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Research Article

Design of H_∞ Filter for a Class of Switched Linear Neutral Systems

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This paper is concerned with the H_∞ filtering problem for a class of switched linear neutral systems with time-varying delays. The time-varying delays appear not only in the state but also in the state derivatives. Based on the average dwell time approach and the piecewise Lyapunov functional technique, sufficient conditions are proposed for the exponential stability of the filtering error dynamic system. Then, the corresponding solvability condition for a desired filter satisfying a weighted H_∞ performance is established. All the conditions obtained are delay-dependent. Finally, two numerical examples are given to illustrate the effectiveness of the proposed theory.

1. Introduction

Switched time-delay systems have been attracting considerable attention during the recent years [1–9], due to the significance both in theory development and practical applications. However, it is worth noting that only the state time delay is considered, and the time delay in the state derivatives is largely ignored in the existing literature. If each subsystem of a switched system has time delay in the state derivatives, then the switched system is called switched neutral system [10]. Switched neutral systems exist widely in engineering and social systems, many physical plants can be modeled as switched neutral systems, such as distributed networks and heat exchanges. For example, in [11], a switched neutral type delay equation with nonlinear perturbations was exploited to model the drilling system. Compared with the switched systems with state time delay, switched neutral systems are much more complicated [12–15]. As effective tools, the common Lyapunov function method, dwell time approaches, and average dwell time approaches have been extended to study the switched neutral systems, and many valuable results have been obtained for switched neutral systems. On the research of stability analysis for switched neutral systems, the asymptotically stable problem of switched neutral systems was considered in [16]. If there exists a Hurwitz linear convex combination of state matrices, and subsystems are not necessarily stable, switching rules can be designed to guarantee the asymptotical stability of the switched neutral system. The method of Lyapunov-Metzler linear matrix inequalities in [17] was extended to switched neutral systems [18], and some less conservative stability results were obtained.

In contrast with the traditional Kalman filtering, the H_∞ filtering does not require the exact knowledge of the statistics of the external noise signals, and it is insensitive to the uncertainties both in the exogenous signal statistics and in dynamic models [19, 20]. Because of these advantages, the H_∞ filtering has attracted much attention in the past decade for nonswitched neutral systems [21–24]. In [22], some sufficient conditions for the existence of an H_∞ filter of a Luenberger observer type have been provided. However, to the authors' best knowledge, the H_∞ filtering for switched neutral systems has been rarely investigated and still remains challenging. This motivates our research.

The contribution of this paper lies in three aspects. First, we address the delay-dependent H_∞ filtering problem for switched linear neutral systems with time-varying delays, which appear not only in the state, but also in the

state derivatives. The resulting filter is of the Luenberger-observer type. Second, by using average dwell time approach and the piecewise Lyapunov function technique, we derive a delay-dependent sufficient condition, which guarantees exponential stability of the filtering error system. Then, the corresponding solvability condition for a desired filter satisfying a weighted H_∞ performance is established. Here, to reduce the conservatism of the delay-dependent condition, we introduce some slack matrix variables and a new integral inequality recently proposed in [25]. Finally, we succeed in transforming the filter design problem into the feasibility problem of some linear matrix inequalities. To show the efficiency of the obtained results, we present two relevant examples.

The remainder of this paper is organized as follows. The H_∞ filtering problem for switched neutral systems is formulated in Section 2. Section 3 presents our main results. Numerical examples are given in Section 4, and we conclude this paper in Section 5.

Notation. Throughout this paper, R^n denotes n-dimensional Euclidean space; $R^{n\times m}$ is the set of all $n \times m$ real matrices; $P > 0$ means that P is positive definite; L_2 denotes the space of square integrable vector functions on $[0,\infty)$ with norm $\|\cdot\| = (\int_0^\infty \|\cdot\|^2 dt)^{1/2}$, where $\|\cdot\|$ denotes the Euclidean vector norm; I is the identity matrix with appropriate dimensions; the symmetric terms in a symmetric matrix are denoted by $*$ as for example

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}. \tag{1}$$

2. Problem Statement

Consider the following switched linear neutral system:

$$\begin{aligned} \dot{x}(t) &= A_{0\sigma(t)}x(t) + A_{1\sigma(t)}x(t-h(t)) \\ &\quad + F_{\sigma(t)}\dot{x}(t-\tau(t)) + B_{\sigma(t)}\omega(t), \\ y(t) &= C_{0\sigma(t)}x(t) + C_{1\sigma(t)}x(t-h(t)) + D_{\sigma(t)}\omega(t), \\ z(t) &= L_{\sigma(t)}x(t), \\ x(\theta) &= \psi(\theta), \quad \forall\theta \in [-H,0], \ H = \max\{h,\tau\}, \end{aligned} \tag{2}$$

where $x(t) \in R^n$ is the state vector; $y(t) \in R^m$ is the measurements vector; $\omega(t) \in R^p$ is the noise signal vector, which belongs to $L_2[0,\infty)$; $z(t) \in R^q$ is the signal to be estimated; $\psi(t)$ is the initial vector function that is continuously differentiable on $[-H,0]$; $\sigma(t) : [0,\infty) \to M = \{1,2,\ldots,m\}$ is a piecewise constant function of time t called switching signal. Corresponding to the switching signal $\sigma(t)$, we have the switching sequence $\{x_{t_0} : (i_0,t_0),\ldots,(i_k,t_k),\ldots, | \ i_k \in M, k = 0,1,\ldots\}$, which means that the i_kth subsystem is active when $t \in [t_k, t_{k+1})$. The system coefficient matrices A_{0i_k}, A_{1i_k}, F_{i_k}, B_{i_k}, C_{0i_k}, C_{1i_k}, D_{i_k}, and L_{i_k} are known real constant matrices of appropriate dimensions. $h(t)$ and $\tau(t)$ are time-varying delays satisfying

$$\begin{aligned} 0 \le h(t) \le h, &\qquad \dot{h}(t) \le \bar{h} < 1, \\ 0 \le \tau(t) \le \tau, &\qquad \dot{\tau}(t) \le \bar{\tau} < 1. \end{aligned} \tag{3}$$

