# Nonfragile Guaranteed Cost Control and Optimization for Interconnected Systems of Neutral Type 

Heli Hu, ${ }^{1}$ Dan Zhao, ${ }^{2}$ and Qingling Zhang ${ }^{3}$<br>${ }^{1}$ Key Laboratory of Manufacturing Industrial Integrated Automation, Shenyang University, Shenyang 110044, China<br>${ }^{2}$ Department of Fundamental Teaching, Shenyang Institute of Engineering, Shenyang 110136, China<br>${ }^{3}$ Institute of Systems Science, Northeastern University, Shenyang, Liaoning 110004, China

Correspondence should be addressed to Heli Hu; huheli2002@yahoo.com.cn
Received 4 April 2013; Accepted 19 June 2013
Academic Editor: Ming Cao
Copyright © 2013 Heli Hu et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.


#### Abstract

The design and optimization problems of the nonfragile guaranteed cost control are investigated for a class of interconnected systems of neutral type. A novel scheme, viewing the interconnections with time-varying delays as effective information but not disturbances, is developed to decrease the conservatism. Many techniques on decomposing and magnifying the matrices are utilized to obtain the guaranteed cost of the considered system. Also, an algorithm is proposed to solve the nonlinear problem of the interconnected matrices. Based on this algorithm, the minimization of the guaranteed cost of the considered system is obtained by optimization. Further, the state feedback control is extended to the case in which the underlying system is dependent on uncertain parameters. Finally, two numerical examples are given to illustrate the proposed method, and some comparisons are made to show the advantages of the schemes of dealing with the interconnections.


## 1. Introduction

Time delays often arise in the processing state, input or related variables of dynamic systems. Particularly, when the state derivative also contains time delay, the considered systems are called neutral systems [1]. The outstanding characteristic of neutral systems is the fact that such systems contain the same highest order derivatives for the state vector $x(t)$, at both time $t$ and past time(s) $t_{s} \leq t$. Many engineering systems can be represented as neutral equation [2-10], such as heat exchangers, robots in contact with rigid environments [11], distributed networks containing lossless transmission lines [12], and population ecology [13]. Therefore, great interest has been devoted to analysis and synthesis of a class of neutral delay systems. The delay-dependent stability criteria for stochastic systems of neutral type are studied in [3, 6]. The difference between them is that the exponential stability problem is investigated in the former, and the robust stochastic stability, stabilization, and $H_{\infty}$ control problems are considered in the other. Furthermore, the improved stability criteria for neutral systems are established by the method of a memory state feedback control [2] and by the
method of a robust $H_{\infty}$ reduced order filter in [4]. In the context of infinite-dimensional linear systems modeled by neutral functional differential equations, a periodic output feedback is studied in [14] and the stabilization of neutral systems with delayed control is the main work. As the further results, in [15-17], the stability and $H_{\infty}$ performance analysis, the finite-time $H_{\infty}$ control, and the reliable stabilization for uncertain switched systems of neutral type are investigated, respectively.

On the other hand, interconnected systems appear in a variety of engineering applications including power systems, large structures and manufacturing systems, and their applications, such as [18-21]. In [18], Mukaidani investigates the stability of a class of nonlinear large-scale systems and proposes a suboptimal guaranteed cost control instead of solving the nonconvex optimization problem. But the time delays are invariant and not involved in the interconnections. Furthermore, the scheme of counteracting the interconnections to simplify the problem may add conservatism in some cases. In [19], Mahmoud and Xia propose a generalized approach to stabilization of systems which are composed of
linear time delay subsystems coupled by linear time-varying interconnections. The decentralized structure of dissipative state-feedback controllers is designed to render the closedloop interconnected system delay-dependent asymptotically stable with strict dissipativity. However, the optimization problem for the dissipativity $\beta_{j}$ is not taken into account. In [20], a decentralized control scheme for a class of linear large-scale interconnected systems with norm-bounded time-varying parameter uncertainties is designed under a class of control failures. It is worth noting that the considered systems do not include any time delay, and the optimization problem for the guaranteed cost $J(x, u)$ is not investigated.

To the best of the authors' knowledge, the nonfragile guaranteed cost control and optimization for neutral interconnected systems have not yet been investigated, which motivates the present study. One contribution of this paper is that a novel scheme, viewing the interconnections with time-varying delays as effective information but not disturbances, is developed to decrease the conservatism. The other contribution lies in the fact that an algorithm is proposed to solve the nonlinear constraint problem caused by the interconnected matrices. In this paper, the designed control is the state feedback control with gain perturbations. Also, the guaranteed cost of the considered system can be obtained by solving the corresponding matrix inequality. Based on the proposed algorithm, the minimization of the guaranteed cost of the considered system can be obtained by optimization. particuraly, the matrix $E_{i}^{1 / 2}$ is introduced to denote the square root matrix of symmetric positive semidefinite matrix $E_{i} \geq 0$, that is, $E_{i}^{1 / 2}=V_{i} H_{i}^{1 / 2} V_{i}^{T}$ with $V_{i}$ the eigenvector matrix of $E_{i}$ satisfying $V_{i} V_{i}^{T}=I$ and $H_{i}$ the diagonal eigenvalues matrix of $E_{i}$.

The remainder of the paper is organized as follows. The nonfragile control problem formulation is described in Section 2. In Section 3, the guaranteed cost control with gain perturbations and optimization are investigated for unperturbed and uncertain neutral interconnected systems. The numerical examples, the simulation results, and some comparisons are presented in Section 4. The conclusion is provided in Section 5.

## 2. Problem Formulation

Consider the following uncertain neutral interconnected systems composed of $N$ subsystems:

$$
\begin{aligned}
\dot{x}_{i}(t)- & \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right) \\
= & {\left[A_{i}+\Delta A_{i}(t)\right] x_{i}(t) } \\
& +\left[A_{i \sigma_{i}}+\Delta A_{i \sigma_{i}}(t)\right] x_{i}\left(t-\sigma_{i}(t)\right) \\
& +\left[A_{i \eta_{i}}+\Delta A_{i \eta_{i}}(t)\right] \dot{x}_{i}\left(t-\eta_{i}(t)\right) \\
& +\left[B_{i}+\Delta B_{i}(t)\right] u_{i}(t) \\
& +\left[B_{i \delta_{i}}+\Delta B_{i \delta_{i}}(t)\right] u_{i}\left(t-\delta_{i}(t)\right)
\end{aligned}
$$

$$
\begin{equation*}
x_{i}(t)=\phi_{i}(t), \quad t \in[-l, 0], i=1,2, \ldots, N \tag{1}
\end{equation*}
$$

where $x_{i}(t) \in \Re^{n_{i}}$ and $u_{i}(t) \in \Re^{m_{i}}$ are the state vector and the input vector of the $i$ th subsystem, respectively. $\sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right)$ is the interconnections between the $i$ th subsystem and the other $N-1$ subsystems, in which $A_{i j}$ is known interconnected matrices of appropriate dimensions, and $x_{j}\left(t-\tau_{i j}(t)\right)$ implies the interconnections between the $i$ th subsystem and the other $N-1$ subsystems have different time-varying delays $\tau_{i j}(t), j=1,2, \ldots, N, j \neq i . A_{i}, A_{i \sigma_{i}}$, $A_{i \eta_{i}}, B_{i}$, and $B_{i \delta_{i}}$ are known constant matrices of appropriate dimensions. $\phi_{i}(t)$ is the initial condition. Assume that there exist constants $f_{i 0}, g_{i 0}, h_{i 0}, l_{i 0}, f_{i}, g_{i}, h_{i}, l_{i}$, and $l$ satisfying

$$
\begin{gather*}
0 \leq \sigma_{i}(t) \leq f_{i 0}, \\
0 \leq \eta_{i}(t) \leq g_{i 0} \\
0 \leq \delta_{i}(t) \leq h_{i 0},  \tag{2}\\
0 \leq \tau_{i j}(t) \leq l_{i 0} \\
\dot{\sigma}_{i}(t) \leq f_{i}<1, \quad \dot{\eta}_{i}(t) \leq g_{i}<1, \\
\dot{\delta}_{i}(t) \leq h_{i}<1, \quad \dot{\tau}_{i j}(t) \leq l_{i}<1, \\
l=\max \left\{f_{i 0}, g_{i 0}, h_{i 0}, l_{i 0}\right\}, \\
i, j=1,2 \ldots, N, j \neq i
\end{gather*}
$$

