rules that managers can use to minimize the cost (maximize the benefit) associated with balancing the materials' supply and demand for different users. Supposing that one material should be purchased or produced and its shortage is not allowed, the demand is D units per unit time. The relative costs include S (setup cost for ordering one batch (\$)), C (unit cost for purchasing or producing each unit (\$/unit)), and C_1 (holding cost per unit per unit of time held in inventory (\$/month)). In detail, setup cost means all the costs for ordering one batch to replenish the storage, including the handling charge, communication expenses, and travelling expenses encountered in the ordering process; unit cost is the purchase or produce cost for one unit; holding cost represents all the costs associated with the storage of the inventory until it is used, including the cost of capital tied up, space, insurance, protection, and taxes attributed to storage. The objective is to determine when and how much to replenish inventory in order to minimize the sum of the produce or purchase costs per unit time [21]. Correspondingly, a basic EOQ model can be formulated as follows:

$$f(Q) = \frac{C_1 Q}{2} + \frac{SD}{Q} + CD, \tag{3a}$$

where Q is the purchasing or producing batch in the period of t; $C_1 Q/2$ is the holding cost per period; SD/Q is the ordering cost per period; CD is the purchase or produce cost per period; $f(Q)$ is the total cost, which is a function of Q. By setting the first derivative of $f(Q)$ to zero (and noting that the second derivative is positive), the economic order quantity (batch) can be obtained as follows:

$$Q^* = \sqrt{\frac{2SD}{C_1}}. \tag{3b}$$

Accordingly, the purchasing or producing period $t$can be obtained by the following equation:

$$t = \frac{Q}{D}. \tag{3c}$$

In this case, the total cost $f(Q)$ would reach its minimum value. Model (3a)–(3c) is the actual economic order quantity (EOQ) model, which can effectively tackle the complexities of inventory theory issues associated with purchasing or producing batch and period. Actually, the similar inventory problems exist in water resources management system. For

example, the excessive transferring water could bring on risks for reservoir capacity and water waste. Too short a transferring period could increase the transferring cost and bring on inconvenience in management; too long a transferring period may not meet the water demand, and both of these could cause economic losses. Consequently, a comprehensive method including both the advantages of EOQ model and IMSP model is needed, which leads to an inventory-theory-based inexact multistage stochastic programming (IB-IMSP) model. Concretely, IB-IMSP can be formulated as follows:

$$\text{Max} \quad f^{\pm} = \sum_{t=1}^{T} U_t^{\pm} X_t^{\pm} - \sum_{t=1}^{T} \sum_{h=1}^{h_t} p_{th} \left(\frac{1}{2} C_{1t}^{\pm} Q_{th}^{\pm} + C_t^{\pm} D_{th}^{\pm} + \frac{S_t^{\pm} D_{th}^{\pm}}{Q_{th}^{\pm}} \right) \tag{4a}$$

$$\text{s.t.} \quad A_{rt}^{\pm} X_t^{\pm} \le B_{rt}^{\pm}, \quad r = 1, 2, \ldots, m_1; \quad t = 1, 2, \ldots, T, \tag{4b}$$

$$A_{it}^{\pm} X_t^{\pm} + \left(A'_{ith} \right)^{\pm} D_{th}^{\pm} \ge w_{ith}^{\pm}, \quad i = 1, 2, \ldots, m_2; \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{4c}$$

$$x_{jt}^{\pm} \ge 0, \; x_{jt}^{\pm} \in X_t^{\pm}, \quad j = 1, 2, \ldots, n_1; \; t = 1, 2, \ldots, T, \tag{4d}$$

$$D_{th}^{\pm}, \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t. \tag{4e}$$

According to (3b), the batch size can be replaced by a function of D. Therefore, model (4a)–(4e) can be transferred as follows:

$$\text{Max} \quad f^{\pm} = \sum_{t=1}^{T} U_t^{\pm} X_t^{\pm} - \sum_{t=1}^{T} \sum_{h=1}^{h_t} p_{th} \left(\frac{1}{2} C_{1t}^{\pm} \sqrt{\frac{2 S_t^{\pm} D_{th}^{\pm}}{C_{1t}^{\pm}}} + C_t^{\pm} D_{th}^{\pm} + \sqrt{\frac{S_t^{\pm} D_{th}^{\pm} C_{1t}^{\pm}}{2}} \right) \tag{5a}$$

$$\text{s.t.} \quad A_{rt}^{\pm} X_t^{\pm} \le B_{rt}^{\pm}, \quad r = 1, 2, \ldots, m_1; \quad t = 1, 2, \ldots, T, \tag{5b}$$

$$A_{it}^{\pm} X_t^{\pm} + \left(A'_{ith} \right)^{\pm} D_{th}^{\pm} \ge w_{ith}^{\pm}, \quad i = 1, 2, \ldots, m_2; \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{5c}$$

$$x_{jt}^{\pm} \ge 0, \; x_{jt}^{\pm} \in X_t^{\pm}, \quad j = 1, 2, \ldots, n_1; \; t = 1, 2, \ldots, T, \tag{5d}$$

$$D_{th}^{\pm}, \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{5e}$$

$$Q_{th}^{\pm} = \sqrt{\frac{2 S_t^{\pm} D_{th}^{\pm}}{C_{1t}^{\pm}}}. \tag{5f}$$

In (5a), let $f^{\pm} = f_1^{\pm} - f_2^{\pm}$, in which $f_1^{\pm} = \sum_{t=1}^{T} U_t^{\pm} X_t^{\pm}$, and $f_2^{\pm} = \sum_{t=}^{T} \sum_{h=1}^{h_t} p_{th} (1/2 C_{1t}^{\pm} \sqrt{2 S_t^{\pm} D_{th}^{\pm} / C_{1t}^{\pm}} + C_t^{\pm} D_{th}^{\pm} + \sqrt{S_t^{\pm} D_{th}^{\pm} C_{1t}^{\pm} / 2})$. Since $C_{1t}^{\pm}$, $S_t^{\pm}$, and $C_t^{\pm}$ are parameters about different costs and $D_{th}^{\pm}$ is variable about transferring water, all of them would be greater than zero. In addition, p_{th} is probability, and thus, it can be easily obtained that $f_2^{\pm} \ge 0$ and $f_2^{\pm}$ is an increasing function of $C_{1t}^{\pm}$, $S_t^{\pm}$, $C_t^{\pm}$, and $D_{th}^{\pm}$. In this case, $|f_2|^{-} \text{Sign}(f_2^{-}) = f_2^{-}$ and $|f_2|^{+} \text{Sign}(f_2^{+}) = f_2^{+}$. Accordingly, model (5a)–(5f) can be converted into two deterministic submodels based on a two-step interactive algorithm [47]. The submodel for f^{+} can be formulated in the first step when the system objective is to be maximized; the other submodel (corresponding to f^{-}) can then be formulated based on the solution of the first submodel. Therefore, the first submodel is

$$\text{Max} \quad f^{+} = \sum_{t=1}^{T} \left(\sum_{j=1}^{j_1} u_{jt}^{+} x_{jt}^{+} + \sum_{j=j_1+1}^{n_1} u_{jt}^{+} x_{jt}^{-} \right) - \sum_{t=1}^{T} \sum_{h=1}^{h_t} p_{th} \left(\frac{1}{2} C_{1t}^{-} \sqrt{\frac{2 S_t^{-} D_{th}^{-}}{C_{1t}^{-}}} + C_t^{-} D_{th}^{-} + \sqrt{\frac{S_t^{-} D_{th}^{-} C_{1t}^{-}}{2}} \right) \tag{6a}$$

