

Research Article

H_∞ Observer-Based Sliding Mode Control for Uncertain Stochastic Systems with Time-Varying Delays

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The paper is concerned with sliding mode control for uncertain time-delay systems subjected to input nonlinearity and stochastic perturbations. Using the sliding mode control, a robust law is derived to guarantee the reachability of the sliding surface in a finite time interval. The sufficient conditions on asymptotic stability of the error system and sliding mode dynamics with disturbance attenuation level are presented in terms of linear matrix inequalities (LMIs). Finally, an example is provided to illustrate the efficiency and effectiveness of the proposed method.

1. Introduction

In real industrial fields, such as chemical processes, vehicle control systems, transmission line systems, and hydraulic systems, the time-delayed perturbations may induce system instability, oscillations, and degraded performance. Therefore, control of time-delay systems is of both practical and theoretical importance [1–3]. Recently, H_{∞} control concept has been proposed to reduce the effect of the disturbance input on the measured output to within a prescribed level [4-14]. Besides, as an effective robust control strategy for uncertain time-delay systems, sliding mode control (SMC) has been successfully applied to a wide variety of practical engineering systems such as robot manipulators, aircrafts, underwater vehicles, spacecrafts, flexible space structures, electrical motors, power systems, and automotive engines [15-27], because SMC has attractive features such as fast response and good transient response, and it is also insensitive to variations in system parameters and external disturbances. Xia et al. [15] considered sliding mode H_{∞} control for a class of uncertain nonlinear state-delayed systems without stochastic effects and input nonlinearity.

As is known, input nonlinearity is often found in practice systems and can cause a serious degradation of the system

performance [16]. Therefore, the effects of input nonlinearity must be taken into account when analyzing and implementing a SMC scheme. Some papers about input nonlinearity have been presented [28-38], but there are few works discussing SMC for time-varying delay stochastic systems with unknown state subjected to input nonlinearity. Moreover, observer is of great importance for control systems. Recently, observer-based sliding mode control has attracted more and more attention [16, 24, 34]. Liu et al. [16] have discussed H_{∞} nonfragile observer-based sliding mode control for uncertain time-delay systems without stochastic effects. Wu et al. [24] have probed observer-based sliding mode control of a class of uncertain nonlinear neutral delay systems without stochastic effects. Niu and Ho [34] have considered robust observer design for stochastic time-delay systems without input nonlinearity.

Motivated by the aforementioned reasons, the purpose of this paper lies in the development of an SMC for the uncertain time-delay systems with input nonlinearity and stochastic perturbations. Based on a nonfragile observer, a robust law is established to guarantee the reachability of the sliding surface in finite time. The sufficient condition for the asymptotic stability of the overall closed-loop system with disturbance attenuation level is derived via LMI. Finally, an example illustrates the validity of the proposed method. Notations. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $L^p_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ is the family of all \mathcal{F}_0 -measurable $C([-\tau, 0]; \mathbb{R}^n)$ -valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau \leq \theta \leq 0} \mathcal{E} \|\xi(\theta)\|_2^2 < \infty$, where $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure \mathcal{P} . \mathbb{R}^n and $\mathbb{R}^{n\times m}$ denote, respectively, the *n*-dimensional Euclidean space and the set of $n \times m$ real matrices. The superscript *T* denotes the transpose and the notation *X*, *Y* (resp. X > Y), where *X* and *Y* are symmetric matrices, and means that X - Y is positive semidefinite (resp., positive definite). L^2 stands for the space of square integral vector functions. $\|\cdot\|$ will refer to the Euclidean vector norm, and * represents the symmetric form of matrix.