The objective of this paper is to design a family of filters of Luenberger observer type as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{0\sigma(t)}\hat{x}(t) + A_{1\sigma(t)}\hat{x}(t-h(t)) + F_{\sigma(t)}\dot{\hat{x}}(t-\tau(t)) \\ &\quad + K_{\sigma(t)}\left[y(t) - C_{0\sigma(t)}\hat{x}(t) - C_{1\sigma(t)}\hat{x}(t-h(t))\right], \\ \hat{z}(t) &= L_{\sigma(t)}\hat{x}(t), \\ \hat{x}(\theta) &= \hat{\psi}(\theta), \quad \forall\theta \in [-H,0], \ H = \max\{h,\tau\}, \end{aligned} \tag{4}$$

where K_{i_k} are the filter parameters, which are to be determined.

Now, we introduce the estimation errors: $x_e(t) = x(t) - \hat{x}(t)$, $z_e(t) = z(t) - \hat{z}(t)$.

Combining (2) with (4) gives the following filtering error dynamic system:

$$\begin{aligned} \dot{x}_e(t) &= \widetilde{A}_{0\sigma(t)}x_e(t) + \widetilde{A}_{1\sigma(t)}x_e(t-h(t)) \\ &\quad + F_{\sigma(t)}\dot{x}_e(t-\tau(t)) + \widetilde{B}_{\sigma(t)}\omega(t), \\ z_e(t) &= L_{\sigma(t)}x_e(t), \\ x_e(\theta) &= \psi(\theta) - \hat{\psi}(\theta), \quad \forall\theta \in [-H,0], \ H = \max\{h,\tau\}, \end{aligned} \tag{5}$$

where $\widetilde{A}_{0\sigma} = A_{0\sigma} - K_\sigma C_{0\sigma}$, $\widetilde{A}_{1\sigma} = A_{1\sigma} - K_\sigma C_{1\sigma}$, $\widetilde{B}_\sigma = B_\sigma - K_\sigma D_\sigma$.

The following definitions are introduced, which will play key roles in deriving our main results.

Definition 1 (see [26]). The equilibrium $x_e^* = 0$ of the filtering error system (5) is said to be exponentially stable under $\sigma(t)$ if the solution $x_e(t)$ of system (5) with $\omega(t) = 0$ satisfies $\|x_e(t)\| \le \Gamma e^{-\lambda(t-t_0)}\|x_e(t_0)\|_H$, for all $t \ge t_0$ for constants $\Gamma > 0$ and $\lambda > 0$, where $\|\cdot\|$ denotes the Euclidean norm, and $\|x_e(t)\|_H = \sup_{-H\le\theta\le 0}\{x_e(t+\theta), \dot{x}_e(t+\theta)\}$.

Definition 2 (see [26]). For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1,T_2)$ denote the number of switching of $\sigma(t)$ over (T_1,T_2). If $N_\sigma(T_1,T_2) \le N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \ge 0$, then T_a is called average dwell time. As commonly used in the literature, we choose $N_0 = 0$.

The filtering problem addressed in this paper is to seek for suitable filter gain K_i such that the filtering error system (5) for any switching signal with average dwell time has a prescribed H_∞ performance γ; that is,

(1) the error system (5) with $\omega(t) = 0$ is exponentially stable;

(2) under the zero initial conditions, that is, $x_e(\theta) = 0$, for all $\theta \in [-H, 0]$, the weighted H_∞ performance $\int_0^\infty e^{-\alpha s} z_e^T(s) z_e(s) ds \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s) ds$ is guaranteed for all nonzero $\omega(t) \in L_2[0,\infty)$ and a prescribed level of noise attenuation $\gamma > 0$.

Before concluding this section, we introduce three lemmas which are essential for the development of the results.

Lemma 3 (see [25]). *Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any matrices M_1, $M_2 \in R^{n\times n}$, and $X = X^T > 0$, and a scalar $h \ge 0$,*

$$\begin{aligned} &-\int_{t-h}^{t} \dot{x}^T(s) X \dot{x}(s)\, ds \\ &\le \xi^T(t) \begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_2 \end{bmatrix} \xi(t) \\ &+ h\xi^T(t) \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} X^{-1} \begin{bmatrix} M_1 & M_2 \end{bmatrix} \xi(t), \end{aligned} \tag{6}$$

where $\xi^T(t) = \left[x^T(t) \quad x^T(t-h)\right]$.

Lemma 4 (see [27]). *For any constant matrix $0 < R = R^T \in R^{n\times n}$, scalar $r > 0$, vector function $\omega : [0, r] \to R^n$ such that the integrations concerned are well defined; then, $(\int_0^r \omega(s)ds)^T R (\int_0^r \omega(s)ds) \le r \int_0^r \omega^T(s) R \omega(s) ds$.*

Lemma 5 (*Schur complement*). *For given $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} < 0$, where $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T$, the following is equivalent:*

$$\begin{aligned} &(1)\ S_{11} < 0, \qquad S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0; \\ &(2)\ S_{22} < 0, \qquad S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0. \end{aligned} \tag{7}$$

3. Main Results

In this section, we first present a sufficient condition for exponential stability of the filtering error system (5) with $\omega(t) = 0$. Then, it is applied to formulate an approach to design the desired H_∞ filters for switched neutral system (2).