Time-varying parametric uncertainties $\Delta A_{i}(t), \Delta A_{i \sigma_{i}}(t)$, $\Delta A_{i \eta_{i}}(t), \Delta B_{i}(t)$, and $\Delta B_{i \delta_{i}}(t)$ are assumed to be of the following form:

$$
\begin{gather*}
{\left[\begin{array}{lllll}
\Delta A_{i}(t) & \Delta A_{i \sigma_{i}}(t) & \Delta A_{i \eta_{i}}(t) & \Delta B_{i}(t) & \Delta B_{i \delta_{i}}(t)
\end{array}\right]}  \tag{3}\\
=C_{i} F_{i}(t)\left[\begin{array}{lllll}
D_{i 1} & D_{i \sigma_{i}} & D_{i \eta_{i}} & D_{i 2} & D_{i \delta_{i}}
\end{array}\right]
\end{gather*}
$$

where $C_{i}, D_{i 1}, D_{i \sigma_{i}}, D_{i \eta_{i}}, D_{i 2}$, and $D_{i \delta_{i}}$ are constant matrices of appropriate dimensions, and $F_{i}(t)$ is the unknown matrix function satisfying $F_{i}^{T}(t) F_{i}(t) \leq I_{n_{i}}$, for all $t \geq 0$.

Construct the following state feedback control with gain perturbations:

$$
\begin{equation*}
u_{i}(t)=-\left(K_{i}+\Delta K_{i}\right) x_{i}(t), \tag{4}
\end{equation*}
$$

where $K_{i} \in \Re^{m_{i} \times n_{i}}$ is the control gain to be designed, and $\Delta K_{i}$ is a perturbed matrix satisfying $\Delta K_{i}=M_{i} \bar{F}_{i}(t) N_{i}$, where $M_{i}$ and $N_{i}$ are known matrices of appropriate dimensions, and $\bar{F}_{i}(t)$ satisfies $\bar{F}_{i}^{T}(t) \bar{F}_{i}(t) \leq I_{m_{i}}$, for all $t \geq 0$; the resulting
closed-loop uncertain neutral interconnected systems are obtained:

$$
\begin{align*}
\dot{x}_{i}(t)- & \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right) \\
= & {\left[A_{i}-B_{i}\left(K_{i}+\Delta K_{i}\right)+\Delta A_{i}(t)\right.} \\
& \left.-\Delta B_{i}(t)\left(K_{i}+\Delta K_{i}\right)\right] x_{i}(t) \\
+ & {\left[A_{i \sigma_{i}}+\Delta A_{i \sigma_{i}}(t)\right] x_{i}\left(t-\sigma_{i}(t)\right) }  \tag{5}\\
+ & {\left[A_{i \eta_{i}}+\Delta A_{i \eta_{i}}(t)\right] \dot{x}_{i}\left(t-\eta_{i}(t)\right) } \\
+ & {\left[-B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right)-\Delta B_{i \delta_{i}}(t)\left(K_{i}+\Delta K_{i}\right)\right] } \\
& \times x_{i}\left(t-\delta_{i}(t)\right) .
\end{align*}
$$

Define the following quadratic cost function:

$$
\begin{equation*}
J=\sum_{i=1}^{N} \int_{0}^{\infty}\left[x_{i}^{T}(t) S_{i 1} x_{i}(t)+u_{i}^{T}(t) S_{i 2} u_{i}(t)\right] d t \tag{6}
\end{equation*}
$$

where $S_{i 1} \in \Re^{n_{i} \times n_{i}}$ and $S_{i 2} \in \Re^{m_{i} \times m_{i}}$ are two given symmetric positive definite matrices.

One objective of this paper is to design a control (4) and determine a scalar $J_{u}$ satisfying the following two conditions:
(a) the closed-loop system (5) is asymptotically stable,
(b) $J \leq J_{u}$.

If the aforementioned control gain $K_{i}$ and constant $J_{u}$ exist, control (4) is the decentralized nonfragile guaranteed cost control and $J_{u}$ is the guaranteed cost for the considered system.

The other is to find out $J^{*}$, the minimization of the guaranteed cost $J_{u}$.

Lemma 1 (see [8]). Let $Z, X, S$, and $Y$ be matrices of appropriate dimensions. Assuming that $Z$ is symmetric and $S^{T} S \leq I$, then $Z+X S Y+Y^{T} S^{T} X^{T}<0$ if and only if there exists a scalar $\varepsilon>0$ satisfying

$$
\begin{align*}
Z+\varepsilon X X^{T}+\varepsilon^{-1} Y^{T} Y= & Z+\varepsilon^{-1}(\varepsilon X)(\varepsilon X)^{T} \\
& +\varepsilon^{-1} Y^{T} Y<0 \tag{7}
\end{align*}
$$

Lemma 2 (see [8]). For any constant matrix $P>0$ and differentiable vector function $x_{i}(t)$ with appropriate dimensions, one has

$$
\begin{aligned}
& {\left[\int_{t-\sigma_{i}(t)}^{t} \dot{x}_{i}(s) d s\right]^{T} P\left[\int_{t-\sigma_{i}(t)}^{t} \dot{x}_{i}(s) d s\right]} \\
& \quad \leq f_{i 0} \int_{t-\sigma_{i}(t)}^{t} \dot{x}_{i}^{T}(s) P \dot{x}_{i}(s) d s \\
& \quad \leq f_{i 0} \int_{t-f_{i 0}}^{t} \dot{x}_{i}^{T}(s) P \dot{x}_{i}(s) d s
\end{aligned}
$$

## 3. Main Result

3.1. Nonfragile Guaranteed Cost Control and Optimization for Unperturbed Neutral Interconnected Systems. For convenience, firstly consider the following unperturbed neutral interconnected systems with time-varying delays:

$$
\begin{align*}
& \dot{x}_{i}(t)-\sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right) \\
& =A_{i} x_{i}(t)+A_{i \sigma_{i}} x_{i}\left(t-\sigma_{i}(t)\right) \\
& +A_{i \eta_{i}} \dot{x}_{i}\left(t-\eta_{i}(t)\right)  \tag{9}\\
& +B_{i} u_{i}(t)+B_{i \delta_{i}} u_{i}\left(t-\delta_{i}(t)\right), \\
& x_{i}(t)=\phi_{i}(t), \quad t \in[-l, 0], i=1,2, \ldots, N .
\end{align*}
$$

Now a sufficient condition for existence of the decentralized nonfragile guaranteed cost control (4) for unperturbed neutral interconnected systems (9) with cost function (6) is presented in the following results.