2. System Description and Definitions

Consider a class of uncertain time-delay systems subjected to input nonlinearity described by:

$$dx (t) = [(A + \Delta A (t)) x (t) + (A_1 + \Delta A_1 (t)) x (t - \tau (t)) + B\phi (u) + f (x) + Gv (t)] dt + g (x (t), t) dw (t),$$
(1)
$$y (t) = Cx (t) + Dx (t - \tau (t)), x (t) = \phi (t), \quad t \in [-\tau, 0],$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $\varphi(t) \in L^p_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$ is a compatible vector-valued continuous function; $\varphi(u) \in \mathbf{R}^m$ is the control input; f(x) is the nonlinear disturbance input; v(t) is the exogenous noise; $g(x(t), t) \in \mathbf{R}^{n \times m}$ is the stochastic perturbations. $\omega(t) = (\omega_1(t), \dots, \omega_n(t))^{\mathsf{T}} \in \mathbb{R}^m$ is a Brownian motion defined on a completely space. *A*, *A*₁, *B*, *G*, and *C* are real constant matrices of appropriate dimensions. $\Delta A(t)$ and $\Delta A_1(t)$ are the uncertainties which are assumed to be the form of

$$\left[\Delta A\left(t\right), \Delta A_{1}\left(t\right)\right] = MF\left(t\right)\left[S, S_{1}\right],\tag{2}$$

where M, S, and S_1 are the real constant matrices and F(t): $R \mapsto R^{k \times l}$ is the unknown time-varying matrix function satisfying

$$F^{T}(t) F(t) \le I. \tag{3}$$

 $\tau(t)$ denotes the time-varying delay and satisfies

$$0 \le \tau(t) \le \tau, \quad \dot{\tau}(t) \le h < 1. \tag{4}$$

Remark 1. In (4), the constraint of the delay derivative $\dot{\tau}(t) \le h < 1$ is strong. The condition can be relaxed. In fact, now, many papers have no longer required the delay derivative less than 1 [39]. However, in real world, time delays in some of the systems change slowly: therefore, the constraint of the delay

derivative $\dot{\tau}(t) \leq h < 1$ is still of significance. Of course, we also can use the similar method in [39] to deal with the relaxed condition.

The following assumptions and lemmas are necessary for the sake of convenience. We will assume the following to be valid.

Assumption 2. (A, B) is completely controllable; (A, C) is completely observable.

Assumption 3. The nonlinear input $\phi(u)$ applied to the system satisfies the following property:

$$u^{T}\phi\left(u\right) \geq \alpha u^{T}u,\tag{5}$$

where α is a nonzero positive constant and $\phi(0) = 0$.

Assumption 4. For all $x_1, x_2, f(x)$ satisfies Lipschitz condition; that is,

$$\|f(x_1) - f(x_2)\| \le \rho \|x_1 - x_2\|,$$
 (6)

where ρ is a positive constant.

Assumption 5. For g(x(t), t) there is a constant matrix H such that

trace
$$\left[g^{T}(x(t),t)g(x(t),t)\right] \leq \left|Hx(t)\right|^{2}$$
 (7)

for all $t \ge 0$.

Lemma 6 (see [21]). Let $Q = Q^T$, S, $R = R^T$ be matrices of appropriate dimensions, and then R < 0, and $Q - SR^{-1}S^T < 0$ is equivalent to

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} < 0.$$
 (8)

Lemma 7 (see [21]). Let D, E, and F(t) be real matrices of appropriate dimensions with F(t) satisfying $F^{T}(t)F(t) \leq I$ and scalar $\varepsilon > 0$, and the following inequality

$$DF(t)E + E^{T}F^{T}(t)D^{T} \le \varepsilon DD^{T} + \frac{1}{\varepsilon}E^{T}E$$
 (9)

is always satisfied.

Lemma 8. Let $X \in \mathbb{R}^n$, $Y \in \mathbb{R}^n$, and $\varepsilon > 0$. Then one has $X^TY + Y^TX \le \varepsilon Y^TY + (1/\varepsilon)X^TX$.

3. Slide Mode Control

3.1. H_{∞} Nonfragile Observer Design. First, the following nonfragile state observer is utilized to estimate the state of uncertain time-delay systems (1)

$$\dot{\hat{x}}(t) = A\hat{x}(t) + A_1\hat{x}(t - \tau(t)) + B\phi(u) + f(\hat{x}) + (L + \Delta L(t))(y - C\hat{x}),$$
(10)

where $L \in \mathbb{R}^{n \times q}$ is the observer gain to be designed later and $\Delta L(t)$ is a nonlinear function matrix satisfying

$$\|\Delta L(t)\| \le \delta,\tag{11}$$

where δ is a positive constant.