$$\text{s.t.} \quad \sum_{j=1}^{j_1} \left| a_{rjt} \right|^{-} \text{sign} \left(a_{rjt}^{-} \right) x_{jt}^{+} + \sum_{j=j_1+1}^{n_1} \left| a_{rjt} \right|^{+} \text{sign} \left(a_{rjt}^{+} \right) x_{jt}^{-} \le B_{rt}^{+}, \quad r = 1, 2, \ldots, m_1; \; t = 1, 2, \ldots, T, \tag{6b}$$

$$\sum_{j=1}^{j_1} \left| a_{ijt} \right|^{-} \text{sign} \left(a_{ijt}^{-} \right) x_{jt}^{+} + \sum_{j=j_1+1}^{n_1} \left| a_{ijt} \right|^{+} \text{sign} \left(a_{ijt}^{+} \right) x_{jt}^{-} + \left| A'_{ith} \right|^{+} \text{sign} \left(A'^{+}_{ith} \right) D_{th}^{-} \ge w_{ith}^{-}, \quad i = 1, 2, \ldots, m_2; \; t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{6c}$$

$$x_{jt}^{+} \ge 0, \quad j = 1, 2, \ldots, j_1; \; t = 1, 2, \ldots, T, \tag{6d}$$

$$x_{jt}^{-} \ge 0, \quad j = j_1 + 1, j_1 + 2, \ldots, n_1; \; t = 1, 2, \ldots, T, \tag{6e}$$

$$D_{th}^{-}, \quad t = 1, 2, \ldots, T; \; h = 1, 2, \ldots, h_t, \tag{6f}$$

$$Q_{th}^{-} = \sqrt{\frac{2 S_t^{-} D_{th}^{-}}{C_{1t}^{-}}}, \tag{6g}$$

where $x_{jt}^{\pm}$ $(j = 1, 2, \ldots, j_1)$ are interval variables with positive coefficients in the objective function; $x_{jt}^{\pm}$ $(j = j_1 + 1, j_1 + 2, \ldots, n_1)$ are interval variables with negative coefficients. Solutions of $x_{jt\,\text{opt}}^{+}$ $(j = 1, 2, \ldots, j_1)$, $x_{jt\,\text{opt}}^{-}$ $(j = j_1 + 1, j_1 +$

$2, \ldots, n_1$), $D^-_{th\,\text{opt}}$, and $Q^-_{th\,\text{opt}}$ can be obtained from submodel (6a)–(6g). Based on the above solutions, the second submodel for f^- can be formulated as follows:

$$\text{Max} \quad f^- = \sum_{t=1}^{T}\left(\sum_{j=1}^{j_1} u^-_{jt}x^-_{jt} + \sum_{j=j_1+1}^{n_1} u^-_{jt}x^+_{jt}\right) - \sum_{t=1}^{T}\sum_{h=1}^{h_t} p_{th}\left(\frac{1}{2}C^+_{1t}\sqrt{\frac{2S^+_t D^+_{th}}{C^+_{1t}}} + C^+_t D^+_{th} + \sqrt{\frac{S^+_t D^+_{th} C^+_{1t}}{2}}\right) \tag{7a}$$

$$\text{s.t.} \quad \sum_{j=1}^{j_1}\left|a_{rjt}\right|^+ \text{sign}\left(a^+_{rjt}\right) x^-_{jt} + \sum_{j=j_1+1}^{n_1}\left|a_{rjt}\right|^- \text{sign}\left(a^-_{rjt}\right) x^+_{jt} \le B^-_{rt}, \quad r = 1, 2, \ldots, m_1;\ t = 1, 2, \ldots, T, \tag{7b}$$

$$\sum_{j=1}^{j_1}\left|a_{ijt}\right|^+ \text{sign}\left(a^+_{ijt}\right) x^-_{jt} + \sum_{j=j_1+1}^{n_1}\left|a_{ijt}\right|^- \text{sign}\left(a^-_{ijt}\right) x^+_{jt} + \left|A'_{ith}\right|^- \text{sign}\left(A'^-_{ith}\right) D^+_{th} \ge w^+_{ith}, \quad i = 1, 2, \ldots, m_2;\ t = 1, 2, \ldots, T;\ h = 1, 2, \ldots, h_t, \tag{7c}$$

$$0 \le x^-_{jt} \le x^+_{jt\,\text{opt}}, \quad j = 1, 2, \ldots, j_1;\ t = 1, 2, \ldots, T, \tag{7d}$$

$$x^+_{jt} \ge x^-_{jt\,\text{opt}},\ j = j_1 + 1, j_1 + 2, \ldots, n_1;\ t = 1, 2, \ldots, T, \tag{7e}$$

$$D^-_{th} \le D^+_{th\,\text{opt}}, \quad t = 1, 2, \ldots, T;\ h = 1, 2, \ldots, h_t, \tag{7f}$$

$$Q^+_{th} = \sqrt{\frac{2S^+_t D^+_{th}}{C^+_{1t}}}. \tag{7g}$$

Solutions of $x^-_{jt\,\text{opt}}$ ($j = 1, 2, \ldots, j_1$), $x^+_{jt\,\text{opt}}$ ($j = j_1 + 1, j_1 + 2, \ldots, n_1$), $D^+_{th\,\text{opt}}$, and $Q^+_{th\,\text{opt}}$ can be obtained by solving submodel (7a)–(7g). Then, the expected objective function value can be calculated as follows:

$$f^+_{\text{opt}} = \sum_{t=1}^{T}\left(\sum_{j=1}^{j_1} u^+_{jt}x^+_{jt} + \sum_{j=j_1+1}^{n_1} u^+_{jt}x^-_{jt}\right) - \sum_{t=1}^{T}\sum_{h=1}^{h_t} p_{th}\left(\frac{1}{2}C^-_{1t}\sqrt{\frac{2S^-_t D^-_{th}}{C^-_{1t}}} + C^-_t D^-_{th} + \sqrt{\frac{S^-_t D^-_{th} C^-_{1t}}{2}}\right), \tag{8a}$$

$$f^-_{\text{opt}} = \sum_{t=1}^{T}\left(\sum_{j=1}^{j_1} u^-_{jt}x^-_{jt} + \sum_{j=j_1+1}^{n_1} u^-_{jt}x^+_{jt}\right) - \sum_{t=1}^{T}\sum_{h=1}^{h_t} p_{th}\left(\frac{1}{2}C^+_{1t}\sqrt{\frac{2S^+_t D^+_{th}}{C^+_{1t}}} + C^+_t D^+_{th} + \sqrt{\frac{S^+_t D^+_{th} C^+_{1t}}{2}}\right). \tag{8b}$$

Consequently, through combining solutions of submodels (6a)–(6g) and (7a)–(7g), the solution for IB-IMSP model can be obtained as follows:

$$x^\pm_{jt\,\text{opt}} = \left[x^-_{jt\,\text{opt}}, x^+_{jt\,\text{opt}}\right], \quad \forall j, t, \tag{8c}$$

$$D^\pm_{th\,\text{opt}} = \left[D^-_{th\,\text{opt}}, D^+_{th\,\text{opt}}\right], \quad \forall t, h = 1, 2, \ldots, h_t, \tag{8d}$$

$$Q^\pm_{th\,\text{opt}} = \left[Q^-_{th\,\text{opt}}, Q^+_{th\,\text{opt}}\right], \quad \forall t, h = 1, 2, \ldots, h_t, \tag{8e}$$

$$f^\pm_{\text{opt}} = \left[f^-_{\text{opt}}, f^+_{\text{opt}}\right]. \tag{8f}$$

Figure 1 shows the framework of the IB-IMSP model, which is based on EOQ and IMSP techniques. The introduction of EOQ model makes the IB-IMSP can provide the transferring batch size and period, avoiding unnecessary waste of capital and time as well as solving water shortage problem. The application of IMSP enhances the IB-IMSP's capacities in handling the uncertainties and dynamic complexities. For example, the proposed IB-IMSP can tackle uncertainties expressed as random variables, interval parameters as well as their combinations. In addition, the IB-IMSP can identify dynamics of not only the uncertainties but also the related decisions. Therefore, the method can be used for generating decision alternatives and help decision makers to identify desired policies under different flow levels, and analyze all possible policy scenarios that are associated with different transferring schemes.

