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Research Article

Adaptive Stabilization for Nonholonomic Systems with Unknown Time Delays

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This paper presents an adaptive control strategy for a class of nonholonomic systems in chained form with virtual control coefficients, nonlinear uncertainties, and unknown time delays. State scaling technique and backstepping recursive approach are applied to design a nonlinear state feedback controller, which can guarantee the stabilization of the closed-loop systems. The simulation results are provided to show the effectiveness of the proposed method.

1. Introduction

In the last few decades, considerable efforts have been devoted to the research of nonholonomic system, which is a particular class of nonlinear systems and widespread in real world, such as mobile robots, car-like vehicle, underactuated satellites, and knife-edge. It is well known that the control of nonholonomic systems is extremely challenging, largely due to the impossibility of asymptotically stabilizing nonholonomic systems via smooth time-invariant state feedback, a well-recognized fact pointed out in [1, 2]. In order to overcome this obstruction, several approaches have been proposed, such as discontinuous feedback, time-varying feedback, and hybrid stabilization.

The discontinuous feedback stabilization was first proposed in [3], and then further discussion was made in [4–7]; especially an elegant discontinuous coordinate transformation approach was presented in [5] for the stabilization problem of nonholonomic systems. Meanwhile, the smooth time-varying feedback control strategies also have drawn much attention [8–11]. To date, there have been several controller design approaches for the asymptotic stabilization or exponential regulation of nonholonomic control systems [4–14].

As pointed out in [9], many nonlinear mechanical systems with nonholonomic constraints can be transformed, either locally or globally, into the nonholonomic systems in the so-called chained form. Therefore, a number of research literature resources [8–23] for such chained nonholonomic systems are provided. Recently, some new adaptive control strategies have been proposed to stabilize the nonholonomic systems. For instance, state feedback control is studied in [15–20] and output feedback control in [21–25].

From a practical point of view, when modeling a mechanical system, time delay should be taken into account, and there are a few literature resources [19, 20, 25] for the nonholonomic systems with time delay. In [19, 20, 25], the problem of stabilization is studied for delayed nonholonomic systems; however, the virtual control coefficients and unknown parameter vector are not considered.

In this paper, we introduce a new class of chained nonholonomic systems with unknown virtual control coefficients, uncertain nonlinearities, and unknown time delays and then study the problem of adaptive state feedback stabilization. Since the nonholonomic system considered in this paper contains the delayed terms, it cannot be handled by existing conventional methods. The proposed constructive design method is based on a combined application of the state

scaling technique, the recursive backstepping approach, and the novel Lyapunov-Krasovskii functionals. The switching control strategy for the first subsystem is employed to achieve the asymptotic stabilization.

2. Problem Formulation and Preliminaries

In this paper, we present an adaptive stabilization control design procedure for the following nonholonomic systems with nonlinear uncertainties and unknown time delays:

$$\begin{aligned}
\dot{x}_0(t) &= d_0 u_0(t) + \phi_0(t, x_0(t), \theta),\\
\dot{x}_1(t) &= d_1 u_0(t) x_2(t) + \varphi_1(y(t), y(t-\tau_1))\\
&\quad + \phi_1(t, u_0(t), x_0(t), x(t), \theta),\\
&\vdots\\
\dot{x}_i(t) &= d_i u_0(t) x_{i+1}(t) + \varphi_i(y(t), y(t-\tau_i))\\
&\quad + \phi_i(t, u_0(t), x_0(t), x(t), \theta),\\
&\vdots\\
\dot{x}_n(t) &= d_n u_1(t) + \varphi_n(y(t), y(t-\tau_n))\\
&\quad + \phi_n(t, u_0(t), x_0(t), x(t), \theta),\\
y(t) &= [x_0(t), x_1(t)]^T,
\end{aligned} \tag{1}$$

where $[x_0(t), x(t)]^T = [x_0(t), x_1(t), \ldots, x_n(t)]^T \in R^{n+1}$ and $u(t) = [u_0(t), u_1(t)]^T \in R^2$ are system states and control input, respectively. $\theta \in R^m$ is an unknown bounded parameter vector. $d_i (0 \le i \le n)$ are disturbed virtual control coefficients, and the individual signs are known. $\varphi_i^d(y(t), y(t-\tau_i))'s$ denote the delayed terms, which contain output delays. $\tau_i (1 \le i \le n)$ are unknown constants; $\phi_i \in R^m$ are vectors of smooth nonlinear functions and represent unmodeled dynamic and external disturbances.

Assumption 1. For nonlinear functions ϕ_0 and $\phi_i (1 \le i \le n)$, there exist (known) smooth nonnegative functions $\overline{\phi}_0$ and $\overline{\phi}_{ij} (1 \le j \le i \le n)$ such that

$$\begin{aligned}
\phi_0(t, x_0(t), \theta) &= x_0 \overline{\phi}_0(x_0, \theta),\\
|\phi_i(t, u_0, x_0, x_i, \theta)| &\le |x_0|^{n-i+1} \overline{\phi}_{i0}(x_0, x_1, \theta)\\
&\quad + \sum_{j=1}^{i} |x_j| \overline{\phi}_{ij}(u_0, x_0, \overline{x}_i, \theta),
\end{aligned} \tag{2}$$

where $\overline{x}_i = [x_1, \cdot, x_i]^T$.

Assumption 2. The nonlinear functions $\varphi_i(y(t), y(t-\tau_i)) (1 \le i \le n)$ satisfy

$$\begin{aligned}
&|\varphi_i(y(t), y(t-\tau_i))|\\
&\quad \le |x_1(t) x_1(t-\tau_i)| \overline{\varphi}_i(y(t), y(t-\tau_i)),
\end{aligned} \tag{3}$$

in which $\overline{\varphi}_i$ are (known) smooth nonnegative nonlinear functions.

Remark 3. It is clear that system (1) covers a number of important classes of uncertain nonholonomic systems that have been investigated in some existing literature resources. For instance, when $d_i = 1$ and $\phi_i = 0, \phi_{i=0}$ system (1) reduces the standard form of nonholonomic system which has been widely studied in the literature [15, 18–20]. Moreover, in Ge et al. [15], not only the virtual control coefficients $d_i = 1$ and the dynamics ϕ_i satisfying $\phi_i = \widetilde{\phi}_i^T \theta$ are assumed but also the modeled dynamics φ_i do not exist. In Liu and Zhang [22], the virtual control coefficients and time delays have not been considered, and the expression $\phi_i = \widetilde{\phi}_i^T \theta$ is also required. While $d_i = 1$ and φ_i and unknown parameters θ are not existent, system (1) degenerates to the one studied in Xi et al. [21]. When $\varphi_i = 0$, together with $\phi_i = \widetilde{\phi}_i^T \theta$, system (1) becomes the considered system in Ju et al. [23].

Remark 4. Note that here we only use the sign of d_i without any knowledge of individual virtual control coefficient $d_i (1 \le i \le n)$. Moreover, Assumptions 1 and 2 are imposed on the nonlinear functions ϕ_i and the delayed terms φ_i of system (1), respectively. It can be seen that some similar conditions are implied in [22].

Lemma 5. *For any real-valued continuous function $f(x, y)$, where $x \in R^n$ and $y \in R^m$, there are smooth functions $a(x) \ge 0$, $b(y) \ge 0$, $c(x) \ge 1$ and $d(y) \ge 1$ such that*

$$|f(x, y)| \le a(x) + b(y), \qquad |f(x, y)| \le c(x) d(y). \tag{4}$$

By Lemma 5 and Assumption 1, we know that there exist smooth functions $\omega_0 \ge 1, \omega_{i0} \ge 1, \omega_{ij} \ge 1, \zeta_0 \ge 1, \zeta_{i0} \ge 1$, and $\zeta_{ij} \ge 1$ such that

$$\begin{aligned}
|\phi_0(t, x_0(t), \theta)| &\le |x_0| \omega_0(x_0) \zeta_0(\theta),\\
|\phi_i(t, u_0, x_0, x_i, \theta)| &\le |x_0|^{n-i+1} \omega_{i0}(x_0, x_1) \zeta_{i0}(\theta)\\
&\quad + \sum_{j=1}^{i} |x_j| \omega_{ij}(u_0, x_0, \overline{x}_i) \zeta_{ij}(\theta).
\end{aligned} \tag{5}$$

Denote that $\vartheta = \zeta_0 + \sum_{i=1}^{n} \sum_{j=0}^{i} \zeta_{ij}(\theta)$, then the above inequalities can be rewritten as follows:

$$\begin{aligned}
|\phi_0(t, x_0(t), \theta)| &\le |x_0| \omega_0(x_0) \vartheta,\\
|\phi_i(t, u_0, x_0, x_i, \theta)| &\le |x_0|^{n-i+1} \omega_{i0}(x_0, x_1) \vartheta\\
&\quad + \sum_{j=1}^{i} |x_j| \omega_{ij}(u_0, x_0, \overline{x}_i) \vartheta.
\end{aligned} \tag{6}$$

3. Adaptive Stabilization Control Design

In this section, we will design an adaptive stabilization controller for the case that $x_0(t) \ne 0$, and the case that $x_0(t) = 0$ will be considered in the next section. Now, we use two

separate stages to globally asymptotically stabilize the system (1). Firstly, the control $u_0(t)$ should be designed for x_0-subsystem; in the second stage, we design $u_1(t)$ to guarantee all states of the rest in system (1) converge to zero.

