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Research Article

Optimal Homotopy Asymptotic Method to Nonlinear Damped Generalized Regularized Long-Wave Equation

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A new semianalytical technique optimal homology asymptotic method (OHAM) is introduced for deriving approximate solution of the homogeneous and nonhomogeneous nonlinear Damped Generalized Regularized Long-Wave (DGRLW) equation. We tested numerical examples designed to confine the features of the proposed scheme. We drew 3D and 2D images of the DGRLW equations and the results are compared with that of variational iteration method (VIM). Results reveal that OHAM is operative and very easy to use.

1. Introduction

Partial differential equations used in modeling different problems in physics, biology, chemical reactions, and engineering sciences problems are frequently too difficult to be solved exactly, and even if an exact solution is possible, the required calculations may be too difficult.

The DGRLW equation is a partial differential equation that describes the amplitude of long-wave, which takes the following form:

$$u_t - (\varphi(x,t)\,u_{xt})_x - \alpha u_{xx} + u_x + u^p u_x. \tag{1}$$

Here $\alpha > 0$, $p \geq 1$ is an integer, $\varphi(x,t)$ is known function, and $u(x,t)$ is the amplitude of the long-wave at the position x and time t. For $\alpha \neq 0$, (1) features a balance between nonlinear and dispersive effects but also takes into account mechanisms of dissipation. In the physical sense, (1) with the dissipative term αu_{xx} is suggested if the good predictive power is preferred; such type of problem arises in the bore propagation as well as for water waves [1].

The Equal Width (EW) Wave, Regularized Long-Wave (RLW), and Generalized Regularized Long-Wave (GRLW) equations are special cases of the DGRLW equation [2–4].

The EW equation corresponds to $\alpha = 0$, $\varphi(x,t) = 1$, whereas the RLW or Benjamin-Bona-Mahony equation corresponds to $\alpha = 0$, $\varphi(x,t) = 1$, and $p = 1$. On the other hand, the GRLW equation corresponds to $\alpha = 0$, $\varphi(x,t) = 1$, and $p \geq 1$.

Different methods have been used for numerical solutions of DGRLW equations. VIM was presented by Demir et al. for numerical solutions for the DGRLW equation [5]. Yousefi et al. used Bernstein Ritz-Galerkin Method for solving the DGRLW equation [6]. Achouri et al. worked on an article called *"A fully Galerkin method for the damped generalized regularized long-wave (DGRLW) equation,"* namely, in [7]. For the mathematical theory and physical significance of DGRLW equation see [8–16] and references therein.

Marinca and Herişanu proposed semianalytical technique OHAM for deriving approximate solution of nonlinear problems of thin film flow of a fourth grade fluid down a vertical cylinder [17–19]. The method has been used by many researchers for obtaining numerical approximations of linear and nonlinear differential equations [20–24]. The author has successfully applied the OHAM for deriving approximate solution of Equal Width Wave equation, Burger equations, and tenth order boundary value problems [25, 26]. The convergence criterion of proposed method is similar to that of homotopy analysis method (HAM) and homotopy perturbation method (HPM), but this method is more efficient and flexible. To improve the efficiency and accuracy of OHAM,

Herişanu and Marinca introduced more generalized and new advances in OHAM which shows that the auxiliary function includes the functions of physical parameters in addition to the convergence control parameter [27, 28].

Here, we investigate the approximate solution of the DGRLW equation with a variable coefficient using OHAM. The whole paper is divided into 3 sections. Section 2 is devoted to the analysis of the proposed method. In Section 3, solution of homogeneous and non-homogenous (DGRLW) equations is presented by OHAM, and absolute errors are also compared with VIM. The 3D and 2D images of the approximate solution and exact solution are also drawn. In all cases, the proposed method yields very encouraging results.

2. Fundamental Theory of OHAM

Consider the partial differential equation of the following form:

$$\begin{gathered} L(u(x,t)) + N(u(x,t)) + g(x,t) = 0, \quad x \in \Omega \\ B\left(u, \frac{\partial u}{\partial t}\right) = 0, \end{gathered} \tag{2}$$

where L is a linear operator and N is nonlinear operator. B is boundary operator, $u(x,t)$ is an unknown function, x and t denote spatial and time variables, respectively, Ω is the problem domain, and $g(x,t)$ is a known function.

Using the basic idea of OHAM, the optimal homotopy $\psi(x,t;q) : \Omega \times [0,1] \rightarrow R$ is constructed which satisfies the following condition:

$$\begin{aligned} &(1-q)\{L(\psi(x,t;q)) + g(x,t)\} \\ &\quad = H(q)\{L(\psi(x,t;q)) + N(\psi(x,t;q)) + g(x,t)\}, \end{aligned} \tag{3}$$

where $q \in [0,1]$ is an embedding parameter and $H(q)$ is a nonzero auxiliary function for $q \neq 0$, $H(0) = 0$. In such a case, (3) is called optimal homotopy equation. Clearly, we have

$$\begin{aligned} q = 0 &\Longrightarrow H(\psi(x,t;0),0) \\ &\quad = L(\psi(x,t;0)) + g(x,t) = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} q = 1 &\Longrightarrow H(\psi(x,t;1),1) \\ &\quad = H(1)\{L(\psi(x,t;q)) \\ &\qquad + N(\psi(x,t;q)) + g(x,t)\} = 0, \end{aligned} \tag{5}$$

when $q = 0$ and $q = 1$, then $\psi(x,t;0) = u_0(x,t)$ and $\psi(x,t;1) = u(x,t)$ hold. Thus, as q varies from 0 to 1, the solution $\psi(x,t;q)$ approaches from $u_0(x,t)$ to $u(x,t)$, where $u_0(x,t)$ is obtained from (3) for $q = 0$:

$$L(u_0(x,t)) + g(x,t) = 0, \qquad B\left(u_0, \frac{\partial u_0}{\partial t}\right) = 0. \tag{6}$$

Next, we choose auxiliary function $H(q)$ of the following general form:

$$H(q) = qc1 + q^2 c2 + \cdots. \tag{7}$$

Here $c1, c2, \ldots$ are constants to be determined later.

To get an approximate solution, we expand $\psi(x,t;q,ci)$ in Taylor's series about q in the following manner,

$$\begin{aligned} &\psi(x,t;q,ci) \\ &\quad = u_0(x,t) + \sum_{k=1}^{\infty} u_k(x,t;ci)\ q^k, i = 1,2,\ldots \end{aligned} \tag{8}$$

Substituting (8) into (3) and equating the coefficient of like powers of q, we obtain Zeroth order problem, given by (6); the first and second order problems are given by (9) and (10), respectively, and the general governing equations for $u_k(x,t)$ are given by (11) as follows:

$$L(u_1(x,t)) = c1N_0(u_0(x,t)), \qquad B\left(u_1, \frac{\partial u_1}{\partial t}\right) = 0 \tag{9}$$

$$\begin{aligned} &L(u_2(x,t)) - L(u_1(x,t)) \\ &\quad = c2N_0(u_0(x,t)) \\ &\qquad + c1[L(u_1(x,t)) + N_1(u_0(x,t), u_1(x,t))], \\ &\qquad B\left(u_2, \frac{\partial u_2}{\partial t}\right) = 0 \end{aligned} \tag{10}$$

$$\begin{aligned} &L(u_k(x,t)) - L(u_{k-1}(x,t)) \\ &\quad = ckN_0(u_0(x,t)) \\ &\qquad + \sum_{i=1}^{k-1} ci[L(u_{k-i}(x,t)) \\ &\qquad\quad + N_{k-i}(u_0(x,t), u_1(x,t), \ldots, u_{k-i}(x,t))], \\ &B\left(u_k, \frac{\partial u_k}{\partial t}\right) = 0, \quad k = 2,3,\ldots, \end{aligned} \tag{11}$$

where $N_{k-i}(u_0(x,t), u_1(x,t), \ldots, u_{k-i}(x,t))$ is the coefficient of q^{k-i} in the expansion of $N(\psi(x,t;q))$ about the embedding parameter q:

$$\begin{aligned} N(\psi(x,t;q,ci)) = {} & N_0(u_0(x,t)) \\ & + \sum_{k\geq 1} N_k(u_0, u_1, u_2, \ldots, u_k)\, q^k. \end{aligned} \tag{12}$$

Here u_k for $k \geq 0$ are a set of linear equations with the linear boundary conditions, which can be easily solved.