3. Application

The following water resources management problem will be used to demonstrate the applicability of the proposed IB-IMSP model. A manager is responsible for delivering water from an unregulated reservoir during three planning periods (with each one being five years) to three users: a municipality, an industrial concern, and an agricultural sector. All users want to know how much water they can expect over the three periods. If the promised water is delivered, a net benefit to the local economy will be generated for each unit of water allocated. However, if the promised water is not delivered, they will try to transfer water from other abundant water sources to ensure the local normal life and economic growth. Correspondingly, transferring water will be decided based on the available water resources and target demands from the three users. In addition, although transferring water can solve the water shortage problem, it will result in a reduced net system benefit from three main aspects: setup cost, unit cost,

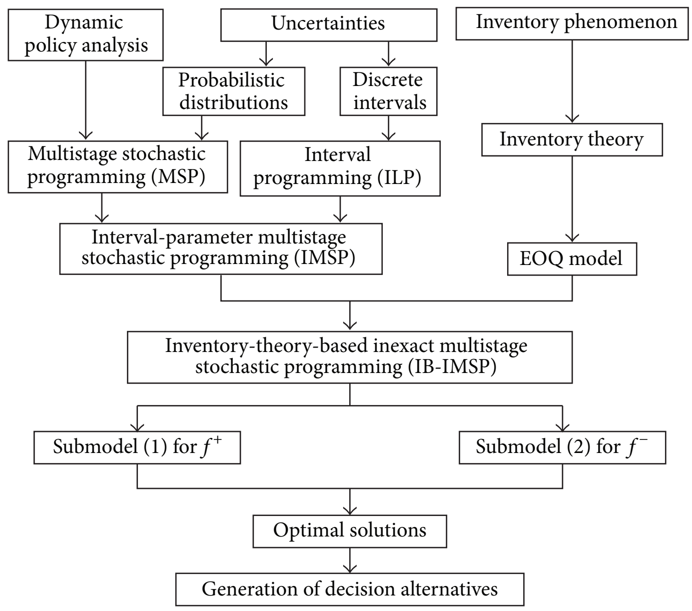


Figure 1: Framework of the IB-IMSP model.

Table 1: Stream flows in the three periods (supply).

Stream flow level	Probability	Stream flow (10^6 m^3)		
		$t = 1$	$t = 2$	$t = 3$
Low (L)	0.2	[4.2, 5.8]	[4.7, 6.5]	[4.3, 5.9]
Medium (M)	0.6	[8.0, 9.6]	[7.9, 8.9]	[8.3, 9.2]
High (H)	0.2	[12.3, 13.6]	[11.8, 13.9]	[12.6, 13.4]

Table 2: Water allocation targets for users.

	Time period		
	$t = 1$	$t = 2$	$t = 3$
Water allocation target (10^6 m^3):			
$W_{1t}^{\pm}$ (to municipal)	[4.1, 5.1]	[5.5, 6.4]	[6.4, 7.5]
$W_{2t}^{\pm}$ (to industrial)	6.2	[7.2, 8.3]	[7.8, 8.9]
$W_{3t}^{\pm}$ (to agricultural)	7.8	9.1	[8.7, 9.1]
Maximum allowable allocation (10^6 m^3):			
$W_{1t\ \max}^{\pm}$ (to municipal)	[4.1, 4.5]	[5.4, 6.0]	[5.4, 6.1]
$W_{2t\ \max}^{\pm}$ (to industrial)	[5.1, 5.5]	[6.8, 7.4]	[7.2, 9.4]
$W_{3t\ \max}^{\pm}$ (to agricultural)	[6.5, 8.3]	[7.0, 8.1]	[7.3, 8.1]

Table 3: Related economic data.

	Time period		
	$t = 1$	$t = 2$	$t = 3$
Net benefit when water demand is satisfied, $\mathrm{NB}_{it}^{\pm}$ (\$/m^3):			
Municipal ($i = 1$)	[90, 110]	[95, 115]	[100, 120]
Industrial ($i = 2$)	[45, 55]	[55, 70]	[65, 85]
Agricultural ($i = 3$)	[30, 35]	[35, 50]	[35, 50]
Costs when water is transferred:			
$C_{1t}^{\pm}$ (holding cost, \$/m^3)	[18.0, 22.0]	[19.0, 23.0]	[20.0, 25.0]
$C_t^{\pm}$ (unit cost, \$/m^3)	[1.2, 1.6]	[2.3, 3.0]	[3.4, 4.0]
$S_t^{\pm}$ (setup cost, \$)	[6.5, 8.0]	[7.0, 9.0]	[7.5, 9.0]

and holding cost for water transferring. Tables 1 and 2 denote the available water resources from the local region and target demands. Table 3 provides the associated economic data. The objective is to maximize the expected value of the net benefits over the planning horizon.

Under this condition, random variables (available water supply) with probability p_{th} to construct three scenario trees for the planning horizon with a branching structure of 1-3-3-3 can be applied. Therefore, a three-period (four-stage) scenario tree can be generated for each of the three water users. All of the scenario trees have the same structure with one initial node at time 0 and three succeeding ones in period 1; each node in period 1 has three succeeding nodes in period 2, and so on for each node in period 3. These result in 27 nodes in period 3 (and thus 81 scenarios since here are three water users). Figure 2 shows the formulation of the scenario tree.

In this study, the random stream flow under each scenario may be expressed as discrete interval. Moreover, the relevant water allocation plan would be of dynamic feature, where the related decisions must be made at discrete points in time under multiple probability levels. In addition, different transferring water under each probability level will affect the system benefit. For example, if too much water is transferred per unit time, the holding cost will increase; conversely, if too little water is transferred per unit time, the transferring frequency and setup cost will increase. Therefore, to identify these uncertainties and dynamics, as well as solving the water

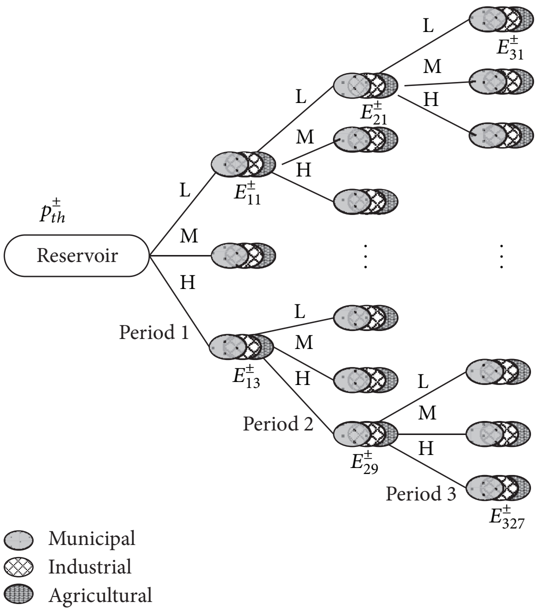


Figure 2: Formulation of scenario tree.

