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Research Article

Study on the Nonsingular Problem of Fractional-Order Terminal Sliding Mode Control

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An improved type of control strategy combining the fractional calculus with nonsingular terminal sliding mode control named non-singular fractional terminal sliding mode control (NFOTSM) is proposed for the nonlinear tire-road friction control system of vehicle in this paper. A fractional-order switching manifold is proposed, and the corresponding control law is formulated based on the Lyapunov stability theory to guarantee the sliding condition. The proposed controller ensures the finite time stability of the closed-loop system. Then, a terminal attractor is introduced to solve the singularity problem of fractional terminal sliding mode control (FOTSM). Finally, the performance of the NFOTSM is fully investigated compared with other related algorithms to find the effectiveness for the tire-road friction system. The results show that the NFOTSM has better performance than other related algorithms.

1. Introduction

Fractional calculus is an old mathematical with a 300 years old history [1, 2]. For many years, this branch of science was considered as a sole mathematical and theoretical subject with nearly no applications [3]. But, in recent decades, fractional calculus has become an interesting topic among researchers, and different applications have been proposed for this field of science [4–6]. Designing fractional-order controllers is one of these applications.

The idea of designing a fractional-order (FO) controller was firstly proposed by Oustaloup in 1988 [7]. He introduced a robust fractional-order control scheme which is called Commande Robuste d'Ordre Non-Entier (CRONE) [8, 9]. A frequency domain approach has been introduced in [10]. Variable order fractional controllers are introduced by Valério et al. [11]. Tuning FO-PID controller is introduced by Padula et al. [12]. Model Predictive Control of Fractional-Order Nonlinear Discrete-Time Systems is proposed by Domek [13]. The adaptive FO controllers [14] and the effect of fractional order in variable structure control are investigated and verified by Tenreiro Machado [15].

Sliding mode control (SMC) is a well-known efficient control scheme which has been widely applied for both linear and nonlinear systems [16]. This control is also considered as an effective approach for control of the dynamical systems with uncertainties. In general, an arbitrary linear manifold is considered as a sliding surface, which can guarantee the asymptotic stability and desired performance of the closed-loop control system. However, the main disadvantage of SMC scheme is that the system states cannot reach the equilibrium point in finite time. But, in recent years, a new control scheme called terminal sliding mode control (TSMC) is proposed to overcome this drawback utilizing nonlinear sliding surface [17, 18]. Nonlinear switching hyperplanes in TSMC can improve the transient performance substantially. Besides, compared with the conventional SMC with linear sliding manifold, TSMC offers some superior properties such as faster, finite time convergence and higher control precision. And a nonsingular terminal sliding mode (NTSM) method is proposed to avoid the singularity in TSM control systems [19, 20].

However, TSMC designs are restricted to integer-order controllers, and there is only few papers in which utilizing the means of FO calculus in the design procedure of a fractional-order TSMC (FOTSMC) is considered, especially for the nonsingular of the controllers [20]. Therefore, designing an NFOTSMC for dynamical systems is still an open problem.

This paper is organized as follows: Section 2 presents nonsingular fractional terminal sliding mode control strategy. The adopted control structure of tire-road friction Ext-ABS is introduced and realized in Section 3. At last, numerical simulation results are shown in Section 4.

2. Nonsingular Fractional-Order Terminal Sliding Mode Control

In this section, nonsingular fractional terminal sliding mode control strategy is introduced and realized for the tire-road friction servocontrol system.

2.1. Basic Definition and Preliminaries of Fractional-Order Calculus. Fractional-order integration and differentiation are the generalization of the integer-order ones. Efforts to extend the specific definitions of the traditional integer-order to the more general arbitrary order context led to different definitions for fractional derivatives [1, 2, 19]. Two of the most commonly used definitions are Riemann-Liouville and Caputo definitions.

Definition 1. The αth-order Riemann-Liouville fractional derivative of function $f(t)$ with respect to t and the terminal value t_0 is given by

$$D^{\alpha} f(t) = \frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau \tag{1}$$

and the Riemann-Liouville definition of the αth-order fractional integration is given by

$${}_{t_0}I_t^{\alpha} f(t) = I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \frac{d^m}{dt^m} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \tag{2}$$

where m is the first integer larger than α and Γ is the Gamma function.