3.1. State Scaling. The following state scaling discontinuous transformation is introduced:

$$z_i(t) = \frac{x_i(t)}{x_0^{n-i}(t)}, \quad 1 \le i \le n. \tag{7}$$

Under the new z-coordinates, the system (1) is transformed into

$$\begin{aligned}
\dot{x}_0(t) &= d_0 u_0(t) + \phi_0(t, x_0(t), \theta), \\
\dot{z}_i(t) &= \frac{u_0(t)}{x_0(t)} d_i z_{i+1}(t) - (n-i)\frac{u_0(t)}{x_0(t)} d_0 z_i(t) + \widetilde{\phi}_i + \widetilde{\varphi}_i, \\
\dot{z}_n(t) &= d_n u_1(t) + \widetilde{\phi}_n + \widetilde{\varphi}_n,
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\widetilde{\phi}_i = {} & \frac{\phi_i^d(t, u_0(t), x_0(t), x(t), \theta)}{x_0^{n-i}(t)} \\
& - \frac{(n-i)\phi_0(t, u_0(t), x_0(t), x(t), \theta)}{x_0(t)} z_i(t),
\end{aligned} \tag{9}$$

$$\widetilde{\varphi}_i = \frac{\varphi_i(y(t), y(t-\tau_i))}{x_0^{n-i}(t)}. \tag{10}$$

In order to obtain the estimation for the nonlinear functions $\widetilde{\phi}_i$ and $\widetilde{\varphi}_i$, the following lemmas are given.

Lemma 6. *For every $1 \le i \le n$, there exist smooth nonnegative functions $\widetilde{\omega}_{i0}(x_0, z_1)$ and $\widetilde{\omega}_{ij}(u_0, x_0, \overline{z}_i)$ such that*

$$\begin{aligned}
\left|\widetilde{\phi}_i\right| \le {} & |x_0(t)| \widetilde{\omega}_{i0}(x_0(t), z_1(t))\vartheta \\
& + \sum_{j=1}^{i} \left|z_j(t)\right| \widetilde{\omega}_{ij}(u_0(t), x_0(t), \overline{z}_i(t))\vartheta,
\end{aligned} \tag{11}$$

where $\overline{z}_i = [z_1, \ldots, z_i]^T$.

Proof. By the above inequalities (6), it can be deduced that

$$\begin{aligned}
\left|\widetilde{\phi}_i\right| &\le \frac{\left|\phi_i(t, u_o, x_0, x, \theta)\right|}{\left|x_0^{n-i}\right|} + \frac{\left|(n-i)\phi_0(t, x_0, \theta)\right|}{\left|x_0\right|}\left|z_i\right| \\
&\le \left|x_0\right| \omega_{i0}(x_0, x_1)\vartheta + \sum_{j=1}^{i}\left|z_j\right| \\
&\qquad \cdot \left|x_0\right|^{i-j}\omega_{ij}(u_0, x_0, \overline{x}_i)\vartheta \\
&\qquad + (n-i)\left|z_i\right|\omega_0(x_0)\vartheta \\
&= \left|x_0\right|\omega_{i0}\left(x_0, z_1 x_0^{n-i}\right)\vartheta \\
&\quad + \sum_{j=1}^{i}\left|z_j\right| \cdot \left|x_0\right|^{i-j}\omega_{ij}\left(u_0, x_0, z_1 x_0^{n-i}, \ldots, z_i x_0^{n-i}\right)\vartheta \\
&\quad + (n-i)\left|z_i\right|\omega_0(x_0)\vartheta.
\end{aligned} \tag{12}$$

Introduce the notation

$$\begin{aligned}
\widetilde{\omega}_{i0} &= \omega_{i0}\left(x_0, z_1 x_0^{n-i}\right), \\
\widetilde{\omega}_{ij} &= \left|x_0\right|^{i-j}\omega_{ij} \\
&\quad \times \left(u_0, x_0, z_1 x_0^{n-i}, \ldots, z_i x_0^{n-i}\right), \quad 1 \le i \le j-1, \\
\widetilde{\omega}_{ii} &= \omega_{ii}\left(u_0, x_0, z_1 x_0^{n-i}, \ldots, z_i x_0^{n-i}\right) + (n-i)\omega_0(x_0).
\end{aligned} \tag{13}$$

It is clear that $\widetilde{\omega}_{ij}(0 \le j \le i)$ are smooth nonnegative functions, and inequality (11) holds. □

Lemma 7. *For every $1 \le i \le n-1$, the following inequality holds:*

$$\begin{aligned}
\left|\widetilde{\varphi}_i\right| &\le \left|z_1(t) z_1(t-\tau_i)\right| \Gamma_i(y(t), y(t-\tau_i)) \\
&\le \left|z_1(t) z_1(t-\tau_i)\right| f_{i1}(y(t)) f_{i2}(y(t-\tau_i)),
\end{aligned} \tag{14}$$

where Γ_i, f_{i1}, and f_{i2} are smooth nonnegative functions.

Proof. According to Assumption 2, the nonlinear functions $\widetilde{\varphi}_i$ yield that

$$\begin{aligned}
\left|\widetilde{\varphi}_i\right| &= \frac{\left|\varphi_i(y(t), y(t-\tau_i))\right|}{\left|x_0^{n-i}(t)\right|} \\
&\le \frac{\left|x_1(t) x_1(t-\tau_i)\right|}{\left|x_0^{n-i}(t)\right|}\overline{\varphi}_i(y(t), y(t-\tau_i)) \\
&= \left|z_1(t) x_0^{i-1}(t)\right| \\
&\quad \cdot \left|z_1(t-\tau_i) x_0^{n-1}(t-\tau_i)\right| \overline{\varphi}_i(y(t), y(t-\tau_i)).
\end{aligned} \tag{15}$$

Let $\Gamma_i(y(t), y(t-\tau_i)) = |[1,0]y(t)|^{i-1}|[1,0]y(t-\tau_i)|^{n-1}\overline{\varphi}_i(y(t), y(t-\tau_i))$; then the above inequality can be expressed as

$$\left|\widetilde{\varphi}_i\right| \le \left|z_1(t) z_1(t-\tau_i)\right| \Gamma_i(y(t), y(t-\tau_i)). \tag{16}$$

It is seen that $\Gamma_i(y(t), y(t-\tau_i))$ are smooth functions. Then using Lemma 5, there exist smooth functions $f_{i1} \geq 1$ and $f_{i2} \geq 1$ such that

$$\left|\widetilde{\varphi}_i\right| \leq \left|z_1(t) z_1(t-\tau_i)\right| f_{i1}(y(t)) f_{i2}(y(t-\tau_i)). \quad (17)$$

□

3.2. Control Design. In this section, we design the control inputs $u_0(t)$ and $u_1(t)$ subject to $x_0(t_0) \neq 0$. The case that the initial condition $x_0(t_0) = 0$ will be treated in Section 4. The design of the control inputs here is based on the backstepping method for the transformed system (10). The recursive procedure stops once the true system inputs occur.

Step 1. For the x_0-subsystem

$$\dot{x}_0(t) = d_0 u_0(t) + \phi_0(t, x_0(t), \theta), \quad (18)$$

define new variables $r_0 = 1/|d_0|$, $\widetilde{r}_0 = r_0 - \widehat{r}_0$, and $\widetilde{\vartheta}_0 = \vartheta - \widehat{\vartheta}_0$, where $\widehat{r}_0$ and $\widehat{\vartheta}_0$ are the estimates of r_0 and ϑ, respectively.

Consider the Lyapunov function candidate

$$V_0 = \frac{1}{2}x_0^2(t) + \frac{1}{2}\widetilde{\vartheta}_0^2 + \frac{1}{2r_0}\widetilde{r}_0^2. \quad (19)$$

Calculating the time derivative of V_0 along the system (18)

$$\dot{V}_0 = x_0(t)\left[d_0 u_0(t) + \phi_0(t, x_0(t), \theta)\right] - \widetilde{\vartheta}_0\dot{\widehat{\vartheta}}_0 - \frac{1}{r_0}\widetilde{r}_0\dot{\widehat{r}}_0. \quad (20)$$

The controller $u_0(t)$ can be chosen as

$$u_0(t) = g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) x_0(t), \quad (21)$$

where

$$g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) = \operatorname{sign}(d_0)\,\widehat{r}_0 \pi_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right), \quad (22)$$

$$\pi_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) = -\frac{1+\sqrt{1+\left(k_0/\widehat{r}_0\right)^2}}{2} \times \left[\omega_0(x_0)\widehat{\vartheta}_0 + \sqrt{c_0^2 + \left(\omega_0(x_0)\widehat{\vartheta}_0\right)^2}\right]. \quad (23)$$

With the controller $u_0(t)$ in (21), the time derivative of V_0 satisfies

$$\begin{aligned}\dot{V}_0 &= \left(1 - \frac{\widetilde{r}_0}{r_0}\right)\pi_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) x_0^2(t) \\ &\quad + x_0(t)\phi_0(t, x_0(t), \theta) - \widetilde{\vartheta}_0\dot{\widehat{\vartheta}}_0 - \frac{1}{r_0}\widetilde{r}_0\dot{\widehat{r}}_0 \\ &\leq -\sqrt{c_0^2 + \left(\omega_0(x_0)\widehat{\vartheta}_0\right)^2}\, x_0^2(t) - \frac{\widetilde{r}_0}{r_0}\pi_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) x_0^2(t) \\ &\quad + x_0^2(t)\,\omega_0(x_0)\,\widetilde{\vartheta}_0 \\ &\quad - \widetilde{\vartheta}_0\dot{\widehat{\vartheta}}_0 - \frac{1}{r_0}\widetilde{r}_0\dot{\widehat{r}}_0.\end{aligned} \quad (24)$$

Choosing the following update laws $\dot{\widehat{\vartheta}}_0$ and $\dot{\widehat{r}}_0$ as

$$\dot{\widehat{\vartheta}}_0 = \omega_0(x_0)\, x_0^2(t), \quad (25)$$

$$\dot{\widehat{r}}_0 = -\pi_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) x_0^2(t), \quad (26)$$

we have

$$\dot{V}_0 \leq -c_0 x_0^2(t), \quad (27)$$

where $c_0 > 0$ is a positive design parameter. Therefore, it implies that $x_0(t)$, $\widehat{\vartheta}_0$, and $\widehat{r}_0$ are bound. By LaSalle's Invariant Theorem, we can further achieve that $x_0(t) \to 0$, as $t \to \infty$.