shortage problem, an IB-IMSP model can be formulated as follows:

$$\text{Max} \quad f^{\pm} = \sum_{i=1}^{I}\sum_{t=1}^{T} NB^{\pm}_{it} W^{\pm}_{it} - \sum_{t=1}^{T}\sum_{h=1}^{h_t} p_{th}\left(\frac{1}{2}C^{\pm}_{1t}\sqrt{\frac{2S^{\pm}_{t}D^{+}_{th}}{C^{\pm}_{1t}}} + C^{\pm}_{t}D^{\pm}_{th} + \sqrt{\frac{S^{\pm}_{t}D^{\pm}_{th}C^{\pm}_{1t}}{2}}\right) \tag{9a}$$

Subject to
(constraint of available water flow)

$$\sum_{i=1}^{I} W^{\pm}_{it} \leq q^{\pm}_{th} + D^{\pm}_{th} + E^{\pm}_{(t-1)h}, \quad \forall t = 1,2,3;\ h = 1,2,\ldots,h_t \tag{9b}$$

$$q^{\pm}_{th} + D^{\pm}_{th} + E^{\pm}_{(t-1)h} \leq \sum_{i=1}^{I} W^{\pm}_{it\,\max}, \quad \forall t = 1,2,3;\ h = 1,2,\ldots,h_t, \tag{9c}$$

(constraint of surplus water)

$$E^{\pm}_{th} = q^{\pm}_{tk} + D^{\pm}_{th} - \sum_{i=1}^{I} W^{\pm}_{it} + E^{\pm}_{(t-1)h}, \ E^{\pm}_{0h} = 0, \quad \forall t = 1,2,3;\ h = 1,2,\ldots,h_t, \tag{9d}$$

(constraint of reservoir capacity)

$$q^{\pm}_{th} + E^{\pm}_{(t-1)h} + D^{\pm}_{th} - \sum_{i=1}^{I} W^{\pm}_{it} \leq RC^{\pm}, \quad \forall t;\ h = 1,2,\ldots,h_t, \tag{9e}$$

(constraint of transferring water batch)

$$Q^{\pm}_{th} = \sqrt{\frac{2S^{\pm}_{t}D^{\pm}_{th}}{C^{\pm}_{1t}}}, \tag{9f}$$

(non-negative constraint)

$$D^{\pm}_{th} \geq 0, \quad \forall t;\ h = 1,2,\ldots,h_t, \tag{9g}$$

where

i = water user, $i = 1, 2, 3$,
t = planning time period, $t = 1, 2, 3$,
h = available flow level, $h = 1, 2, \ldots, h_t$,
p_{th} = probability level of available water during period t with $p_{th} > 0$ and $\sum_{h=1}^{h_t} p_{th} = 1$,
$q^{\pm}_{th}$ = random variable of total water availability during period t with probability level of p_{th} (m^3),
$f^{\pm}$ = net system benefit over the planning horizon (\$),
$C^{\pm}_{t}$ = unit cost for transferring water during period t (\$/m^3),
$C^{\pm}_{1t}$ = holding cost per unit per unit of time held in reservoir (\$/m^3),
$D^{\pm}_{th}$ = transferring water amount from other abundant water sources when the total water-allocation target is not met under the flow of $q^{\pm}_{th}$ (m^3),
$E^{\pm}_{(t-1)h}$ = surplus flow when water is delivered in period $t-1$ (m^3),
$E^{\pm}_{(t-2)h}$ = surplus flow when water is delivered in period $t-2$ (m^3),
$Q^{\pm}_{th}$ = transferring water batch when the flow is $q^{\pm}_{th}$ (m^3),
$S^{\pm}_{t}$ = setup cost for transferring one batch (\$),
$\text{NB}^{\pm}_{it}$ = net benefit (i.e. revenue minus expense) to user i per unit of water allocated during period t (\$/m^3),
$RC^{\pm}$ = storage capacity of the reservoir (m^3),
$W^{\pm}_{it}$ = fixed allocation target for water that is promised to user i during period t (m^3),
$W^{\pm}_{it\,\max}$= maximum amount that should be allocated to user i during period t (m^3).

In model (9a)–(9g), $D^{\pm}_{th}$ are decision variables, which are affected by local flow levels. Random variables (available water supplies) under different probability levels (p_{th}) are used to construct the scenario tree. Correspondingly, the IB-IMSP model can identify nonanticipativity of the water

Table 4: Solution of the IB-IMSP model (period 1).

Water flow level	Probability	Optimized transferring water (10^6 m^3)	Optimized transferring batch (10^3 m^3)	Optimized transferring period (hour)
L	0.2	[13.30, 13.90]	[3.10, 3.18]	[9.88, 10.06]
M	0.6	[9.50, 10.10]	[2.62, 2.71]	[11.59, 11.91]
H	0.2	[5.50, 5.80]	[1.99, 2.05]	[15.30, 15.65]

Table 5: Solution of the IB-IMSP model (period 2).

Water flow level	Probability	Joint flow	Optimized transferring water (10^6 m^3)	Optimized transferring batch (10^3 m^3)	Optimized transferring period (hour)
L	0.04	L-L	[11.50, 12.90]	[2.91, 3.18]	[10.64, 10.94]
L	0.12	L-M	[9.10, 9.70]	[2.59, 2.76]	[12.27, 12.29]
L	0.04	L-H	[4.10, 5.80]	[1.74, 2.13]	[15.87, 18.31]
M	0.12	M-L	[7.70, 9.10]	[2.38, 2.67]	[12.67, 13.36]
M	0.36	M-M	[5.30, 5.90]	[1.98, 2.15]	[15.73, 16.11]
M	0.12	M-H	[0.30, 2.00]	[0.47, 1.25]	[27.02, 67.70]
H	0.04	H-L	[3.70, 4.80]	[[1.65, 1.94]	[17.44, 19.28]
H	0.12	H-M	[1.30, 1.60]	[0.98, 1.12]	[30.21, 32.52]
H	0.04	H-H	0	0	0

resources management system, where a decision must be made in each stage without the knowledge of the realizations of random variables in the future stages. Based on the method depicted in Section 2, the IB-IMSP model can be converted into two deterministic submodels. Interval solutions can then be obtained by solving the two submodels sequentially. The specific solution process can be summarized as follows.

Step 1. Formulate IB-IMSP model (9a)–(9g).

Step 2. Transform model (9a)–(9g) into two submodels, where the upper bound (f^+) is first solved because the objective is to maximize $f^{\pm}$.

Step 3. Solve the f^+ submodel and obtain solutions of $D^-_{th\,\mathrm{opt}}$, $Q^-_{th\,\mathrm{opt}}$, and f^+_{opt}.

Step 4. Formulate the objective function and relevant constraints of the f^- submodel.

Step 5. Solve the f^- submodel and obtain solutions of $D^+_{th\,\mathrm{opt}}$, $Q^+_{th\,\mathrm{opt}}$, and f^-_{opt}.

Step 6. Make the second programming using ILP based on the solution of $D^{\pm}_{th\ \mathrm{opt}}$, and obtain the actual allocation $W^{\pm}_{ith\,\mathrm{opt}}$.