Define the error $e(t) = x(t) - \hat{x}(t)$, and then it follows from systems (1) and (10) that

$$de(t) = \left[(A - LC + \Delta A) e(t) - \Delta LCe(t) + (A_1 + \Delta A_1) e(t - \tau(t)) + \Delta A_1 \hat{x}(t - \tau(t)) + \Delta A \hat{x}(t) - (L + \Delta L) Dx(t - \tau(t)) + f(x) - f(\hat{x}) + Gv(t) \right] dt + g(x(t), t) d\omega(t),$$

$$y_e(t) = Ce(t) + De(t - \tau(t)).$$
(12)

where $y_e(t)$ is the output of error system.

Then we introduce H_{∞} performance measure as follows:

$$J = \mathscr{C} \int_0^\infty \left[y_e^T(t) \ y_e(t) - \gamma^2 v^T(t) \ v(t) \right] dt, \qquad (13)$$

Therefore, the problem is to determine the error e(t) within the upper bound; that is,

$$\mathscr{E}\left(\sup_{\nu(t)\in L_2}\frac{\left\|y_e(t)\right\|_2}{\left\|\nu(t)\right\|_2}\right) < \gamma.$$
(14)

A novel switching function is chosen as

$$s(t) = \sigma(t) + B^T \hat{x}(t), \qquad (15)$$

with

$$\dot{\sigma}(t) = B^T B K \hat{x}(t) - B^T A_1 \hat{x}(t - \tau(t)) - B^T f(\hat{x}), \quad (16)$$

where the matrix K is to be chosen later; obliviously, $B^T B$ is nonsingularity.

Remark 9. The novel sliding mode surface functional (15) is established, which is different from the existing references [16, 19, 20, 22–25, 28] due to the additional term $\sigma(t)$ that is a differentiable functional with respect to nonfragile state observers and time-varying delays. As a result, it will be more convenient for the controller design.

Control input $\phi(u)$ in system (1) should be appropriately designed such that the estimated state in system (10) can be driven to the sliding surface even when the input nonlinearity presented. The SMC law is derived as follows:

$$u(t) = -\frac{s(t)}{\|s(t)\|} \Psi(\hat{x}), \qquad (17)$$

where $\Psi(\hat{x}) = (1/\alpha)(\|K\hat{x}(t)\| + \|(B^TB)^{-1}B^TL(y - C\hat{x})\| + \delta\|(B^TB)^{-1}B^T\| \cdot \|y - C\hat{x}\| + \|(B^TB)^{-1}B^TA\hat{x}(t)\| + \beta)$, and β is an arbitrarily positive scalar.

This proposed control scheme above will drive the estimate state to approach the sliding mode surface s(t) = 0 in a finite time, and it is stated in the following theorem.

Theorem 10. If the control input u(t) is designed as (17), then the trajectories of the observer system (10) converge to the sliding surface s(t) = 0 in a finite time. Proof. From system (10) and (15), we have

$$\dot{s}(t) = B^T B K \hat{x}(t) + B^T B \phi(u)$$

$$+ B^T (L + \Delta L) (y - C \hat{x}) + B^T A \hat{x}(t).$$
(18)