Definition 2. The Caputo fractional derivative of order α of a continuous function $f : R^{+} \to R$ is defined as follows:

$$D^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1 < \alpha < m, \\ \dfrac{d^m}{dt^m} f(t), & \alpha = m, \end{cases} \tag{3}$$

where m is the first integer larger than α.

Property 1. If the fractional derivative ${}_{\alpha}D_t^{\alpha} y(t)$ $(k-1 \le \alpha < k)$ of a function $y(t)$ is integrable, then

$${}_{\alpha}I_t^{\alpha} \left({}_{\alpha}D_t^{\alpha} y(t)\right) = y(t) - \sum_{j=1}^{k} \left[{}_{\alpha}D_t^{\alpha-j} y(t)\right]_{t=\alpha} \frac{(t-\alpha)^{\alpha-j}}{\Gamma(\alpha-j+1)}. \tag{4}$$

Lemma 3. *The fractional integration operator ${}_{\alpha}I_t^{\alpha}$ with $[\alpha] > 0$ is bounded*

$$\left\| I_t^{\alpha} y \right\|_p \le K \left\| y \right\|_p, \quad 1 \le p \le \infty. \tag{5}$$

Theorem 4. *Let $x = 0$ be an equilibrium point for the nonautonomous fractional-order system*

$$D^{\alpha} x(t) = f(x,t), \tag{6}$$

where $f(x,t)$ satisfies the Lipschitz condition with Lipschitz constant $l > 0$. Assume that there exists a Lyapunov candidate $V(t,x(t))$ satisfying

$$\alpha_1 \|x\|^{\alpha} \le V(t,x) \le \alpha_2 \|x\|, \quad \dot{V}(t,x) \le -\alpha_3 \|x\|, \tag{7}$$

where $\alpha > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_3 > 0$. Then the equilibrium point of system (6) *is Mittag-Leffler stable, and the stability implies asymptotic stability.*

Theorem 5. *Let $x = 0$ be an equilibrium point for the nonautonomous fractional-order system* (6)*. Assume that there exist a Lyapunov function $V(t,x(t))$ and class-K functions α_i $(i = 1,2,3)$ satisfying*

$$\alpha_1 \|x\| \le V(t,x) \le \alpha_2 \|x\|, \quad D^{\beta} V(t,x) \le -\alpha_3 \|x\|, \tag{8}$$

where $\beta \in (0,1)$. Then the equilibrium point of system (6) *is asymptotic stability.*

2.2. Review of Integer-Order Nonsingular Terminal Sliding Mode Control. In this paper, the *control problem* is to get the system to track a desired vector X_d $(X_d = [x_d, \dot{x}_d]^T)$. So, the fractional-order system can be defined as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f(x_1, x_2, t) + u, \end{aligned} \tag{9}$$

where u is the control effort, $x = [x_1, x_2]^T$.

And the tracking error can be defined as

$$e(t) = x(t) - x_d(t). \tag{10}$$

The control goal is to design a robust nonsingular fractional-order terminal sliding mode controller to stabilize state variables and track the desired vector X_d in finite time.

Here, definition and some properties of NTSM are briefly reviewed for essential preparation of the fractional-order NTSMC design.

Definition 6. Terminal sliding manifold can be defined as the following nonlinear differential equation:

$$s = \dot{e}(t) + \xi e(t)^{p/q}, \tag{11}$$

where $\xi > 0$ is a constant, p, q are the given positive odd integers, and $p < q < 2p$.

Considering the constraint $\dot{s} = 0$, the control input can be defined as

$$u(t) = \ddot{x}_d - f(x_1, x_2, t) - \frac{\xi p}{q}\dot{e}(t)\, e(t)^{(p/q-1)} + K \operatorname{sign}(s). \tag{12}$$

According to finite time stability theory, the equilibrium point $e = 0$ of error system is globally finite time stable, and the time is

$$T_{tsm} = \frac{q}{\xi(q-p)} e(0)^{(1-p/q)}. \tag{13}$$

2.3. Nonsingular Fractional Terminal Sliding Mode Controller. In this section we will develop nonsingular fractional-order terminal sliding mode control for trajectory tracing of a class of dynamical systems, for example, the servosystem. For this purpose, the forms introduced in the previous section are used. As a result, the high precision tracking can be acquired with faster convergence speed compared with the integer-order terminal sliding mode control.