Theorem 3. Assume $\left\|A_{i \eta_{i}}\right\|<1$. If there exist a positive number $\varepsilon_{i 1}$, some symmetric positive definite matrices $Q_{i k}(k=$ $0,1,2), \bar{W}_{j i}, W_{i j}$, and matrix $X_{i}$ such that the following inequality holds:

$$
\Gamma_{i}=\left[\begin{array}{cccccc}
\Gamma_{11}^{i} & \Gamma_{12}^{i} & 0 & \Gamma_{14}^{i} & \Gamma_{15}^{i} & \Gamma_{16}^{i}  \tag{10}\\
* & \Gamma_{22}^{i} & 0 & \Gamma_{24}^{i} & 0 & \Gamma_{26}^{i} \\
* & * & \Gamma_{33}^{i} & 0 & 0 & 0 \\
* & * & * & \Gamma_{44}^{i} & 0 & \Gamma_{46}^{i} \\
* & * & * & * & \Gamma_{55}^{i} & 0 \\
* & * & * & * & * & \Gamma_{66}^{i}
\end{array}\right]<0 ;
$$

then control (4) with $K_{i}=X_{i} Q_{i 0}^{-1}$ is the decentralized nonfragile guaranteed cost control of unperturbed neutral interconnected systems (9) with the following guaranteed cost:

$$
\begin{align*}
J_{u}=\sum_{i=1}^{N}[ & \phi_{i}^{T}(0) Q_{i 0}^{-1} \phi_{i}(0) \\
& +\int_{-\sigma_{i}(0)}^{0} \phi_{i}^{T}(s) Q_{i 0}^{-1} Q_{i 1} Q_{i 0}^{-1} \phi_{i}(s) d s \\
& +\frac{1}{1-g_{i}} \int_{-\eta_{i}(0)}^{0} \dot{\phi}_{i}^{T}(s) \dot{\phi}_{i}(s) d s \\
& +f_{i 0} \int_{-f_{i 0}}^{0}\left(s+f_{i 0}\right) \dot{\phi}_{i}^{T}(s) \dot{\phi}_{i}(s) d s  \tag{11}\\
& +\int_{-\delta_{i}(0)}^{0} \phi_{i}^{T}(s) Q_{i 0}^{-1} Q_{i 2} Q_{i 0}^{-1} \phi_{i}(s) d s \\
& \left.+\frac{1}{1-l_{i}} \sum_{j=1, j \neq i}^{N} \int_{-\tau_{i j}(0)}^{0} \phi_{j}^{T}(s) W_{i j} \phi_{j}(s) d s\right]
\end{align*}
$$

where

$$
\begin{aligned}
& \Gamma_{11}^{i}=A_{i} Q_{i 0}+Q_{i 0} A_{i}^{T}-B_{i} X_{i}-X_{i}^{T} B_{i}^{T}+Q_{i 1}+Q_{i 2}+E_{i}, \\
& \Gamma_{12}^{i}=\left[\begin{array}{llll}
A_{i \sigma_{i}} Q_{i 0} & A_{i \eta_{i}} & 0 & -B_{i \delta_{i}} X_{i}
\end{array}\right], \quad \Gamma_{16}^{i}=\left[\begin{array}{llll}
\varepsilon_{i 1} B_{i} M_{i} & \varepsilon_{i 1} B_{i \delta_{i}} M_{i} & -Q_{i 0} N_{i}^{T} & 0
\end{array}\right] \text {, } \\
& \Gamma_{14}^{i}=\left[Q_{i 0} A_{i}^{T}-X_{i}^{T} B_{i}^{T} Q_{i 0}-X_{i}^{T} \quad Q_{i 0} A_{i}^{T} E_{i}^{1 / 2}-X_{i}^{T} B_{i}^{T} E_{i}^{1 / 2} \quad 0 \quad 0 \quad 0\right], \\
& \Gamma_{22}^{i}=\operatorname{diag}\left\{-\left(1-f_{i}\right) Q_{i 1},-I_{n_{i}},-I_{n_{i}},-\left(1-h_{i}\right) Q_{i 2}\right\}, \\
& \Gamma_{24}^{i}=\left[\begin{array}{cccccccc}
Q_{i 0} A_{i \sigma_{i}}^{T} & 0 & 0 & 0 & 0 & Q_{i 0} A_{i \sigma_{i}}^{T} E_{i}^{1 / 2} & 0 & 0 \\
A_{i \eta_{i}}^{T} & 0 & 0 & 0 & 0 & 0 & A_{i \eta_{i}}^{T} E_{i}^{1 / 2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-X_{i}^{T} B_{i \delta_{i}}^{T} & 0 & 0 & 0 & 0 & 0 & 0 & -X_{i}^{T} B_{i \delta_{i}}^{T} E_{i}^{1 / 2}
\end{array}\right], \\
& \Gamma_{15}^{i}=[\underbrace{Q_{i 0} \cdots Q_{i 0}}_{N-1}], \quad \Gamma_{26}^{i}=\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -Q_{i 0} N_{i}^{T}
\end{array}\right], \\
& E_{i}=\sum_{j=1, j \neq i}^{N} A_{i j} A_{i j}^{T}, \quad \Gamma_{33}^{i}=\left[\begin{array}{cccccc}
\Psi_{11}^{i} & \ldots & \Psi_{1 i-1}^{i} & \Psi_{1 i+1}^{i} & \cdots & \Psi_{1 N}^{i} \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
* & * & \Psi_{i-1 i-1}^{i} & \Psi_{i-1 i+1}^{i} & \cdots & \Psi_{i-1 N}^{i} \\
* & * & * & \Psi_{i+1 i+1}^{i} & \cdots & \Psi_{i+1 N}^{i} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & * & * & \Psi_{N N}^{i}
\end{array}\right], \\
& \Gamma_{44}^{i}=\operatorname{diag}\left\{-e_{i 1}^{-1} I_{n_{i}},-S_{i 1}^{-1},-S_{i 2}^{-1},-e_{i 1}^{-1} I_{n_{i}},-e_{i 1}^{-1} I_{n_{i}},-e_{i 1}^{-1} I_{n_{i}},-e_{i 1}^{-1} I_{n_{i}},-e_{i 1}^{-1} I_{n_{i}}\right\}, \\
& \Gamma_{46}^{i}=\left[\begin{array}{cccc}
\varepsilon_{i 1} B_{i} M_{i} & \varepsilon_{i 1} B_{i \delta_{i}} M_{i} & 0 & 0 \\
0 & 0 & 0 & 0 \\
\varepsilon_{i 1} M_{i} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\varepsilon_{i 1} E_{i}^{1 / 2} B_{i} M_{i} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & \varepsilon_{i 1} E_{i}^{1 / 2} B_{i \delta_{i}} M_{i} & 0 & 0
\end{array}\right], \\
& e_{i 1}=\frac{1}{1-g_{i}}+f_{i 0}^{2}, \quad e_{i 2}=1+\frac{5}{1-g_{i}}+5 f_{i 0}^{2}, \\
& \Gamma_{55}^{i}=\operatorname{diag}\left\{-\left(1-l_{1}\right) \bar{W}_{1 i}, \ldots,-\left(1-l_{i-1}\right) \bar{W}_{i-1 i},-\left(1-l_{i+1}\right) \bar{W}_{i+1 i}, \ldots,-\left(1-l_{N}\right) \bar{W}_{N i}\right\}, \\
& \Gamma_{66}^{i}=\operatorname{diag}\left\{-\varepsilon_{i 1} I_{n_{i}},-\varepsilon_{i 1} I_{n_{i}},-\varepsilon_{i 1} I_{n_{i}},-\varepsilon_{i 1} I_{n_{i}}\right\}, \\
& \Psi_{11}^{i}=e_{i 2} I_{n_{1}}-W_{i 1}+e_{i 1} A_{i 1}^{T} A_{i 1}, \\
& \Psi_{1 i-1}^{i}=e_{i 1} A_{i 1}^{T} A_{i i-1}, \quad \Psi_{1 i+1}^{i}=e_{i 1} A_{i 1}^{T} A_{i i+1}, \\
& \Psi_{1 N}^{i}=e_{i 1} A_{i 1}^{T} A_{i N}, \quad \Psi_{i-1 i-1}^{i}=e_{i 2} I_{n_{i-1}}-W_{i i-1}+e_{i 1} A_{i i-1}^{T} A_{i i-1}, \\
& \Psi_{i-1 i+1}^{i}=e_{i 1} A_{i i-1}^{T} A_{i i+1}, \quad \Psi_{i-1 N}^{i}=e_{i 1} A_{i i-1}^{T} A_{i N}, \\
& \Psi_{i+1 i+1}^{i}=e_{i 2} I_{n_{i+1}}-W_{i i+1}+e_{i 1} A_{i i+1}^{T} A_{i i+1},
\end{aligned}
$$