Let $V_1(t) = (1/2)s^T(t)(B^T B)^{-1}s(t)$. It follows from (18) that

$$\begin{split} \dot{V}_{1} &= s^{T} \left(t \right) \left(B^{T} B \right)^{-1} \left(B^{T} B K \hat{x} \left(t \right) + B^{T} B \phi \left(u \right) \right. \\ &+ B^{T} \left(L + \Delta L \right) \left(y - C \hat{x} \right) + B^{T} A \hat{x} \left(t \right) \right) \\ &= s^{T} \left(t \right) K \hat{x} \left(t \right) + s^{T} \left(t \right) \phi \left(u \right) \\ &+ s^{T} \left(t \right) \left(B^{T} B \right)^{-1} B^{T} \left(L + \Delta L \right) \left(y - C \hat{x} \right) \\ &+ s^{T} \left(t \right) \left(B^{T} B \right)^{-1} B^{T} A \hat{x} \left(t \right) \\ &\leq s^{T} \phi \left(u \right) + \| s \left(t \right) \| \left(\| K \hat{x} \left(t \right) \| \right. \\ &+ \left\| \left(B^{T} B \right)^{-1} B L \left(y - C \hat{x} \right) \right\| \\ &+ \left\| \left(B^{T} B \right)^{-1} B^{T} A \hat{x} \left(t \right) \right\| \\ &+ \delta \left\| \left(B^{T} B \right)^{-1} B^{T} \| \cdot \| \left(y - C \hat{x} \right) \| \right). \end{split}$$
(19)

Using (17) and Assumption 3, we have $u^T \phi(u) = -(s^T(t)/||s(t)||)\Psi(\hat{x})\phi(u) \ge \alpha \Psi^2(\hat{x})$, and then

$$s^{T}\phi(u) \le -\alpha \Psi(\hat{x}) \| s(t) \|.$$
⁽²⁰⁾

Substituting (20) into (19) yields

$$\dot{V}_1 \le -\beta \|s(t)\| < 0, \quad \forall \|s(t)\| \ne 0.$$
 (21)

From (21), we prove the finite time convergence of system (10) toward the surface s(t) = 0. Then the proof is completed.

From $\dot{s}(t) = 0$, the following equivalent control law can be obtained:

$$\phi_{\text{eq}}(u) = -K\hat{x}(t) - (B^{T}B)^{-1}B^{T}(L + \Delta L)(y - C\hat{x})$$

$$- (B^{T}B)^{-1}B^{T}A\hat{x}(t).$$
(22)

Substituting (22) into the observer system (10) and noting $\overline{B} = I - B(B^T B)^{-1} B^T$, the sliding mode dynamics in the state estimation space can be obtained as follows:

$$\widehat{x}(t) = (BA - BK) \widehat{x}(t) + A_1 \widehat{x}(t - \tau(t)) + \overline{B}(L + \Delta L) Ce(t) + \overline{B}(L + \Delta L) D[e(t - \tau(t)) + \widehat{x}(t - \tau(t))] + f(\widehat{x}).$$
(23)

Hence, the stability of the overall closed-loop system with (1) and (4) will be analyzed through the error system (12) and the sliding mode dynamics (23).

3.2. Analysis of Asymptotic Stability. In the following theorem, the sufficient condition for the asymptotic stability of the overall closed-loop system with disturbance attenuation level is given in terms of LMIs.

Theorem 11. Consider that the systems (12) and (23). Given a scalar $\gamma > 0$, the switching function is chosen as (15), and the SMC law is chosen as (17). If there exist matrices $Q_1 > 0$, $Q_2 > 0$, L, K, and scalars $\varepsilon_i > 0$, (i = 1, 2, 3, 4, 5, 6, 7, 8, 9) satisfying the following linear matrix inequality (LMI):

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & C^T L^T \overline{B}^T + H^T H & -LD & G & N_1 \\ * & \Gamma_{22} & D^T L^T \overline{B}^T & 0 & 0 & N_2 \\ * & * & \Gamma_{33} & \Gamma_{34} & 0 & N_3 \\ * & * & * & \Gamma_{44} & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & N_4 \end{bmatrix} < 0,$$
(24)