3.1. Stability Analysis

Theorem 6. *Given $\alpha > 0$, $\|F_{i_k}\| < 1$, for all $i_k \in M$. If there exist matrices $P_{i_k} > 0$, $Q_{i_k} > 0$, $R_{i_k} > 0$, $M_{i_k} > 0$, $N_{i_k} > 0$, and T_{1i_k}, T_{2i_k}, N_{gi_k} $(g = 1, 2, \ldots, 7)$ of appropriate dimensions, and $\mu \ge 1$, such that for all $i_k \in M$,*

$$\overline{\Sigma}_{i_k} = \begin{bmatrix} \overline{\Sigma}_{11} & \overline{\Sigma}_{12} & \overline{\Sigma}_{13} & \overline{\Sigma}_{14} & \overline{\Sigma}_{15} & \widetilde{A}_{0i_k}^T N_{6i_k} & \overline{\Sigma}_{17} \\ * & \overline{\Sigma}_{22} & \overline{\Sigma}_{23} & -N_{4i_k} & \overline{\Sigma}_{25} & -N_{6i_k} & -N_{7i_k} \\ * & * & \overline{\Sigma}_{33} & \widetilde{A}_{1i_k}^T N_{4i_k} & \overline{\Sigma}_{35} & \widetilde{A}_{1i_k}^T N_{6i_k} & \widetilde{A}_{1i_k}^T N_{7i_k} \\ * & * & * & \overline{\Sigma}_{44} & N_{4i_k}^T F_{i_k} & 0 & \tau T_{2i_k}^T \\ * & * & * & * & \overline{\Sigma}_{55} & F_{i_k}^T N_{6i_k} & F_{i_k}^T N_{7i_k} \\ * & * & * & * & * & -\dfrac{1}{h} e^{-\alpha h} M_{i_k} & 0 \\ * & * & * & * & * & * & -\tau e^{\alpha\tau} N_{i_k} \end{bmatrix} < 0, \tag{8}$$

$$P_{i_k} \le \mu P_{i_j}, \quad Q_{i_k} \le \mu Q_{i_j}, \quad R_{i_k} \le \mu R_{i_j}, \quad M_{i_k} \le \mu M_{i_j}, \quad N_{i_k} \le \mu N_{i_j}, \quad \forall i_k, i_j \in M, \tag{9}$$

where

$$\begin{aligned} \overline{\Sigma}_{11} &= hM_{i_k} + \alpha P_{i_k} + Q_{i_k} + e^{-\alpha\tau} T_{1i_k}^T + e^{-\alpha\tau} T_{1i_k} \\ &\quad + N_{1i_k}^T \widetilde{A}_{0i_k} + \widetilde{A}_{0i_k}^T N_{1i_k}, \\ \overline{\Sigma}_{12} &= P_{i_k} - N_{1i_k}^T + \widetilde{A}_{0i_k}^T N_{2i_k}, \\ \overline{\Sigma}_{13} &= N_{1i_k}^T \widetilde{A}_{1i_k} + \widetilde{A}_{0i_k}^T N_{3i_k}, \\ \overline{\Sigma}_{14} &= -e^{-\alpha\tau} T_{1i_k}^T + e^{-\alpha\tau} T_{2i_k} + \widetilde{A}_{0i_k}^T N_{4i_k}, \\ \overline{\Sigma}_{15} &= N_{1i_k}^T F_{i_k} + \widetilde{A}_{0i_k}^T N_{5i_k}, \\ \overline{\Sigma}_{17} &= \tau T_{1i_k}^T + \widetilde{A}_{0i_k}^T N_{7i_k}, \\ \overline{\Sigma}_{22} &= R_{i_k} + \tau N_{i_k} - N_{2i_k}^T - N_{2i_k}, \\ \overline{\Sigma}_{23} &= -N_{3i_k} + N_{2i_k}^T \widetilde{A}_{1i_k}, \\ \overline{\Sigma}_{25} &= -N_{5i_k} + N_{2i_k}^T F_{i_k}, \\ \overline{\Sigma}_{33} &= -\left(1 - \overline{h}\right) e^{-\alpha h} Q_{i_k} + N_{3i_k}^T \widetilde{A}_{1i_k} + \widetilde{A}_{1i_k}^T N_{3i_k}, \\ \overline{\Sigma}_{35} &= N_{3i_k}^T F_{i_k} + \widetilde{A}_{1i_k}^T N_{5i_k}, \end{aligned}$$

$$\overline{\Sigma}_{44} = -e^{-\alpha\tau}T_{2i_k}^T - e^{-\alpha\tau}T_{2i_k},$$
$$\overline{\Sigma}_{55} = -(1-\overline{\tau})e^{-\alpha\tau}R_{i_k} + N_{5i_k}^T F_{i_k} + F_{i_k}^T N_{5i_k}, \tag{10}$$

then the error dynamic system (5) *with* $\omega(t) = 0$ *is exponentially stable for any switching signal with average dwell time satisfying* $T_a > T_a^* = \ln\mu/\alpha$.

Proof. Define the piecewise Lyapunov-Krasovskii functional candidate

$$V(t) = V_{\sigma(t)}(t) = \sum_{j=1}^{5} V_{j\sigma(t)}(t), \tag{11}$$

where

$$V_{1i_k}(t) = x_e^T(t)P_{i_k}x_e(t),$$
$$V_{2i_k}(t) = \int_{t-h(t)}^{t} x_e^T(s)e^{\alpha(s-t)}Q_{i_k}x_e(s)\,ds,$$
$$V_{3i_k}(t) = \int_{t-\tau(t)}^{t} \dot{x}_e^T(s)e^{\alpha(s-t)}R_{i_k}\dot{x}_e(s)\,ds, \tag{12}$$
$$V_{4i_k}(t) = \int_{-h}^{0}\int_{t+\theta}^{t} x_e^T(s)e^{\alpha(s-t)}M_{i_k}x_e(s)\,ds\,d\theta,$$
$$V_{5i_k}(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_e^T(s)e^{\alpha(s-t)}N_{i_k}\dot{x}_e(s)\,ds\,d\theta.$$