Step 7. Integrate solutions of the two submodels and $W^{\pm}_{ith\,\mathrm{opt}}$, and the optimal results can be expressed as $D^{\pm}_{th\ \mathrm{opt}} = [D^-_{th\,\mathrm{opt}}, D^+_{th\,\mathrm{opt}}]$, $Q^{\pm}_{th\,\mathrm{opt}} = [Q^-_{th\,\mathrm{opt}}, Q^+_{th\,\mathrm{opt}}]$, $W^{\pm}_{ith\,\mathrm{opt}} = [W^-_{ith\,\mathrm{opt}}, W^+_{ith\,\mathrm{opt}}]$, and $f^{\pm}_{\mathrm{opt}} = [f^-_{\mathrm{opt}}, f^+_{\mathrm{opt}}]$.

Step 8. Stop.

4. Results Analysis

Tables 4–6 denote the optimized transferring water schemes under different flow levels during the planning horizon. It is shown that the solutions for the objective function value and most of the nonzero decision variables are intervals. Commonly, solutions expressed as intervals indicate that the associated decisions should be sensitive to the uncertain modeling inputs. For instance, the interval solutions for $D^{\pm}_{th}$ under the given targets reveal potential system-condition variations caused by uncertain inputs of $NB^{\pm}_{it}$, $C^{\pm}_t$, $C^{\pm}_{1t}$, $S^{\pm}_t$, and $q^{\pm}_{th}$. The demands and shortages are associated to water availability. Shortages would happen if the available water resources cannot satisfy the users' demands. Under the condition of insufficient water, more water needs to be transferred from other abundant water sources to guarantee the local normal life and economic development.

Tables 4–6 show the solution of water transferring schemes obtained from the IB-IMSP model, including transferring amount, transferring batch, and transferring period. Different transferring water is associated with varied flow levels and the targeted water demands. If the manager is optimistic of water supply to users and thus promises an upper-bound water quantity, the shortages probability would be small, and thus less water should be transferred; conversely, if the manager has a conservative attitude towards water supply, more water should be transferred to meet the users' basic demands. The transferring batch means the quantity of transferring water in every transferring scheme, which is mainly influenced by the transferring amount. The transferring period represents a cycle length from one the transferring scheme to the next scheme, and equals to transferring batch divided by demand and then multiplied by each planning period.

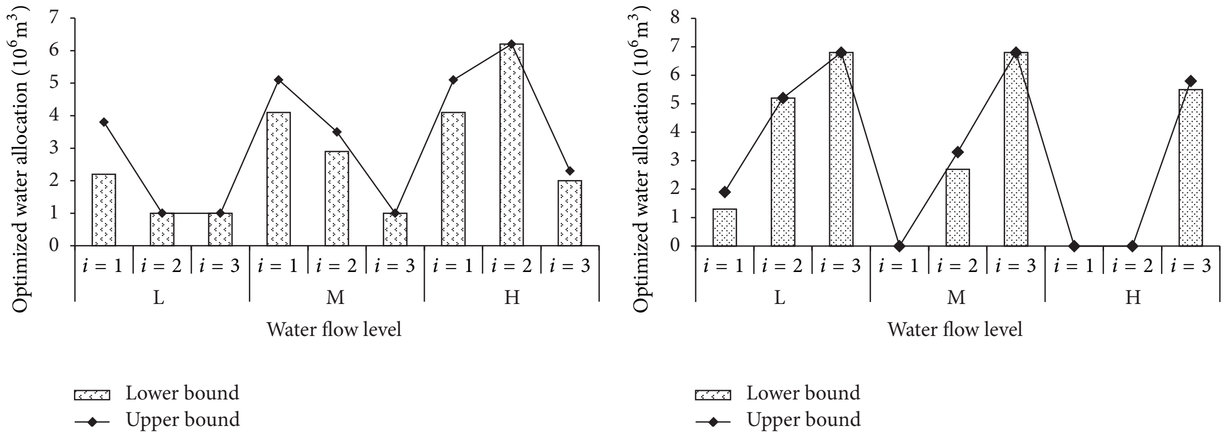
Table 6: Solution of the IB-IMSP model (period 3).

Water flow level	Probability	Joint flow	Optimized transferring water (10^6 m^3)	Optimized transferring batch (10^3 m^3)	Optimized transferring period (hour)
L	0.008	L-L-L	[7.30, 9.70]	[2.40, 2.64]	[11.77, 13.85]
L	0.024	L-L-M	[4.00, 5.70]	[1.73, 2.02]	[15.35, 18.71]
L	0.008	L-L-H	[0, 1.40]	[0, 1.03]	[0, 30.98]
L	0.024	L-M-L	[4.90, 6.50]	[1.92, 2.16]	[14.38, 16.90]
L	0.072	L-M-M	[1.60, 2.50]	[1.10, 1.34]	[23.18, 29.58]
L	0.024	L-M-H	0	0	0
L	0.008	L-H-L	[0, 2.60]	[0, 1.37]	[0, 22.73]
L	0.024	L-H-M	0	0	0
L	0.008	L-H-H	0	0	0
M	0.024	M-L-L	[3.50, 5.90]	[1.62, 2.06]	[15.09, 20.00]
M	0.072	M-L-M	[0.20, 1.90]	[0.39, 1.17]	[26.59, 83.66]
M	0.024	M-L-H	0	0	0
M	0.072	M-M-L	[1.10, 2.70]	[0.91, 1.39]	[24.17, 27.89]
M	0.216	M-M-M	0	0	0
M	0.072	M-M-H	0	0	0
M	0.024	M-H-L	0	0	0
M	0.072	M-H-M	0	0	0
M	0.024	M-H-H	0	0	0
H	0.008	H-L-L	[0, 1.60]	[0, 1.07]	[0, 28.98]
H	0.024	H-L-M	0	0	0
H	0.008	H-L-H	0	0	0
H	0.024	H-M-L	0	0	0
H	0.072	H-M-M	0	0	0
H	0.024	H-M-H	0	0	0
H	0.008	H-H-L	0	0	0
H	0.024	H-H-M	0	0	0
H	0.008	H-H-H	0	0	0

Total expected value of net benefit (\$$10^9$): $f^{\pm}_{\text{opt}} = [3.53, 5.03]$.

Table 4 provides the optimized water transferring schemes under three different flow levels in the first period. For example, the solutions of $D^{\pm}_{11\,\text{opt}} = [13.30, 13.90] \times 10^6\ \text{m}^3$ means much more water should be transferred under low flow level (probability = 20%), which leads to larger transferring batch ($Q^{\pm}_{11\,\text{opt}} = [3.10, 3.18] \times 10^3\ \text{m}^3$). Correspondingly, the transferring period ([9.88, 10.06] hour) would be smaller, which is calculated by (3c) and means that the transferring frequency would become more frequent. When the flow level is medium (probability = 60%), the transferring water ($D^{\pm}_{12\,\text{opt}} = [9.50, 10.10] \times 10^6\ \text{m}^3$) and transferring batch ($Q^{\pm}_{12\,\text{opt}} = [2.62, 2.71] \times 10^3\ \text{m}^3$) are smaller than the solutions of low flow level, which are associated with a wide transferring period ([11.59, 11.91] hour). Under high flow level (probability = 20%), the related transferring water and transferring batch are the smallest, while the transferring period is the widest, being $[5.50, 5.80] \times 10^6\ \text{m}^3$, $[1.99, 2.05] \times 10^3\ \text{m}^3$, and [15.30, 15.65] hour, individually. This implies that the water shortage under high flow is the least serious compared with low and medium flow levels under the same demand condition.