Remark 8. The closed-loop dynamics of x_0-subsystem is

$$\dot{x}_0(t) = x_0(t)\left[d_0 g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) + \overline{\phi}_0(x_0, \theta)\right]. \quad (28)$$

It is seen that $\psi(t) = d_0 g_0(x_0, \widehat{\vartheta}_0, \widehat{r}_0) + \overline{\phi}_0(x_0, \theta)$ is bounded as $x_0(t)$, $\widehat{\vartheta}_0$, $\widehat{r}_0$, and θ are bounded. On the other hand, the solution of x_0-system can be computed as

$$x_0(t) = x_0(t_0)\, e^{\int_{t_0}^{t}\psi(s)ds}. \quad (29)$$

Obviously, for $x_0(t_0) \neq 0$ and $t \geq t_0$, the solution $x_0(t)$ exists and satisfies $|x_0(t)| > 0$. That is, $x_0(t)$ does not become zero at any time instant for $x_0(t_0) \neq 0$. Therefore, the introduced state scaling above is effective.

Under the controller $u_0(t)$ in (25), the z-system can be rewritten as

$$\begin{aligned}\dot{z}_i(t) &= d_i g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_{i+1}(t) \\ &\quad - (n-i)\, d_0 g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_i(t) + \widetilde{\phi}_i + \widetilde{\varphi}_i, \\ \dot{z}_n(t) &= d_n u_1(t) + \widetilde{\phi}_n + \widetilde{\varphi}_n.\end{aligned} \quad (30)$$

Step 2. For $z_1(t)$-subsystem in (30)

$$\begin{aligned}\dot{z}_1(t) &= d_1 g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_2(t) \\ &\quad - (n-1)\, d_0 g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_1(t) + \widetilde{\phi}_1 + \widetilde{\varphi}_1,\end{aligned} \quad (31)$$

let $r_1 = 1/|d_1|$, and $\widetilde{d}_0 = d_0 - \widehat{d}_0$, $\widetilde{d}_1 = d_1 - \widehat{d}_1$, $\widetilde{r}_1 = r_1 - \widehat{r}_1$, $\widetilde{\vartheta}_1 = \vartheta - \widehat{\vartheta}_1$, and $\widetilde{v} = \vartheta^2 - \widehat{v}$, where $\widehat{d}_0$, $\widehat{d}_1$, $\widehat{r}_1$, $\widehat{\vartheta}_1$, and $\widehat{v}$ are the estimates of unknown parameters d_0, d_1, r_1, ϑ, and ϑ^2, respectively. Introduce the coordinate transformations $\xi_1(t) = z_1(t)$ and $\xi_2(t) = z_2(t) - \alpha_1$, where α_1 is regarded as the virtual control input. Construct the following Lyapunov-Krasovskii functional:

$$\begin{aligned}V_1 &= V_0 + \frac{1}{2}\xi_1^2(t) + \frac{1}{2}\widetilde{d}_0^2 + \frac{1}{2}\widetilde{d}_1^2 + \frac{1}{2r_1}\widetilde{r}_1^2 + \frac{1}{2}\widetilde{\vartheta}_1^2 + \frac{1}{2}\widetilde{v}^2 \\ &\quad + \sum_{l=1}^{n}\sum_{k=1}^{l}\int_{t-\tau_k}^{t}\frac{\varepsilon_{lk}}{2}\xi_1^2(\sigma)\, f_{k2}^2(y(\sigma))\, d\sigma,\end{aligned} \quad (32)$$

where $\varepsilon_{lk} > 0$ $(1 \le k \le l \le n)$ are scalars. Along (31), the time derivative of V_1 gives

$$\begin{aligned}\dot{V}_1 = {} & \dot{V}_0 + \xi_1(t)\Big[\left(\hat{d}_1 + \tilde{d}_1\right) g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\xi_2(t) \\ & + d_1 g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\alpha_1 - (n-1)\left(\hat{d}_0 + \tilde{d}_0\right) g_0 \\ & \times \left(x_0, \hat{\vartheta}_0, \hat{r}_0\right) z_1(t) + \tilde{\varphi}_1 + \tilde{\phi}_1\Big] - \tilde{d}_0\dot{\hat{d}}_0 - \tilde{d}_1\dot{\hat{d}}_1 \\ & - \frac{1}{r_1}\tilde{r}_1\dot{\hat{r}}_1 - \tilde{\vartheta}_1\dot{\hat{\vartheta}}_1 - \tilde{v}\dot{\hat{v}} + \sum_{l=1}^{n}\sum_{k=1}^{l}\frac{\varepsilon_{lk}}{2}\xi_1^2(t) f_{k2}^2(y(t)) \\ & - \sum_{l=1}^{n}\sum_{k=1}^{l}\frac{\varepsilon_{lk}}{2}\xi_1^2(t-\tau_k) f_{k2}^2(y(t-\tau_k)).\end{aligned} \tag{33}$$

By Lemmas 6 and 7, the following inequalities hold:

$$\begin{aligned}\left|\xi_1(t)\tilde{\phi}_1\right| &\le x_0^2(t) + \xi_1^2(t)\gamma_{10}(x_0, z_1)\vartheta^2 \\ &\quad + \xi_1^2(t)\gamma_{11}(u_0, x_0, z_1)\vartheta, \\ \left|\xi_1(t)\tilde{\varphi}_1\right| &\le \left|\xi_1(t) z_1(t) z_1(t-\tau_1)\right. \\ &\quad \left.\times f_{11}(y(t)) f_{12}(y(t-\tau_1))\right| \\ &\le \frac{1}{2\varepsilon_{11}}\xi_1^2(t) z_1^2(t) f_{11}^2(y(t)) \\ &\quad + \frac{\varepsilon_{11}}{2}\xi_1^2(t-\tau_1) f_{12}^2(y(t-\tau_1)),\end{aligned} \tag{34}$$

where $\gamma_{10} = (1/4)\tilde{\omega}_{10}^2(x_0, z_1)$ and $\gamma_{11} = \tilde{\omega}_{11}(u_0, x_0, z_1)$. Choose a virtual control function $\alpha_1(x_0, z_1, \hat{\vartheta}_0, \hat{\vartheta}_1, \hat{v}, \hat{r}_0, \hat{r}_1, \hat{d}_0)$ as follows:

$$\begin{aligned}\alpha_1 &= \operatorname{sign}(d_1)\hat{r}_1\pi_1, \\ \pi_1 &= -\frac{1}{g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)}\Big[c_1\xi_1(t) + \xi_1(t)\gamma_{10}(x_0, z_1)\hat{v} \\ &\qquad + \xi_1(t)\gamma_{11}(u_0, x_0, z_1)\hat{\vartheta}_1 \\ &\qquad + \frac{1}{2\varepsilon_{11}}\xi_1(t) z_1^2(t) f_{11}^2(y(t)) \\ &\qquad + \sum_{l=1}^{n}\sum_{k=1}^{l}\frac{\varepsilon_{lk}}{2}\xi_1(t) f_{k2}^2(y(t))\Big] \\ &\quad + (n-1)\hat{d}_0 z_1(t),\end{aligned} \tag{35}$$

where c_1 is a positive design parameter. With the choice of the update law

$$\dot{\hat{r}}_1 = -g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\pi_1\xi_1(t), \tag{36}$$

we can obtain

$$\begin{aligned}\dot{V}_1 \le {} & -(c_0 - 1)x_0^2(t) - c_1\xi_1^2(t) + \hat{d}_1 g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\xi_1(t)\xi_2(t) \\ & - \tilde{d}_0\left(\dot{\hat{d}}_0 - \tau_{10}\right) - \tilde{d}_1\left(\dot{\hat{d}}_1 - \tau_{11}\right) \\ & - \tilde{v}\left(\dot{\hat{v}} - \eta_{10}\right) - \tilde{\vartheta}_1\left(\dot{\hat{\vartheta}}_1 - \eta_{11}\right) \\ & - \sum_{l=2}^{n}\sum_{k=1}^{l}\frac{\varepsilon_{lk}}{2}\xi_1^2(t-\tau_k) f_{k2}^2(y(t-\tau_k)),\end{aligned} \tag{37}$$

where

$$\begin{aligned}\tau_{10} &= \xi_1(t) h_{10}, \quad h_{10} = -(n-1) g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\xi_1(t), \\ \tau_{11} &= \xi_1(t) h_{11}, \quad h_{11} = g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\xi_2(t), \\ \eta_{10} &= \gamma_{10}(x_0, z_1)\xi_1^2(t), \quad \eta_{11} = \gamma_{11}(u_0, x_0, z_1)\xi_1^2(t).\end{aligned} \tag{38}$$

Step i $(2 \le i \le n-1)$. Assume that, at step $i-1$, a virtual control function α_{i-1} and a Lyapunov functional V_{i-1} have been designed for the $(z_1, \ldots, z_{i-1})$-subsystem of (30) in such a way that

$$\begin{aligned}\dot{V}_{i-1} \le {} & -(c_0 - i + 1)x_0^2(t) - \sum_{j=1}^{i-1}\left[c_j - i + j + 1\right]\xi_j^2(t) \\ & + \hat{d}_{i-1} g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right)\xi_{i-1}(t)\xi_i(t) \\ & - \sum_{j=0}^{i-1}\left(\tilde{d}_j + \sum_{l=j+2}^{i}\frac{\partial\alpha_{l-1}}{\partial\hat{d}_j}\xi_l(t)\right)\left(\dot{\hat{d}}_j - \tau_{ij}\right) \\ & - \left(\tilde{v} + \sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{v}}\xi_j(t)\right)\left(\dot{\hat{v}} - \eta_{i0}\right) \\ & - \left(\tilde{\vartheta}_1 + \sum_{j=2}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\vartheta}_1}\xi_j(t)\right)\left(\dot{\hat{\vartheta}}_1 - \eta_{i1}\right) \\ & - \sum_{l=i}^{n}\sum_{k=1}^{l}\frac{1}{2}\xi_1^2(t-\tau_k) f_{k2}^2(y(t-\tau_k)).\end{aligned} \tag{39}$$