The convergence of the series in (8) depends upon the auxiliary constants $c1, c2, \ldots$. If it is convergent at $q = 1$, then

$$\tilde{u}(x,t;ci) = u_0(x,t) + \sum_{k\geq 1} u_k(x,t;ci). \tag{13}$$

Substituting (13) into (1), it results with the following expression for residual:

$$R(x,t;ci) = L(\tilde{u}(x,t;ci)) + g(x,t) + N(\tilde{u}(x,t;ci)). \tag{14}$$

If $R(x,t;Ci) = 0$, then $\tilde{u}(x,t;ci)$ will be the exact solution.

For computing the optimal values of auxiliary constants, $ci, i = 1, 2, \ldots, m$, there are many methods available like Galerkin's, Ritz, Least Squares, and Collocation method. One can apply the method of Least Squares as under the following:

$$J(C_i) = \int_0^t \int_\Omega R^2(x, t, ci)\, dx\, dt, \tag{15}$$

where R is the residual,

$$R(x, t; C_i) = L(\tilde{u}(x, t; C_i)) + g(x, t) + N(\tilde{u}(x, t; C_i)), \tag{16}$$

$$\frac{\partial J}{\partial c1} = \frac{\partial J}{\partial c2} = \cdots = \frac{\partial J}{\partial c3} = 0. \tag{17}$$

The constants ci can also be determined by another method as under:

$$R(h_1; ci) = R(h_2; ci) = \cdots = R(h_m; ci) = 0, \quad i = 1, 2, \ldots, m \tag{18}$$

at any time t, where $h_i \in \Omega$. The convergence depends upon constants $c1, c2, \ldots$, which can be optimally identified and minimized by (18).

Example 1. Consider (1) with $\alpha = 1$, $p = 2$, and $\varphi(x, t) = -(1/6)e^{-(2x+4t)} f(x, t) = 0$ which in the simplest form is given as

$$u_t + \frac{1}{6}\left(\left(e^{-(2x+4t)}\right) u_{xt}\right)_x - u_{xx} + u_x + u^2 u_x = 0. \tag{19}$$

The initial condition is $u(x, 0) = e^{-x}$ and exact solution given by

$$u(x, t) = e^{(-x+2t)}. \tag{20}$$

Zeroth Order Problem. Consider the following:

$$(u_0)_t = 0, \qquad u_0(x, 0) = e^{-x}. \tag{21}$$

Its solution is given as d under

$$u_0(x, t) = e^{-x}. \tag{22}$$

First Order Problem. Consider the following:

$$\begin{aligned} &-(u_0)_t - c(u_0)_t + (u_1)_t - c1(u_0)_x \\ &\quad - c1u_0^2(u_0)_x + \frac{1}{3}c1e^{4t-2x}(u_0)_{xt} \\ &\quad + c1(u_0)_{xx} + c1e^{4t-2x}(u_0)_{xxt} = 0. \end{aligned} \tag{23}$$

Its solution is as follows:

$$u_1(x, t, c1) = -c1e^{-3x}\left(1 + 2e^{2x}\right)t. \tag{24}$$

Second Order Problem. Consider the following:

$$\begin{aligned} &-c2(u_0)_t - (u_1)_t - c1(u_1)_t + (u_2)_t \\ &- c2(u_0)_x - c2(u_0)^2(u_0)_x - 2c1u_0u_1(u_0)_x \\ &- c1(u_1)_x - c1(u_0)^2(u_1)_x + \frac{1}{3}c2e^{4t-2x}(u_0)_{x,t} \\ &+ \frac{1}{3}c1e^{4t-2x}(u_1)_{x,t} = 0. \end{aligned} \tag{25}$$

Its solution is under

$$\begin{aligned} &u_2(x, t, c1, c2) \\ &= \frac{1}{8}e^{-5x}\left(5c1^2 - 5c1^2e^{4t} + 2c1^2e^{2x}\right. \\ &\qquad - 2c1^2e^{4t+2x} - 8c1e^{2x}t - 8c1^2e^{2x}t \\ &\qquad - 8c2e^{2x}t - 16c1e^{4x}t - 16c1^2e^{4x}t \\ &\qquad - 16c2e^{4x}t + 20c1^2t^2 \\ &\qquad \left. + 72c1^2e^{2x}t^2 + 16c1^2e^{4x}t^2\right). \end{aligned} \tag{26}$$

Third Order Problem. Consider the following:

$$\begin{aligned} &-c3(u_0)_t - c2(u_1)_t - (u_2)_t - c1(u_2)_t \\ &+ (u_3)_t - c3(u_0)_x - c3(u_0)^2(u_0)_x \\ &- 2c2u_0u_1(u_0)_x - c1(u_1)^2(u_0)_x \\ &- 2c1u_0u_2(u_0)_x - c2(u_1)_x - c2(u_0)^2(u_1)_x \\ &- 2c1u_0u_1(u_1)_x - c1(u_2)_x - c1(u_0)^2(u_2)_x \\ &+ \frac{1}{3}c3e^{4t-2x}(u_0)_{x,t} + \frac{1}{3}c2e^{4t-2x}(u_1)_{x,t} \\ &+ \frac{1}{3}c1e^{4t-2x}(u_2)_{x,t} + c3(u_2)_{x,t} + c2(u_1)_{x,x} \\ &+ c1(u_2)_{x,x} - \frac{1}{6}c3e^{4t-2x}(u_0)_{x,x,t} \\ &- \frac{1}{6}c2e^{4t-2x}(u_1)_{x,x,t} - \frac{1}{6}c1e^{4t-2x}(u_2)_{x,x,t} = 0. \end{aligned} \tag{27}$$

Its solution is given as follows:

$$\begin{aligned} &u_3(x, t, c1, c2, c3) \\ &= \frac{1}{96}e^{-7x}\left(245c1^3 - 70c1^3e^{4t} - 175c1^3e^{8t}\right. \\ &\qquad + 120c1^2e^{2x} - 60c1^3e^{2x} + 120c1c2e^{2x} \\ &\qquad + 48c1^2e^{4x} - 12c1^3e^{4x} + 48c1c2e^{4x} \\ &\qquad - 120c1^2e^{4t+2x} + 90c1^3e^{4t+2x} - 120c1c2e^{4t+2x} \\ &\qquad - 30c1^3e^{8t+2x} - 48c1^2e^{4t+4x} + 12c1^3e^{4t+4x} \\ &\qquad - 48c1c2e^{4t+4x} - 420c1^3t + 700c1^3e^{4t}t \\ &\qquad - 1920c1^3e^{2x}t - 96c1e^{4x}t - 192c1^2e^{4x}t \\ &\qquad - 384c1^3e^{4x}t - 96c2e^{4x}t - 192c1c2e^{4x}t \end{aligned}$$

$$
\begin{aligned}
&- 96c3e^{4x}t - 192c1e^{6x}t - 384c1^2e^{6x}t \\
&- 192c1^3e^{6x}t - 192c2e^{6x}t - 384c1c2e^{6x}t \\
&- 192c3e^{6x}t + 1080c1^3e^{4t+2x}t + 48c1^3e^{4t+4x}t \\
&+ 480c1^2e^{2x}t^2 + 480c1^3e^{2x}t^2 + 480c1c2e^{2x}t^2 \\
&+ 1728c1^2e^{4x}t^2 + 1728c1^3e^{4x}t^2 \\
&+ 1728c1c2e^{4x}t^2 + 384c1^2e^{6x}t^2 + 384c1^3e^{6x}t^2 \\
&+ 384c1c2e^{6x}t^2 - 784c1^3t^3 - 4480c1^3e^{2x}t^3 \\
&- 4032c1^3e^{4x}t^3 - 128c1^3e^{6x}t^3\Big).
\end{aligned} \tag{28}
$$