Table 5 presents the optimized water transferring schemes under all possible scenarios in period 2. Under low-, medium-, high-flow levels in period 2 (following a low flow in period 1), the needing transferring water would be $[11.50, 12.90] \times 10^6\ \text{m}^3$, $[9.10, 9.70] \times 10^6\ \text{m}^3$, and $[4.10, 5.80] \times 10^6\ \text{m}^3$, respectively (with probability levels of 4%, 12%, and 4%). Accordingly, the transferring batches would be $[2.91, 3.18] \times 10^3\ \text{m}^3$, $[2.59, 2.76] \times 10^3\ \text{m}^3$, and $[1.74, 2.13] \times 10^3\ \text{m}^3$, individually, associated with the transferring periods being [10.64, 10.94] hour, [12.27, 12.29] hour, and [15.87, 18.31] hour. This transferring scheme demonstrates that although the water flow is high in period 2, water transferring is still needed since low flow is in period 1. On the contrast, the solutions of $D^{\pm}_{27\,\text{opt}} = [3.70, 4.80] \times 10^6\ \text{m}^3$, $D^{\pm}_{28\,\text{opt}} = [1.30, 1.60] \times 10^6\ \text{m}^3$,



(a) (b)

Figure 3: (a) Optimized water allocation from local water source in period 1. (b) Optimized water allocation from water transferring in period 1.

and $D^{\pm}_{29\,\mathrm{opt}} = 0$ imply that there would be less water shortages even without shortages during the second period if the water flow is high in the first period.

Table 6 shows the optimized water transferring alternatives under all possible scenarios in period 3. The solutions of $D^{\pm}_{31\,\mathrm{opt}} = [7.30, 9.70] \times 10^6$ m^3, $D^{\pm}_{32\,\mathrm{opt}} = [4.00, 5.70] \times 10^6$ m^3 and $D^{\pm}_{33\,\mathrm{opt}} = [0, 1.40] \times 10^6$ m^3 mean that, if the flows are low in the previous two periods, there would be $[7.30, 9.70] \times 10^6$ m^3, $[4.00, 5.70] \times 10^6$ m^3 and $[0, 1.40] \times 10^6$ m^3 of transferring water under low, medium and high water-flow scenarios, individually (probability = 0.8%, 24% and 0.8%), followed with the transferring batch being $[2.40, 2.64] \times 10^3$ m^3, $[1.73, 2.02] \times 10^3$ m^3 and $[0, 1.03] \times 10^3$ m^3, individually. If the flow is low in period 1 and high in period 2, then there would be $[0, 2.60] \times 10^6$ m^3, 0, and 0 of water transferring needed under low-, medium- and high-flow scenarios in period 3. The water shortage in period 3 would become less if there is some surplus in the reservoir due to the high-flow condition during period 2.

The solutions also indicate that under the worst-case scenario (probability = 0.8%) when the water flows are low during the entire planning period, the total of transferring water would be $[7.30, 9.70] \times 10^6$ m^3 associated with the total water demand from the three users being $[22.9, 25.5] \times 10^6$ m^3, implying a serious shortage in water supply. Therefore, the users need to transfer water from the other more expensive sources and ensure their demands. Under the best scenario (probability = 0.8%) when the water flows are high during the entire planning period, the total of transferring water would be 0, indicating that the water demands of the three users could generally be satisfied. Although the probability of the worst-case scenario is low (0.8%), the deficits due to the occurrence of such an extreme event are high. Consequently, an optimal policy that is formulated based on the analyses of not only the system benefits but also the related deficits would be desired.

Two extreme expected values of the net system benefit over the planning period are provided by the solution of the objective function ($f^{\pm}_{\mathrm{opt}} = \$\,[3.53, 5.03] \times 10^9$). With the actual value of every continuous variable changing within its lower and upper bounds, the desired system benefit would fluctuate accordingly between f^{-}_{opt} and f^{+}_{opt} with a range of dependability levels. Given varied water availability conditions and underlying probability distributions, the consequential plans of water transferring (and system benefit) would differ between their related solution intervals. A plan with lower system benefit might need more water transferring, while that of higher benefit would link to smaller transferring water under advantageous situations.

With water transferring, users' demands can be well satisfied under various water flow levels, and meanwhile water shortage risks can be avoided. Figures 3–6 show the optimized water allocations from the local water source and the transferring under different flow levels over the planning horizon (except the optimized water allocation from H-L-L to H-H-H in period 3). With only considering local water availability, the allotment to the agricultural sector would be first decreased, followed by that to industrial sector in the case of water insufficiency. The municipal use would be guaranteed because it brings the highest benefit when its water demand is met. In comparison, the industrial and agricultural uses match to lower benefits (see Table 3).

Figures 3(a) and 3(b) describe the optimized water allocations to three users from the local water source and the transferring under three flow levels in the first period, respectively. Under low flow, the solution of water allocation to municipal sector, industrial sector, and agricultural sector would be $[2.2, 3.8] \times 10^6$ m^3, 1×10^6 m^3, and 1×10^6 m^3 from local water source, which implies that encountering water shortage, the industrial sector and agricultural sector only obtain the minimum amount to guarantee their necessary uses while the municipal sector can gain more water (Figure 3(a)). Correspondingly, the water allocation from the transferring to agricultural sector would be the largest (6.8×10^6 m^3), followed by industrial sector (5.2×10^6 m^3) and municipal sector ($[1.3, 1.9] \times 10^6$ m^3), which are shown