Now, taking the derivative of $V_{ji_k}(t)$, $j = 1,2,\ldots,5$ with respect to t along the trajectory of the error system (5) with $\omega(t) = 0$, according to (3) and Lemma 4, we have

$$\begin{aligned}
&\dot{V}_{i_k}(t) + \alpha V_{i_k}(t)\\
&\quad\le 2x_e^T(t)P_{i_k}\dot{x}_e(t) + x_e^T(t)Q_{i_k}x_e(t) + \dot{x}_e^T(t)R_{i_k}\dot{x}_e(t)\\
&\qquad - \left(1-\overline{h}\right)x_e^T(t-h(t))e^{-\alpha h}Q_{i_k}x_e(t-h(t))\\
&\qquad + hx_e^T(t)M_{i_k}x_e(t) + \tau\dot{x}_e^T(t)N_{i_k}\dot{x}_e(t) + \alpha x_e^T(t)P_{i_k}x_e(t)\\
&\qquad - (1-\overline{\tau})\dot{x}_e^T(t-\tau(t))e^{-\alpha\tau}R_{i_k}\dot{x}_e(t-\tau(t))\\
&\qquad - \frac{e^{-\alpha h}}{h}\int_{t-h}^{t} x_e^T(s)\,ds M_{i_k}\int_{t-h}^{t} x_e(s)\,ds\\
&\qquad - \int_{t-\tau}^{t} \dot{x}_e^T(s)e^{\alpha(s-t)}N_{i_k}\dot{x}_e(s)\,ds.
\end{aligned} \tag{13}$$

From Lemma 3, it holds

$$\begin{aligned}
&-\int_{t-\tau}^{t} \dot{x}_e^T(s)e^{\alpha(s-t)}N_{i_k}\dot{x}_e(s)\,ds\\
&\quad\le -\int_{t-\tau}^{t} \dot{x}_e^T(s)e^{-\alpha\tau}N_{i_k}\dot{x}_e(s)\,ds\\
&\quad\le e^{-\alpha\tau}\left[x_e^T(t)\quad x_e^T(t-\tau)\right]\begin{bmatrix} T_{1i_k}^T + T_{1i_k} & -T_{1i_k}^T + T_{2i_k}\\ * & -T_{2i_k}^T - T_{2i_k}\end{bmatrix}\\
&\qquad\times\begin{bmatrix} x_e(t)\\ x_e(t-\tau)\end{bmatrix} + \tau e^{-\alpha\tau}\left[x_e^T(t)\quad x_e^T(t-\tau)\right]\\
&\qquad\times\begin{bmatrix} T_{1i_k}^T\\ T_{2i_k}^T\end{bmatrix}N_{i_k}^{-1}\left[T_{1i_k}\quad T_{2i_k}\right]\begin{bmatrix} x_e(t)\\ x_e(t-\tau)\end{bmatrix}.
\end{aligned} \tag{14}$$

Define

$$\xi_e^T(t) = \left[\zeta_e^T(t)\quad \dot{x}_e^T(t-\tau(t))\quad \int_{t-h}^{t} x_e^T(s)\,ds\right], \tag{15}$$

where $\zeta_e^T(t) = \left[x_e^T(t)\quad \dot{x}_e^T(t)\quad x_e^T(t-h(t))\quad x_e^T(t-\tau)\right]$.

By some algebraic manipulations, it is easy to show that

$$\dot{V}_{i_k}(t) + \alpha V_{i_k}(t) \le \xi_e^T(t)\Sigma_{i_k}\xi_e(t), \tag{16}$$

where

$$\Sigma_{i_k} = \begin{bmatrix} \Sigma_{11} & P_{i_k} & 0 & \Sigma_{14} & 0 & 0\\ * & R_{i_k} + \tau N_{i_k} & 0 & 0 & 0 & 0\\ * & * & -\left(1-\overline{h}\right)e^{-\alpha h}Q_{i_k} & 0 & 0 & 0\\ * & * & * & \Sigma_{44} & 0 & 0\\ * & * & * & * & \Sigma_{55} & 0\\ * & * & * & * & * & \Sigma_{66}\end{bmatrix}, \tag{17}$$

where

$$\begin{aligned}
\Sigma_{11} &= hM_{i_k} + \alpha P_{i_k} + Q_{i_k} + e^{-\alpha\tau}T_{1i_k}^T + e^{-\alpha\tau}T_{1i_k}\\
&\quad + \tau e^{-\alpha\tau}T_{1i_k}^T N_{i_k}^{-1}T_{1i_k},\\
\Sigma_{14} &= -e^{-\alpha\tau}T_{1i_k}^T + e^{-\alpha\tau}T_{2i_k} + \tau e^{-\alpha\tau}T_{1i_k}^T N_{i_k}^{-1}T_{2i_k},\\
\Sigma_{55} &= -(1-\overline{\tau})e^{-\alpha\tau}R_{i_k},\\
\Sigma_{44} &= -e^{-\alpha\tau}T_{2i_k}^T - e^{-\alpha\tau}T_{2i_k} + \tau e^{-\alpha\tau}T_{2i_k}^T N_{i_k}^{-1}T_{2i_k},\\
\Sigma_{66} &= -\frac{1}{h}e^{-\alpha h}M_{i_k}.
\end{aligned} \tag{18}$$

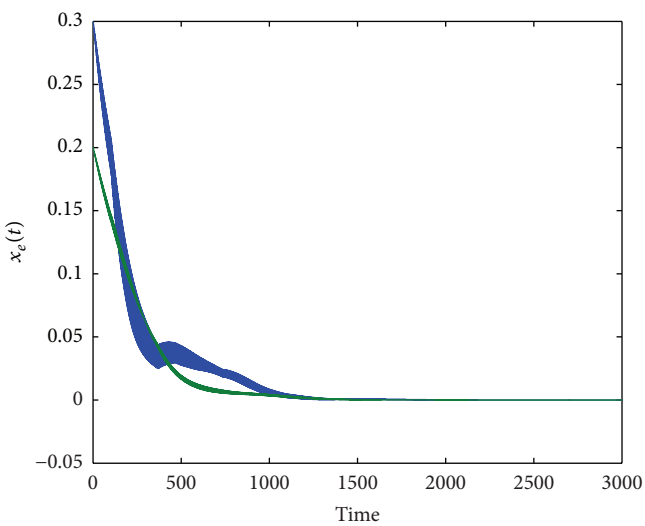

FIGURE 1: The state responses of the filtering error dynamic system with $\omega(t) = 0$.