Now, we examine the $(z_1, \ldots, z_i)$-subsystem of (30). Define $r_i = 1/|d_i|$, and $\tilde{d}_i = d_i - \hat{d}_i$ and $\tilde{r}_i = r_i - \hat{r}_i$, where $\hat{d}_i$ and $\hat{r}_i$ are the estimates of unknown parameters d_i and r_i, respectively. Introduce the coordinate transformations $\xi_{i+1}(t) = z_{i+1}(t) - \alpha_i$, where α_i is regarded as a virtual control input, and construct the following Lyapunov-Krasovskii functional

$$V_i = V_{i-1} + \frac{1}{2}\xi_i^2(t) + \frac{1}{2}\tilde{d}_i^2 + \frac{1}{2r_i}\tilde{r}_i^2. \tag{40}$$

Based on (39), the time derivative of V_i along the solutions of (30) satisfies

$$\begin{aligned}\dot{V}_i = {} & \dot{V}_{i-1} + \xi_i(t) \\ & \times \Bigg\{ \left(\widehat{d}_i + \widetilde{d}_i\right) g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) \xi_{i+1}(t) \\ & + d_i g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) \alpha_i(t) + \widetilde{\varphi}_i + \widetilde{\phi}_i \\ & - (n-i)\left(\widehat{d}_0 + \widetilde{d}_0\right) g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_i(t) \\ & - \frac{\partial \alpha_{i-1}}{\partial x_0}\left[-\left(\widehat{d}_0 + \widetilde{d}_0\right) g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) x_0(t) + \phi_0\right] \\ & - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial z_j}\Big[-\left(\widehat{d}_j + \widetilde{d}_j\right) g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_{j+1}(t) \\ & \qquad - (n-j)\left(\widehat{d}_0 + \widetilde{d}_0\right) g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) z_j(t) \\ & \qquad + \widetilde{\varphi}_j + \widetilde{\phi}_j\Big] \\ & - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{r}_j} \dot{\widehat{r}}_j - \sum_{j=0}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \widehat{d}_j} \dot{\widehat{d}}_j \\ & - \frac{\partial \alpha_{i-1}}{\partial \widehat{\vartheta}_0} \dot{\widehat{\vartheta}}_0 - \frac{\partial \alpha_{i-1}}{\partial \widehat{\vartheta}_1} \dot{\widehat{\vartheta}}_1 - \frac{\partial \alpha_{i-1}}{\partial \widehat{v}} \dot{\widehat{v}} \Bigg\} \\ & - \widetilde{d}_i \dot{\widehat{d}}_i - \frac{1}{r_i} \widetilde{r}_i \dot{\widehat{r}}_i. \end{aligned} \tag{41}$$

By Lemmas 5 and 7 and Young inequality, the following inequality holds:

$$\begin{aligned} & \left|\xi_i(t)\left[\widetilde{\varphi}_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial z_j} \widetilde{\varphi}_j\right]\right| \\ & \quad \le \frac{1}{2\varepsilon_{ii}} \xi_i^2(t) z_1^2(t) f_{i1}^2(y(t)) \\ & \qquad + \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}} \xi_i^2(t) \left(\frac{\partial \alpha_{i-1}}{\partial z_j}\right)^2 z_1^2(t) f_{j1}^2(y(t)) \\ & \qquad + \sum_{j=1}^{i} \frac{\varepsilon_{ij}}{2} \xi_1^2(t-\tau_j) f_{j2}^2\left(y(t-\tau_j)\right). \end{aligned} \tag{42}$$

Using Lemma 6 and Young inequality, there are nonnegative smooth functions $\gamma_{i0}(x_0, \overline{z}_i, \widehat{\vartheta}_0, \widehat{\vartheta}_1, \widehat{v}, \widehat{r}_0, \ldots, \widehat{r}_{i-1}, \widehat{d}_0, \ldots, \widehat{d}_{i-2})$ and $\gamma_{i1}(u_0, x_0, \overline{z}_i)$ such that

$$\begin{aligned} & \left|\xi_i(t)\left[\widetilde{\phi}_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial z_j} \widetilde{\phi}_j - \frac{\partial \alpha_{i-1}}{\partial x_0} \phi_0\right]\right| \\ & \quad \le x_0^2(t) + \sum_{j=1}^{i-1} \xi_j^2(t) + \xi_i^2(t) \gamma_{i0} \vartheta^2 + \xi_i^2(t) \gamma_{i1} \vartheta, \end{aligned} \tag{43}$$

where γ_{i0} and γ_{i1} are known nonnegative functions.

Choose the following virtual control function α_i:

$$\begin{aligned} \alpha_i = {} & \operatorname{sign}(d_i) \widehat{r}_i \pi_i, \\ \pi_i = {} & -\widehat{d}_{i-1} \xi_{i-1}(t) + (n-i) \widehat{d}_0 z_i(t) \\ & + \sum_{j=1}^{i-1} \widehat{d}_j \frac{\partial \alpha_{i-1}}{\partial z_j} z_{j+1}(t) - \widehat{d}_0 \sum_{j=1}^{i-1} (n-j) \frac{\partial \alpha_{i-1}}{\partial z_j} z_j(t) \\ & + \widehat{d}_0 \frac{\partial \alpha_{i-1}}{\partial x_0} x_0(t) - \frac{1}{g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right)} \\ & \times \Bigg[c_i \xi_i(t) + \xi_i(t) \gamma_{i0} \widehat{v} + \xi_i(t) \gamma_{i1} \widehat{\vartheta}_1 \\ & - \frac{\partial \alpha_{i-1}}{\partial \widehat{\vartheta}_0} \dot{\widehat{\vartheta}}_0 - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{r}_j} \dot{\widehat{r}}_j \\ & - \sum_{j=0}^{i-2} \frac{\partial \alpha_{i-1}}{\partial \widehat{d}_j} \tau_{ij} - \frac{\partial \alpha_{i-1}}{\partial \widehat{v}} \eta_{i0} - \frac{\partial \alpha_{i-1}}{\partial \widehat{\vartheta}_1} \eta_{i1} \\ & + \frac{1}{2\varepsilon_{ii}} \xi_i(t) z_1^2(t) f_{i1}^2(y(t)) \\ & + \sum_{j=1}^{i-1} \frac{1}{2\varepsilon_{ij}} \xi_i(t) \left(\frac{\partial \alpha_{i-1}}{\partial z_j}\right)^2 z_1^2(t) f_{j1}^2(y(t)) \\ & - \sum_{l=2}^{i-1} \sum_{j=0}^{l-2} \xi_l(t) \frac{\partial \alpha_{l-1}}{\partial \widehat{d}_j} h_{ij} - \sum_{j=2}^{i-1} \xi_j(t) \frac{\partial \alpha_{j-1}}{\partial \widehat{v}} \gamma_{i0} \\ & - \sum_{j=2}^{i-1} \xi_j(t) \frac{\partial \alpha_{j-1}}{\partial \widehat{\vartheta}_1} \gamma_{i1} \Bigg], \end{aligned} \tag{44}$$

where c_i is a positive design parameter and τ_{ij} $(0 \le j \le i-2)$, η_{i0} and η_{i1} are pending nonnegative functions to be specified in (47). Moreover, construct the following update law:

$$\dot{\widehat{r}}_i = -g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) \pi_i \xi_i(t). \tag{45}$$

Substituting inequalities (42)–(45) into (41) yields

$$\begin{aligned} \dot{V}_i \le {} & -(c_0 - i) x_0^2(t) - \sum_{j=1}^{i} \left[c_j - i + j\right] \xi_j^2(t) \\ & + \widehat{d}_i g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) \xi_i(t) \xi_{i+1}(t) \\ & - \sum_{j=0}^{i} \left(\widetilde{d}_j + \sum_{l=j+2}^{i} \frac{\partial \alpha_{l-1}}{\partial \widehat{d}_j} \xi_l(t)\right)\left(\dot{\widehat{d}}_j - \tau_{ij}\right) \\ & - \left(\widetilde{v} + \sum_{j=2}^{i} \frac{\partial \alpha_{j-1}}{\partial \widehat{v}} \xi_j(t)\right)\left(\dot{\widehat{v}} - \eta_{i0}\right) \end{aligned}$$

$$-\left(\tilde{\vartheta}_1+\sum_{j=2}^{i}\frac{\partial\alpha_{j-1}}{\partial\hat{\vartheta}_1}\xi_j(t)\right)\left(\dot{\hat{\vartheta}}_1-\eta_{i1}\right)$$
$$-\sum_{l=i+1}^{n}\sum_{k=1}^{l}\frac{\varepsilon_{lk}}{2}\xi_1^2(t-\tau_k)f_{k2}^2(y(t-\tau_k)),\tag{46}$$

where

$$\begin{aligned}
\tau_{i0} &= \tau_{i-1,0}+h_{i0}\xi_i(t),\\
h_{i0} &= -(n-i)g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)z_i(t)\\
&\quad-\frac{\partial\alpha_{i-1}}{\partial x_0}g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)x_0(t)\\
&\quad+\sum_{j=1}^{i-1}(n-j)\frac{\partial\alpha_{i-1}}{\partial z_j}g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)z_j(t),\\
\tau_{il} &= \tau_{i-1,l}+h_{il}\xi_i(t),\\
h_{il} &= -\frac{\partial\alpha_{i-1}}{\partial z_l}g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)z_{l+1}(t),\\
&\qquad 1\le l\le i-1,\\
\tau_{ii} &= h_{ii}\xi_i(t),\qquad h_{ii}=g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)\xi_{i+1}(t),\\
\eta_{i0} &= \eta_{i-1,0}+\gamma_{i0}\xi_i(t),\quad \eta_{i1}=\eta_{i-1,1}+\gamma_{i1}\xi_i(t).
\end{aligned}\tag{47}$$