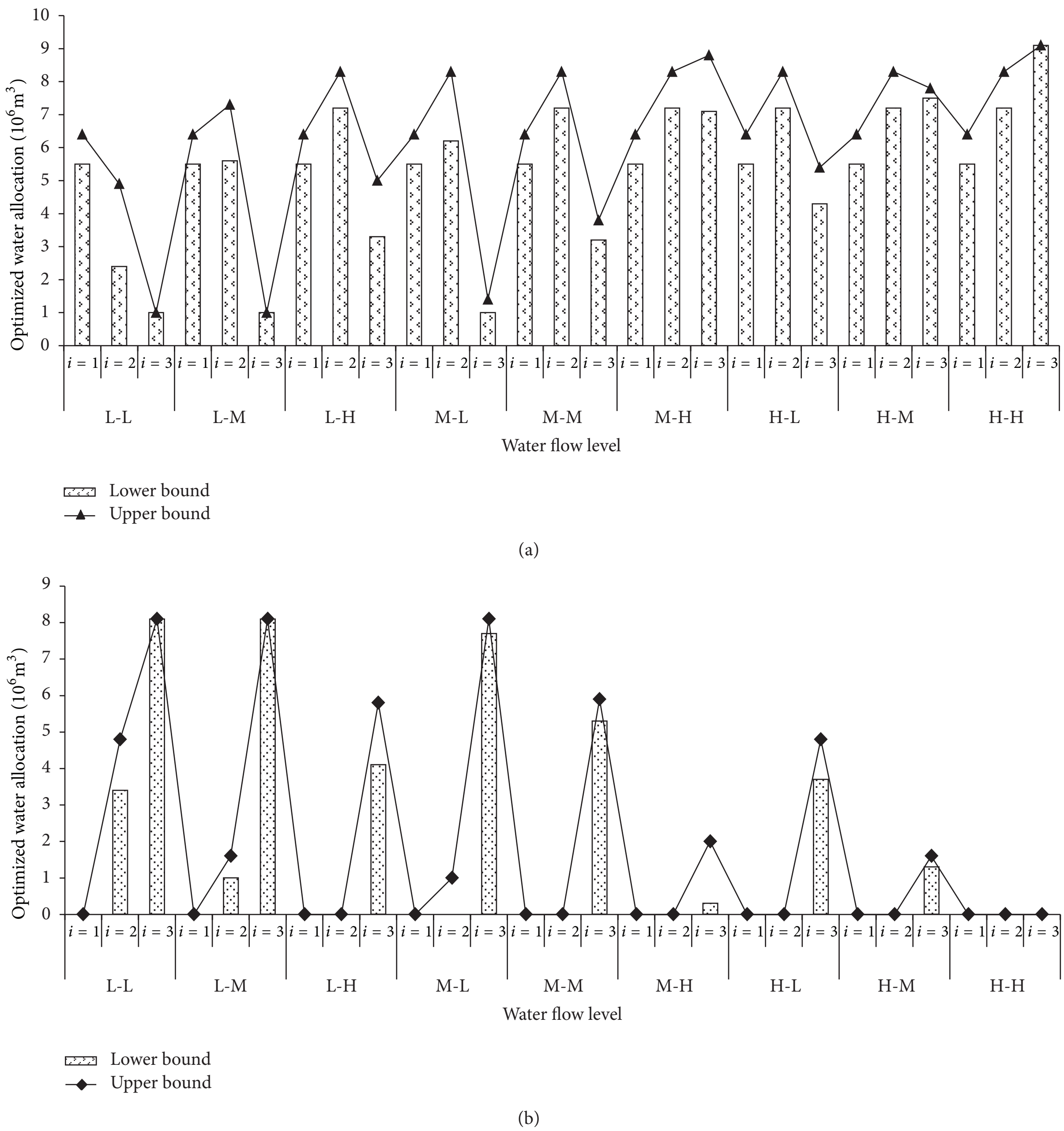


Figure 4: (a) Optimized water allocation from local water source in period 2. (b) Optimized water allocation from water transferring in period 2.

in Figure 3(b). Under high flow, both the targeted water demands from municipal and industrial users can be satisfied under the local water supply, while the agricultural sector still needs water transferring to reach its targeted demand.

Figures 4(a) and 4(b) present the optimized water allocations to all users from the local water source and the transferring under all possible flow scenarios in period 2, individually. Under low, medium, and high flow levels in period 2 (following a low flow in period 1), the solution of water allocation to agricultural sector would be $1.0 \times 10^6 m^3$, $1.0 \times 10^6 m^3$, and $[3.3, 5.0] \times 10^6 m^3$ from the local water supply, respectively (with probability levels of 4%, 12%, and 4%); correspondingly, it would be $8.1 \times 10^6 m^3$, $8.1 \times 10^6 m^3$, and $[4.1, 5.8] \times 10^6 m^3$ from transferring, individually. This allocation implies that although the water flow is high in period 2, the agricultural sector still cannot be satisfied by local water supply since low flow in period 1. In comparison, the water allocation of $[4.3, 5.4] \times 10^6 m^3$, $[7.5, 7.8] \times 10^6 m^3$, and $9.1 \times 10^6 m^3$ from local water supply, and $[3.7, 4.8] \times 10^6 m^3$, $[1.3, 1.6] \times 10^6 m^3$, and 0 from transferring to agricultural sector under the last three flow scenarios implies that less water transferring would be needed during the second period if the water flow is high in the first period.

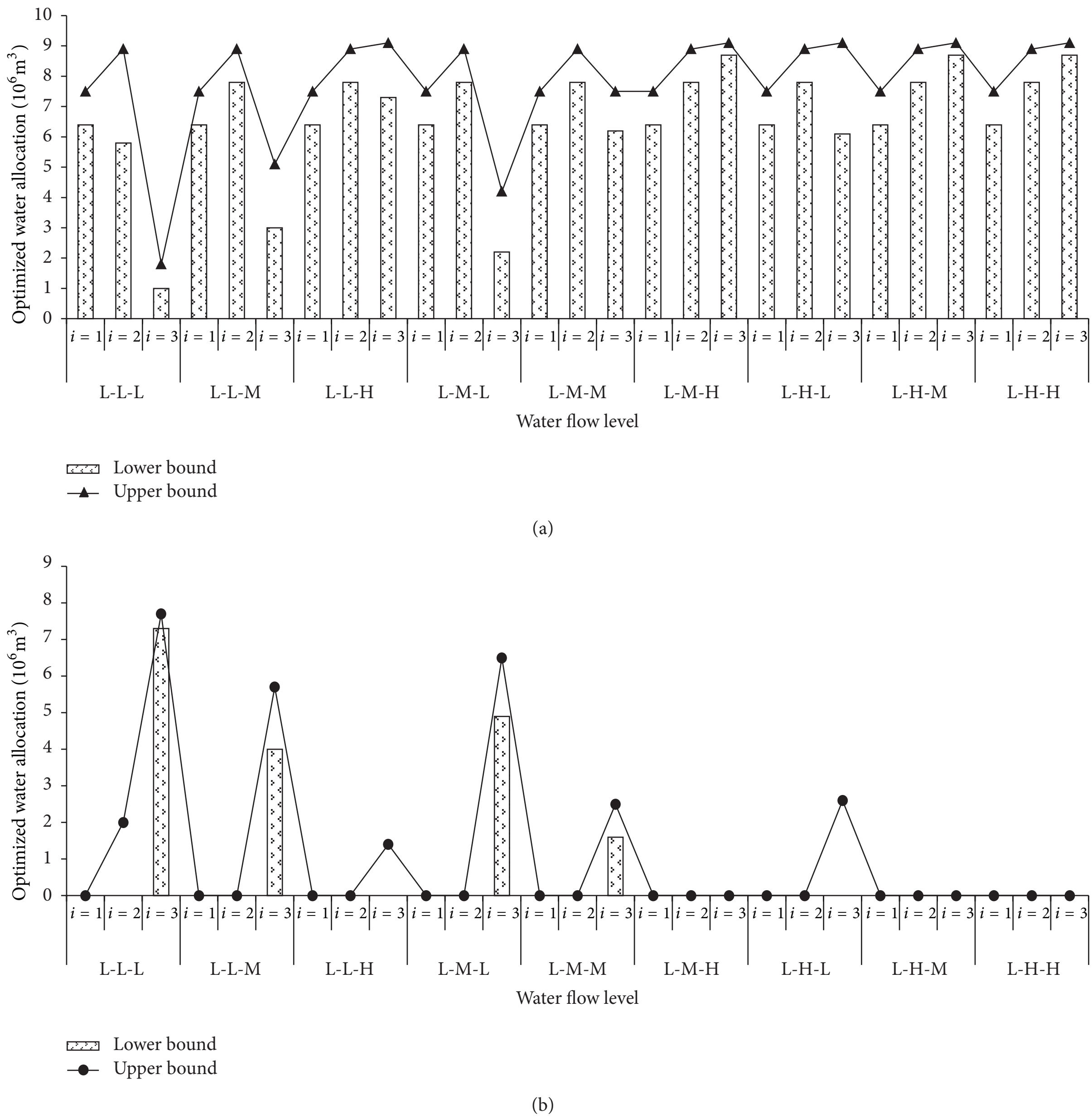
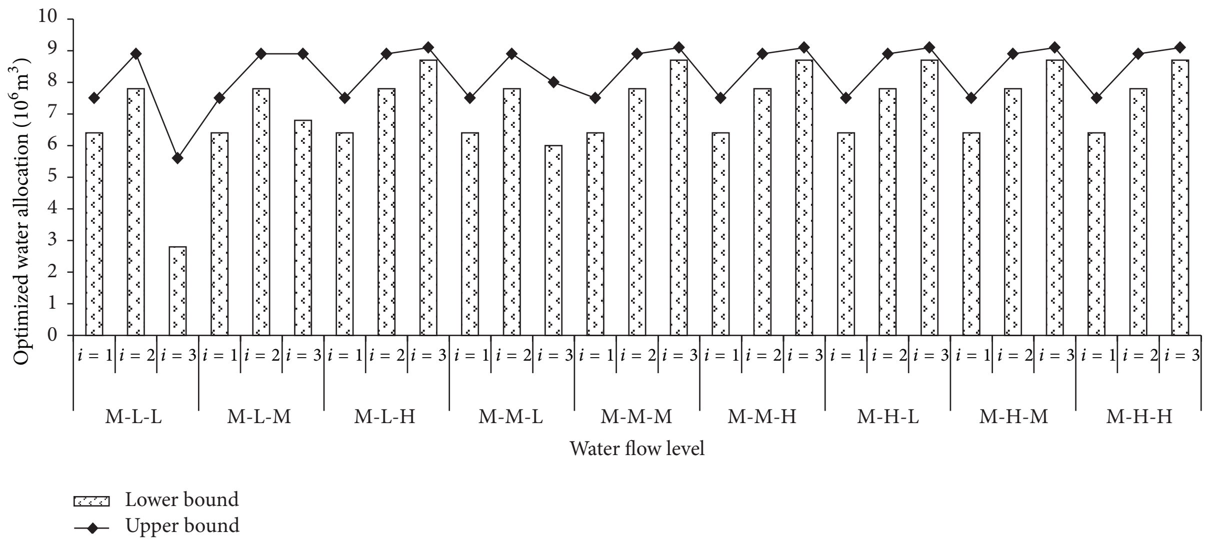