From Lemma 5, $\Sigma_{i_k} < 0$ is equivalent to

$$\widetilde{\Sigma}_{i_k} = \begin{bmatrix} \widetilde{\Sigma}_{11} & P_{i_k} & 0 & \widetilde{\Sigma}_{14} & 0 & 0 & \tau T_{1i_k}^T \\ * & R_{i_k} + \tau N_{i_k} & 0 & 0 & 0 & 0 & 0 \\ * & * & \widetilde{\Sigma}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \widetilde{\Sigma}_{44} & 0 & 0 & \tau T_{2i_k}^T \\ * & * & * & * & \widetilde{\Sigma}_{55} & 0 & 0 \\ * & * & * & * & * & \widetilde{\Sigma}_{66} & 0 \\ * & * & * & * & * & * & -\tau e^{\alpha\tau} N_{i_k} \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} \widetilde{\Sigma}_{11} &= hM_{i_k} + \alpha P_{i_k} + Q_{i_k} + e^{-\alpha\tau} T_{1i_k}^T + e^{-\alpha\tau} T_{1i_k}, \\ \widetilde{\Sigma}_{14} &= -e^{-\alpha\tau} T_{1i_k}^T + e^{-\alpha\tau} T_{2i_k}, \qquad \widetilde{\Sigma}_{33} = -\left(1 - \overline{h}\right) e^{-\alpha h} Q_{i_k}, \\ \widetilde{\Sigma}_{44} &= -e^{-\alpha\tau} T_{2i_k}^T - e^{-\alpha\tau} T_{2i_k}, \qquad \widetilde{\Sigma}_{55} = \Sigma_{55}, \qquad \widetilde{\Sigma}_{66} = \Sigma_{66}. \end{aligned} \tag{20}$$

In addition, the following is true from (5) with $\omega(t) = 0$:

$$\begin{aligned} &2\overline{\xi}_e^T(t)\left[N_{1i_k} \;\; N_{2i_k} \;\; N_{3i_k} \;\; N_{4i_k} \;\; N_{5i_k} \;\; N_{6i_k} \;\; N_{7i_k}\right]^T \\ &\quad \times \left[\widetilde{A}_{0i_k} \;\; -I \;\; \widetilde{A}_{1i_k} \;\; 0 \;\; F_{i_k} \;\; 0 \;\; 0\right] \overline{\xi}_e(t) = 0, \end{aligned} \tag{21}$$

where $\overline{\xi}_e^T(t) = \left[\xi_e^T(t) \;\; 0\right]$.

Then, (19) along with (21) gives $\overline{\Sigma}_{i_k} < 0$, which yields $\overline{\Sigma}_{i_k} < 0$; thus,

$$\dot{V}_{i_k}(t) + \alpha V_{i_k}(t) \le 0. \tag{22}$$

Combining (9) with (22), for any $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} V(t) &= V_{i_k}(t) \\ &\le e^{-\alpha(t-t_k)} V_{i_k}(t_k) \\ &\le \mu e^{-\alpha(t-t_k)} V_{\sigma(t_k^-)}(t_k^-) \\ &\le \mu e^{-\alpha(t-t_k)} e^{-\alpha(t_k - t_{k-1})} V_{\sigma(t_{k-1})}(t_{k-1}) \\ &\le \cdots \\ &\le \mu^k e^{-\alpha(t-t_0)} V(t_0) \\ &\le e^{-(\alpha - (\ln\mu/T_a))(t-t_0)} V(t_0). \end{aligned} \tag{23}$$

According to (11), we have

$$a\|x_e(t)\|^2 \le V(t) \le b\|x_e(t_0)\|_H^2, \tag{24}$$

where

$$\begin{aligned} a &= \min_{\forall i_k \in M} \left\{\lambda_{\min}\left(P_{i_k}\right)\right\}, \\ b &= \max_{\forall i_k \in M} \left\{\lambda_{\max}\left(P_{i_k}\right)\right\} + h \max_{\forall i_k \in M} \left\{\lambda_{\max}\left(Q_{i_k}\right)\right\} \\ &\quad + \tau \max_{\forall i_k \in M} \left\{\lambda_{\max}\left(R_{i_k}\right)\right\} + \frac{h^2}{2} \max_{\forall i_k \in M} \left\{\lambda_{\max}\left(M_{i_k}\right)\right\} \\ &\quad + \frac{\tau^2}{2} \max_{\forall i_k \in M} \left\{\lambda_{\max}\left(N_{i_k}\right)\right\}. \end{aligned} \tag{25}$$

Considering (23) and (24), it holds $\|x_e(t)\| \le \sqrt{(b/a)} e^{-(1/2)(\alpha - (\ln\mu/T_a))(t-t_0)} \|x_e(t_0)\|_H$.

Therefore, if $\alpha - (\ln\mu/T_a) > 0$, that is $T_a > (\ln\mu/\alpha)$, then error dynamic system (5) is exponentially stable. □

Remark 7. When $\mu = 1$, we have $T_a^* = 0$, which means that the switching signal $\sigma(t)$ can be arbitrary. In this case, condition (9) implies that there exists a common Lyapunov functional for all subsystems. Moreover, setting $\alpha = 0$ in (8) gives asymptotic stability of the filtering error system (5) under arbitrary switching.

Remark 8. The filters of Luenberger observer type has been adopted in the literatures, see [17]. The Luenberger-type observer can produce an approximation to the system state that is independent of the system trajectory, and it only depends on the initial value of the system state.

Remark 9. The condition $\|F_{i_k}\| < 1$ guarantees that Lipschitz constant for the right hand of (2) with respect to $\dot{x}(t - \tau(t))$ is less than one.