Step n. At the last step, we study the whole z-subsystem (30), and the true input $u_1(t)$ will be designed on the basis of the virtual control α_i's and the Lyapunov function V_{n-1} introduced before. Here, let us consider a Lyapunov-Krasovskii function V_n as follows:

$$V_n=V_{n-1}+\frac{1}{2}\xi_n^2(t)+\frac{1}{2r_n}\tilde{r}_n^2.\tag{48}$$

Denote $r_n=1/|d_n|$ and $\tilde{r}_n=r_n-\hat{r}_n$, where $\hat{r}_n$ is the estimate of unknown parameter r_n. Recall that $\xi_n(t)=z_n(t)-\alpha_{n-1}$, with α_{n-1} being a virtual control input, then

$$\begin{aligned}
\dot{V}_{n-1}\le{}&-(c_0-n+1)x_0^2(t)-\sum_{j=1}^{n-1}\left[c_j-n+j+1\right]\xi_j^2(t)\\
&+\hat{d}_{n-1}g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)\xi_{n-1}(t)\xi_n(t)\\
&-\sum_{j=0}^{n-1}\left(\tilde{d}_j+\sum_{l=j+2}^{n-1}\frac{\partial\alpha_{l-1}}{\partial\hat{d}_j}\xi_l(t)\right)\left(\dot{\hat{d}}_j-\tau_{n-1,j}\right)\\
&-\left(\tilde{v}+\sum_{j=2}^{n-1}\frac{\partial\alpha_{j-1}}{\partial\hat{v}}\xi_j(t)\right)\left(\dot{\hat{v}}-\eta_{n-1,0}\right)\\
&-\left(\tilde{\vartheta}_1+\sum_{j=2}^{n-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\vartheta}_1}\xi_j(t)\right)\left(\dot{\hat{\vartheta}}_1-\eta_{n-1,1}\right)\\
&-\sum_{k=1}^{n}\frac{\varepsilon_{lk}}{2}\xi_1^2(t-\tau_k)f_{k2}^2(y(t-\tau_k)).
\end{aligned}\tag{49}$$

Differentiating V_n along the solution of (30) gives

$$\begin{aligned}
\dot{V}_n={}&\dot{V}_{n-1}+\xi_n(t)\\
&\times\Bigg\{d_nu_1(t)+\tilde{\varphi}_n+\tilde{\phi}_n-\frac{\partial\alpha_{n-1}}{\partial x_0}\\
&\quad\times\left[\left(\hat{d}_0+\tilde{d}_0\right)g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)x_0(t)+\phi_0\right]\\
&\quad-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial z_j}\left[\left(\hat{d}_j+\tilde{d}_j\right)g_0\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)z_{j+1}(t)\right.\\
&\qquad\qquad-(n-j)\left(\hat{d}_0+\tilde{d}_0\right)g_0\\
&\qquad\qquad\left.\times\left(x_0,\hat{\vartheta}_0,\hat{r}_0\right)z_j(t)+\tilde{\varphi}_j+\tilde{\phi}_j\right]\\
&\quad-\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{r}_j}\dot{\hat{r}}_j-\sum_{j=0}^{n-2}\frac{\partial\alpha_{i-1}}{\partial\hat{d}_j}\dot{\hat{d}}_j\\
&\quad-\frac{\partial\alpha_{n-1}}{\partial\hat{\vartheta}_0}\dot{\hat{\vartheta}}_0-\frac{\partial\alpha_{n-1}}{\partial\hat{\vartheta}_1}\dot{\hat{\vartheta}}_1-\frac{\partial\alpha_{n-1}}{\partial\hat{v}}\dot{\hat{v}}\Bigg\}\\
&-\frac{1}{r_i}\tilde{r}_n\dot{\hat{r}}_n.
\end{aligned}\tag{50}$$

Similarly, by Lemmas 5–7 and Young inequality, we can easily obtain that there are scalars $\varepsilon_{nj}>0$ $(1\le j\le n)$ and smooth nonnegative functions γ_{n0} and γ_{n1} such that

$$\begin{aligned}
&\left|\xi_n(t)\left[\tilde{\varphi}_n-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial z_j}\tilde{\varphi}_j\right]\right|\\
&\quad\le\frac{1}{2\varepsilon_{nn}}\xi_n^2(t)z_1^2(t)f_{n1}^2(y(t))\\
&\qquad+\sum_{j=1}^{n-1}\frac{1}{2\varepsilon_{nj}}\xi_n^2(t)\left(\frac{\partial\alpha_{n-1}}{\partial z_j}\right)^2z_1^2(t)f_{j1}^2(y(t))\\
&\qquad+\sum_{j=1}^{n}\frac{\varepsilon_{nj}}{2}\xi_1^2(t-\tau_j)f_{j2}^2(y(t-\tau_j)),
\end{aligned}$$

$$\left|\xi_n(t)\left[\widetilde{\phi}_n-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial z_j}\widetilde{\phi}_j-\frac{\partial\alpha_{n-1}}{\partial x_0}\phi_0\right]\right| \le x_0^2(t)+\sum_{j=1}^{n-1}\xi_j^2(t)+\xi_n^2(t)\gamma_{n0}\vartheta^2 + \xi_n^2(t)\gamma_{n1}(u_0,x_0,\overline{z}_n)\vartheta. \tag{51}$$

Next, we can design the control input $u_1(t)$ as follows:

$$\begin{aligned} u_1(t) &= \operatorname{sign}(d_n)\widehat{r}_n\pi_n, \\ \pi_n &= -g_0\left(x_0,\widehat{\vartheta}_0,\widehat{r}_0\right) \\ &\quad\times\left\{\widehat{d}_{n-1}\xi_{n-1}(t)-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial z_j}\right. \\ &\quad\times\left[\widehat{d}_j z_{j+1}(t)-(n-j)\widehat{d}_0 z_j(t)\right] \\ &\quad\left.-\widehat{d}_0\frac{\partial\alpha_{n-1}}{\partial x_0}x_0(t)\right\}-c_n\xi_n(t)-\xi_n(t)\gamma_{n0}\widehat{v} \\ &\quad-\xi_n(t)\gamma_{n1}\widehat{\vartheta}_1+\frac{\partial\alpha_{n-1}}{\partial\widehat{\vartheta}_0}\dot{\widehat{\vartheta}}_0+\sum_{j=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\widehat{r}_j}\dot{\widehat{r}}_j \\ &\quad+\sum_{j=0}^{n-2}\frac{\partial\alpha_{n-1}}{\partial\widehat{d}_j}\tau_{nj}+\frac{\partial\alpha_{n-1}}{\partial\widehat{v}}\eta_{n0}+\frac{\partial\alpha_{n-1}}{\partial\widehat{\vartheta}_1}\eta_{n1} \\ &\quad-\frac{1}{2\varepsilon_{nn}}\xi_n(t)z_1^2(t)f_{n1}^2(y(t)) \\ &\quad-\sum_{j=1}^{n-1}\frac{1}{2\varepsilon_{nj}}\xi_n(t)\left(\frac{\partial\alpha_{n-1}}{\partial z_j}\right)^2 z_1^2(t)f_{j1}^2(y(t)) \\ &\quad+\sum_{l=2}^{n-1}\sum_{j=0}^{l-2}\xi_l(t)\frac{\partial\alpha_{l-1}}{\partial\widehat{d}_j}h_{nj}+\sum_{j=2}^{n-1}\xi_j(t)\frac{\partial\alpha_{j-1}}{\partial\widehat{v}}\eta_{n0} \\ &\quad+\sum_{j=2}^{n-1}\xi_j(t)\frac{\partial\alpha_{j-1}}{\partial\widehat{\vartheta}_1}\eta_{n1}, \end{aligned} \tag{52, 53}$$

where c_n is a positive design parameter and τ_{nj} $(0 \le j \le n-1)$, η_{n0} and η_{n1} are smooth nonnegative functions to be specified in (56). With the choice of the update law

$$\dot{\widehat{r}}_n = -\pi_n\xi_i(t), \tag{54}$$

it renders

$$\begin{aligned} \dot{V}_n \le& -(c_0-n)x_0^2(t)-\sum_{j=1}^{n}\left[c_j-n+j\right]\xi_j^2(t) \\ &-\sum_{j=0}^{n-1}\left(\widetilde{d}_j+\sum_{l=j+2}^{n-1}\frac{\partial\alpha_{l-1}}{\partial\widehat{d}_j}\xi_l(t)\right)\left(\dot{\widehat{d}}_j-\tau_{nj}\right) \\ &-\left(\widetilde{v}+\sum_{j=2}^{n}\frac{\partial\alpha_{j-1}}{\partial\widehat{v}}\xi_j(t)\right)\left(\dot{\widehat{v}}-\eta_{n0}\right) \\ &-\left(\widetilde{\vartheta}_1+\sum_{j=2}^{n}\frac{\partial\alpha_{j-1}}{\partial\widehat{\vartheta}_1}\xi_j(t)\right)\left(\dot{\widehat{\vartheta}}_1-\eta_{n1}\right), \end{aligned} \tag{55}$$

where

$$\begin{aligned} \tau_{n0} &= \tau_{n-1,0}+h_{n0}\xi_n(t), \\ h_{n0} &= -\frac{\partial\alpha_{n-1}}{\partial x_0}g_0\left(x_0,\widehat{\vartheta}_0,\widehat{r}_0\right)x_0(t) \\ &\quad+\sum_{j=1}^{n-1}(n-j)\frac{\partial\alpha_{n-1}}{\partial z_j}g_0\left(x_0,\widehat{\vartheta}_0,\widehat{r}_0\right)z_j(t), \\ \tau_{nl} &= \tau_{n-1,l}+h_{nl}\xi_i(t), \\ h_{nl} &= -\frac{\partial\alpha_{n-1}}{\partial z_l}g_0\left(x_0,\widehat{\vartheta}_0,\widehat{r}_0\right)z_{l+1}(t), \\ &\qquad 1\le l\le n-1, \\ \eta_{n0} &= \eta_{n-1,0}+\gamma_{n0}\xi_n(t), \qquad \eta_{n1}=\eta_{n-1,1}+\gamma_{n1}\xi_n(t). \end{aligned} \tag{56}$$