Definition 7. Fractional terminal sliding manifold can be defined as

$$s = D^{\alpha}\ddot{e}(t) + \xi e(t)^{p/q}, \quad \alpha \in (0,1). \tag{14}$$

If the terminal sliding mode control law is designed as (15), then the tracking error converges to zero in finite time [20]. Consider

$$\begin{aligned} u(t) = {} & \ddot{x}_d - f(x_1, x_2, t) - D^{-1-\alpha}\left[\frac{\xi p}{q}\dot{e}(t)\, e(t)^{(p/q-1)}\right] \\ & + D^{-1-\alpha}\left[Ks + K\operatorname{sign}(s) + \delta\right]. \end{aligned} \tag{15}$$

But this system has a singularity problem because of $(p/q - 1) < 0$. Consequently, to avoid this phenomenon, a new "terminal attractor" is proposed to develop a singularity free TSM control.

Definition 8. Nonsingular fractional terminal sliding manifold can be defined as follows:

$$s = D^{\alpha}\ddot{e}(t) + \xi_1 \dot{e}(t)^{p/q} + \xi_2 e(t)^{g/h}, \quad \alpha \in (0,1) \tag{16}$$

where g, h are the given positive odd integers and $1 < g/h < 2$.

And the "terminal attractor" can be defined as

$$\dot{s} = \left(-\phi s - \gamma s^{\zeta_1/\zeta_2}\right)\dot{e}^{p/q-1}, \tag{17}$$

where $\phi \in R^{+}$; $\gamma \in R^{+}$; ζ_1 and ζ_2 are the given positive odd integers and $0 < \zeta_1/\zeta_2 < 1$.

From (9), (10), (16), and (17), the NFOTSM control law can be obtained as follows:

$$\begin{aligned} u(t) = {} & \ddot{x}_d - f(x_1, x_2, t) \\ & + \frac{q}{\xi_1 p}\left[\left(-\phi s - \gamma s^{\zeta_1/\zeta_2}\right)\right. \\ & \left. - D^{\alpha+1}\ddot{e}\dot{e}^{1-p/q} - \frac{\xi_2 g}{h} e^{g/h-1}\dot{e}^{2-p/q}\right]. \end{aligned} \tag{18}$$

Proof. Firstly, for $g/h - 1 > 0$ and $2 - p/q > 1 - p/q > 0$, we can get that the proposed NFOTSM control law is a singularity free TSM control.

Secondly, the Lyapunov function is defined in the form

$$V = \frac{1}{2}s^2. \tag{19}$$

Then, we have

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s\left(D^{\alpha+1}\ddot{e} + \frac{\xi_1 p}{q}\dot{e}^{p/q-1}\ddot{e} + \frac{\xi_2 g}{h}e^{g/h-1}\dot{e}\right) \\ &= s\left(D^{\alpha+1}\ddot{e} + \frac{\xi_1 p}{q}\dot{e}^{p/q-1}\left(f(x_1, x_2, t) + u - \ddot{x}_d\right)\right. \\ &\qquad \left. + \frac{\xi_2 g}{h}e^{g/h-1}\dot{e}\right). \end{aligned} \tag{20}$$

Substituting (18) into (20) results in

$$\dot{V} = s\left(-\phi s - \gamma s^{\zeta_1/\zeta_2}\right) = -\phi s^2 - \gamma s^{(\zeta_1+\zeta_2)/\zeta_2} \le -\phi s^2. \tag{21}$$

So, the sliding condition is satisfied.

According to (16), we get that the tracking error converges to zero. To show that this happening (exact tracking, $e = 0$) occurs in finite time, we can define a stopping time as follows:

$$\tau_s = \inf\left\{t \ge t_r : e(t) = 0\right\}, \tag{22}$$

where t_r is the reaching time, the time for the system states to reach the sliding surface. So, all we need is to prove that there exists a $t_r \le t_s < \infty$, such that $\tau_s \le t_s$.