3.2. Filter Design. Now, we design the desired H_∞ filter for the switched neutral system (2).

Theorem 10. *Given $\alpha > 0$, if $\|F_{i_k}\| < 1$, for all $i_k \in M$, and if there exists matrices $P_{i_k} > 0$, $Q_{i_k} > 0$, $R_{i_k} > 0$, $M_{i_k} > 0$, $N_{i_k} > 0$, and T_{1i_k}, T_{2i_k}, W_{i_k}, X_{i_k} of appropriate dimensions, and $\mu \ge 1$, such that, for any $i_k \in M$,*

$$\Omega_{i_k} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & W_{i_k}^T F_{i_k} & 0 & \tau T_{1i_k}^T & \Omega_{18} \\ * & \Omega_{22} & \Omega_{23} & 0 & W_{i_k}^T F_{i_k} & 0 & 0 & \Omega_{28} \\ * & * & \Omega_{33} & 0 & W_{i_k}^T F_{i_k} & 0 & 0 & \Omega_{38} \\ * & * & * & \Omega_{44} & 0 & 0 & \tau T_{2i_k}^T & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 \\ * & * & * & * & * & * & -\tau e^{\alpha\tau} N_{i_k} & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{26}$$

$$P_{i_k} \le \mu P_{i_j}, \quad Q_{i_k} \le \mu Q_{i_j}, \quad R_{i_k} \le \mu R_{i_j}, \quad M_{i_k} \le \mu M_{i_j}, \quad N_{i_k} \le \mu N_{i_j}, \quad \forall i_k, i_j \in M, \tag{27}$$

where

$$\begin{aligned}
\Omega_{11} &= hM_{i_k} + \alpha P_{i_k} + Q_{i_k} + e^{-\alpha\tau} T_{1i_k}^T + e^{-\alpha\tau} T_{1i_k} \\
&\quad + W_{i_k}^T A_{0i_k} + A_{0i_k}^T W_{i_k} - X_{i_k} C_{0i_k} - C_{0i_k}^T X_{i_k}^T + L_{i_k}^T L_{i_k}, \\
\Omega_{12} &= P_{i_k} - W_{i_k}^T + A_{0i_k}^T W_{i_k} - C_{0i_k}^T X_{i_k}^T, \\
\Omega_{13} &= W_{i_k}^T A_{1i_k} + A_{0i_k}^T W_{i_k} - X_{i_k} C_{1i_k} - C_{0i_k}^T X_{i_k}^T, \\
\Omega_{14} &= -e^{-\alpha\tau} T_{1i_k}^T + e^{-\alpha\tau} T_{2i_k}, \\
\Omega_{18} &= W_{i_k}^T B_{i_k} - X_{i_k} D_{i_k}, \\
\Omega_{22} &= R_{i_k} + \tau N_{i_k} - W_{i_k}^T - W_{i_k}, \\
\Omega_{23} &= W_{i_k}^T A_{1i_k} - W_{i_k} - X_{i_k} C_{1i_k}, \\
\Omega_{28} &= W_{i_k}^T B_{i_k} - X_{i_k} D_{i_k}, \\
\Omega_{33} &= -\left(1 - \overline{h}\right) e^{-\alpha h} Q_{i_k} + W_{i_k}^T A_{1i_k} + A_{1i_k}^T W_{i_k} \\
&\quad - X_{i_k} C_{1i_k} - C_{1i_k}^T X_{i_k}^T, \\
\Omega_{38} &= W_{i_k}^T B_{i_k} - X_{i_k} D_{i_k}, \\
\Omega_{44} &= -e^{-\alpha\tau} T_{2i_k}^T - e^{-\alpha\tau} T_{2i_k}, \\
\Omega_{55} &= -\left(1 - \overline{\tau}\right) e^{-\alpha\tau} R_{i_k}, \qquad \Omega_{66} = -\frac{1}{h} e^{-\alpha h} M_{i_k},
\end{aligned} \tag{28}$$

then the filter problem for the system (2) *is solvable for any switching signal with average dwell time satisfying* $T_a > T_a^* = \ln \mu / \alpha$.

Moreover, the filter gain K_{i_k} *are given by* $K_{i_k} = W_{i_k}^{-T} X_{i_k}$.

Proof. Consider the piecewise Lyapunov-krasovskii functional candidate as (11) and introduce the vector $\eta_e^T(t) = \left[\overline{\xi}_e^T(t) \;\; \omega^T(t)\right]$, where $\overline{\xi}_e(t)$ is defined in (21). Then, replace (21) with the following

$$\begin{aligned}
&2\eta_e^T(t)\left[W_{i_k} \;\; W_{i_k} \;\; W_{i_k} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right]^T \\
&\quad \times \left[\widetilde{A}_{0i_k} \;\; -I \;\; \widetilde{A}_{1i_k} \;\; 0 \;\; F_{i_k} \;\; 0 \;\; 0 \;\; \widetilde{B}_{i_k}\right] \eta_e(t) = 0.
\end{aligned} \tag{29}$$

Let $X_{i_k} = W_{i_k}^T K_{i_k}$ and $T(t) = z_e^T(t) z_e(t) - \gamma^2 \omega^T(t) \omega(t)$, similar to the proof of Theorem 6, we have

$$\dot{V}(t) + \alpha V(t) + T(t) \le \eta_e^T(t) \Omega_{i_k} \eta_e(t). \tag{30}$$

If $\Omega_{i_k} < 0$, it has

$$\dot{V}(t) \le -\alpha V(t) - T(t). \tag{31}$$

Integrating both sides of (31) from t_k to t, for any $t \in [t_k, t_{k+1})$, gives

$$V(t) \le e^{-\alpha(t-t_k)} V(t_k) - \int_{t_k}^{t} e^{-\alpha(t-s)} T(s)\, ds. \tag{32}$$

Therefore, similar to the proof method of Theory 2 in [13], we have

$$\begin{aligned}
V(t) &\le e^{-\alpha(t-t_0) + N_\sigma(t_0,t) \ln \mu} V(t_0) \\
&\quad - \int_{t_0}^{t} e^{-\alpha(t-s) + N_\sigma(s,t) \ln \mu} T(s)\, ds.
\end{aligned} \tag{33}$$

