Research Article

Tool Wear Detection Using Lipschitz Exponent and Harmonic Wavelet

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The paper researches a novel engineering application of Lipschitz exponent function and harmonic wavelet for detecting tool condition. Tool wear affects often the quality grade of products and is gradually formed during cutting process. Meanwhile, since cutting noise is very strong, we think tool wear belongs to detecting weak singularity signals in strong noise. It is difficult to obtain a reliable worn result by raw sampled data. We propose singularity analysis with harmonic wavelet for data processing and a new concept of Lipschitz exponent function. The method can be quantitative tool condition and make maintaining decision. Test result was validated with 27 kinds of cutting conditions with the sharp tool and the worn tool; 54 group data are sampled by acoustic emission (AE).

1. Introduction

In recent years, great achievements in tool wear monitoring have been made based on the advanced mathematical models [1–8], even though none of these methods was successful online monitoring due to the complex performance of the machining processes. How to be quantitative tool condition monitoring is very important for practical engineering applications. This is a signal process problem that detects weak singularity signals in strong noise. However, the classical signal process is Fourier theory which has four defects [9–11]: (1) suits stationary stochastic signal, this kind of signal does not almost exists in engineering; (2) for discrete Fourier, Fourier analysis is precise with time series long from zero to infinite, otherwise truncation error is made; (3) weak singularity signal, relatively strange signal, is omitted in FFT however, weak singularity signal often contains important fault information; (4) when signal frequency happen a sudden change with adjacent time intervals, Fourier method only obtains two frequencies, it cannot display the moment two frequency changes or occurs (i.e., the singularities) in frequency domain. In order to overcome the above four defects to improve the measuring precision of weak singular signal in strong noise, we propose the combination between Lipschitz exponent index and complex wavelet for the detection of quantitative tool wear.