The third order approximate solution is given by the following equation:

$$
\begin{aligned}
&u(x, t, c1, c2, c3) \\
&\quad = u_0(x, t) + u_1(x, t, c1) + u_2(x, t, c1, c2) \\
&\quad\quad + u_3(x, t, c1, c2, c3), \\
&u = e^{-x} - c1e^{-3x}\left(1 + 2e^{2x}\right)t \\
&\quad + \frac{1}{8}e^{-5x}\Big(5c1^2 - 5c1^2e^{4t} + 2c1^2e^{2x} \\
&\quad\quad - 2c1^2e^{4t+2x} - 8c1e^{2x}t - 8c1^2e^{2x}t \\
&\quad\quad - 8c2e^{2x}t - 16c1e^{4x}t - 16c1^2e^{4x}t \\
&\quad\quad - 16c2e^{4x}t + 20c1^2t^2 \\
&\quad\quad + 72c1^2e^{2x}t^2 + 16c1^2e^{4x}t^2\Big) \\
&\quad + \frac{1}{96}e^{-7x}\Big(245c1^3 - 70c1^3e^{4t} \\
&\quad\quad - 175c1^3e^{8t} + 120c1^2e^{2x} - 60c1^3e^{2x} \\
&\quad\quad + 120c1c2e^{2x} + 48c1^2e^{4x} - 12c1^3e^{4x} \\
&\quad\quad + 48c1c2e^{4x} - 120c1^2e^{4t+2x} + 90c1^3e^{4t+2x} \\
&\quad\quad - 120c1c2e^{4t+2x} - 30c1^3e^{8t+2x} \\
&\quad\quad - 48c1^2e^{4t+4x} + 12c1^3e^{4t+4x} \\
&\quad\quad - 48c1c2e^{4t+4x} - 420c1^3t \\
&\quad\quad + 700c1^3e^{4t}t - 1920c1^3e^{2x}t \\
&\quad\quad - 96c1e^{4x}t - 192c1^2e^{4x}t - 384c1^3e^{4x}t \\
&\quad\quad - 96c2e^{4x}t - 192c1c2e^{4x}t - 96c3e^{4x}t \\
&\quad\quad - 192c1e^{6x}t - 384c1^2e^{6x}t - 192c1^3e^{6x}t \\
&\quad\quad - 192c2e^{6x}t - 384c1c2e^{6x}t - 192c3e^{6x}t \\
&\quad\quad + 1080c1^3e^{4t+2x}t + 48c1^3e^{4t+4x}t \\
&\quad\quad + 480c1^2e^{2x}t^2 + 480c1^3e^{2x}t^2 \\
&\quad\quad + 480c1c2e^{2x}t^2 + 1728c1^2e^{4x}t^2 \\
&\quad\quad + 1728c1^3e^{4x}t^2 + 1728c1c2e^{4x}t^2 \\
&\quad\quad + 384c1^2e^{6x}t^2 + 384c1^3e^{6x}t^2 \\
&\quad\quad + 384c1c2e^{6x}t^2 - 784c1^3t^3 - 4480c1^3e^{2x}t^3 \\
&\quad\quad - 4032c1^3e^{4x}t^3 - 128c1^3e^{6x}t^3\Big).
\end{aligned} \tag{29}
$$

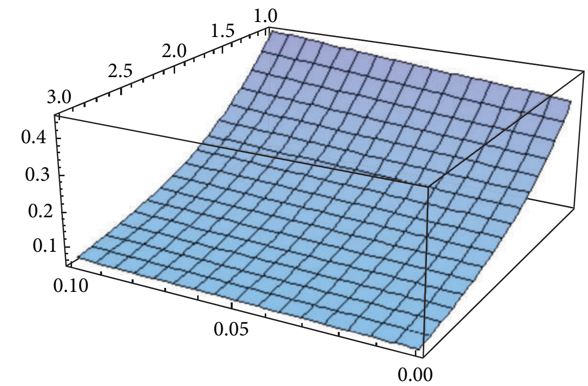

Figure 1: Plot of of 3rd order approximate solution for the homogeneous DGRLW equation (2).

The constants $c1$, $c2$, and $c3$ are calculated using the Least Squares, we have their optimal values as follows:

$$
\begin{aligned}
c1 &= -0.8410486261349961, \\
c2 &= -0.022653709109618163, \\
c3 &= 0.0031930834039587894.
\end{aligned} \tag{30}
$$

The 3rd order OHAM solution yields very encouraging results after comparing with 3rd order VIM solution [5]. Tables 1(a–c), and Figures 1, 2, 3, and 4 show the effectiveness of OHAM for $x = 1.2$, $x = 1.4$ and $x = 1.6$.

Example 2. Consider (1) with $\alpha = 1$, $p = 2$ and $\varphi(x, t) = -(1/6)e^{-(2x+4t)}$ which in the simplest form is given as

$$
u_t + \frac{1}{6}\left(\left(e^{-(2x+4t)}\right)u_{xt}\right)_x - u_{xx} + u_x + u^2u_x = 0. \tag{31}
$$

The initial condition is $u(x, 0) = \operatorname{sech}[x/4]^2$ and exact solution given by

$$
u(x, t) = \operatorname{sech}\left[\frac{x}{4} - \frac{t}{3}\right]^2. \tag{32}
$$

Zeroth Order Problem. Consider the following:

$$
(u_0)_t = 0, \qquad u_0(x, 0) = \operatorname{sech}\left[\frac{x}{4}\right]^2. \tag{33}
$$

Table 1: (a) Comparison of absolute errors of 3rd order OHAM solution and 3rd order VIM solution for Example 1 at $x = 1.2$ and various values of t. (b) Comparison of absolute errors of 3rd order OHAM solution and 3rd order VIM solution for Example 1 at $x = 1.4$ and various values of t. (c) Comparison of absolute errors of 3rd order OHAM solution and 3rd order VIM solution for Example 1 at $x = 1.6$ and various values of t.

(a)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.313486	0.313486	1.92728×10^{-5}	3.146×10^{-7}
0.04	0.32628	0.32627	3.39912×10^{-5}	1.01891×10^{-5}
0.06	0.339596	0.339571	1.301192×10^{-4}	2.46667×10^{-5}
0.08	0.353455	0.353416	2.401071×10^{-4}	3.87876×10^{-5}
0.10	0.367879	0.367829	3.356920×10^{-4}	5.08783×10^{-5}

(b)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.256661	0.256661	1.331×10^{-7}	3.29357×10^{-8}
0.04	0.267135	0.267135	2.11230×10^{-5}	5.45591×10^{-7}
0.06	0.278037	0.278037	4.57127×10^{-5}	1.19768×10^{-7}
0.08	0.289384	0.289387	5.71818×10^{-5}	2.34162×10^{-6}
0.10	0.301194	0.301199	3.93977×10^{-5}	4.79793×10^{-6}

(c)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.210136	0.210136	1.4946×10^{-6}	4.27298×10^{-7}
0.04	0.218712	0.218712	7.2701×10^{-6}	1.00227×10^{-7}
0.06	0.227638	0.227635	7.6219×10^{-6}	2.32488×10^{-6}
0.08	0.236928	0.23692	6.6969×10^{-6}	7.99082×10^{-6}
0.10	0.246597	0.246577	4.44429×10^{-5}	1.99024×10^{-5}

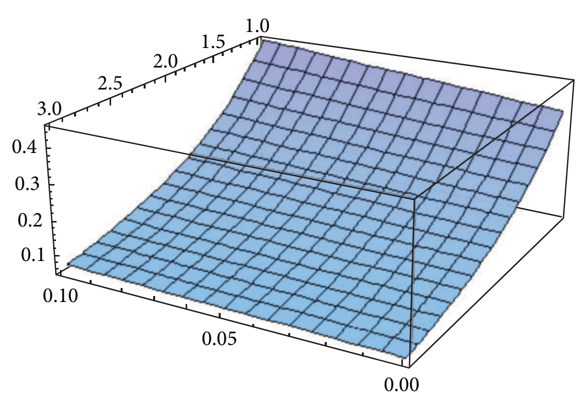

Figure 2: Variation profile of exact solution for the homogeneous DGRLW equation (2).