Taking the concept of fractional integral and derivative operators into account and using (16), assume the reaching time is t_r; one obtains

$$D^{-\alpha}\left(D^{\alpha}\ddot{e}(t)\right) = -D^{-\alpha}\left(\xi_1 \dot{e}(t)^{p/q} + \xi_2 e(t)^{g/h}\right). \tag{23}$$

So, one can conclude that

$$\begin{aligned} & \ddot{e}(t) - \left[{}_{t_r}D^{\alpha-1}\ddot{e}(t)\right]_{t=t_r}\frac{(t-t_r)^{\alpha-1}}{\Gamma(\alpha)} \\ & \quad = -D^{-\alpha}\left(\xi_1 \dot{e}(t)^{p/q} + \xi_2 e(t)^{g/h}\right), \end{aligned} \tag{24}$$

where

$$\left[{}_{t_r}D^{\alpha-1}\ddot{e}(t)\right]_{t=t_r}\frac{(t-t_r)^{\alpha-1}}{\Gamma(\alpha)} = 0. \tag{25}$$

So, it can be concluded that

$$\begin{aligned} & e(t) - \left[{}_{t_r}D_t e(t)\right]_{t=t_r}\frac{(t-t_r)}{2} - \left[e(t)\right]_{t=t_r} \\ & \quad = -D^{-\alpha-2}\left(\xi_1 \dot{e}(t)^{p/q} + \xi_2 e(t)^{g/h}\right). \end{aligned} \tag{26}$$

According to Lemma 3, we have

$$\begin{aligned} {}_{t_r}D^{-\alpha-2}\left(\dot{e}(t)^{p/q}+e(t)^{g/h}\right) &= {}_{t_r}I_t^{\alpha+2}\left(\dot{e}(t)^{p/q}+e(t)^{g/h}\right) \\ &\le \left\| {}_{t_r}I_t^{\alpha+2}\left(\dot{e}(t)^{p/q}+e(t)^{g/h}\right)\right\| \\ &\le N\left\|\dot{e}(t)^{p/q}+e(t)^{g/h}\right\|. \end{aligned} \tag{27}$$

Substituting (27) into (26) results in

$$\begin{aligned} &e(t)-\left[{}_{t_{r1}}D_t e(t)\right]_{t=t_{r1}}\frac{(t-t_{r1})}{2}-[e(t)]_{t=t_{r1}} \\ &\quad\le N\left\|\xi_1\dot{e}(t)^{p/q}+\xi_2 e(t)^{g/h}\right\|. \end{aligned} \tag{28}$$

So, we get

$$\begin{aligned} &\left\|e(t)-\left[{}_{t_{r1}}D_t e(t)\right]_{t=t_{r1}}\frac{(t-t_{r1})}{2}\right\|-\left\|[e(t)]_{t=t_{r1}}\right\| \\ &\quad\le N\left\|\xi_1\dot{e}(t)^{p/q}+\xi_2 e(t)^{g/h}\right\|. \end{aligned} \tag{29}$$

Noting that $e(t)=\dot{e}(t)=0$ at $t=t_s$, it yields

$$\left\|\frac{\left[{}_{t_r}D_t e(t)\right]_{t=t_r}}{2}\right\|(t-t_r)\le\left\|[e(t)]_{t=t_r}\right\|. \tag{30}$$

Consequently, if $[{}_{t_r}D_t e(t)]_{t=t_r}$, then $t_s=t_r$. Else, we have

$$t_s\le\frac{2\left\|[e(t)]_{t=t_r}\right\|}{\left\|\left[{}_{t_{r1}}D_t e(t)\right]_{t=t_r}/2\right\|}+t_{r1}=\frac{2\left\|e(t_r)\right\|}{\left\|\dot{e}(t_r)\right\|}+t_r. \tag{31}$$

Therefore, it can be concluded that the system trajectories attain to reference trajectory in finite time.