Lipschitz functions appear almost everywhere in mathematics [12]. Lipschitz functions are an important class of strong variations originating from smooth deformations of corresponding nonsmooth function [13]; the local regularity is often measured by Lipschitz exponents (LE); the singular variations can be viewed as combinations of weak increasing and decreasing variations; it means the case of discontinuity occurence on smooth function. Since the paper [14] is published in 1992 to detect signal's singularities, which proposed that the instantaneous frequencies can be approximately identified from the modulus maxima of a real wavelet based on general maxima, which are the locations of the largest modulus along maxima lines through an extension of [14], the complex-wavelet modulus maxima can detect and characterize singularities [15, 16] and has taken excellent result for measuring weak singularities of signal. Harmonic wavelet is one kind of complex wavelet; it is very sensitive to singularity feature of the signal that contains weak faults [17–20] and satisfies multiwavelets frames with arbitrary integer dilation factor [21]. The paper combines the modulus maxima of using a harmonic wavelet, and Lipschitz functions can be quantitative tool condition during cutting process. By 27 kinds of cutting conditions and turning experiments on HL-32 NC turning center, 54 group data are sampled by acoustic emission (AE). We have demonstrated
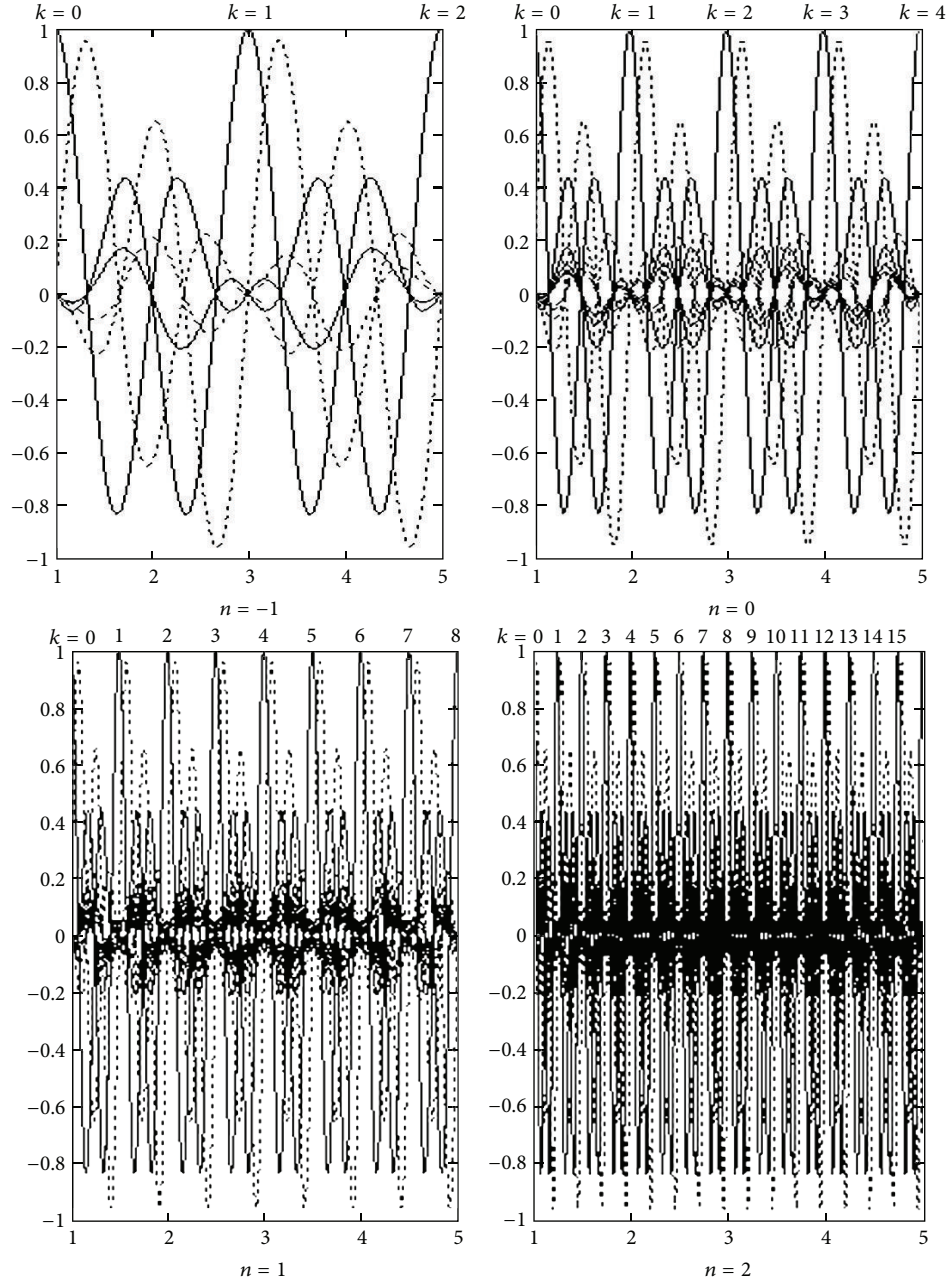
FIGURE 1: Real (thick line) and imaginary (thin line).

that combining the modulus maxima of harmonic wavelet and Lipschitz functions is better for detecting and estimating tool wear when a worn tool generates a singular signal in a strong noise.

2. Harmonic Wavelet Transform Algorithm

Discrete harmonic wavelets are given by

$$\psi_k^n \left(2^n x - k\right) \overset{\text{def}}{=} \frac{\exp\left(4\pi i \left(2^n x - k\right)\right) - \exp\left(2\pi i \left(2^n x - k\right)\right)}{2\pi i \left(2^n x - k\right)},$$

(1)

where $n = -1, 0, 1, 2, \ldots, N$. n is different levels (scale), and k is different steps (time). The functions of its scale and time are given by Figure 1.