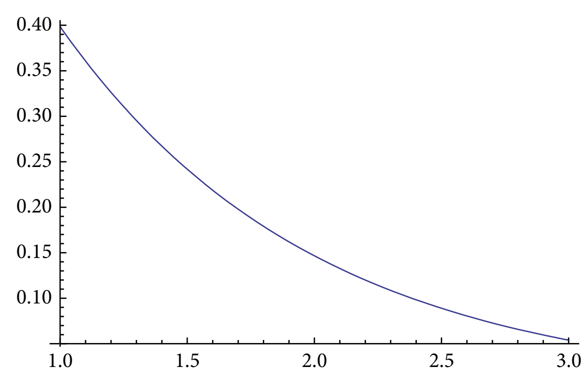

Figure 3: Approximate solution plot for $t = 0.04$.

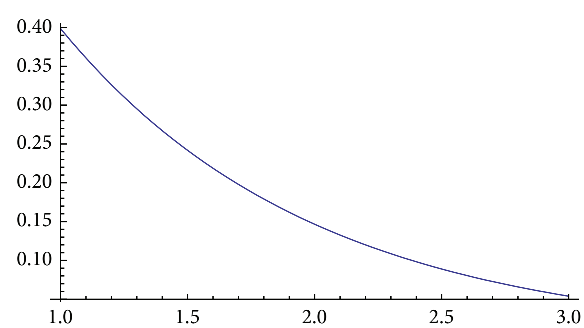

Figure 4: Exact solution plot for $t = 0.04$.

Its solution is given by

$$u_0(x,t) = \operatorname{sech}\left[\frac{x}{4}\right]^2. \tag{34}$$

First Order Problem. Consider the following:

$$\begin{aligned} &-(u_0)_t - c1(u_0)_t + (u_1)_t - c1(u_0)_x \\ &\quad - c1u_0^2(u_0)_x + \frac{1}{3}c1e^{4t-2x}(u_0)_{xt} + c1(u_0)_{xx} \\ &\quad + c1e^{4t-2x}(u_0)_{xxt} = 0. \end{aligned} \tag{35}$$

Table 2: (a) Absolute error of the solution of Example 2 by optimal homotopy asymptotic method (OHAM) at $x = 15$, $x = 20$, and $x = 25$ and various values of t. (b) Absolute error of the solution of Example 2 by optimal homotopy asymptotic method (OHAM) at various values of x and t.

(a)

t	$x = 15$	$x = 20$	$x = 25$
0.01	2.7905×10^{-6}	2.30504×10^{-7}	1.89307×10^{-8}
0.02	2.83577×10^{-6}	2.35074×10^{-7}	1.93117×10^{-8}
0.03	8.32571×10^{-6}	6.87803×10^{-7}	5.6488×10^{-8}
0.04	5.60061×10^{-6}	4.64385×10^{-7}	3.81506×10^{-8}
0.05	1.37955×10^{-5}	1.13979×10^{-6}	9.36098×10^{-8}

(b)

x	$t = 0.1$	$t = 0.2$	$t = 0.3$	$t = 0.4$	$t = 0.5$
15	2.24995×10^{-5}	4.36594×10^{-5}	6.27619×10^{-5}	9.29866×10^{-5}	9.16758×10^{-5}
16	1.36916×10^{-5}	26576×10^{-5}	3.82161×10^{-5}	5.66063×10^{-5}	5.58636×10^{-5}
17	8.32101×10^{-6}	1.61544×10^{-5}	2.32343×10^{-5}	3.441×10^{-5}	3.39789×10^{-5}
18	5.05308×10^{-6}	9.81109×10^{-6}	1.41127×10^{-5}	2.0899×10^{-5}	2.06447×10^{-5}
19	3.0671×10^{-6}	5.95551×10^{-6}	8.56725×10^{-6}	1.26863×10^{-5}	1.25347×10^{-5}
20	1.86112×10^{-6}	3.61396×10^{-6}	5.19906×10^{-6}	7.69844×10^{-6}	7.60747×10^{-6}
21	1.12913×10^{-6}	2.19263×10^{-6}	3.1544×10^{-6}	4.67075×10^{-6}	4.61593×10^{-6}
22	6.84967×10^{-7}	1.33013×10^{-6}	1.91361×10^{-6}	2.83347×10^{-6}	2.80036×10^{-6}
23	4.15495×10^{-7}	8.06854×10^{-7}	1.1608×10^{-6}	1.71878×10^{-6}	1.69874×10^{-6}
24	2.52026×10^{-7}	4.89414×10^{-7}	7.04113×10^{-7}	1.04256×10^{-6}	1.03043×10^{-6}
25	1.52867×10^{-7}	2.96856×10^{-7}	4.27085×10^{-7}	6.32372×10^{-7}	6.25018×10^{-7}

Its solution is

$$
\begin{aligned}
u_1(x,t,c1) &= -\frac{1}{8}t\left(-c1\,\text{sech}\left[\frac{x}{4}\right]^4 + 4c1\text{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right]\right. \\
&\quad + 4c1\,\text{sech}\left[\frac{x}{4}\right]^6\tanh\left[\frac{x}{4}\right] \\
&\quad \left. + 2c1\,\text{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right]^2\right).
\end{aligned}
\tag{36}
$$

Second Order Problem

$$
\begin{aligned}
&-c2(u_0)_t - (u_1)_t - c1(u_1)_t + (u_2)_t \\
&\quad - c2(u_0)_x - c2u_0^2(u_0)_x - 2c1u_0u_1(u_0)_x \\
&\quad - c1(u_1)_x - c1u_0^2(u_1)_x + \frac{1}{3}c2e^{4t-2x}(u_0)_{x,t} \\
&\quad + \frac{1}{3}c1e^{4t-2x}(u_1)_{x,t} + c2(u_0)_{x,x} + c1(u_1)_{x,x} \\
&\quad - \frac{1}{6}c2e^{4t-2x}(u_0)_{x,x,t} - \frac{1}{6}c1e^{4t-2x}(u_1)_{x,x,t} = 0.
\end{aligned}
\tag{37}
$$

Its approximate solution $u_2(x,t,c1,c2)$ is obtained in similar manner. The second order approximate solution is given by

$$
\begin{aligned}
u(x,t,c1,c2) &= u_0(x,t) + u_1(x,t,c1) + u_2(x,t,c1,c2) \\
&= \text{sech}\left[\frac{x}{4}\right]^2 - \frac{1}{8}t\left(-c1\,\text{sech}\left[\frac{x}{4}\right]^4\right. \\
&\qquad + 4c1\,\text{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right] \\
&\qquad + 4c1\,\text{sech}\left[\frac{x}{4}\right]^6\tanh\left[\frac{x}{4}\right] \\
&\qquad \left. + 2c1\,\text{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right]^2\right) \\
&\quad + u_2(x,t,c1,c2).
\end{aligned}
\tag{38}
$$

Using method of Least Squares, the optimal values of constants $c1$ and $c2$ are computed and are given as under

$$
\begin{aligned}
c1 &= -0.8122876966282708, \\
c2 &= -0.011502183336221636.
\end{aligned}
\tag{39}
$$

Table 2(a) shows the effectiveness of OHAM for $x = 15$, $x = 20$, and $x = 25$, while Table 2(b), and Figures 5, 6, 7, and 8 shows the effectiveness of OHAM for various values of x and t.