The proof is complete. □

3. Control Structure of Ext-ABS

In this section, Ext-ABS, which is designed to track any slip ratio based on the exiting antilock system (ABS), is chosen as illustrative example to verify the effectiveness of the proposed controllers. Here, we only consider the design of its control law using the proposed NFOTSM.

3.1. Quarter-Car Model. Tire friction estimation is a prerequisite for longitudinal friction control. For an electric vehicle with in-wheel motors, the brake and drive torque are measured by transducers mounted in motors. And there has been an attempt to measure the brake torque of conventional vehicles by force transducer mounted on the brake caliper support. In this paper, assume that the brake torque can be measured, and then the tire friction can be estimated using the method discussed below.

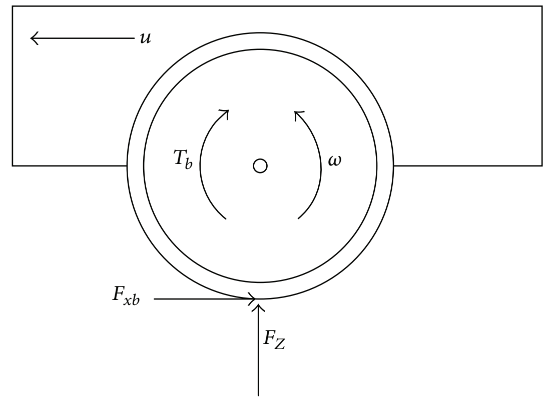

Figure 1: Quarter-car model.

A quarter-car model is shown in Figure 1. The dynamic equations of motion are as follows:

$$m\frac{du}{dt}=-F_{xb} \tag{32}$$

$$J\frac{d\omega}{dt}=r_bF_{xb}-T_b \tag{33}$$

$$F_Z=mg, \tag{34}$$

where m is the mass of the quarter car, u is the vehicle horizontal speed, F_{xb} is the tire friction, J is the wheel inertia, ω is the wheel angular speed, r_b is the wheel radius, T_b is the brake torque, F_Z is the vertical force, and g is the acceleration of gravity.

Then the tire friction can be estimated by the following equation:

$$\widehat{F}_{xb}=\frac{J}{r_b}\frac{d\omega}{dt}+\frac{T_b}{r_b}. \tag{35}$$

And the longitudinal slip ratio $\widehat{\lambda}_{xb}$ with respect to $\widehat{F}_{xb}$ is estimated by

$$\widehat{\lambda}_{xb}=\frac{u-\omega r_b}{u}. \tag{36}$$

The following equation is to describe the nonlinear relationship of the tire friction F_{xb} and the corresponding λ_{xb}

$$F_{xb}=F_Z\cdot\mu_{xb}(\lambda_{xb}), \tag{37}$$

where μ_{xb} is friction coefficient.

The desired longitudinal slip ratio λ_{Ref}, which is corresponding to the desired tire friction F_{Ref}, can be calculated by using the numerical method of nonlinear equations as follows:

$$F_{\text{Ref}}-F_Z\cdot\mu^*_{\text{Ref}}(\lambda_{\text{Ref}})=0. \tag{38}$$

3.2. Tire Model. The magic formula is applied in this paper, which is widely used to calculate steady-state tire force and

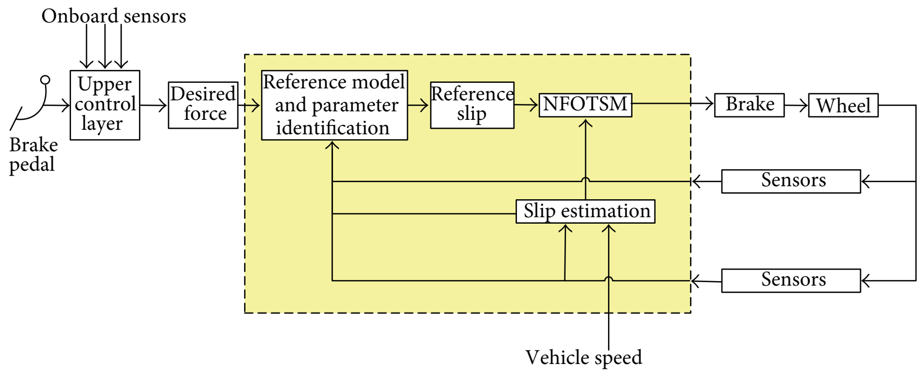

FIGURE 2: Control architecture of the Ext-ABS.