Furthermore, employing the following update laws

$$\dot{\widehat{d}}_j = \tau_{nj}, \quad 0\le j\le n-1, \quad \dot{\widehat{v}} = \eta_{n0}, \quad \dot{\widehat{\vartheta}}_1 = \eta_{n1} \tag{57}$$

eventually achieves

$$\dot{V}_n \le -(c_0-n)x_0^2(t)-\sum_{j=1}^{n}\left[c_j-n+j\right]\xi_j^2(t). \tag{58}$$

This together with (48) implies that x_0, ξ_i, $\widetilde{v}$, $\widetilde{\vartheta}_0$, $\widetilde{\vartheta}_1$ and $\widetilde{d}_i$, $\widetilde{r}_i$ $(0 \le i \le n)$ are bounded. Since θ and d_i are constant vector and constant, respectively, we know that $\widehat{\vartheta}_0$, $\widehat{\vartheta}_1$ and $\widehat{d}_i$, $\widehat{r}_i$ $(0 \le i \le n)$ are also bounded. According to the definitions of the virtual control input α_i in the above design procedure, z_i are bounded as α_i are bounded. It indicates that all signals of the closed loop system are bounded.

LaSalle Invariant Theorem further achieves that $x_0, \xi_i, \widetilde{v}, \widetilde{\vartheta}_0, \widetilde{\vartheta}_1, \widetilde{d}_i, \widetilde{r}_i \to 0$ as $t \to \infty$. The boundedness of all signals and the choice of virtual control functions α_i imply that α_i converge to zero, which shows that z_i also tend to zero. From the transformation $x_i = z_i x_0^{n-i}$, we can prove that $x_0 \to 0$, as $t \to 0$.

The above analysis is summarized into the following theorem.

Theorem 9. *For the system* (1)*, under Assumptions 1 and 2, if the control strategies* (21) *and* (52) *are applied with an appropriate choice of the design parameters* c_i $(0 \le i \le n)$*, the global asymptotic stabilization of the closed loop system is achieved for* $x_0(t_0) \neq 0$.

In the next section, we will deal with the stability analysis of the closed loop system with our control laws (21) and (52) as long as the initial condition $x_0(t_0)$ is zero.

4. Switching Controller

Several switching controllers have been proposed in some existing literature resources. As well known, the choice of a constant feedback for $u_0(t)$ may lead to a finite escape. That is, the solution $x_0(t)$ issued from the origin may blow up before the switch. Usually, the phenomenon occurs for systems with non-Lipschitz nonlinearities. In this paper, the term $\phi_0(t, x_0(t), \theta)$ in $x_0(t)$-subsystem does not satisfy the Lipschitz conditions; then we should apply a novel switching control design. When the initial state $x_0(t_0) = 0$, choose controller $u_0(t)$ as

$$u_0(t) = g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) x_0(t) + \operatorname{sign}(d_0)\lambda, \quad t \in [t_0, t_s], \tag{59}$$

where $\lambda > 0$ is a constant, $g_0(x_0, \widehat{\vartheta}_0, \widehat{r}_0)$ is defined in (22), and update laws of the parameters $\widehat{\vartheta}_0$ and $\widehat{r}_0$ are chosen as in (25) and (26), respectively.

With controller $u_0(t)$ in (59), the derivative of the Lyapunov function V_0 in (19) along x_0-subsystem gives that

$$\begin{aligned}\dot{V}_0 &\le -c_0 x_0^2(t) + \lambda |d_0| x_0(t) \\ &= -c_0\left[x_0(t) - \frac{\lambda |d_0|}{2c_0}\right]^2 + \frac{\lambda^2 |d_0|^2}{4c_0^2}.\end{aligned} \tag{60}$$

The above inequality indicates that $x_0(t)$, $\widehat{\vartheta}_0$, and $\widehat{r}_0$ are bounded; then state $x_0(t)$ cannot blow up during the time period $[t_0, t_s)$.

For $x_0(t) = 0$ and Assumption 2, we have

$$\begin{aligned}\dot{x}_0(t) &= d_0 u_0(t) + \phi_0(t, x_0(t), \theta) \\ &\ge -\left[-d_0 g_0\left(x_0, \widehat{\vartheta}_0, \widehat{r}_0\right) + \omega_0(x_0)\vartheta\right] x_0(t) + \lambda |d_0| \\ &= -\varpi_0(t) + \lambda |d_0|,\end{aligned} \tag{61}$$

where $\varpi = -d_0 g_0(x_0, \widehat{\vartheta}_0, \widehat{r}_0) + \omega_0(x_0)\vartheta$. Obviously, ϖ is bounded.

The above inequality indicates that

$$\begin{aligned}x_0(t) &\ge e^{-\int_{t_0}^{t} \varpi(s)ds} \int_{t_0}^{t} \lambda |d_0| e^{-\int_{t_0}^{s} \varpi(\tau)d\tau} ds \\ &\quad + x_0(t_0) e^{-\int_{t_0}^{t} \varpi(s)ds} \\ &= e^{-\int_{t_0}^{t} \varpi(s)ds} \int_{t_0}^{t} \lambda |d_0| e^{-\int_{t_0}^{s} \varpi(\tau)d\tau} ds.\end{aligned} \tag{62}$$

It is clear that when $0 < t \le t_s$, $|x_0(t)| > 0$. Therefore, the state scaling coordinate transformation in (8) is effective, and we can use the following switching control strategy for $x_0(t_0) = 0$.

During the time period $[t_0, t_s]$, using the controller $u_0(t)$ in (59), when $t = t_0$ and $u_0(t)$ is a constant, the controller $u_1(t)$ can be designed implying the simple nonlinear backstepping method. When $t_0 < t \le t_s$, choose the controller $u_1(t)$ as the iterative procedure in Section 3. Since $x_0(t_s) \neq 0$ at t_s, we switch the control law $u_0(t)$ and $u_1(t)$ into (21) and (52), respectively.

Theorem 10. *For the system* (1)*, under Assumptions 1 and 2, if the above switching control strategy is applied with an appropriate choice of the design parameters* $c_i (0 \le i \le n)$*, then the closed-loop system is globally asymptotically regulated at the origin for* $x_0(t_0) = 0$.

5. Simulation Example

In this section, a numerical example will be given to illustrate that the proposed systematic control law design method is effective.

Example 1. Consider the following system:

$$\begin{aligned}\dot{x}_0(t) &= d_0 u_0(t) + (x_0(t))^{\theta_0}, \\ \dot{x}_1(t) &= d_1 u_0(t) x_2(t) + \frac{1}{2}\ln\left(1 + x_1^2(t)\right) e^{x_0(t)} x_1^2(t - 0.3) \\ &\quad + x_1(t)\theta_1^{x_1(t)}, \\ \dot{x}_2(t) &= d_2 u_1(t) + x_1(t) e^{x_0(t-0.2)} x_1^{3/2}(t - 0.2) \\ &\quad + \ln\left(1 + (\theta_2 x_2(t))^2\right),\end{aligned} \tag{63}$$

where d_0, d_1, and d_2 are unknown virtual control coefficients, and θ_0, θ_1, and θ_2 are unknown bounded parameters. Our purpose is to design controllers $u_0(t)$ and $u_1(t)$ such that the states of the closed-loop system tend to zero when $t \to \infty$.

To apply the proposed design method, we make the estimation of nonlinear functions in system (62) as follows:

$$\begin{aligned}(x_0(t))^{\theta_0} &\le |x_0(t)| e^{(\theta_0 - 1)\ln|x_0(t)|} \\ &\le |x_0(t)| e^{(1/2)(\theta_0-1)^2 + (1/2)\ln^2|x_0(t)|} \\ &\le |x_0(t)| e^{(1/8)\ln^2(1+x_0^2(t))} \cdot e^{(1/2)(\theta_0-1)}, \\ x_1(t)\theta_1^{x_1(t)} &\le |x_1(t)| e^{x_1(t)\ln\theta_1} \le |x_1(t)| e^{(1/2)x_1^2(t)} e^{(1/2)\ln^2\theta_1}, \\ \ln\left(1 + (\theta_2 x_2(t))^2\right) &\le |x_2(t)| \cdot |\theta_2|.\end{aligned} \tag{64}$$

Let $\vartheta = e^{(1/2)(\theta_0-1)} + e^{(1/2)(\theta_0-1)} + |\theta_2|$; then the above inequalities can be deduced as

$$\begin{aligned} &\left(x_0(t)\right)^{\theta_0} \le \left|x_0(t)\right| e^{(1/8)\ln^2(1+x_0^2(t))}\vartheta, \\ &x_1(t)\,\theta_1^{x_1(t)} \le \left|x_1(t)\right| e^{(1/2)x_1^2(t)}\vartheta, \\ &\ln\left(1+\left(\theta_2 x_2(t)\right)^2\right) \le \left|x_2(t)\right|\vartheta. \end{aligned} \tag{65}$$

Introduce the following coordinate transformation:

$$z_1(t) = \frac{x_1(t)}{x_0(t)}, \qquad z_2(t) = x_2(t); \tag{66}$$

then system (62) can be rewritten as

$$\begin{aligned} \dot{x}_0(t) &= d_0 u_0(t) + \left(x_0(t)\right)^{\theta_0}, \\ \dot{z}_1(t) &= \frac{u_0(t)}{x_0(t)} d_1 z_2(t) - \frac{u_0(t)}{x_0(t)} d_0 z_1(t) \\ &\quad + \frac{x_1(t)\,\theta_1^{x_1(t)}}{x_0(t)} - \frac{\left(x_0(t)\right)^{\theta_0}}{x_0(t)} z_1(t) \\ &\quad \times \frac{(1/2)\ln\left(1+x_1^2(t)\right) e^{x_0(t)} x_1^2(t-0.3)}{x_0(t)}, \\ \dot{z}_2(t) &= d_2 u_1(t) + x_1(t)\, e^{x_0(t-0.2)} x_1^3(t-0.2) \\ &\quad + \ln\left(1+\left(\theta_2 x_2(t)\right)^2\right). \end{aligned} \tag{67}$$