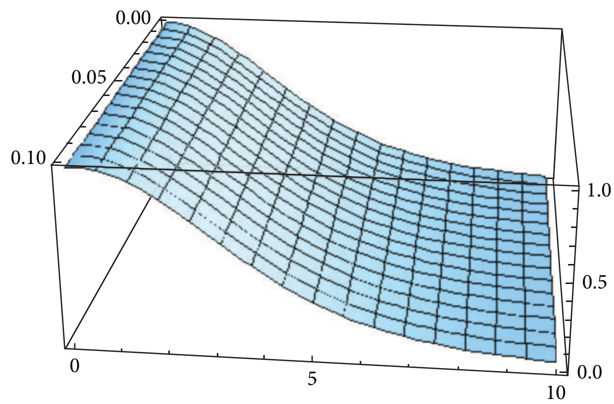

FIGURE 5: Surface 2nd order approximate solution for (4.12).

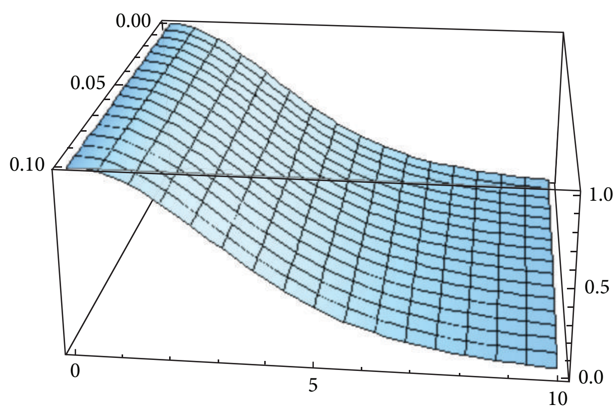

FIGURE 6: Exact solution for (4.12).

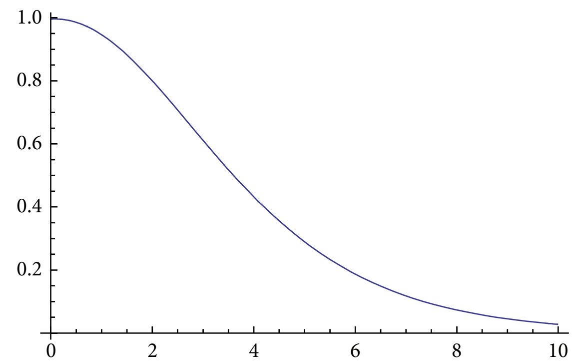

FIGURE 7: Approximate solution plot for $t = 0.05$.

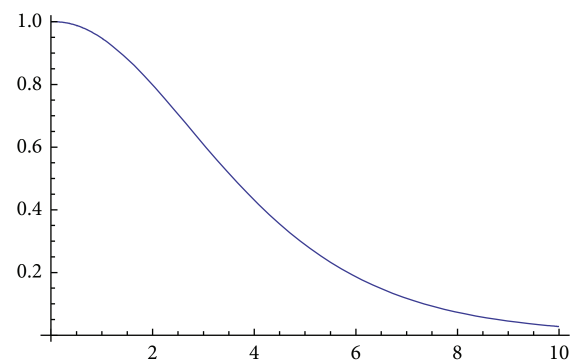

FIGURE 8: Exact solution plot for $t = 0.05$.

Example 3. Consider (1) with $\alpha = 1$, $p = 1$, and $\varphi(x,t) = 1 - (3\cosh[x/4-t/3]\sinh[x/4-t/3]/(-3+2\cosh[t/3-x/4]^2))$ which in the simplest form is given as

$$u_t + \left(1 - \frac{3\cosh[x/4-t/3]\sinh[x/4-t/3]}{-3+2\cosh[t/3-x/4]^2}u_{xt}\right)_x - u_{xx} + u_x + uu_x = 0. \tag{40}$$

The initial condition is $u(x,0) = \operatorname{sech}[x/4]^2$ and exact solution given by

$$u(x,t) = \operatorname{sech}\left[\frac{x}{4} - \frac{t}{3}\right]^2. \tag{41}$$

Zeroth Order Problem. Consider the following:

$$(u_0)_t = 0, \qquad u_0(x,0) = \operatorname{sech}\left[\frac{x}{4}\right]^2. \tag{42}$$

Its solution is as follows:

$$u_0(x,t) = \operatorname{sech}\left[\frac{x}{4}\right]^2. \tag{43}$$

First Order Problem. Consider the following:

$$\begin{aligned}
&-(u_0)_t - c1(u_0)_t + (u_1)_t - c1(u_0)_x - c1u_0(u_0)_x \\
&- c1\left(\frac{3\cosh[t/3-x/4]^2}{4\left(-3+2\cosh[t/3-x/4]^2\right)}\right. \\
&\qquad - \frac{3\cosh[t/3-x/4]^2\sinh[t/3-x/4]^2}{\left(-3+2\cosh[t/3-x/4]^2\right)^2} \\
&\qquad \left. + \frac{3\sinh[t/3-x/4]^2}{4\left(-3+2\cosh[t/3-x/4]^2\right)}\right)(u_0)_{x,t} + c1(u_0)_{x,x} \\
&- c1\left(-1 - \frac{3\cosh[t/3-x/4]\sinh[t/3-x/4]}{-3+2\cosh[t/3-x/4]^2}\right) \\
&\quad \times (u_0)_{x,x,t} = 0.
\end{aligned} \tag{44}$$

Its solution is

$$\begin{aligned}
u_1&(x,t,c_1) \\
&= -\frac{1}{8}t - c_1\operatorname{sech}\left[\frac{x}{4}\right]^4 + 4c_1\operatorname{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right] \\
&\quad + 4c_1\operatorname{sech}\left[\frac{x}{4}\right]^4\tanh\left[\frac{x}{4}\right] + 2c_1\operatorname{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right]^2.
\end{aligned} \tag{45}$$

The first order approximate solution is given by

$$u(x,t,c_1) = u_0(x,t) + u_1(x,t,c_1) \tag{46}$$

$$\begin{aligned} u(x,t,c_1) &= \operatorname{sech}\left[\frac{x}{4}\right]^2 - \frac{1}{8}t\left(-c_1\operatorname{sech}\left[\frac{x}{4}\right]^4\right. \\ &\quad + 4c_1\operatorname{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right] \\ &\quad + 4c_1\operatorname{sech}\left[\frac{x}{4}\right]^4\tanh\left[\frac{x}{4}\right] \\ &\quad \left. + 2c_1\operatorname{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right]^2\right). \end{aligned} \tag{47}$$

The constants $c1$ is calculated using the Least Squares that we have its optimal values as follows:

$$c_1 = -0.802767563787412. \tag{48}$$

The first order optimum solution using OHAM is as follows:

$$\begin{aligned} u(x,t) &= \operatorname{sech}\left[\frac{x}{4}\right]^2 \\ &\quad - \frac{1}{8}t\left(0.802767563787412\operatorname{sech}\left[\frac{x}{4}\right]^4\right. \\ &\quad - 3.211070255149648\operatorname{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right] \\ &\quad - 3.211070255149648\operatorname{sech}\left[\frac{x}{4}\right]^4\tanh\left[\frac{x}{4}\right] \\ &\quad \left. - 1.605535127574824\operatorname{sech}\left[\frac{x}{4}\right]^2\tanh\left[\frac{x}{4}\right]^2\right). \end{aligned} \tag{49}$$

The first order OHAM solution yields very encouraging results after comparing with 2nd order VIM solution [5]. Tables 3(a–d), and Figures 9, 10, 11, and 12 show the effectiveness of OHAM for $x = -2.5$, $x = 0$, $x = 2.5$ and $x = 5$.