$$
\begin{gather*}
\Psi_{i+1 N}^{i}=e_{i 1} A_{i i+1}^{T} A_{i N}, \quad \Psi_{N N}^{i}=e_{i 2} I_{n_{N}}-W_{i N}+e_{i 1} A_{i N}^{T} A_{i N} \\
\bar{W}_{j i}=W_{j i}^{-1}, \quad i, j=1,2 \ldots, N, j \neq i \tag{12}
\end{gather*}
$$

Proof. Choose $P_{i 0}=Q_{i 0}^{-1}, P_{i 1}=Q_{i 0}^{-1} Q_{i 1} Q_{i 0}^{-1}$, and $P_{i 2}=$ $Q_{i 0}^{-1} Q_{i 2} Q_{i 0}^{-1}$, and construct the following Lyapunov functional:

$$
\begin{aligned}
& V(x(t), t) \\
& =\sum_{i=1}^{N}\left[x_{i}^{T}(t) P_{i 0} x_{i}(t)\right. \\
& \\
& \quad+\int_{t-\sigma_{i}(t)}^{t} x_{i}^{T}(s) P_{i 1} x_{i}(s) d s \\
& \\
& \quad+\frac{1}{1-g_{i}} \int_{t-\eta_{i}(t)}^{t} \dot{x}_{i}^{T}(s) \dot{x}_{i}(s) d s \\
& \\
& \quad+f_{i 0} \int_{t-f_{i 0}}^{t}\left(s-\left(t-f_{i 0}\right)\right) \dot{x}_{i}^{T}(s) \dot{x}_{i}(s) d s \\
& \\
& \quad+\int_{t-\delta_{i}(t)}^{t} x_{i}^{T}(s) P_{i 2} x_{i}(s) d s \\
& \\
&
\end{aligned}
$$

Obviously, $V(x(t), t)>0$ for all $x_{i}(t) \neq 0$. Differentiating $V(x(t), t)$ along the trajectories of the unperturbed neutral interconnected systems (9) with control (4) and applying (2) and Lemma 2 yield

$$
\begin{aligned}
& \dot{V}(x(t), t) \\
& \qquad \begin{array}{l}
\leq \sum_{i=1}^{N}\left\{x_{i}^{T}(t)\left(P_{i 0} A_{i}+A_{i}^{T} P_{i 0}\right) x_{i}(t)\right. \\
\\
\\
+2 x_{i}^{T}(t) P_{i 0} A_{i \sigma_{i}} x_{i}\left(t-\sigma_{i}(t)\right) \\
\\
+2 x_{i}^{T}(t) P_{i 0} A_{i \eta_{i}} \dot{x}_{i}\left(t-\eta_{i}(t)\right) \\
\\
\quad-2 x_{i}^{T}(t) P_{i 0} B_{i}\left(K_{i}+\Delta K_{i}\right) x_{i}(t) \\
\\
\quad-2 x_{i}^{T}(t) P_{i 0} B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right) \\
\\
\quad \times x_{i}\left(t-\delta_{i}(t)\right)+2 x_{i}^{T}(t) P_{i 0}
\end{array}
\end{aligned}
$$

$$
\begin{align*}
& \times \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right) \\
& +x_{i}^{T}(t) P_{i 1} x_{i}(t)-\left(1-f_{i}\right) x_{i}^{T} \\
& \times\left(t-\sigma_{i}(t)\right) P_{i 1} x_{i}\left(t-\sigma_{i}(t)\right) \\
& +\left(\frac{1}{1-g_{i}}+f_{i 0}^{2}\right) \\
& \times\left[x_{i}^{T}(t) A_{i}^{T}+x_{i}^{T}\left(t-\sigma_{i}(t)\right) A_{i \sigma_{i}}^{T}\right. \\
& +\dot{x}_{i}^{T}\left(t-\eta_{i}(t)\right) A_{i \eta_{i}}^{T} \\
& -x_{i}^{T}(t)\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T}-x_{i}^{T} \\
& \times\left(t-\delta_{i}(t)\right)\left(K_{i}+\Delta K_{i}\right)^{T} B_{i \delta_{i}}^{T} \\
& \left.+\sum_{j=1, j \neq i}^{N} x_{j}^{T}\left(t-\tau_{i j}(t)\right) A_{i j}^{T}\right] \\
& \times\left[A_{i} x_{i}(t)+A_{i \sigma_{i}} x_{i}\left(t-\sigma_{i}(t)\right)\right. \\
& +A_{i \eta_{i}} \dot{x}_{i}\left(t-\eta_{i}(t)\right) \\
& -B_{i}\left(K_{i}+\Delta K_{i}\right) x_{i}(t) \\
& -B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right) x_{i}\left(t-\delta_{i}(t)\right) \\
& \left.+\sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right)\right] \\
& -\dot{x}_{i}^{T}\left(t-\eta_{i}(t)\right) \dot{x}_{i}\left(t-\eta_{i}(t)\right) \\
& -\left[\int_{t-\sigma_{i}(t)}^{t} \dot{x}_{i}(s) d s\right]^{T}\left[\int_{t-\sigma_{i}(t)}^{t} \dot{x}_{i}(s) d s\right] \\
& +x_{i}^{T}(t) P_{i 2} x_{i}(t)-\left(1-h_{i}\right) x_{i}^{T} \\
& \times\left(t-\delta_{i}(t)\right) P_{i 2} x_{i}\left(t-\delta_{i}(t)\right) \\
& +\frac{1}{1-l_{i}} \sum_{j=1, j \neq i}^{N} x_{j}^{T}(t) W_{i j} x_{j}(t) \\
& \left.-\sum_{j=1, j \neq i}^{N} x_{j}^{T}\left(t-\tau_{i j}(t)\right) W_{i j} x_{j}\left(t-\tau_{i j}(t)\right)\right\} . \tag{14}
\end{align*}
$$