For $x_0(t)$-subsystem, design the following controller $u_0(t)$:

$$u_0(t) = g_0\left(x_0, \hat{\vartheta}_0, \hat{r}_0\right) x_0(t), \tag{68}$$

with

$$\begin{aligned} g_0 &= \operatorname{sign}(d_0)\,\hat{r}_0 \pi_0, \\ \pi_0 &= -\frac{1+\sqrt{1+\left(k_0/\hat{r}_0\right)^2}}{2}\left[\omega_0\hat{\vartheta}_0 + \sqrt{c_0^2+\left(\omega_0\hat{\vartheta}_0\right)^2}\right], \end{aligned} \tag{69}$$

where $\omega_0 = e^{(1/8)\ln^2(1+x_0^2(t))}$ and k_0, and c_0 are design constants. Moreover, the adaptation laws of $\hat{r}_0$ and $\hat{\vartheta}_0$ are chosen as

$$\begin{aligned} \dot{\hat{\vartheta}}_0 &= \omega_0 x_0^2(t), \\ \dot{\hat{r}}_0 &= -\pi_0 x_0^2(t). \end{aligned} \tag{70}$$

Define $\xi_1(t) = z_1(t)$ and $\xi_2(t) = z_2(t) - \alpha_1$; according to the design procedure in Section 3, the following controller $u_1(t)$ can be given:

$$u_1(t) = \operatorname{sign}(d_2)\,\hat{r}_2 \pi_2, \tag{71}$$

with

$$\begin{aligned} \pi_2 = &-c_2\xi_2(t) - \hat{d}_1 g_0 \xi_1(t) + \frac{\partial\alpha_1}{x_0(t)}\hat{d}_0 g_0 x_0(t) \\ &+ \frac{\partial\alpha_1}{z_1(t)} g_0\left[\hat{d}_1 z_2(t) - \hat{d}_0 z_1(t)\right] + \frac{\partial\alpha_1}{\hat{\vartheta}_0}\dot{\hat{\vartheta}}_0 \\ &- \xi_2(t)\,\gamma_{20}\hat{v} - \xi_2(t)\,\gamma_{21}\hat{\vartheta}_1 \\ &- \frac{1}{2\varepsilon_{21}}\xi_2(t)\left(\frac{\partial\alpha_1}{z_1(t)}\right)^2 \xi_1^2(t)\, e^{2x_0(t)} \\ &- \frac{1}{2\varepsilon_{22}}\xi_2(t)\,\xi_1^2(t)\, x_0^2(t) \\ &+ \frac{\partial\alpha_1}{\hat{r}_0}\dot{\hat{r}}_0 + \frac{\partial\alpha_1}{\hat{r}_1}\dot{\hat{r}}_1 \\ &+ \frac{\partial\alpha_1}{\hat{d}_0}\left[-g_0\xi_1^2(t) - \frac{\partial\alpha_1}{x_0(t)} g_0 x_0(t)\,\xi_2(t) \right. \\ &\qquad \left. + \frac{\partial\alpha_1}{z_1(t)} g_0 \xi_1(t)\,\xi_2(t)\right] \\ &+ \frac{\partial\alpha_1}{\hat{\vartheta}_1}\left[\xi_1^2(t)\,\tilde{\omega}_{11} + \xi_2^2(t)\,\gamma_{21}\right], \end{aligned} \tag{72}$$

where the virtual control function α_1 can be expressed as follows:

$$\begin{aligned} \alpha_1 &= \operatorname{sign}(d_1)\,\hat{r}_1 \pi_1, \\ \pi_1 &= \hat{d}_0 \xi_1(t) - \frac{1}{g_0}\left[c_1\xi_1(t) + \xi_1(t)\,\tilde{\omega}_{11}\hat{\vartheta}_1 \right. \\ &\qquad \left. + \frac{1}{2\varepsilon_{11}}\xi_1^3(t)\, e^{2x_0(t)} + \frac{\varepsilon_{11}}{8}\xi_1^3(t)\, x_0^4(t)\right]. \end{aligned} \tag{73}$$

It can be seen that ε_{11}, ε_{21}, and ε_{22} are positive constants; $\gamma_{20} = 1$; and $\tilde{\omega}_{11}$ and γ_{21} represent, respectively,

$$\tilde{\omega}_{11} = e^{(1/8)\ln^2(1+x_0^2(t))} + e^{(1/2)z_1^2(t)x_0^2(t)}, \tag{74}$$

$$\begin{aligned} \gamma_{21} = &\frac{1}{2}\left\{\operatorname{sign}(d_1)\,\hat{r}_1\left[\hat{d}_0 - \frac{1}{g_0}\left(c_1 + \tilde{\omega}_{11}\hat{\vartheta}_1 \right.\right.\right. \\ &\qquad + \frac{1}{2\varepsilon_{11}}\xi_1^2(t)\, e^{2x_0(t)} \\ &\qquad \left.\left.\left. + \frac{\varepsilon_{11}}{8}\xi_1^2(t)\, x_0^4(t)\right)\right]\right\}^2 \\ &+ \frac{1}{4}\left(\frac{\partial\alpha_1}{x_0(t)}\right)^2\omega_0^2 + \frac{1}{2}\left(\frac{\partial\alpha_1}{z_1(t)}\right)^2\tilde{\omega}_{11}^2. \end{aligned} \tag{75}$$

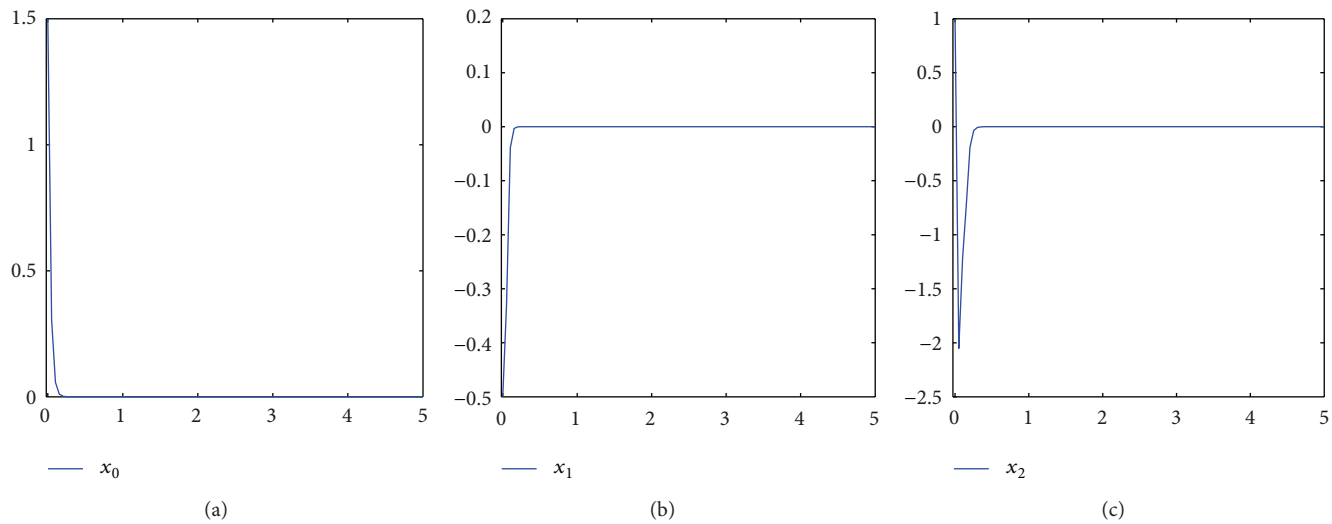


Figure 1: States $x_0(t)$, $x_1(t)$, and $x_2(t)$.

The adaptation laws of the variables $\hat{r}_1$, $\hat{r}_2$, $\hat{d}_0$, and $\hat{d}_1$ in the controller $u_1(t)$ and $\hat{v}$, $\hat{\vartheta}_1$ are chosen as

$$\begin{aligned}
\dot{\hat{r}}_1 &= -g_0\pi_1\xi_1(t), \qquad \dot{\hat{r}}_2 = -\pi_2\xi_2(t), \\
\dot{\hat{d}}_0 &= -g_0\xi_1^2(t) - \frac{\partial\alpha_1}{x_0(t)}g_0x_0(t)\xi_2(t) \\
&\quad + \frac{\partial\alpha_1}{z_1(t)}g_0\xi_1(t)\xi_2(t), \\
\dot{\hat{d}}_1 &= g_0\xi_1(t)\xi_2(t) - \frac{\partial\alpha_1}{z_1(t)}g_0\xi_2(t)z_2(t), \\
\dot{\hat{v}} &= \xi_2^2(t)\gamma_{20}, \qquad \dot{\hat{\vartheta}}_1 = \xi_1^2(t)\tilde{\omega}_{11} + \xi_2^2(t)\gamma_{21}.
\end{aligned} \tag{76}$$

For simulation use, we pick $d_0 = 3.5$, $d_1 = 2.5$, $d_2 = 3$, $\theta_0 = 1.5$, $\theta_1 = 0.5$, $\theta_2 = 0.5$, and the initial state condition $[1.5 - 0.51]^T$, and the initial values of parameter estimates $\hat{d}_0$, $\hat{d}_1$, $\hat{r}_0$, $\hat{r}_1$, $\hat{\vartheta}_0$, $\hat{v}$, and $\hat{\vartheta}_1$ are 0.5, 1.5, 0.1, 0.5, 0.3, 0.2, 0.1, and 0.5, respectively (Figures 3, 4, and 5). In addition, we take the other parameters as $k_0 = 5$, $c_0 = 3$, $c_1 = 2$, $c_2 = 5$, $\varepsilon_{11} = 2$, and $\varepsilon_{21} = \varepsilon_{22} = 1$. Simulation results are shown in Figures 1 and 2. It is obvious that the states $x_0(t)$, $x_1(t)$, and $x_2(t)$ of the system (62) converge to zero, and the control laws $u_0(t)$ and $u_1(t)$ also tend to zero.