The Fourier transform of (1) is as below:

$$\left|\widehat{\psi}_k^n (\omega)\right| = \begin{cases} \dfrac{1}{2\pi 2^n}, & 2\pi 2^n \leq \omega \leq 2\pi 2^{n+1}, \\ 0, & \text{other.} \end{cases}$$

(2)

The amplitude-frequency characteristics of (2) is exactly like a box with band limited, as shown Figure 2. This characteristic is the best than any real wavelets for decomposed signal to difference frequency band.
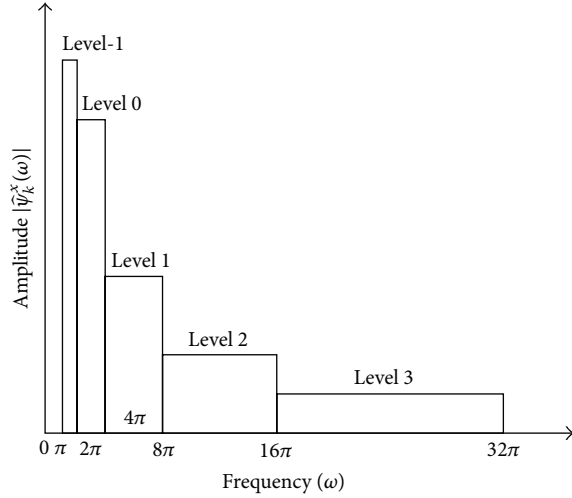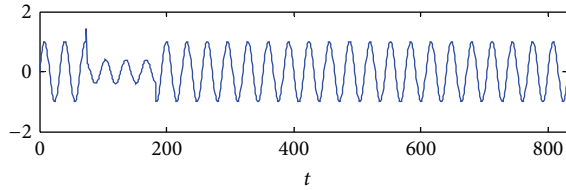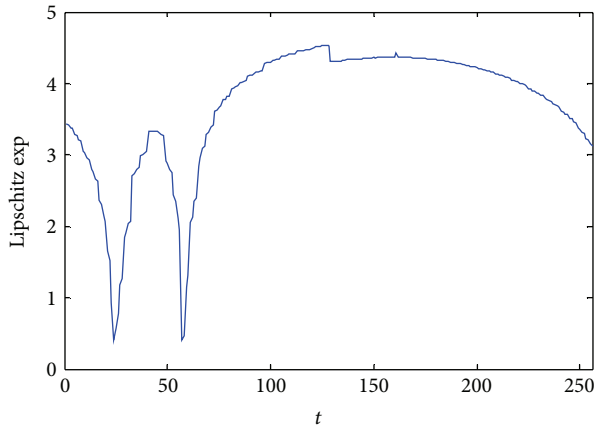
FIGURE 2: Amplitude-frequency characteristics of the discrete harmonic wavelets.

(a) Sine function with two singular points

(b) $\alpha(t)$ along the temporal axis t

FIGURE 3: Extracting Lipschitz exponent function $\alpha(t)$.

3. The Definition of Lipschitz Exponent

Lipschitz is from Taylor formula [16, 22, 23] Suppose that function $f(x)$ is m times differentiable in $[x_0 - h, x_0 + h]$; then, Taylor expands in the neighborhood of x_0:

$$f_{x_0}(x) = \sum_{k=0}^{m-1} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k \qquad (3)$$

exists one approximate error:
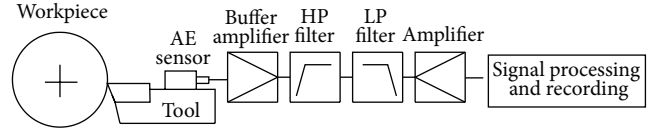
$$e(x) = f(x) - f_{x_0}(x) \qquad (4)$$

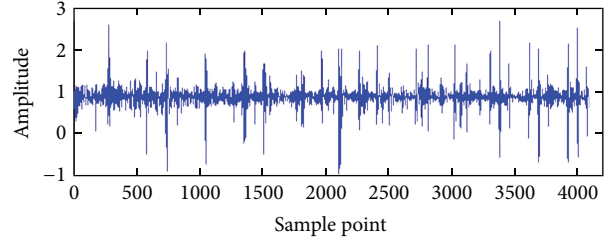FIGURE 4: AE measurement in metal cutting.