According to Lemma 1 and the following the fact:

$$
\begin{align*}
& \sum_{i=1}^{N} \frac{1}{1-l_{i}} \sum_{j=1, j \neq i}^{N} x_{j}^{T}(t) W_{i j} x_{j}(t) \\
& \quad=\sum_{i=1}^{N} x_{i}^{T}(t) \sum_{j=1, j \neq i}^{N} \frac{1}{1-l_{j}} W_{j i} x_{i}(t) \tag{15}
\end{align*}
$$

one can obtain

$$
\begin{aligned}
& \sum_{i=1}^{N}\left\{2 x_{i}^{T}(t) P_{i 0} \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right)+2 e_{i 1}\right. \\
& \times\left[x_{i}^{T}(t) A_{i}^{T}+x_{i}^{T}\left(t-\sigma_{i}(t)\right) A_{i \sigma_{i}}^{T}\right. \\
& +\dot{x}_{i}^{T}\left(t-\eta_{i}(t)\right) A_{i \eta_{i}}^{T}-x_{i}^{T}(t) \\
& \times\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T}-x_{i}^{T}\left(t-\delta_{i}(t)\right) \\
& \left.\times\left(K_{i}+\Delta K_{i}\right)^{T} B_{i \delta_{i}}^{T}\right] \\
& \times \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right)+e_{i 1} \\
& \times \sum_{j=1, j \neq i}^{N} x_{j}^{T}\left(t-\tau_{i j}(t)\right) A_{i j}^{T} \\
& \times \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right) \\
& +\frac{1}{1-l_{i}} \sum_{j=1, j \neq i}^{N} x_{j}^{T}(t) W_{i j} x_{j}(t) \\
& -\sum_{j=1, j \neq i}^{N} x_{j}^{T}\left(t-\tau_{i j}(t)\right) \\
& \left.\times W_{i j} x_{j}\left(t-\tau_{i j}(t)\right)\right\} \\
& \leq \sum_{i=1}^{N}\left\{x _ { i } ^ { T } ( t ) \left[P_{i 0} E_{i} P_{i 0}+e_{i 1} A_{i}^{T} E_{i} A_{i}\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
& \quad+e_{i 1}\left(K_{i}+\Delta K_{i}\right)^{T} \\
& \quad \times B_{i}^{T} E_{i} B_{i}\left(K_{i}+\Delta K_{i}\right) \\
& \left.\quad+\sum_{j=1, j \neq i}^{N} \frac{1}{1-l_{j}} W_{j i}\right] \\
& \times x_{i}(t)+e_{i 1} x_{i}^{T}\left(t-\sigma_{i}(t)\right) \\
& \times A_{i \sigma_{i}}^{T} E_{i} A_{i \sigma_{i}} x_{i}\left(t-\sigma_{i}(t)\right) \\
& +e_{i 1} x_{i}^{T}\left(t-\delta_{i}(t)\right)\left(K_{i}+\Delta K_{i}\right)^{T} \\
& \times B_{i \delta_{i}}^{T} E_{i} B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right) x_{i}\left(t-\delta_{i}(t)\right) \\
& +e_{i 1} \dot{x}_{i}^{T}\left(t-\eta_{i}(t)\right) A_{i \eta_{i}}^{T} E_{i} A_{i \eta_{i}} \dot{x}_{i}\left(t-\eta_{i}(t)\right) \\
& +\sum_{j=1, j \neq i}^{N} x_{j}^{T}\left(t-\tau_{i j}(t)\right) \\
& \times\left(e_{i 2} I-W_{i j}\right) x_{j}\left(t-\tau_{i j}(t)\right)+e_{i 1} \\
& \times \sum_{j=1, j \neq i}^{N} x_{j}^{T}\left(t-\tau_{i j}(t)\right) A_{i j}^{T} \\
& \left.\times \sum_{j=1, j \neq i}^{N} A_{i j} x_{j}\left(t-\tau_{i j}(t)\right)\right\} .
\end{aligned}
$$

Therefore, it follows from (14) and (16) that

$$
\begin{gather*}
\dot{V}(x(t), t)+\sum_{i=1}^{N}\left[x_{i}^{T}(t) S_{i 1} x_{i}(t)+x_{i}^{T}(t)\left(K_{i}+\Delta K_{i}\right)^{T}\right. \\
\left.\quad \times S_{i 2}\left(K_{i}+\Delta K_{i}\right) x_{i}(t)\right]  \tag{17}\\
\leq \sum_{i=1}^{N} \xi_{i}^{T} \Upsilon_{i} \xi_{i}
\end{gather*}
$$

where

$$
\begin{gathered}
\xi_{i}^{T}=\left[\begin{array}{lll}
x_{i}^{T}(t) & x_{i}^{T}\left(t-\sigma_{i}(t)\right) & \dot{x}_{i}^{T}\left(t-\eta_{i}(t)\right) \int_{t-\sigma_{i}(t)}^{t} \dot{x}_{i}^{T}(s) d s \\
x_{i}^{T}\left(t-\delta_{i}(t)\right) & x_{1}^{T}\left(t-\tau_{i 1}(t)\right) \\
& \cdots x_{i-1}^{T}\left(t-\tau_{i i-1}(t)\right) x_{i+1}^{T}\left(t-\tau_{i i+1}(t)\right) \cdots x_{N}^{T}\left(t-\tau_{i N}(t)\right)
\end{array}\right] \\
\Upsilon_{i}=\left[\begin{array}{cc}
\Omega_{i} & 0 \\
* & \Gamma_{33}^{i}
\end{array}\right]+\left[\begin{array}{c}
G_{i 1} \\
0
\end{array}\right]\left(e_{i 1} I_{n_{i}}\right)\left[\begin{array}{cc}
G_{i 1}^{T} & 0
\end{array}\right]+\left[\begin{array}{c}
G_{i 2} \\
0
\end{array}\right] S_{i 1}\left[\begin{array}{ll}
G_{i 2}^{T} & 0
\end{array}\right]+\left[\begin{array}{c}
G_{i 3} \\
0
\end{array}\right] S_{i 2}\left[\begin{array}{ll}
G_{i 3}^{T} & 0
\end{array}\right]
\end{gathered}
$$