Under the zero initial condition, (33) gives

$$0 \le -\int_{0}^{t} e^{-\alpha(t-s) + N_\sigma(s,t) \ln \mu} T(s)\, ds. \tag{34}$$

Multiplying both sides of (34) by $e^{-N_\sigma(0,t)\ln\mu}$ yields

$$\int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\mu} z_e^T(s) z_e(s)\, ds \le \int_0^t e^{-\alpha(t-s)-N_\sigma(0,s)\ln\mu} \gamma^2 \omega^T(s)\omega(s)\, ds. \tag{35}$$

Notice that $N_\sigma(0,s) \le (s/T_a)$ and $T_a > T_a^* = (\ln\mu/\alpha)$, we have $N_\sigma(0,s)\ln\mu \le \alpha s$. Thus, (35) implies that

$$\int_0^t e^{-\alpha(t-s)-\alpha s} z_e^T(s) z_e(s)\, ds \le \gamma^2 \int_0^t e^{-\alpha(t-s)} \omega^T(s)\omega(s)\, ds. \tag{36}$$

Integrating both sides of (36) from $t = 0$ to $t = \infty$, we have

$$\int_0^\infty \int_0^t e^{-\alpha t} z_e^T(s) z_e(s)\, ds\, dt \le \gamma^2 \int_0^\infty \int_0^t e^{-\alpha(t-s)} \omega^T(s)\omega(s)\, ds\, dt. \tag{37}$$

Then, we can obtain

$$\int_0^\infty \frac{1}{\alpha} e^{-\alpha s} z_e^T(s) z_e(s)\, ds \le \gamma^2 \int_0^\infty \frac{1}{\alpha} e^{-\alpha s} e^{\alpha s} \omega^T(s)\omega(s)\, ds. \tag{38}$$

Obviously, it follows from (38) that

$$\int_0^\infty e^{-\alpha s} z_e^T(s) z_e(s)\, ds \le \gamma^2 \int_0^\infty \omega^T(s)\omega(s)\, ds. \tag{39}$$

□

Remark 11. Theorem 10 provides a sufficient condition for the solvability of the H_∞ filtering problem for switched neutral system with time-varying delay. If the condition is satisfied, then matrices W_i are nonsingular.

4. Numerical Examples

In this section, we present two numerical examples to illustrate the effectiveness of the results presented previously.

Example 1. Consider the switched neutral system (2) with two subsystems.

Subsystem 1.

$$\begin{gathered} A_{01} = \begin{bmatrix} -1.5 & -0.2 \\ 0.2 & -1.3 \end{bmatrix}, \qquad A_{11} = \begin{bmatrix} -0.3 & 0 \\ 0.1 & -0.4 \end{bmatrix}, \\ D_1 = 0.03, \qquad F_1 = \begin{bmatrix} 0.1 & -0.6 \\ 0 & 0.09 \end{bmatrix}, \\ C_{01} = [-0.2 \;\; 0.26], \qquad C_{11} = [-0.2 \;\; 0.1], \\ B_1 = \begin{bmatrix} 0.2 \\ -0.3 \end{bmatrix}, \qquad L_1 = [0.5 \;\; -0.19], \end{gathered} \tag{40}$$

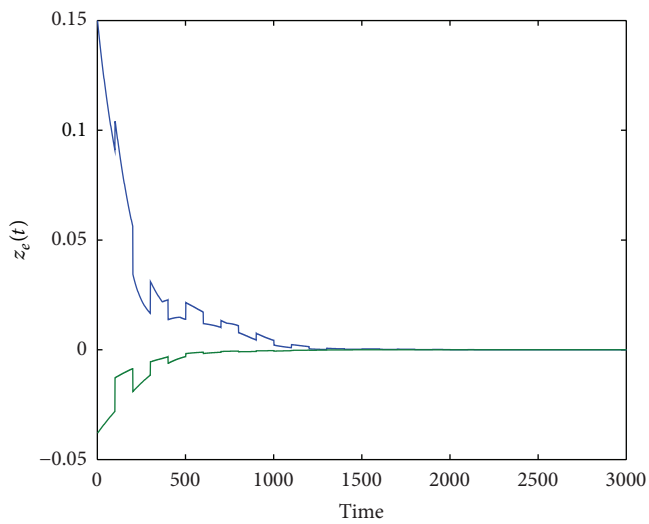

FIGURE 2: z_e(t) of the filtering error dynamic system with $\omega(t) = 0$.

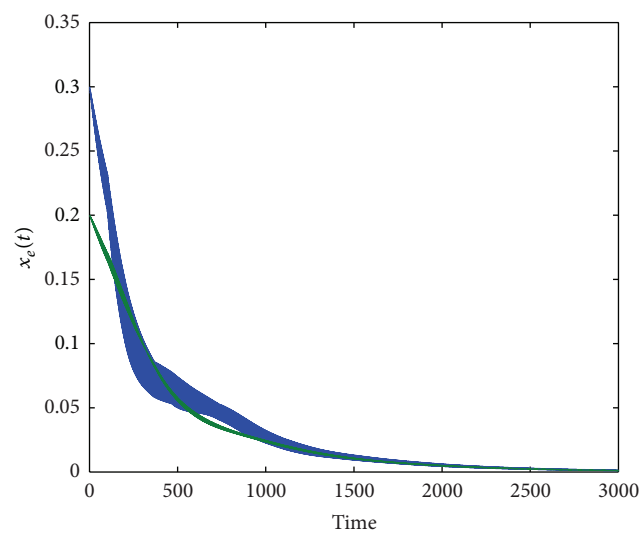

FIGURE 3: The state responses of the filtering error dynamic system with $\omega(t) = 0.1e^{-0.5t}$.