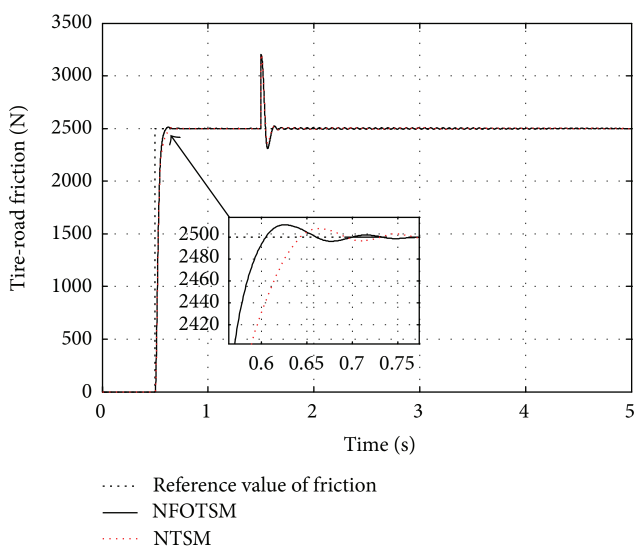

FIGURE 3: Tracking performance of tire friction.

moment in vehicle dynamics studies. For a constant vertical load, the longitudinal friction coefficient is given by

$$\begin{aligned}\mu_L &= \mu\left(\lambda, B_L, C_L, D_L, E_L\right)\\ &= D_L \sin\left[C_L \arctan\left\{B_L\lambda - E_L\left(B_L\lambda - \arctan\left(B_L\lambda\right)\right)\right\}\right],\end{aligned} \quad (39)$$

where μ_L is the longitudinal friction coefficient, λ is the longitudinal slip, B_L is the stiffness factor, C_L is the shape factor, D_L is the peak value, and E_L is the curvature factor.

3.3. NFOTSM Control Law for Ext-ABS. The control architecture of the Ext-ABS is shown in Figure 2. Firstly, let us define

$$e = \lambda - \lambda_{\mathrm{Ref}}. \quad (40)$$

By (32), (33), and (36), we have

$$\dot{\lambda} = \frac{d\left(\left(u - \omega r_b\right)/u\right)}{dt} = \frac{-r_b^2 F_{xb}/J + (1-\lambda)\dot{u}}{u} + \frac{T_b r_b}{uJ}. \quad (41)$$

So, we get

$$\dot{e} = \frac{-r_b^2 F_{xb}/J + (1-\lambda)\dot{u}}{u} + \frac{T_b r_b}{uJ} - \dot{\lambda}_{\mathrm{Ref}}. \quad (42)$$

As it was previously mentioned, we assume that the order of derivatives in the applied controller is $\alpha = 0.9$. So, the NFOTSM control input can be obtained as

$$\begin{aligned}u(t) &= \widehat{T}_b\\ &= \frac{uJ}{r_b}\Bigg\{\dot{\lambda}_{\mathrm{Ref}} - \frac{-r_b^2 F_{xb}/J + (1-\lambda)\dot{u}}{u}\\ &\quad - \frac{q}{\xi_2 p}\Bigg[D^{\alpha+1}\ddot{e}\cdot e^{1-p/q}\\ &\quad + \frac{\xi_1 g}{h}\dot{e}^{g/h-1}\cdot\ddot{e}\cdot e^{1-p/q} - \phi s - \gamma s^{\zeta_1/\zeta_2}\Bigg]\Bigg\}.\end{aligned} \quad (43)$$

4. Simulation Results and Analysis

In this section, simulations are carried out to verify the effectiveness of the proposed controllers. The tracking ability of Ext-ABS is tested, and the superiority of the NFOTSM is compared with nonsingular terminal sliding mode control (NTSM). For the details, the whole performance of the friction servocontrol system (Ext-ABS) is shown under μ-jump road surface and sine input signal.