$$\begin{split} \Gamma_{11} &= (A - LC) + (A - LC)^{T} + \varepsilon_{1}S^{T}S \\ &+ \left(\varepsilon_{3}\delta^{2} + \varepsilon_{6}\delta^{2} + 1 + \varepsilon_{9}\right)C^{T}C + Q_{1} + H^{T}H + 2\rho I, \\ \Gamma_{12} &= \varepsilon_{1}S^{T}S_{1} + (A_{1} - LD), \\ \Gamma_{22} &= \varepsilon_{1}S_{1}^{T}S_{1} - (1 - h)Q_{1} + \left(\varepsilon_{4}\delta^{2} + \varepsilon_{7}\delta^{2}\right)D^{T}D, \\ \Gamma_{33} &= \varepsilon_{2}S^{T}S + \left[\left(\overline{B}A - BK\right) + \left(\overline{B}A - BK\right)^{T}\right] \\ &+ Q_{2} + H^{T}H + 2\rho I, \\ \Gamma_{34} &= A_{1} + \varepsilon_{2}S^{T}S_{1} + D^{T}L^{T}\overline{B}^{T}, \\ \Gamma_{44} &= \varepsilon_{2}S_{1}^{T}S_{1} - (1 - h)Q_{2} + \left(\varepsilon_{5}\delta^{2} + \varepsilon_{8}\delta^{2}\right)D^{T}D, \\ N_{1} &= \left[M, M, I, I, I, 0, 0, 0, 0\right], \\ N_{2} &= \left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, D^{T}\right], \\ N_{3} &= \left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right], \\ N_{4} &= \operatorname{diag}\left[-\varepsilon_{1}I, -\varepsilon_{2}I, -\varepsilon_{3}I, -\varepsilon_{4}I, \\ -\varepsilon_{5}I, -\varepsilon_{6}I, -\varepsilon_{7}I, -\varepsilon_{8}I, -\varepsilon_{9}I\right], \end{split}$$
(25)

the overall closed-loop system with (12) and (23) is asymptotically stable with disturbance attenuation level γ .

Proof. Choose the following Lyapunov function candidate:

$$V_{2}(t) = e^{T}(t) e(t) + \int_{t-\tau(t)}^{t} e^{T}(s) Q_{1}e(s) ds + \hat{x}^{T}(t) \hat{x}(t) + \int_{t-\tau(t)}^{t} \hat{x}^{T}(s) Q_{2}\hat{x}(s) ds.$$
(26)

First, we consider the overall closed-loop system with v(t) = 0, and we have

$$\begin{aligned} \mathscr{D}V_{2}(t) &= e^{T}(t) \left[(A - LC) + (A - LC)^{T} \right] e(t) \\ &+ 2e^{T}(t) \left[\Delta Ae(t) + \Delta A_{1}e(t - \tau(t)) \right] \\ &+ 2e^{T}(t) \left[\Delta Ae(t) + \Delta A_{1}e(t - \tau(t)) \right] \\ &- 2e^{T}(t) \Delta LCe(t) - 2e^{T}(t) \Delta LDe(t - \tau(t)) \\ &- 2e^{T}(t) LD\hat{x}(t - \tau(t)) \\ &+ 2e^{T}(t) \left[\Delta A\hat{x}(t) + \Delta A_{1}\hat{x}(t - \tau(t)) \right] \\ &- 2e^{T}(t) \Delta LD\hat{x}(t - \tau(t)) \\ &+ 2e^{T}(t) \left[f(x) - f(\hat{x}) \right] \\ &+ trace \left[g^{T}(x(t), t) g(x(t), t) \right] + e^{T}(t) Q_{1}e(t) \\ &- (1 - \dot{\tau}(t)) e^{T}(t - \tau(t)) Q_{1}e(t - \tau(t)) \\ &+ \hat{x}^{T}(t) \left[\left(\overline{B}A - BK \right) + \left(\overline{B}A - BK \right)^{T} \right] \hat{x}(t) \\ &+ 2\hat{x}^{T}(t) \overline{B}\Delta LCe(t) + \hat{x}^{T}(t) Q_{2}\hat{x}(t - \tau(t)) \\ &+ 2\hat{x}^{T}(t) f(\hat{x}) + 2\hat{x}^{T}(t) \overline{B}LD \\ &\times \left[e(t - \tau(t)) + \hat{x}(t - \tau(t)) \right] \\ &+ 2\hat{x}^{T}(t) \overline{B}\Delta LD \left[e(t - \tau(t)) + \hat{x}(t - \tau(t)) \right] . \end{aligned}$$