FIGURE 5: Waveform of sharp tool in first condition.

which satisfies:

$$\forall x \in [x_0 - h, x_0 + h],$$
$$|e(x)| \le \frac{|x - x_0|^m}{m!} \sup_{\zeta \in [x_0 - h, x_0 + h]} |f^m(\zeta)|. \qquad (5)$$

The mth order differentiability of $f(x)$ in the neighborhood of x_0 yields an upper bound on the error $e(x)$ when x tends to x_0. The Lipschitz regularity refines this upper bound with noninteger exponents. The definition of Lipschitz exponent is as follows: a function $f(x)$ is of Lipschitz α at x_0, if and only if there exists a constant A such that for all x in a neighborhood of x_0

$$|f(x) - f(x_0)| \le A|x - x_0|^\alpha. \qquad (6)$$

We say that $f(x)$ is uniformly Lipschitz α for all x. Equation (6) is a kind of fractional order [20]. In a neighborhood of x_0, $f(x)$ is singular when $\alpha \ne 1$, and $f(x)$ is continuous when $\alpha > 0$. In other words, If $f(x)$ is discontinuous and bounded in a neighborhood of x_0, then $\alpha = 0$. If $f(x)$ is continuously differentiable then $\alpha = 1$ and $f(x)$ is not singular.

4. Modulus Maxima of Wavelet

Reference [14] depicts the modulus maxima of real wavelet to detect singular signal, and [15] extends to the modulus maxima of complex wavelet, and algorithm is deduced in detail. We say a function $f(x)$ is a modulus maximum of wavelet at the point $(2^n, x_0)$; then,

$$|\psi f(2^n, x)| < |\psi f(2^n, x_0)| \qquad (7)$$

or

$$\frac{\partial \psi f(2^j, x_0)}{\partial x} = 0, \qquad (8)$$

where x belongs to either the right or the left neighborhood of x_0. A maxima line is consisted of a connected curve of the

TABLE 1: Experimental cutting conditions.

| No.	Speed r/min	Depth of cut mm	Feed mm/r	No.	Speed r/min	Depth of cut mm	Feed mm/r
1	1500	1	0.1	15	1000	0.2	0.05
2	1500	0.5	0.1	16	800	1	0.05
3	1500	0.2	0.1	17	800	0.5	0.05
4	1000	1	0.1	18	800	0.2	0.05
5	1000	0.5	0.1	19	1500	1	0.02
6	1000	0.2	0.1	20	1500	0.5	0.02
7	800	1	0.1	21	1500	0.2	0.02
8	800	0.5	0.1	22	1000	1	0.02
9	800	0.2	0.1	23	1000	0.5	0.02
10	1500	1	0.05	24	1000	0.2	0.02
11	1500	0.5	0.05	25	800	1	0.02
12	1500	0.2	0.05	26	800	0.5	0.02
13	1000	1	0.05	27	800	0.2	0.02
14	1000	0.5	0.05				
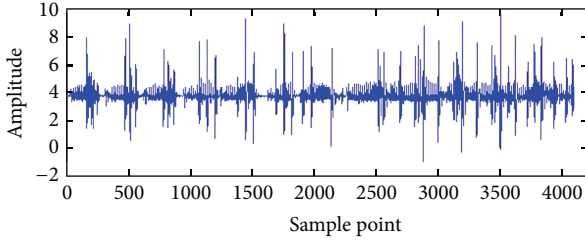
FIGURE 6: Waveform of wear tool in first condition.

modulus maxima in the scale space $n = -1, 0, 1, 2, \ldots, N$. These multiscale modulus maxima are used to locate discontinuities [16], and if $\psi f(2^n, x)$ has no modulus maxima at fine scales, then $f(x)$ is locally regular. Singular point can be recognized by modulus maxima.