Example 4. Let us consider the inhomogeneous DGRLW equation:

$$u_t - (\varphi(x,t)\,u_{xt})_x - u_{xx} + u_x + uu_x = f(x,t), \tag{50}$$

where $\varphi(x,t) = xt$ and $f(x,t) = (-t\cos[x] + xt\sin[x] + \cos[x] + \sin[x]\cos[x]e^{-t})e^{-t}$.

The initial condition is

$$u(x,0) = \sin[x] \tag{51}$$

and exact solution given by

$$u(x,t) = \sin[x]\,e^{-t}. \tag{52}$$

According to OHAM scheme presented in Section 2.

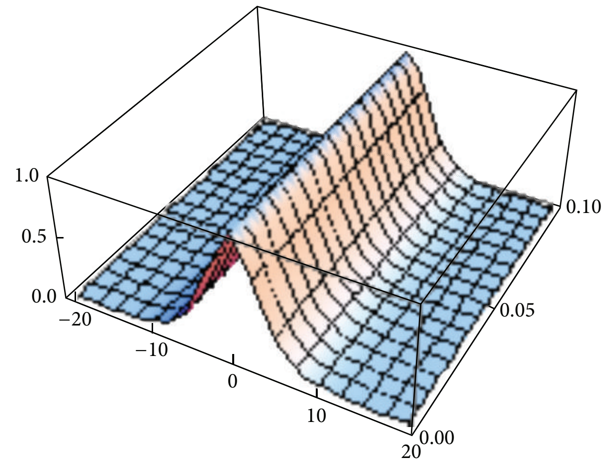

Figure 9: 1st order approximate solution for (4.12).

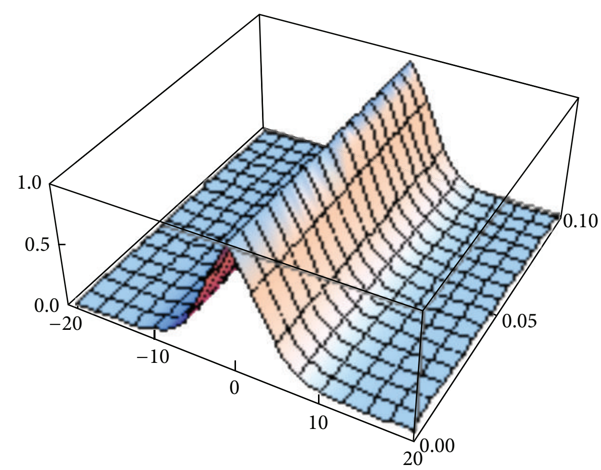

Figure 10: Exact solution plot for (4.12).

Zeroth Order Problem. Consider the following:

$$(u_0)_t = 0, \qquad u_0(x,0) = \sin[x]. \tag{53}$$

Its solution is

$$u_0(x,t) = \sin[x]. \tag{54}$$

First Order Problem. Consider the following:

$$\begin{aligned} &c1e^{-t}\left(\cos[x] - t\cos[x] + tx\sin[x]\right. \\ &\quad \left. + e^{-t}\cos[x]\sin[x]\right) \\ &\quad - (u_0)_t - c1(u_0)_t + (u_1)_t - c1(u_0)_x - c1u_0(u_0)_x \\ &\quad + c1t(u_0)_{x,t} + c1(u_0)_{x,x} + c1tx(u_0)_{x,x,t} = 0. \end{aligned} \tag{55}$$

Table 3: (a) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 3 at $x = -2.5$ and various values of t. (b) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 3 at $x = 0$ and various values of t. (c) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 3 at $x = -2.5$ and various values of t. (d) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 3 at $x = -2.5$ and various values of t.

(a)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.687297	0.687094	1.510466×10^{-3}	2.02246×10^{-4}
0.04	0.682171	0.68177	3.017151×10^{-3}	4.0071×10^{-4}
0.06	0.677041	0.676445	4.521014×10^{-3}	5.96353×10^{-4}
0.08	0.671911	0.67112	6.023003×10^{-3}	7.90121×10^{-4}
0.10	0.666779	0.665796	7.524051×10^{-3}	9.82948×10^{-4}

(b)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.999956	0.997993	2.455556×10^{-3}	1.96248×10^{-3}
0.04	0.999822	0.995986	4.822243×10^{-3}	3.83608×10^{-3}
0.06	0.9996	0.993979	7.100106×10^{-3}	5.62086×10^{-3}
0.08	0.999289	0.991972	9.289225×10^{-3}	7.3169×10^{-3}
0.10	0.99889	0.989965	1.138971×10^{-3}	8.92431×10^{-3}

(c)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.697537	0.697529	1.247750×10^{-3}	7.71606×10^{-6}
0.04	0.702649	0.702639	2.501243×10^{-3}	9.6889×10^{-6}
0.06	0.707754	0.707749	3.761480×10^{-3}	4.91862×10^{-6}
0.08	0.712851	0.712859	5.029472×10^{-3}	7.60727×10^{-6}
0.10	0.71794	0.717969	6.306245×10^{-3}	2.89136×10^{-5}

(d)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.283601	0.283512	6.719964×10^{-4}	889401×10^{-5}
0.04	0.286816	0.286609	1.315023×10^{-3}	20685×10^{-4}
0.06	0.29006	0.289706	1.928998×10^{-3}	353812×10^{-4}
0.08	0.293333	0.292803	2.513842×10^{-3}	529904×10^{-4}
0.10	0.296636	0.295901	3.069485×10^{-3}	735198×10^{-4}

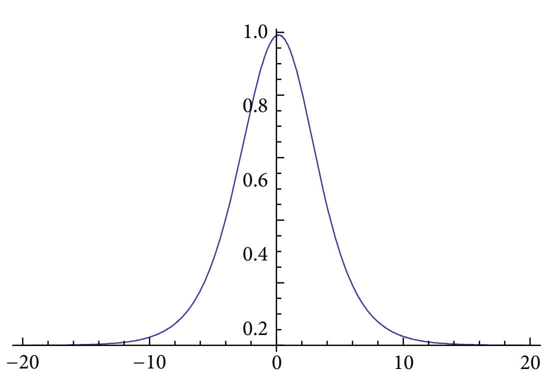

Figure 11: Approximate solution plot for $t = 0.1$.

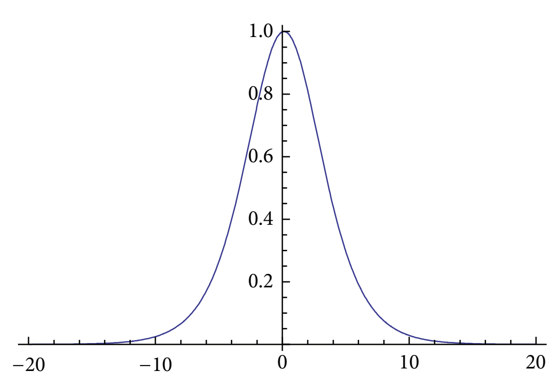

Figure 12: Exact solution plot for $t = 0.1$.