Using Lemma 7 and Assumption 3, we have

$$\begin{split} &2e^{T}\left(t\right)\left[\Delta Ae\left(t\right)+\Delta A_{1}e\left(t-\tau\left(t\right)\right)\right]\\ &\leq\frac{1}{\varepsilon_{1}}e^{T}\left(t\right)MM^{T}e\left(t\right)\\ &+\varepsilon_{1}\left[Se\left(t\right)+S_{1}e\left(t-\tau\left(t\right)\right)\right]^{T}\left[Se\left(t\right)+S_{1}e\left(t-\tau\left(t\right)\right)\right],\\ &2e^{T}\left(t\right)\left[\Delta A\widehat{x}\left(t\right)+\Delta A_{1}\widehat{x}\left(t-\tau\left(t\right)\right)\right]\\ &\leq\frac{1}{\varepsilon_{2}}e^{T}\left(t\right)MM^{T}e\left(t\right)\\ &+\varepsilon_{2}\left[S\widehat{x}\left(t\right)+S_{1}\widehat{x}\left(t-\tau\left(t\right)\right)\right]^{T}\left[S\widehat{x}\left(t\right)+S_{1}\widehat{x}\left(t-\tau\left(t\right)\right)\right]\\ &-2e^{T}\left(t\right)\Delta LCe\left(t\right)\\ &\leq2\delta\left\|e^{T}\left(t\right)\right\|\cdot\left\|Ce\left(t\right)\right\|\\ &\leq\frac{1}{\varepsilon_{3}}e^{T}\left(t\right)e\left(t\right)+\varepsilon_{3}\delta^{2}e^{T}\left(t\right)C^{T}Ce\left(t\right), \end{split}$$

$$-2e^{T}(t) \Delta LDe(t - \tau(t))$$

$$\leq \frac{1}{\epsilon_{4}}e^{T}(t) e(t) + \epsilon_{4}\delta^{2}e^{T}(t - \tau(t)) D^{T}De(t - \tau(t))$$

$$-2e^{T}(t) \Delta LD\hat{x}(t - \tau(t))$$

$$\leq \frac{1}{\epsilon_{5}}e^{T}(t) e(t) + \epsilon_{5}\delta^{2}\hat{x}^{T}(t - \tau(t)) D^{T}D\hat{x}(t - \tau(t)),$$

$$2\hat{x}^{T}(t) \overline{B}\Delta LCe(t) \leq \frac{1}{\epsilon_{6}}\hat{x}^{T}(t) \overline{B}\overline{B}^{T}\hat{x}(t)$$

$$+ \epsilon_{6}\delta^{2}e^{T}(t) C^{T}Ce(t),$$

$$2\hat{x}^{T}(t) \overline{B}\Delta LDe(t - \tau(t))$$

$$\leq \frac{1}{\epsilon_{7}}\hat{x}^{T}(t) \overline{B}\overline{B}^{T}\hat{x}(t) + \epsilon_{7}\delta^{2}e^{T}(t - \tau(t)) D^{T}De(t - \tau(t)),$$

$$2\hat{x}^{T}(t) \overline{B}\Delta LD\hat{x}(t - \tau(t))$$

$$\leq \frac{1}{\epsilon_{8}}\hat{x}^{T}(t) \overline{B}\overline{B}^{T}\hat{x}(t) + \epsilon_{8}\delta^{2}\hat{x}(t - \tau(t)) D^{T}Dx(t - \tau(t)),$$

$$2e^{T}(t) [f(x) - f(\hat{x})] \leq 2 ||e^{T}(t)|| \rho ||e(t)|| = 2\rho e^{T}(t) e(t),$$

$$2\hat{x}^{T}(t) f(\hat{x}) \leq 2 ||\hat{x}^{T}(t)|| \rho ||\hat{x}(t)|| = 2\rho \hat{x}^{T}(t) \hat{x}(t).$$
(28)