$$
\begin{align*}
& \Omega_{i}=\left[\begin{array}{ccccc}
\Omega_{11}^{i} & \Omega_{12}^{i} & \Omega_{13}^{i} & 0 & \Omega_{15}^{i} \\
* & \Omega_{22}^{i} & 0 & 0 & 0 \\
* & * & \Omega_{33}^{i} & 0 & 0 \\
* & * & * & \Omega_{44}^{i} & 0 \\
* & * & * & * & \Omega_{55}^{i}
\end{array}\right], \\
& \Omega_{12}^{i}=P_{i 0} A_{i \sigma_{i}}, \quad \Omega_{13}^{i}=P_{i 0} A_{i \eta_{i}}, \quad \Omega_{44}^{i}=-I_{n_{i}}, \\
& \Omega_{11}^{i}=P_{i 0} A_{i}+A_{i}^{T} P_{i 0}-P_{i 0} B_{i}\left(K_{i}+\Delta K_{i}\right)-\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T} P_{i 0}+P_{i 1}+P_{i 2} \\
& +P_{i 0} E_{i} P_{i 0}+e_{i 1} A_{i}^{T} E_{i} A_{i}+e_{i 1}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T} E_{i} B_{i}\left(K_{i}+\Delta K_{i}\right) \\
& +\sum_{j=1, j \neq i}^{N} \frac{1}{1-l_{j}} W_{j i}, \\
& \Omega_{15}^{i}=-P_{i 0} B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right), \quad \Omega_{22}^{i}=-\left(1-f_{i}\right) P_{i 1}+e_{i 1} A_{i \sigma_{i}}^{T} E_{i} A_{i \sigma_{i}}, \\
& \Omega_{33}^{i}=-I_{n_{i}}+e_{i 1} A_{i \eta_{i}}^{T} E_{i} A_{i \eta_{i}}, \\
& \Omega_{55}^{i}=-\left(1-h_{i}\right) P_{i 2}+e_{i 1}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i \delta_{i}}^{T} E_{i} B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right), \\
& G_{i 1}=\left[A_{i}-B_{i}\left(K_{i}+\Delta K_{i}\right) \quad A_{i \sigma_{i}} A_{i \eta_{i}} 0-B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right)\right]^{T}, \\
& G_{i 2}=\left[\begin{array}{lllll}
I_{n_{i}} & 0 & 0 & 0 & 0
\end{array}\right]^{T}, \\
& G_{i 3}=\left[\begin{array}{llllll}
-\left(K_{i}+\Delta K_{i}\right) & 0 & 0 & 0 & 0
\end{array}\right]^{T} . \tag{18}
\end{align*}
$$

Define

$$
\bar{\Upsilon}_{i}=\left[\begin{array}{ccc}
\Omega_{i} & 0 & \bar{\Upsilon}_{13}^{i}  \tag{19}\\
* & \Gamma_{33}^{i} & 0 \\
* & * & \bar{\Upsilon}_{33}^{i}
\end{array}\right]
$$

where $\bar{\Upsilon}_{13}^{i}=\left[\begin{array}{lll}G_{i 1} & G_{i 2} & G_{i 3}\end{array}\right], \bar{\Upsilon}_{33}^{i}=\operatorname{diag}\left\{-e_{i 1}^{-1} I_{n_{i}},-S_{i 1}^{-1},-S_{i 2}^{-1}\right\}$.
Pre- and postmultiplying the matrix $\bar{Y}_{i}$ in (19) by $U_{i}^{T}$ and $U_{i}$, where $U_{i}=\operatorname{diag}\left\{Q_{i 0}, Q_{i 0}, I_{n_{i}}, I_{n_{i}}, Q_{i 0}, I_{n_{1}}, \ldots\right.$, $\left.I_{n_{i-1}}, I_{n_{i+1}}, \ldots, I_{n_{N}}, I_{n_{i}}, I_{n_{i}}, I_{m_{i}}\right\}$, the following matrix is obtained:

$$
\overline{\bar{Y}}_{i}=\left[\begin{array}{ccc}
\bar{\Omega}_{i} & 0 & \overline{\bar{Y}}_{13}^{i}  \tag{20}\\
* & \Gamma_{33}^{i} & 0 \\
* & * & \bar{\Upsilon}_{33}^{i}
\end{array}\right]
$$

where

$$
\begin{aligned}
& \bar{\Omega}_{i}= {\left[\begin{array}{ccccc}
\bar{\Omega}_{11}^{i} & \bar{\Omega}_{12}^{i} & \bar{\Omega}_{13}^{i} & 0 & \bar{\Omega}_{15}^{i} \\
* & \bar{\Omega}_{22}^{i} & 0 & 0 & 0 \\
* & * & \Omega_{33}^{i} & 0 & 0 \\
* & * & * & \Omega_{44}^{i} & 0 \\
* & * & * & * & \bar{\Omega}_{55}^{i}
\end{array}\right] } \\
& \bar{\Omega}_{12}^{i}=A_{i \sigma_{i}} Q_{i 0}, \quad \bar{\Omega}_{13}^{i}=A_{i \eta_{i}} \\
& \bar{\Omega}_{11}^{i}= A_{i} Q_{i 0}+Q_{i 0} A_{i}^{T}-B_{i}\left(K_{i}+\Delta K_{i}\right) Q_{i 0} \\
&-Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T}+Q_{i 1}+Q_{i 2} \\
&+E_{i}+e_{i 1} Q_{i 0} A_{i}^{T} E_{i} A_{i} Q_{i 0}+e_{i 1} Q_{i 0} \\
& \times\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T} E_{i} B_{i}\left(K_{i}+\Delta K_{i}\right) Q_{i 0} \\
&+Q_{i 0} \sum_{j=1, j \neq i}^{N} \frac{1}{1-l_{j}} W_{j i} Q_{i 0}, \\
& \bar{\Omega}_{15}^{i}=-B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right) Q_{i 0} \\
& \bar{\Omega}_{22}^{i}=-\left(1-f_{i}\right) Q_{i 1}+e_{i 1} Q_{i 0} A_{i \sigma_{i}}^{T} E_{i} A_{i \sigma_{i}} Q_{i 0}
\end{aligned}
$$

$$
\begin{gather*}
\bar{\Omega}_{55}^{i}=-\left(1-h_{i}\right) Q_{i 2}+e_{i 1} Q_{i 0} \\
\times\left(K_{i}+\Delta K_{i}\right)^{T} B_{i \delta_{i}}^{T} E_{i} B_{i \delta_{i}}\left(K_{i}+\Delta K_{i}\right) Q_{i 0} \\
\overline{\bar{Y}}_{13}^{i}=\left[\begin{array}{ccc}
Q_{i 0} A_{i}^{T}-Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T} & Q_{i 0} & -Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} \\
Q_{i 0} A_{i \sigma_{i}}^{T} & 0 & 0 \\
A_{i \eta_{i}}^{T} & 0 & 0 \\
0 & 0 & 0 \\
-Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i \delta_{i}}^{T} & 0 & 0
\end{array}\right] \tag{21}
\end{gather*}
$$

## Define

$$
\Theta_{i}=\left[\begin{array}{ccccc}
\overline{\bar{\Omega}}_{i} & 0 & \overline{\bar{\Upsilon}}_{13}^{i} & \Xi_{i} & \Theta_{15}^{i}  \tag{22}\\
* & \Gamma_{33}^{i} & 0 & 0 & 0 \\
* & * & \bar{\Upsilon}_{33}^{i} & 0 & 0 \\
* & * & * & \Theta_{44}^{i} & 0 \\
* & * & * & * & \Theta_{55}^{i}
\end{array}\right]
$$

where

$$
\begin{gathered}
\overline{\bar{\Omega}}_{i}=\left[\begin{array}{ccccc}
\overline{\bar{\Omega}}_{11}^{i} & \bar{\Omega}_{12}^{i} & \bar{\Omega}_{13}^{i} & 0 & \bar{\Omega}_{15}^{i} \\
* & \overline{\bar{\Omega}}_{22}^{i} & 0 & 0 & 0 \\
* & * & \overline{\bar{\Omega}}_{33}^{i} & 0 & 0 \\
* & * & * & \Omega_{44}^{i} & 0 \\
* & * & * & * & \overline{\bar{\Omega}}_{55}^{i}
\end{array}\right], \\
\Xi_{i}=\left[\begin{array}{ccccc}
\Xi_{11}^{i} & \Xi_{12}^{i} & 0 & 0 & 0 \\
0 & 0 & \Xi_{23}^{i} & 0 & 0 \\
0 & 0 & 0 & \Xi_{34}^{i} & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Xi_{55}^{i}
\end{array}\right], \\
\Theta_{15}^{i}=\left[\begin{array}{c}
\Gamma_{15}^{i} \\
0 \\
0 \\
0 \\
0
\end{array}\right], \\
\Theta_{44}^{i}=\operatorname{diag}\left\{-e_{i 1}^{-1} I_{n_{i}},-e_{i 1}^{-1} I_{n_{i},},-e_{i 1}^{-1} I_{n_{i}},\right. \\
\left.-e_{i 1}^{-1} I_{n_{i}},-e_{i 1}^{-1} I_{n_{i}}\right\}
\end{gathered}
$$