5. Lipschitz Exponent Based on Modulus Maxima of Harmonic Wavelet

It has been shown that the Lipschitz exponent of a local singularity can be characterized by being placed on their local modulus maxima at each scale [14, 16]. References [15, 24] extend to using the modulus maxima of a complex wavelet. Here, we research that the modulus maxima of harmonic wavelet can be used to detect singularities, by means of a Lipschitz exponent of a function, and quantitative analysis on signal's singularities is measured. By examining the asymptotic decay of wavelet modulus maxima from coarser scale to finer scale, the strength of the singularity can be characterized by Lipschitz exponent.

Let $s = 2^n$, $n = -1, 0, \ldots, N$, in terms of [15, 25], function $f(x)$, $x \in (a, b)$, and let nth $(n > 0)$ derivative of $\text{Re}(\psi f(s, x))$ and $\text{Im}(\psi f(s, x))$ be finite deviations for each scale s. If a scale $s_0 > 0$ and constants C and A exist such that for $x \in (a, b)$ and

$s > s_0$, all the modulus maxima of $\psi f(s, x)$ belong to a cone defined by

$$|x - x_0| \leq Cs \qquad (9)$$

such that at each modulus maxima (s, x) in the cone

$$|\psi f(s, x)| \leq As^\alpha \qquad (10)$$

then $f(x)$ is uniformly Lipschitz α at x when $x \neq x_0$, and $f(x)$ is Lipschitz α at x_0 when $\alpha < n$, and α is a noninteger. Equation (10) takes the logarithm on both sides:

$$\log_2 |\psi f(s, x)| \leq \log_2 A + \alpha \log_2 s. \qquad (11)$$

In [15, 24], A and α are computed by

$$\min_{A, \alpha} \sum_s \left[\log_2 |Wf(s, x)| - \log_2 A - \alpha * \log_2 s \right]^2. \qquad (12)$$

The problem of estimating α transforms into optimization based on a nonlinear least squared, and when (12) is minimized at all scale, A and α are obtained.

According to [24] description of the results, we use $s = 2^n$ instead of s, we get the following:

$$
\begin{aligned}
\alpha(x) &= \left(\sum_{n=-1}^{N} \left(\log_2 |Wf(2^n, x)| * \log_2 2^n \right) \right. \\
&\quad \left. - \frac{\left(\sum_{n=-1}^{N} \log_2 |Wf(2^n, x)| \right) \left(\sum_{n=-1}^{N} \log_2 2^n \right)}{N} \right) \\
&\quad \times \left(\sum_{n=-1}^{N} (\log_2 2^n)^2 - \frac{\sum_{n=-1}^{N} (\log_2 2^n)^2}{N} \right)^{-1}.
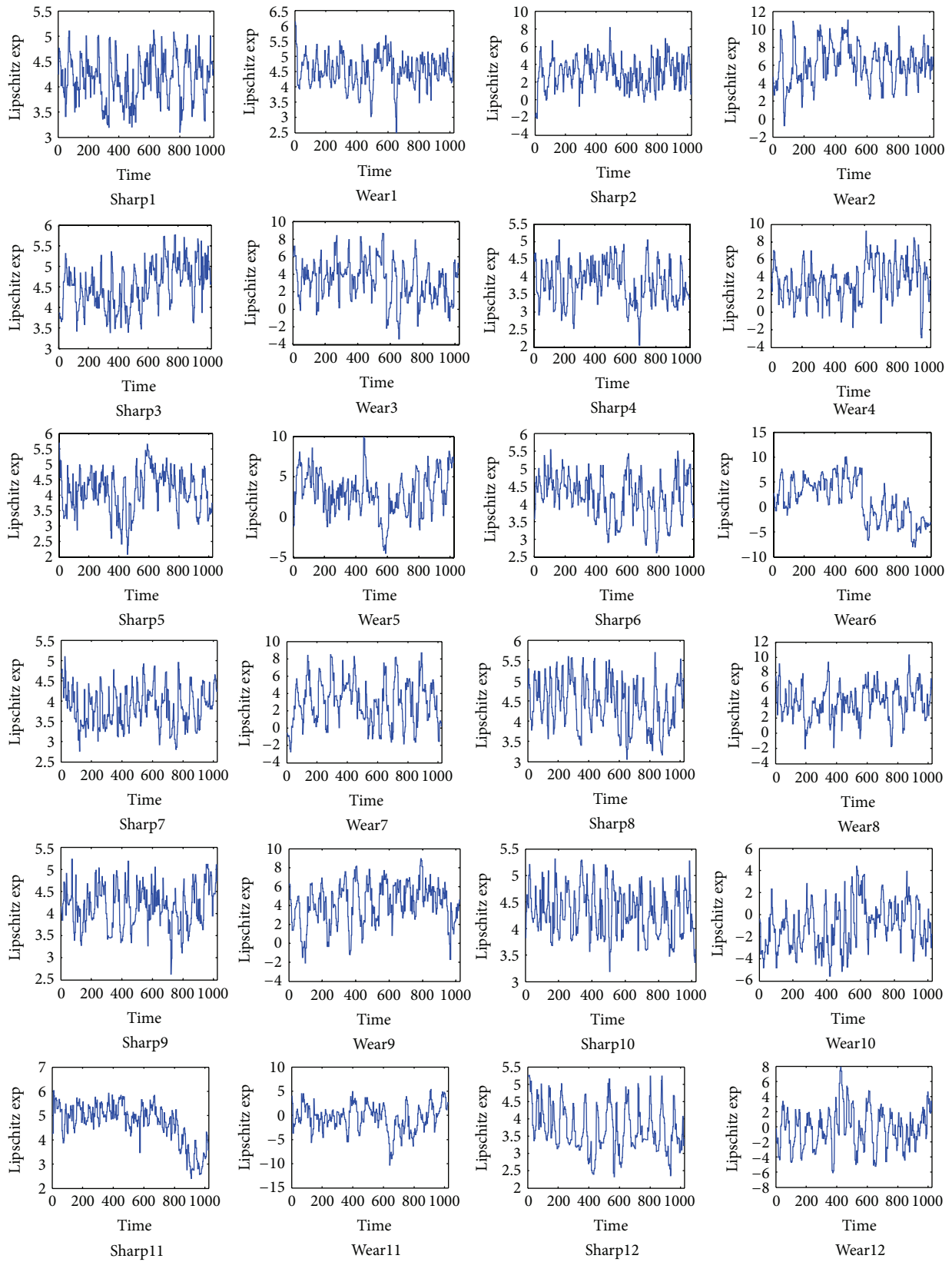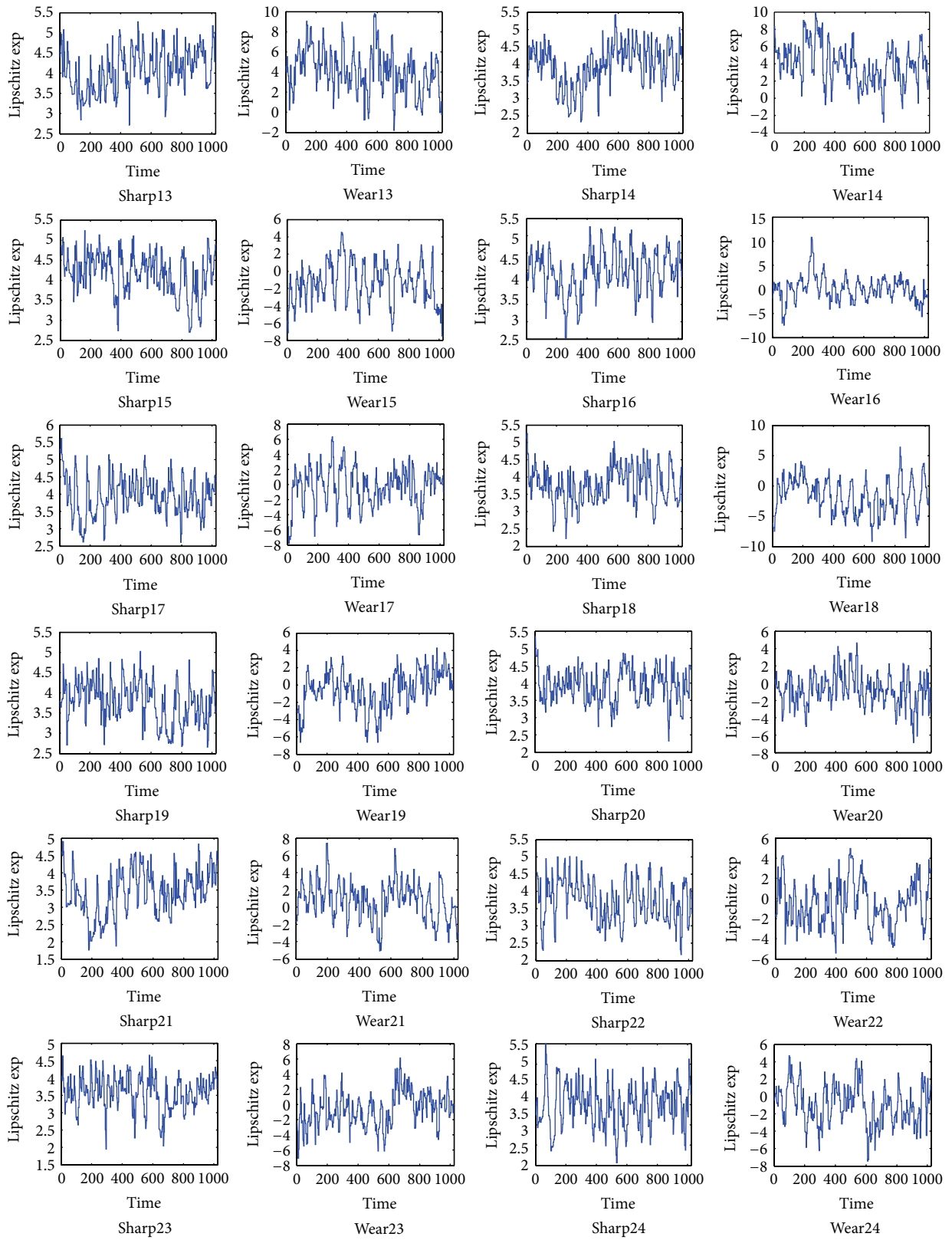\end{aligned} \qquad (13)
$$