Using lemma 8, we have

$$[Ce(t) + De(t - \tau(t))]^{T} [Ce(t) + De(t - \tau(t))]$$

$$\leq (1 + \varepsilon_{9}) e^{T}(t) C^{T} Ce(t) \qquad (29)$$

$$+ (1 + \varepsilon_{9}^{-1}) e^{T}(t - \tau(t)) D^{T} De(t - \tau(t)).$$

Substituting (28)-(29) into (27) results in

$$\mathscr{L}V_{2}(t) \leq \Lambda^{T}(t) \Xi \Lambda(t), \qquad (30)$$

where
$$\Lambda^{T}(t) = [e^{T}(t), e^{T}(t - \tau(t)), \hat{x}^{T}(t), \hat{x}^{T}(t - \tau(t))], \text{ and}$$

$$\Xi = \begin{bmatrix} \Xi_{1} \ \varepsilon_{1}S^{T}S_{1} + (A_{1} - LD) \ C^{T}L^{T}\overline{B}^{T} + H^{T}H \ -LD \\ * \ \Xi_{2} \ D^{T}L^{T}\overline{B}^{T} \ 0 \\ * \ * \ \Xi_{3} \ \Xi_{34} \\ * \ * \ & \Xi_{4} \end{bmatrix},$$
(31)

with

$$\begin{split} \Xi_{1} &= (A - LC) + (A - LC)^{T} + \left(\frac{1}{\varepsilon_{4}} + \frac{1}{\varepsilon_{3}} + \frac{1}{\varepsilon_{5}}\right)I \\ &+ Q_{1} + H^{T}H + 2\rho I + \varepsilon_{1}S^{T}S \\ &+ \left(\frac{1}{\varepsilon_{1}} + \frac{1}{\varepsilon_{2}}\right)MM^{T} + \left(\varepsilon_{3}\delta^{2} + \varepsilon_{6}\delta^{2}\right)C^{T}C, \\ \Xi_{2} &= \varepsilon_{1}S_{1}^{T}S_{1} + \left(\varepsilon_{4}\delta^{2} + \varepsilon_{7}\delta^{2}\right)D^{T}D - (1 - h)Q_{1}, \\ \Xi_{3} &= \varepsilon_{2}S^{T}S + \left[\left(\overline{B}A - BK\right) + \left(\overline{B}A - BK\right)^{T}\right] \\ &+ Q_{2} + \left(\frac{1}{\varepsilon_{6}} + \frac{1}{\varepsilon_{7}} + \frac{1}{\varepsilon_{8}}\right)\overline{B}\overline{B}^{T} + H^{T}H + 2\rho I, \\ \Xi_{4} &= \varepsilon_{2}S_{1}^{T}S_{1} + \left(\varepsilon_{5}\delta^{2} + \varepsilon_{8}\delta^{2}\right)D^{T}D - (1 - h)Q_{2}, \\ \Xi_{34} &= A_{1} + \varepsilon_{2}S^{T}S_{1} + D^{T}L^{T}\overline{B}^{T}. \end{split}$$

It can be shown that if LMI (24) is satisfied, $\Xi < 0$ is held by Lemma 6, and then $\mathscr{L}V_2(t) \le \Lambda^T(t)\Xi\Lambda(t) < 0 \ (\forall \Lambda(t) \ne 0)$, which shows that the closed-loop system is asymptotically stable. The performance index is

$$J = \mathscr{C} \int_{0}^{\infty} \left[y_{e}^{T}(t) y_{e}(t) - \gamma^{2} v^{T}(t) v(t) \right] dt$$

$$\leq \mathscr{C} \int_{0}^{\infty} \left[y_{e}^{T}(s) y_{e}(t) - \gamma^{2} v^{T}(t) v(t) + \mathscr{L} V_{2}(t) \right] dt \quad (33)$$

$$= \mathscr{C} \int_{0}^{\infty} q^{T}(t) \Pi q(t) dt,$$

where
$$q^{T}(t) = [e^{T}(t), e^{T}(t - \tau(t)), \hat{x}^{T}(t), \hat{x}^{T}(t - \tau(t)), v(t)],$$

$$\Pi = \begin{bmatrix} \Xi_1 + (1 + \varepsilon_9) C^T C & \varepsilon_1 S^T S_1 + (A_1 - LD) C^T L^T \overline{B}^T + H^T H & -LD & G \\ & * & \Xi_2 + (1 + \varepsilon_9^{-1}) D^T D & D^T L^T \overline{B}^T & 0 & 0 \\ & * & * & \Xi_3 & \Xi_{34} & 0 \\ & * & * & * & \Xi_4 & 0 \\ & * & * & * & * & -\gamma^2 I \end{bmatrix}.$$
(34)