$$
\begin{gather*}
\overline{\bar{\Omega}}_{11}^{i}=A_{i} Q_{i 0}+Q_{i 0} A_{i}^{T}-B_{i}\left(K_{i}+\Delta K_{i}\right) Q_{i 0} \\
-Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T}+Q_{i 1}+Q_{i 2}+E_{i}, \\
\overline{\bar{\Omega}}_{22}^{i}=-\left(1-f_{i}\right) Q_{i 1}, \quad \overline{\bar{\Omega}}_{33}^{i}=-I_{n_{i}} \\
\overline{\bar{\Omega}}_{55}^{i}=-\left(1-h_{i}\right) Q_{i 2}, \quad \Xi_{11}^{i}=Q_{i 0} A_{i}^{T} E_{i}^{1 / 2} \\
\Xi_{12}^{i}=-Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i}^{T} E_{i}^{1 / 2} \\
\Xi_{23}^{i}=Q_{i 0} A_{i \sigma_{i}}^{T} E_{i}^{1 / 2}, \quad \Xi_{34}^{i}=A_{i \eta_{i}}^{T} E_{i}^{1 / 2} \\
\Xi_{55}^{i}=-Q_{i 0}\left(K_{i}+\Delta K_{i}\right)^{T} B_{i \delta_{i}}^{T} E_{i}^{1 / 2} \tag{23}
\end{gather*}
$$

The following equality is obvious:

$$
\begin{align*}
\Theta_{i}= & {\left[\begin{array}{ccccc}
\Gamma_{11}^{i} & \Gamma_{12}^{i} & 0 & \Gamma_{14}^{i} & \Gamma_{15}^{i} \\
* & \Gamma_{22}^{i} & 0 & \Gamma_{24}^{i} & 0 \\
* & * & \Gamma_{33}^{i} & 0 & 0 \\
* & * & * & \Gamma_{44}^{i} & 0 \\
* & * & * & * & \Gamma_{55}^{i}
\end{array}\right] }  \tag{24}\\
& +\Lambda_{i}^{T} R_{i 1} \$_{i}+\$_{i}^{T} R_{i 1}^{T} \Lambda_{i}
\end{align*}
$$

where

$$
\begin{align*}
& \Lambda_{i}=\left[\begin{array}{ccccc}
M_{i}^{T} B_{i}^{T} & 0 & 0 & \Lambda_{14}^{i} & 0 \\
M_{i}^{T} B_{i \delta_{i}}^{T} & 0 & 0 & \Lambda_{24}^{i} & 0
\end{array}\right], \\
& \$_{i}=\left[\begin{array}{ccccc}
-N_{i} Q_{i 0} & 0 & 0 & 0 & 0 \\
0 & \$_{22}^{i} & 0 & 0 & 0
\end{array}\right], \\
& R_{i 1}=\operatorname{diag}\left\{\bar{F}_{i}(t), \bar{F}_{i}(t)\right\},  \tag{25}\\
& \Lambda_{14}^{i}=\left[\begin{array}{llllllll}
M_{i}^{T} B_{i}^{T} & 0 & M_{i}^{T} & 0 & M_{i}^{T} B_{i}^{T} E_{i}^{1 / 2} & 0 & 0 & 0
\end{array}\right] \text {, } \\
& \Lambda_{24}^{i}=\left[\begin{array}{lllllllll}
M_{i}^{T} & B_{i \delta_{i}}^{T} & 0 & 0 & 0 & 0 & 0 & 0 & M_{i}^{T} B_{i \delta_{i}}^{T} E_{i}^{1 / 2}
\end{array}\right] \text {, } \\
& \$_{22}^{i}=\left[\begin{array}{llll}
0 & 0 & 0 & -N_{i} Q_{i 0}
\end{array}\right] .
\end{align*}
$$

By Lemma 1 and Schur complement formula, the condition $\Gamma_{i}<0$ in (10) is equivalent to $\Theta_{i}<0$ in (24). By Schur complement formula with $\Theta_{i}<0$, one can obtain $\overline{\bar{Y}}_{i}<0$ in (20). The condition $\overline{\bar{Y}}_{i i}<0$ is equivalent to $\bar{Y}_{i}<0$. Again, by Schur complement formula with $\bar{\Upsilon}_{i}<0$, one can obtain $\Upsilon_{i}<0$. From the condition $\Upsilon_{i}<0$ in (17), there exists a constant $\rho_{i}>0$, such that

$$
\begin{equation*}
\dot{V}(x(t), t) \leq \sum_{i=1}^{N}-\rho_{i}\left\|x_{i}(t)\right\|^{2} \tag{26}
\end{equation*}
$$

By conditions (13) and (26) and $\left\|A_{i \eta_{i}}\right\|<1$, one can conclude that system (9) with (2) and (4) is asymptotically stable. From (17) with $\Upsilon_{i}<0$, one can obtain

$$
\begin{align*}
\int_{0}^{\infty} \dot{V} & (x(t), t) d t \\
& =\lim _{t \rightarrow \infty} V(x(t), t)-V(x(0), 0)  \tag{27}\\
& \leq-\sum_{i=1}^{N} \int_{0}^{\infty}\left[x_{i}^{T}(t) S_{i 1} x_{i}(t)+u_{i}^{T}(t) S_{i 2} u_{i}(t)\right] d t .
\end{align*}
$$

Therefore, the following equalities hold:

$$
\begin{gather*}
\lim _{t \rightarrow \infty} V(x(t), t)=0, \\
\sum_{i=1}^{N} \int_{0}^{\infty}\left[x_{i}^{T}(t) S_{i 1} x_{i}(t)+u_{i}^{T}(t) S_{i 2} u_{i}(t)\right] d t  \tag{28}\\
\leq V(x(0), 0)=J_{u} .
\end{gather*}
$$

This completes the proof.
Remark 4. It is obvious that for every subsystem, the corresponding $\Gamma_{i}$ in (10) is an LMI with obtained matrices $\bar{W}_{j i}\left(\bar{W}_{j i}=W_{j i}^{-1}\right)$ and $W_{i j}$ in the last inequality (i.e., the inequality $\Gamma_{i-1}<0$ ). Hence, the decentralized nonfragile control (4) and the guaranteed cost $J_{u}$ in (11) can be obtained by finding feasible set to $\Gamma_{i}<0$ with feasp in [22] one by one.

Remark 5. Obviously, the guaranteed cost $J_{u}$ in (11) cannot be directly optimized by using the toolbox of mincx in [22]. One reason is that inequalities (10) with variable matrices $W_{i j}$ and $\bar{W}_{j i}\left(\bar{W}_{j i}=W_{j i}^{-1}\right)$ are not a group of LMIs but $N$ coupled nonlinear inequalities. Another reason is that $J_{u}$ is a nonconvex function with respect to the optimization variables.