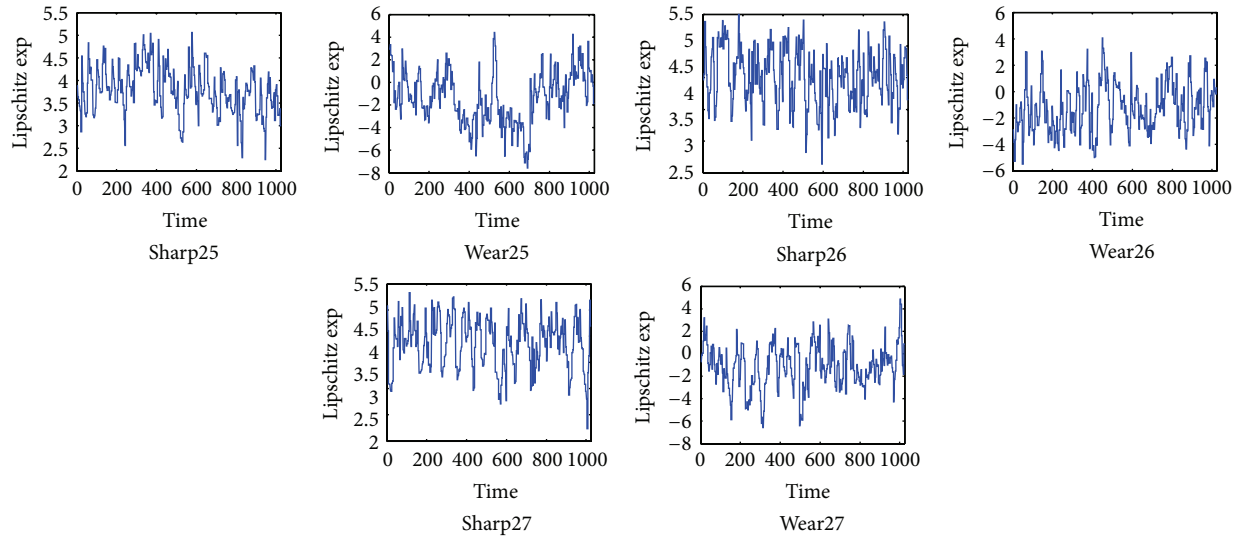FIGURE 7: Continued.