By utilizing lemma 6, it is seen that $\Pi < 0$ is equivalent to (24). This means J < 0 (for $\Pi \neq 0$), so the overall closed-loop

system is asymptotically stable with disturbance attenuation γ . Then the proof is obtained.



FIGURE 1: Trajectories of state x(t) and e(t).



FIGURE 2: : Trajectories of sliding mode variable *s*(*t*).



FIGURE 3: Control signal u(t).

4. Numerical Example

In this section, a numerical example demonstrates the effectiveness of the method mentioned above. Consider the system (12) and (23) with the following matrices:

$$\mathbf{A} = \begin{bmatrix} -5 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -7 \end{bmatrix}, \qquad \mathbf{A}_{1} = \begin{bmatrix} -0.5 & 0.2 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}, \\ \mathbf{B} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \\ \mathbf{D} = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.1 & 0.1 & 0 \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0 \\ 0 & 0.2 & 0.1 \end{bmatrix}, \\ \mathbf{S} = \mathbf{S}_{1} = \begin{bmatrix} 0 & 0 & 0.1 \\ 0 & 0.2 & 0.2 \\ 0 & 0.2 & 0.1 \end{bmatrix}, \qquad \mathbf{G} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \\ 0 \end{bmatrix}, \\ \mathbf{H} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
(35)

where $\phi(u) = [0.5 + 0.2 \sin(u(t))] \cdot u(t)$, $f(x) = \sin(x)$, $F(t) = \sin(t)I_{3\times3}$, $\tau(t) = (1/2)\sin(t)$, $\tau = h = 0.5$, $\rho = 0.5$, $\delta = 0.5$, $\gamma = 0.7$, and $g(x(t), t) = \sin(t) \operatorname{diag}[0.1, 0.2, 0.1]x(t)$. Therefore, by solving the LMI mentioned in Theorem 11, we obtain the following result:

$$\mathbf{L} = \begin{bmatrix} -0.7051 & 23.5131 \\ -3.5327 & 24.8253 \\ 3.6942 & -0.9655 \end{bmatrix},$$
 (36)

 $\mathbf{K} = \begin{bmatrix} 38.9472 & 42.0601 & 2.8093 \end{bmatrix}$.

 $\varepsilon_1 = 9.1178$, $\varepsilon_2 = 7.8570$, $\varepsilon_3 = 4.0186$, $\varepsilon_4 = 45.8969$, $\varepsilon_5 = 21.4368$, $\varepsilon_6 = 3.9560$, $\varepsilon_7 = 42.5322$, $\varepsilon_8 = 21.2533$, and $\varepsilon_9 = 1.7636$. Substitute the above numerical into (12) and (23) and set $[x_1(0), x_2(0), x_3(0), e_1(0), e_2(0), e_3(0)] = [1, -0.6, 0.6, 0.4, -0.3, 0.2]$. The simulation results are given as Figures 1–3, which show the validity of the proposed method.

Remark 12. In this example, because the additional term $\sigma(t)$ is a differentiable functional with respect to nonfragile state observers and time-varying delays, the results in the existing references [16, 19, 20, 22–25, 28] do not work, which shows that our results are new.

5. Conclusions

The problem of H_{∞} sliding mode control for uncertain timedelay systems subjected to input nonlinearity and stochastic perturbations has been addressed. Based on the sliding mode control strategy and LMI technique, some criteria on asymptotic stability of the error system and sliding mode dynamics with disturbance attenuation level are derived in terms of LMIs. Finally, an example is provided to illustrate the validity of the proposed method and the obtained results.

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