Its solution is as follows:

$$\begin{aligned}
u_1(x,t,c1) = -\frac{1}{2}c1e^{-2t}\Big(&2e^{t}t\cos[x]-2e^{2t}t\cos[x]\\
&-2e^{2t}t\sin[x]-2e^{t}x\sin[x]\\
&+2e^{2t}x\sin[x]-2e^{t}tx\sin[x]\\
&-\cos[x]\sin[x]+e^{2t}\cos[x]\sin[x]\\
&-2e^{2t}t\cos[x]\sin[x]\Big).
\end{aligned} \tag{56}$$

Second Order Problem. Consider the following:

$$\begin{aligned}
c2e^{-t}\Big(&\cos[x]-t\cos[x]\\
&+tx\sin[x]+e^{-t}\cos[x]\sin[x]\Big)\\
&-c2(u_0)_t-(u_1)_t-c1(u_1)_t+(u_2)_t\\
&-c2(u_0)_x-c2u_0(u_0)_x-c1u_1(u_0)_x\\
&-c1(u_1)_x-c1u_0(u_1)_x+c2t(u_0)_{x,t}\\
&+c1t(u_1)_{x,x}+c2(u_0)_{x,x}+c1(u_1)_{x,x}\\
&+c2tx(u_0)_{x,x,t}+c1tx(u_0)_{x,x,t}=0.
\end{aligned} \tag{57}$$

Its approximate solution is under

$$\begin{aligned}
u_2(x,t,c1,c2) = \frac{1}{16}e^{-2t}\Big(&-16c1^2e^{t}+16c1^2e^{2t}-8c1^2e^{t}t\\
&-8c1^2e^{2t}t+80c1^2e^{t}\cos[x]\\
&-80c1^2e^{2t}\cos[x]-16c1e^{t}t\cos[x]\\
&+32c1^2e^{t}t\cos[x]-16c2e^{t}t\cos[x]\\
&+16c1e^{2t}t\cos[x]+48c1^2e^{2t}t\cos[x]\\
&+16c2e^{2t}t\cos[x]+8c1^2e^{2t}t^2\cos[x]\\
&-144c1^2e^{t}x\cos[x]+144c1^2e^{2t}x\cos[x]\\
&-128c1^2e^{t}tx\cos[x]-16c1^2e^{2t}tx\cos[x]\\
&-64c1^2e^{t}t^2x\cos[x]+8c1^2e^{2t}t^2x\cos[x]\\
&-8c1^2\cos[2x]+32c1^2e^{t}\cos[2x]\\
&-24c1^2e^{2t}\cos[2x]-8c1^2t\cos[2x]\\
&+24c1^2e^{t}t\cos[2x]+8c1^2e^{2t}t^2\cos[2x]\\
&+c1^2\sin[x]-96c1^2e^{t}\sin[x]\\
&+95c1^2e^{2t}\sin[x]-80c1^2e^{t}t\sin[x]\\
&+16c1e^{2t}t\sin[x]+2c1^2e^{2t}t\sin[x]\\
&+16c2e^{2t}t\sin[x]-32c1^2e^{t}t^2\sin[x]\\
&+6c1^2e^{2t}t^2\sin[x]+16c1e^{t}x\sin[x]\\
&-16c1^2e^{t}x\sin[x]+16c2e^{t}x\sin[x]\\
&-16c1e^{2t}x\sin[x]+16c1^2e^{2t}x\sin[x]\\
&-16c2e^{2t}x\sin[x]+16c1e^{t}tx\sin[x]\\
&+16c2e^{t}tx\sin[x]-16c1^2e^{2t}tx\sin[x]\\
&+8c1^2e^{2t}t^2x\sin[x]+32c1^2e^{t}x^2\sin[x]\\
&-32c1^2e^{2t}x^2\sin[x]+32c1^2e^{t}tx^2\sin[x]\\
&+16c1^2e^{t}t^2x^2\sin[x]+4c1\sin[2x]\\
&-4c1^2\sin[2x]+4c2\sin[2x]\\
&-4c1e^{2t}\sin[2x]+4c1^2e^{2t}\sin[2x]\\
&-4c2e^{2t}\sin[2x]+8c1e^{2t}t\sin[2x]\\
&-8c1^2e^{2t}t\sin[2x]+8c2e^{2t}t\sin[2x]\\
&+24c1^2e^{2t}t^2\sin[2x]+8c1^2x\sin[2x]\\
&-32c1^2e^{t}x\sin[2x]+24c1^2e^{2t}x\sin[2x]\\
&+16c1^2tx\sin[2x]-16c1^2e^{t}tx\sin[2x]\\
&-16c1^2e^{2t}tx\sin[2x]+16c1^2e^{2t}t^2x\sin[2x]\\
&-3c1^2\sin[3x]+3c1^2e^{2t}\sin[3x]\\
&-6c1^2e^{2t}t\sin[3x]+6c1^2e^{2t}t^2\sin[3x]\Big).
\end{aligned} \tag{58}$$

The second order approximate solution is given by the following equation,

$$\begin{aligned}
&u(x,t,c1,c2)\\
&\quad=u_0(x,t)+u_1(x,t,c1)+u_2(x,t,c1,c2).
\end{aligned} \tag{59}$$

Using the method of Least Squares the optimum values of $c1$ and $c2$ are computed which are as follows:

$$\begin{aligned}
c1 &= -1.0433989069917953,\\
c2 &= -0.0018859250375958814.
\end{aligned} \tag{60}$$

The 2nd order OHAM solution yields very encouraging results after comparing with 2nd order VIM solution [5]. Tables 4(a–d), and Figures 13, 14, 15, and 16 show the effectiveness of OHAM for $x = 0.2$, $x = 0.4$, $x = 0.6$, and $x = 1$.

Table 4: (a) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 4 at $x = 0.2$ and various values of t. (b) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 4 at $x = 0.4$ and various values of t. (c) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 4 at $x = 0.6$ and various values of t. (d) Comparison of absolute errors of 1st order OHAM solution and 1st order VIM solution for Example 4 at $x = 1$ and various values of t.

(a)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.194735	0.194388	3.596760×10^{-4}	3.46922×10^{-4}
0.04	0.190879	0.189563	1.374338×10^{-3}	1.31649×10^{-3}
0.06	0.1871	0.184295	2.950802×10^{-3}	2.8045×10^{-3}
0.08	0.183395	0.178685	5.000174×10^{-3}	4.71013×10^{-3}
0.10	0.179763	0.172828	7.437789×10^{-3}	6.93584×10^{-3}

(b)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.381707	0.381449	2.832281×10^{-4}	2.58242×10^{-4}
0.04	0.374149	0.373216	1.043581×10^{-3}	9.32938×10^{-4}
0.06	0.36674	0.364862	2.151953×10^{-3}	1.87854×10^{-3}
0.08	0.359478	0.356523	3.485485×10^{-3}	2.95526×10^{-3}
0.10	0.35236	0.348331	4.927456×10^{-3}	4.02886×10^{-3}

(c)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.553462	0.55333	1.676249×10^{-4}	1.31821×10^{-4}
0.04	0.542503	0.542094	5.651230×10^{-4}	4.08832×10^{-4}
0.06	0.53176	0.531101	1.040306×10^{-3}	6.59364×10^{-4}
0.08	0.521231	0.520511	1.4483802×10^{-3}	7.194×10^{-4}
0.10	0.51091	0.510477	1.651669×10^{-3}	4.32267×10^{-4}

(d)

t	Exact solution	OHAM solution	Absolute error VIM	Absolute error OHAM
0.02	0.703151	0.703174	1.439370×10^{-4}	2.23769×10^{-4}
0.04	0.689228	0.689438	6.672725×10^{-4}	2.10203×10^{-4}
0.06	0.675581	0.676318	1.702709×10^{-3}	7.37797×10^{-4}
0.08	0.662203	0.663974	3.376802×10^{-3}	1.77096×10^{-3}
0.10	0.649091	0.652558	5.810408×10^{-3}	3.46734×10^{-3}