FIGURE 7: Continued.

FIGURE 7: Lipschitz exponent function of 54 group data.

$\alpha(x)$ is called Lipschitz exponent function that describes the change of Lipschitz exponent value α along the temporal axis x. The variable x is sampling point by acoustic emission (AE) during cutting process. Figure 5 is a simulation example of extracting Lipschitz exponent function $\alpha(x)$ on a sine function with singular points. Here, we set $N = 10$ in (13).

Figure 3 demonstrates the potential ability of singularity detection (or fault detection) with Lipschitz exponent function $\alpha(x)$.

6. Application Platform

In turning experiments, the feasibility for tool condition monitoring is demonstrated by 27 kinds of cutting conditions (see Table 1); 54 group data are sampled by AE; they are shaper1~shaper27 and wear1~wear27, respectively. Machining tests were carried out on HL-32 NC turning center [26]. The work material was chosen for ease of machining, allowing for generation of surfaces of varying quality without the use of cutting fluids. The experiment equipment is shown in Figure 4. Currently, AE-based sensing technology is the area of most intense research activity for developing intelligent tool condition systems. The reason is that the sensitivity of AE to tool wear and fracture is coupled with a high response rate of the signal. Sharp1 signal and wear1 signal show Figures 5 and 6, respectively in time domain.

According to (13), we set harmonic wavelet decomposed level $N = 10$, and Lipschitz exponent function $\alpha(x)$ of 54 group data is shown in Figure 7.

In Figure 7, all sharp tools are of positive Lipschitz exponent function $\alpha(t) > 0$, and all wear tools are of a negative Lipschitz exponent function $\alpha(t) < 0$. Wear tool and sharp tool can be distinguished by Lipschitz exponent function $\alpha(t)$. A threshold value $\alpha(t) = 0$ is obtained as a criterion of the tool condition monitoring during cutting process.

7. Conclusion

The proposed research provides a new theoretical basis and a new engineering application on the online tool monitoring which can distinguish between worn tool and sharp tool during cutting process. The results of experiment demonstrate that this method is more precise and robust.

The method which was described in this paper can be used as a valuable method for tool condition monitoring. In comparison to conventional data processing, the advantages of Lipschitz exponent and harmonic wavelet were shown. The zero threshold is determined conveniently which can distinguish between worn tool and sharp tool. For the future development of the presented techniques in laboratory, several approaches will be tested, including new broadband sensors application.

Acknowledgment

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