4.1. Simulation Results under μ-Jump Road Surface. In the simulation of μ-jump road surface, the desired tire friction F_{Ref} input is 2500 N after 0.5 s. And at 1.5 s, the vehicle drives from asphalt, dry road to asphalt, wet road. Simulation results are shown in Figures 3–5.

As is depicted in Figures 3 and 4, with respect to tire friction, since NFOTSM can use the nonlinear fractional-order term $D^{\alpha}\ddot{e}(t)$ to obtain a corrective control force, the values of NFOTSM track the reference value in faster response than that of NTSM after 0.5s and 1.5s. Consequently, the response of vehicle acceleration of NFOTSM is also more sensitive than that of NTSM, as is shown in Figure 5. This proves the

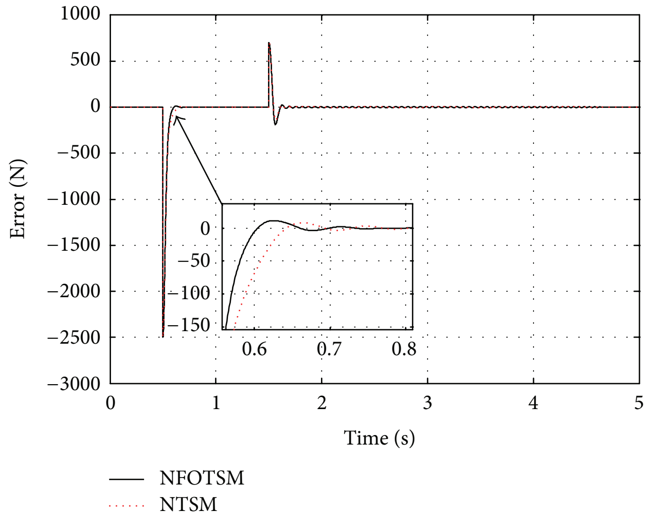

Figure 4: Tracking error of tire friction.

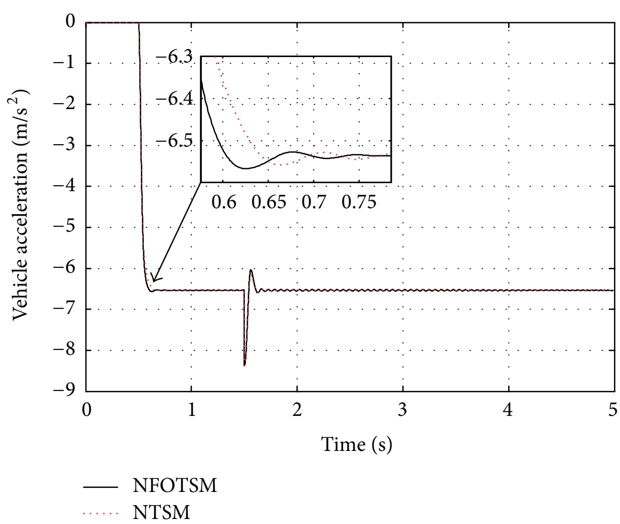

Figure 5: Vehicle acceleration.

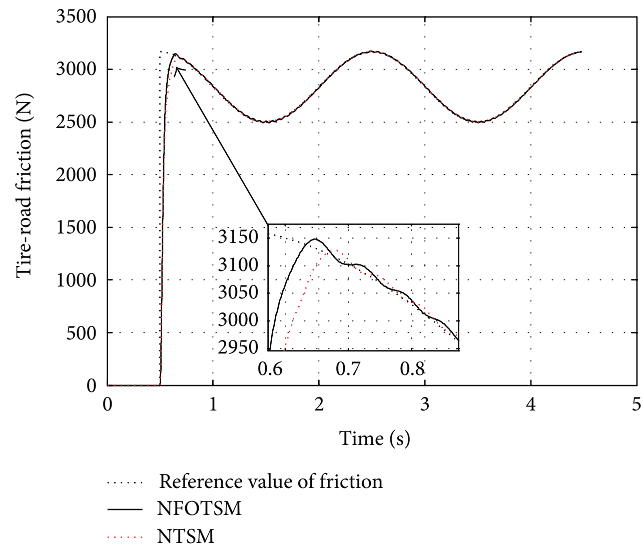

Figure 6: Tracking performance of tire friction.