6. Conclusion

The state feedback adaptive stabilization was investigated for a new class of nonholonomic systems with unknown virtual control coefficients, nonlinear uncertainties, and unknown time delays. It should be mentioned that the stabilization approaches in some existing literature resources may fail to be applied for the concerned systems. In order to overcome the difficulties from the time delay, we introduce novel Lyapunov-Krasovskii functionals, and a recursive technique

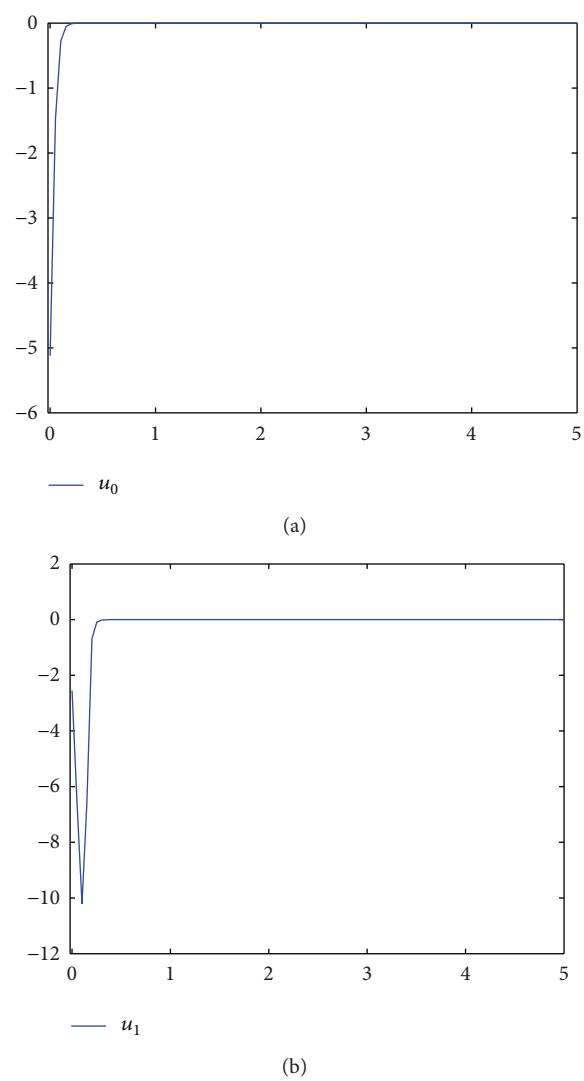
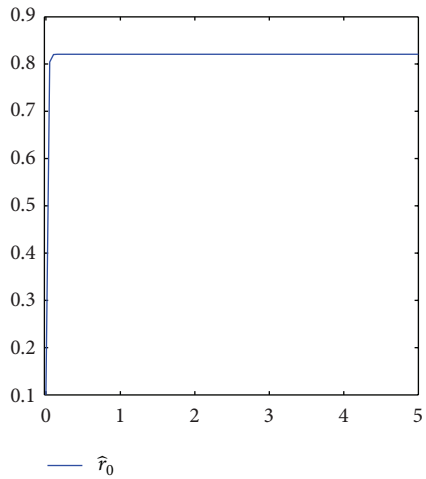
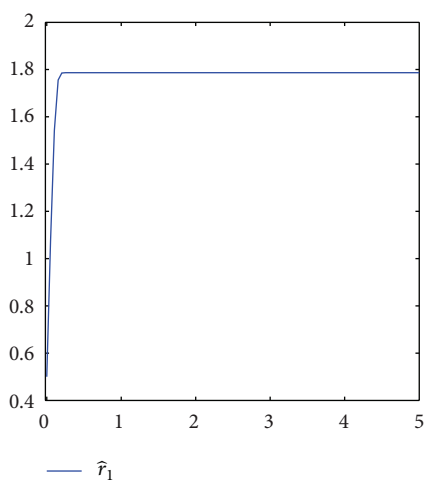


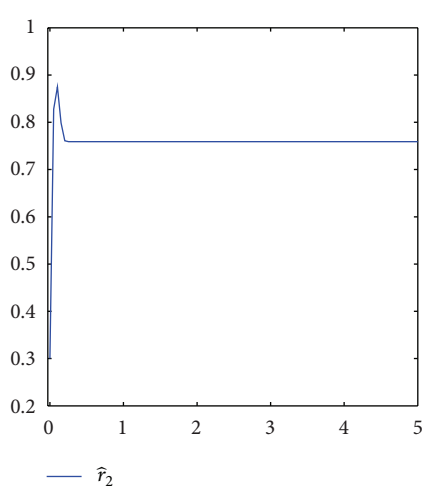
Figure 2: Controllers $u_0(t)$ and $u_1(t)$.



(a)

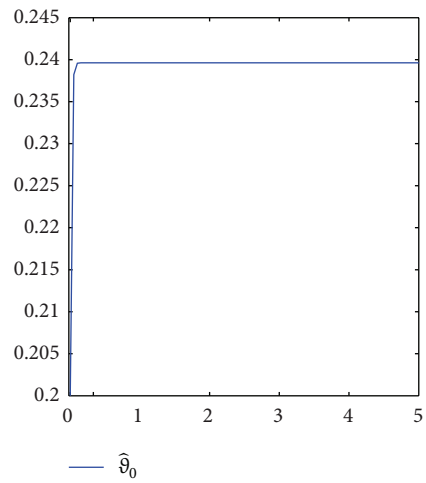


(b)

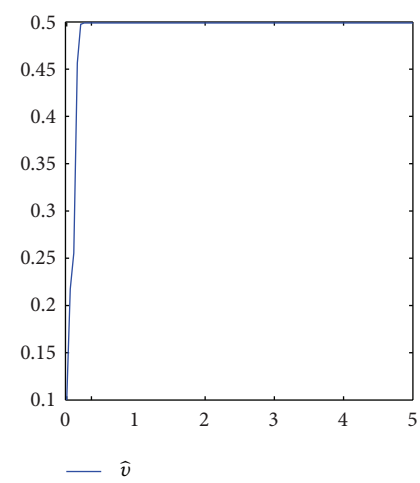


(c)

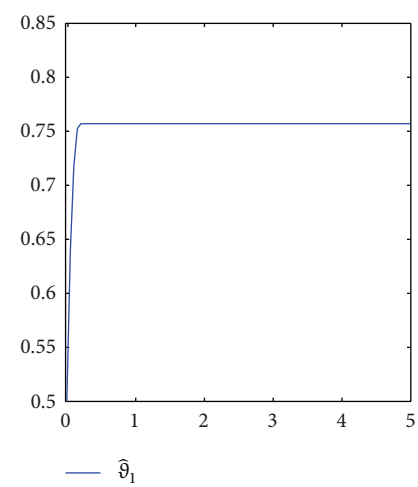
Figure 3: Parameters $\hat{r}_0$, $\hat{r}_1$, and $\hat{r}_2$.



(a)

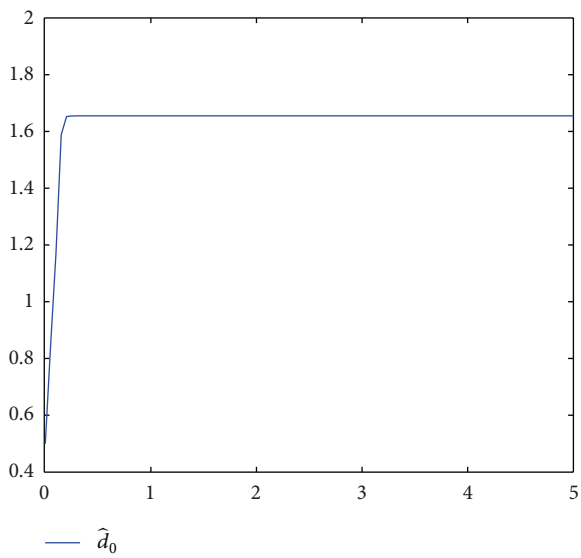


(b)

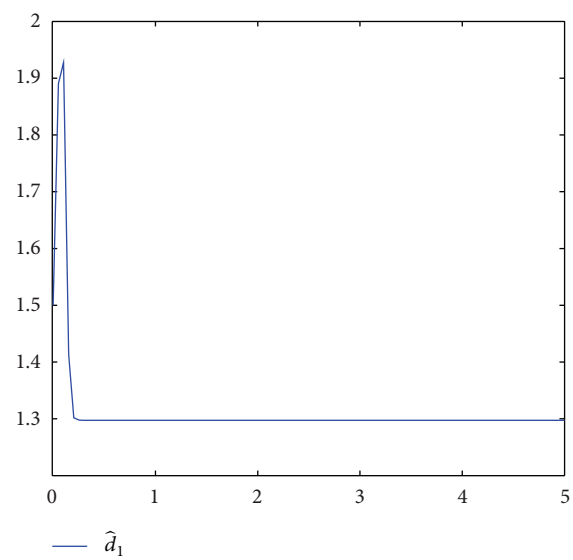


(c)

Figure 4: Parameters $\hat{\vartheta}_0$, $\hat{v}_1$, and $\hat{\vartheta}_1$.



(a)



(b)

Figure 5: Parameters $\hat{d}_0$ and $\hat{d}_1$.

is proposed to design the adaptive controller. To make the state scaling transformation effective, the switching control strategy is employed to achieve the asymptotic stabilization.

Acknowledgments

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