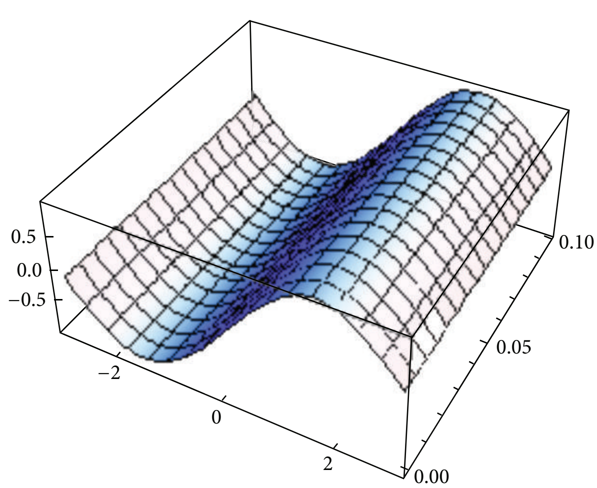

Figure 13: 2nd order approximate solution for (4.12).

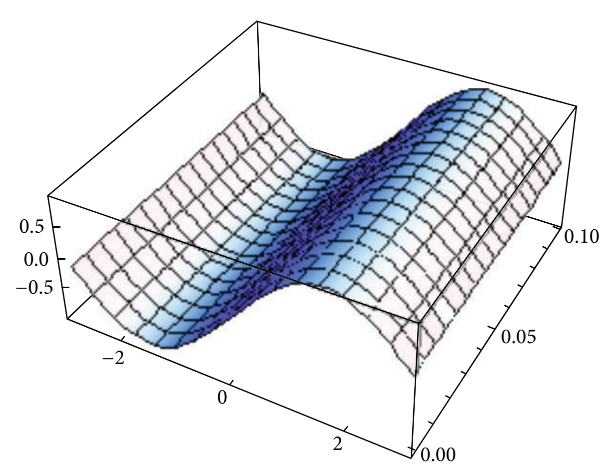

Figure 14: The surface shows Exact solution for (4.12).

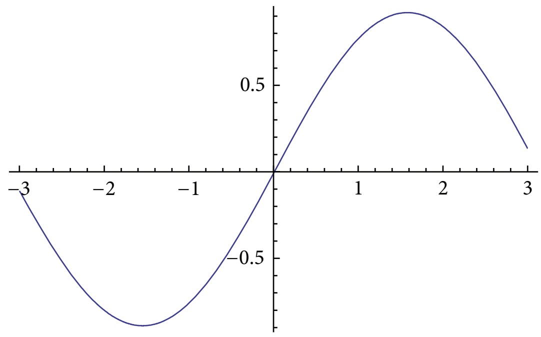

Figure 15: Approximate solution plot for $t = 0.1$ for (4.12).

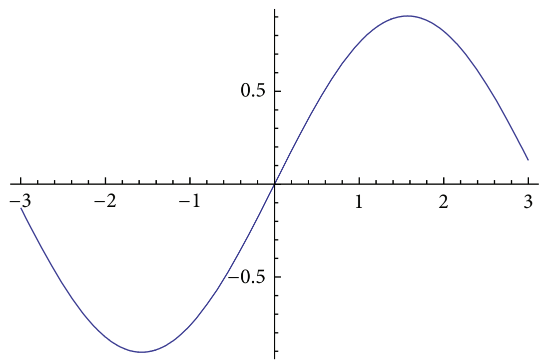

Figure 16: Exact solution plot for $t = 0.1$ for (4.12).

3. Conclusion

In this paper, the OHAM has been successfully implemented for the approximate solution of solutions of the Nonlinear Damped Generalized Regularized Long-Wave (DGRLW) equations. The results obtained by OHAM are very consistent in comparison with VIM.

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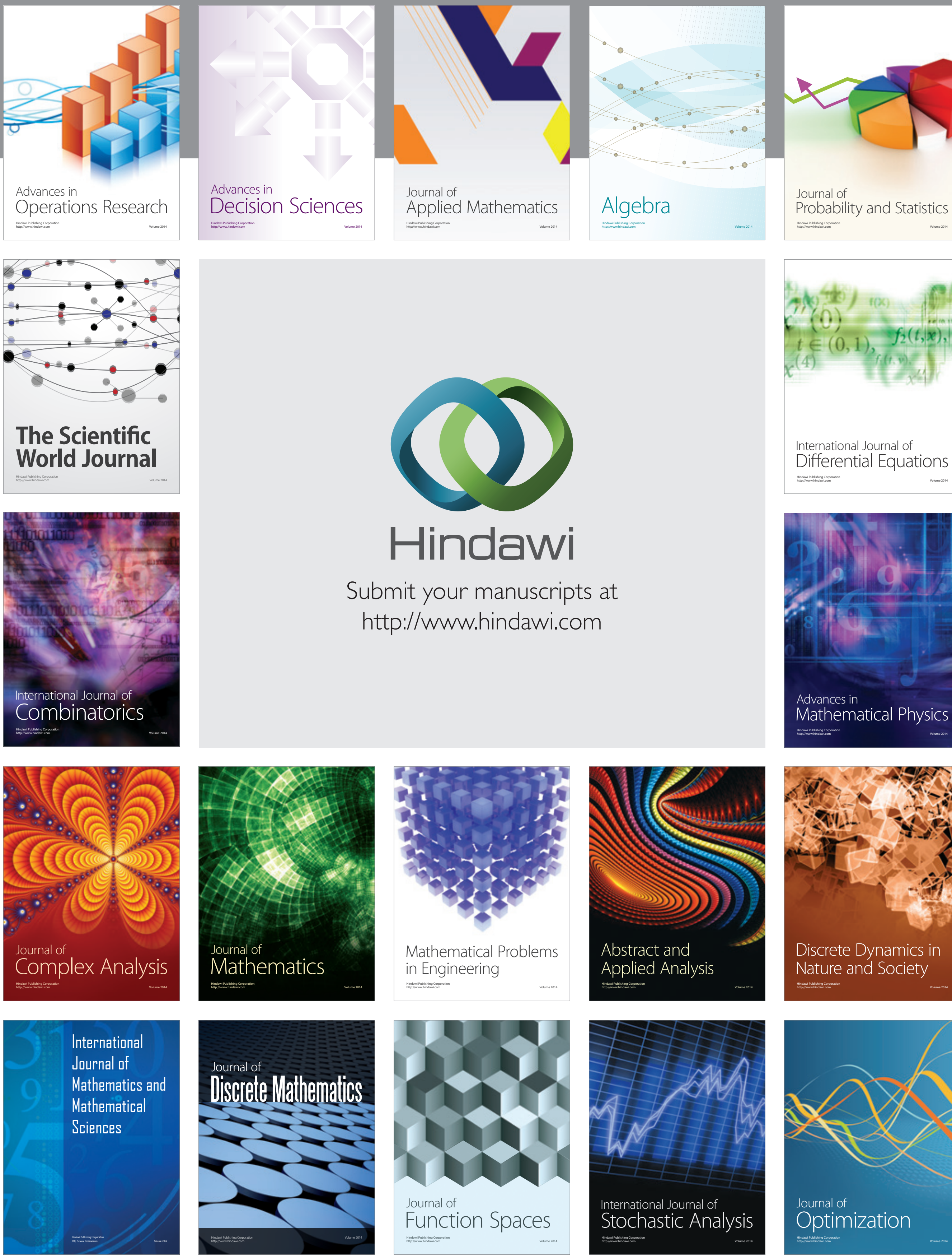
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