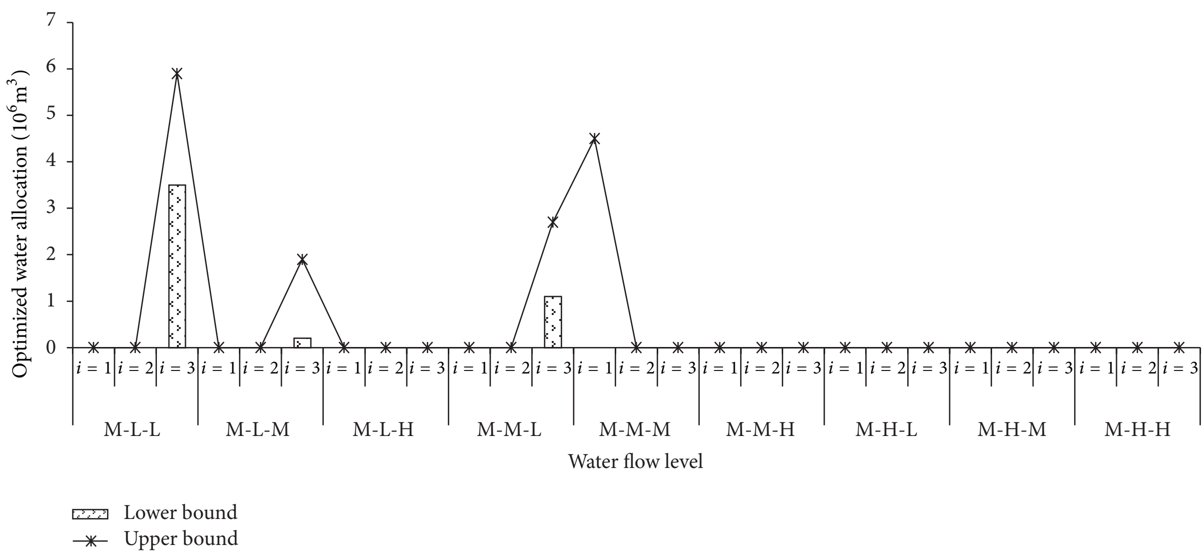
Figure 5: (a) Optimized water allocation from local water source (from L-L-L to L-H-H) in period 3. (b) Optimized water allocation from water transferring (from L-L-L to L-H-H) in period 3.

Figures 5(a), 5(b), 6(a), and 6(b) provide the optimized water allocations to the three users from local water source and the transferring under the flow scenarios from L-L-L to M-H-H in period 3, respectively. The water allocation to the agricultural sector from local water source ($[1.0, 1.8] \times 10^6$ m^3, $[3.0, 5.1] \times 10^6$ m^3, and $[7.3, 9.1] \times 10^6$ m^3) under the scenarios of low-low-low, low-low-medium, and low-low-high mean that if the flows are low in the previous two periods, there would be $[7.3, 7.7] \times 10^6$ m^3, $[4.0, 5.7] \times 10^6$ m^3, and $[0, 1.4] \times 10^6$ m^3 of transferring water under low, medium, and high water-flow scenarios, respectively (probability = 0.8%, 24%, and 0.8%). If the flow is low in period 1 and high in period 2, then the water allocation to agricultural sector would be $[6.1, 9.1] \times 10^6$ m^3, $[8.7, 9.1] \times 10^6$ m^3, and $[8.7, 9.1] \times 10^6$ m^3 from local water source, and $[0, 2.6] \times 10^6$ m^3, 0 and 0 from transferring under low-, medium-, and high-flow scenarios in period 3. The water transferring in period 3 would be less in the case of some surplus existing owing to the high-flow condition in period 2. Under the flow levels of H-L-L to H-H-H, the available water from local water source can basically satisfy the users' demands, and thus a little of water needs to be transferred, which can be seen in Table 3.

Based on the above analysis, it can be obtained that the IB-IMSP has three main advantages. Firstly, it can handle uncertainties existing in water flows by producing scenarios



(a)



(b)

FIGURE 6: (a) Optimized water allocation from local water source (from M-L-L to M-H-H) in period 3. (b) Optimized water allocation from water transferring (from M-L-L to M-H-H) in period 3.

of its future events; these scenarios correspond to varied influences of different water allocations on the economic objectives. Secondly, the IB-IMSP can provide reasonable water transferring schemes (including transferring amount, transferring batch, and transferring period) with respect to all possible flow scenarios, as well as the optimized water allocations to all users from transferring. Thirdly, the IB-IMSP can efficiently identify the dynamics of not only the uncertainties but also the related decisions. With considering all scenarios, a decision can be ascertained at every stage in a real-time manner according to information about the definite realizations of the random variables along with previous decisions; this permits corrective actions to be carried dynamically for the predefined policies and can thus help reduce the deficit.

5. Conclusions

In this study, an inventory-theory-based inexact multistage stochastic programming (IB-IMSP) method has been developed for water resources decision making under uncertainty. This method advanced upon the existing inexact multistage stochastic programming by introducing inventory theory into the optimization framework. The developed IB-IMSP method can not only effectively handle uncertainties represented as probability density functions and discrete intervals, but also efficiently reflect dynamic features of the system conditions through transactions at discrete points in time during the planning horizon. In addition, it can provide reasonable transferring schemes (the transferring amount, batch, and the corresponding transferring period) associated with various flow scenarios for solving water shortage problems.

A hypothetical case study has been provided for demonstrating applicability of the developed method. The solutions obtained have then been analyzed for producing decision alternatives under different system conditions. The results provided the managers with optimal transferring schemes as well as optimized water allocation alternatives from local water availability and transferring to different users for various water shortage problems under all possible flow scenarios over the planning horizon. Therefore, the results can help the managers gain insight into the water resources management with maximizing economic objectives and satisfying targeted water demands from users. Although this study is the first attempt for planning water resources management by the proposal of an IB-IMSP method, the results indicate that this compound technique is effective and can be advanced to other environmental problems that include policy analysis. It can also be incorporated with other optimization techniques to improve their capacities in handling uncertainties presented in multiple forms.

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