Subsystem 2.

$$\begin{gathered} A_{02} = \begin{bmatrix} -1.4 & -0.2 \\ 0.2 & -1.3 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} -0.2 & 0 \\ 0.1 & -0.4 \end{bmatrix}, \\ D_2 = D_1, \qquad F_2 = F_1, \\ C_{02} = [-0.2 \;\; 0.46], \qquad C_{12} = C_{11}, \\ B_2 = B_1, \qquad L_2 = [0.5 \;\; -0.09], \\ h(t) = 0.3, \qquad \tau(t) = 0.3, \qquad \alpha = 0.0376. \end{gathered} \tag{41}$$

By using the LMI toolbox, it can be checked that the conditions given in Theorem 10 are satisfied. Therefore, the previously switched neutral system has the given H_∞ performance γ, when $T_a \ge T_a^* = \ln\mu_{\min}/\alpha = 2.6596e^{-004}$ (here, the allowable minimum of μ is $\mu_{\min} = 1.00001$).

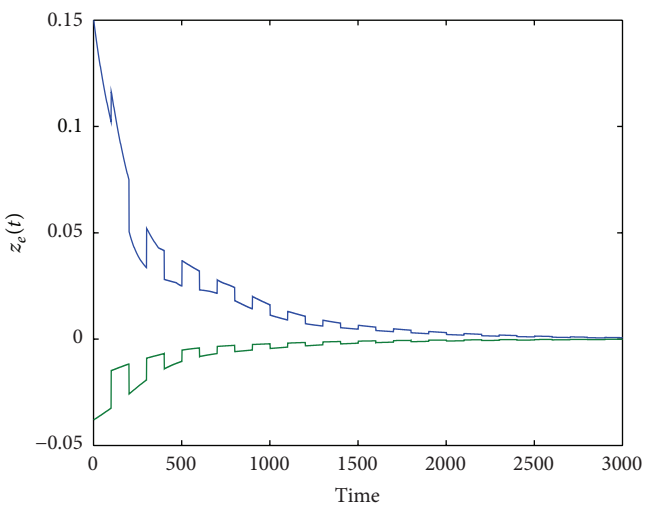

Figure 4: $z_e(t)$ of the filtering error dynamic system with $\omega(t) = 0.1e^{-0.5t}$.

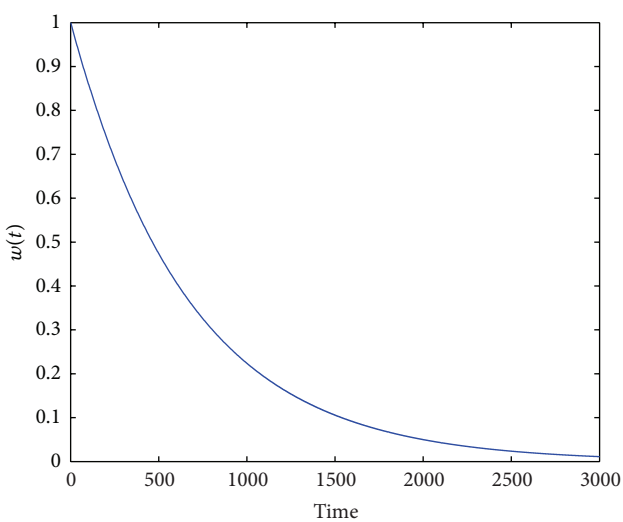

Figure 5: The noise signal $w(t)$.

Setting $\mu = 1.01$ and solving LMIs (26) using the LMI Toolbox in MATLAB, it follows that the minimized feasible γ is $\gamma^* = 2.0$, $T_a = 0.2646$, and the corresponding filter parameters are computed as

$$K_1 = \begin{bmatrix} -5.0768 \\ 1.2264 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -2.8724 \\ 0.9217 \end{bmatrix}. \tag{42}$$

In the following, we illustrate the effectiveness of the designed H_∞ filter through simulation. Let the initial condition be $x_e(t) = \left[\begin{smallmatrix} 0.3 \\ 0.2 \end{smallmatrix}\right]$, $t \in [-0.3, 0]$. Figures 1 and 2 are, respectively, the simulation results on $x_e(t)$ and $z_e(t)$; we can see that the filtering error dynamic system with $\omega(t) = 0$ is stable. $x_e(t)$ and $z_e(t)$ of the filtering error dynamic system with $w(t) = 0.1e^{-0.5t}$ are given in Figures 3 and 4. Figure 5 shows $w(t)$.

Table 1

γ	$h_{\max}$	$\tau_{\max}$	K_1	K_2
0.80	2.0	3.2	$[-4.7398\ 1.1340]^T$	$[-2.8595\ 0.9824]^T$
0.54	1.1	1.9	$[-4.1874\ 1.1606]^T$	$[-2.5316\ 0.9494]^T$
0.48	0.7	0.2	$[-3.9220\ 1.1308]^T$	$[-2.3354\ 0.8774]^T$
0.46	0.5	1.3	$[-3.8076\ 1.0948]^T$	$[-2.2459\ 0.8288]^T$
0.44	0.3	1.1	$[-3.6949\ 1.0686]^T$	$[-2.1560\ 0.7834]^T$

Example 2. Consider the switched neutral system in Example 1 with constant delays; that is, $\bar{h} = 0$, $\bar{\tau} = 0$, $\alpha = 0.0376$, and $\mu = 1.0001$. We calculate the admissible maximum value $h_{\max}$ of h, $\tau_{\max}$ of τ, which ensures that the resulting filtering error system is exponentially stable with a prescribed level γ of noise attenuation. For the different values, γ, the obtained $h_{\max}$, $\tau_{\max}$, and the filter gain K_i are listed in Table 1.

5. Conclusions

We have addressed the H_∞ filtering problem for a class of switched neutral systems with time-varying delays which appear in both the state and the state derivatives. For switched neutral systems with average dwell time scheme, we have provided a condition, in terms of upper bounds on the delays and in terms of a lower bound on the average dwell time, for the solvability of the H_∞ filtering problem. The piecewise Lyapunov functional technique has been used, which makes the proposed conditions are both delay-dependent and neutral delay-dependent. The design of filters for switched neutral systems is a difficult issue that is far from being well explored. Since multiple Lyapunov functional approach is commonly considered less conservative, the design of filters for switching neutral system with an appropriate switching law using multiple Lyapunov functionals is of great significance which deserves further study.

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