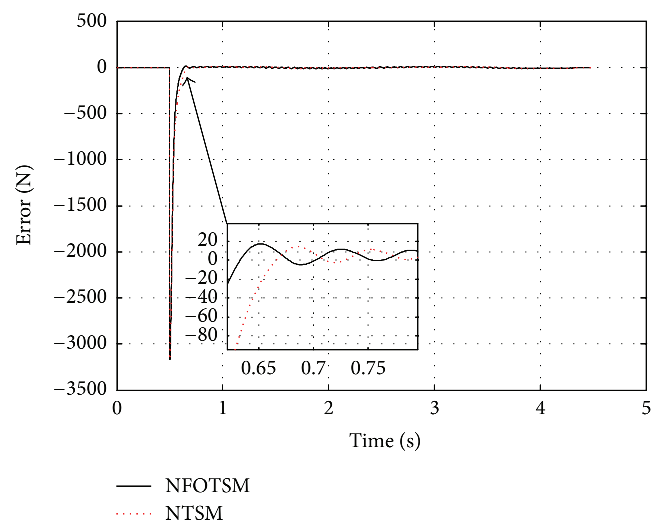

Figure 7: Tracking error of tire friction.

proposed control strategy to be effective in tracking desired tire friction and in adaptation to the variation in road surface.

4.2. Simulation Results under Sine Input Signal. In the simulation of sine input signal, the desired tire friction F_{Ref} input is a sine input. Simulation results are shown in Figures 6–8, under the proposed NFOTSM control law (41) and NTSM control law. Figure 6 represents, respectively, the state time response for tracking a periodic orbit sine input signal. It can be seen that the dynamics of system states are stabilized to a periodic motion. Time responses of the corresponding tracking error of tire friction are depicted in Figure 7. Figure 8 shows the state time responses of the vehicle acceleration. From the above corresponding responses, it can be concluded that the obtained theoretic results are feasible and efficient for controlling dynamical systems, and the proposed controller guarantees the finite time stability of the closed-loop system. Furthermore, it is obvious that the new control law makes the system stable and alleviates the chattering problem.

5. Conclusions

In this paper, the problem of designing NFOTSMC for a class of dynamical systems is investigated. Based on the Lyapunov stability criteria, the nonsingular FO terminal sliding mode control law is designed, and the finite time stability of the closed-loop system is guaranteed under the proposed controller. To further verify the control effect, the Ext-ABS using NFOTSM is designed to realize self-tuning tire friction control system adaptive to variation of road condition. In spite of a higher computation load compared with other control methods, a drastic optimization of the algorithms allows the controller to provide a fast adaptation to the change

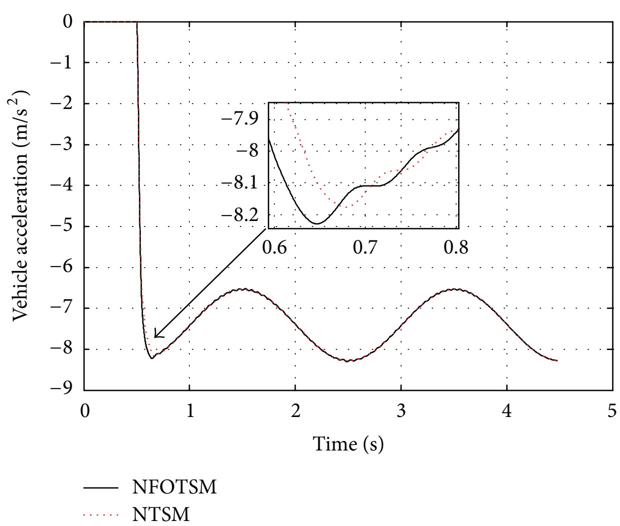

Figure 8: Vehicle acceleration.

in road surface condition, and its stability is proven thanks to a sufficient number of simulation scenarios, such as a sudden change of road condition and a sudden change of reference force condition.

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