The following algorithm is given to solve the nonlinear problem of inequalities (10).

Algorithm 6. Choose constant matrices $W_{i j}>0$ and $\bar{W}_{j i}>$ 0 satisfying $\Psi_{j j}^{i}<0$ in $\Gamma_{i}$, where $\bar{W}_{j i}=W_{j i}^{-1}, i, j=$ $1,2 \ldots, N, j \neq i$.

It is needed to simultaneously select $N \times(N-1)$ constant matrices $W_{i j}>0$ and $\bar{W}_{j i}>0\left(\bar{W}_{j i}=W_{j i}^{-1}\right)$ satisfying $\Psi_{j j}^{i}<$ 0 . For simplicity, one can choose $W_{i j}$ and $\bar{W}_{j i}$ to be positive definite diagonal matrices according to the eigenvalues of $e_{i 2} I_{n_{j}}+e_{i 1} A_{i j}^{T} A_{i j}$ due to $\Psi_{j j}^{i}=e_{i 2} I_{n_{j}}-W_{i j}+e_{i 1} A_{i j}^{T} A_{i j}$. The chosen entries need to be as small as possible, because

$$
\begin{equation*}
\frac{1}{1-l_{i}} \sum_{j=1, j \neq i}^{N} \int_{-\tau_{i j}(0)}^{0} x_{j}^{T}(s) W_{i j} x_{j}(s) d s \tag{29}
\end{equation*}
$$

is involved in $J_{u}$. However, if there is no solution to inequalities (10), the large scalars can be considered.

In the sequel, instead of solving the nonconvex optimization problem, a suboptimal method of minimizing the guaranteed cost $J_{u}$, based on Algorithm 6, is presented.

Theorem 7. Consider unperturbed system (9) with cost function (6), and assume $\left\|A_{i \eta_{i}}\right\|<1$. If the following optimization problem:

$$
\begin{equation*}
\min \sum_{i=1}^{N}\left[\alpha_{i}+\operatorname{Tr}\left(U_{i 1}^{T} \Phi_{i 1} U_{i 1}+U_{i 2}^{T} \Phi_{i 2} U_{i 2}\right)\right] \tag{30}
\end{equation*}
$$

subject to LMI (10) with Algorithm 6, and

$$
\begin{gather*}
{\left[\begin{array}{cc}
-\alpha_{i} & \phi_{i}^{T}(0) \\
* & -Q_{i 0}
\end{array}\right]<0,} \\
{\left[\begin{array}{cc}
-2 Q_{i 0}+Q_{i 1} & I_{n_{i}} \\
* & -\Phi_{i 1}
\end{array}\right]<0,}  \tag{31}\\
{\left[\begin{array}{cc}
-2 Q_{i 0}+Q_{i 2} & I_{n_{i}} \\
* & -\Phi_{i 2}
\end{array}\right]<0}
\end{gather*}
$$

has a solution set $\left(\alpha_{i}>0, \varepsilon_{i 1}>0, Q_{i k}>0(k=0,1,2), \Phi_{i 1}>\right.$ $\left.0, \Phi_{i 2}>0, X_{i}\right)$, where $\int_{-\sigma_{i}(0)}^{0} \phi_{i}(s) \phi_{i}^{T}(s) d s=U_{i 1} U_{i 1}^{T}, \int_{-\delta_{i}(0)}^{0}$ $\phi_{i}(s) \phi_{i}^{T}(s) d s=U_{i 2} U_{i 2}^{T},\left(1 /\left(1-g_{i}\right)\right) \int_{-\eta_{i}(0)}^{0} \dot{\phi}_{i}^{T}(s) \dot{\phi}_{i}(s) d s=$ $L_{i 1}, \quad f_{i 0} \int_{-f_{i 0}}^{0}\left(s+f_{i 0}\right) \dot{\phi}_{i}^{T}(s) \dot{\phi}_{i}(s) d s=L_{i 2},\left(1 /\left(1-l_{i}\right)\right)$ $\sum_{j=1, j \neq i}^{N} \int_{-\tau_{i j}(0)}^{0} \phi_{j}^{T}(s) W_{i j} \phi_{j}(s) d s=L_{i 3}$, then control (4) with $K_{i}=X_{i} Q_{i 0}^{-1}$ is the decentralized nonfragile guaranteed cost control of unperturbed system (9) with the minimization of the guaranteed cost $J_{u}$ as follows:

$$
\begin{align*}
J^{*}= & \min \left(\sum_{i=1}^{N}\left[\alpha_{i}+\operatorname{Tr}\left(U_{i 1}^{T} \Phi_{i 1} U_{i 1}+U_{i 2}^{T} \Phi_{i 2} U_{i 2}\right)\right]\right) \\
& +\sum_{i=1}^{N}\left(L_{i 1}+L_{i 2}+L_{i 3}\right) . \tag{32}
\end{align*}
$$

Proof. Applying the Schur complement formula to LMIs (31) leads to $\phi_{i}^{T}(0) Q_{i 0}^{-1} \phi_{i}(0)<\alpha_{i},-2 Q_{i 0}+Q_{i 1}+\Phi_{i 1}^{-1}<0,-2 Q_{i 0}+$ $Q_{i 2}+\Phi_{i 2}^{-1}<0$, respectively.

Noting that [8]

$$
\begin{align*}
& {\left[Q_{i 0}-\Phi_{i 1}^{-1}\right] \Phi_{i 1}\left[Q_{i 0}-\Phi_{i 1}^{-1}\right]=Q_{i 0} \Phi_{i 1} Q_{i 0}-2 Q_{i 0}+\Phi_{i 1}^{-1} \geq 0} \\
& {\left[Q_{i 0}-\Phi_{i 2}^{-1}\right] \Phi_{i 2}\left[Q_{i 0}-\Phi_{i 2}^{-1}\right]=Q_{i 0} \Phi_{i 2} Q_{i 0}-2 Q_{i 0}+\Phi_{i 2}^{-1} \geq 0} \tag{33}
\end{align*}
$$

the following inequalities are obtained

$$
\begin{align*}
& P_{i 1}=Q_{i 0}^{-1} Q_{i 1} Q_{i 0}^{-1}<\Phi_{i 1}, \\
& P_{i 2}=Q_{i 0}^{-1} Q_{i 2} Q_{i 0}^{-1}<\Phi_{i 2} . \tag{34}
\end{align*}
$$

Further, one can obtain

$$
\begin{align*}
& \int_{-\sigma_{i}(0)}^{0} \phi_{i}^{T}(s) P_{i 1} \phi_{i}(s) d s \\
& =\operatorname{Tr}\left(\int_{-\sigma_{i}(0)}^{0} \phi_{i}(s) \phi_{i}^{T}(s) P_{i 1}\right) d s \\
& =\operatorname{Tr}\left(U_{i 1}^{T} P_{i 1} U_{i 1}\right) \leq \operatorname{Tr}\left(U_{i 1}^{T} \Phi_{i 1} U_{i 1}\right), \\
& \int_{-\delta_{i}(0)}^{0} \phi_{i}^{T}(s) P_{i 2} \phi_{i}(s) d s \\
& =\operatorname{Tr}\left(\int_{-\delta_{i}(0)}^{0} \phi_{i}(s) \phi_{i}^{T}(s) P_{i 2}\right) d s  \tag{35}\\
& =\operatorname{Tr}\left(U_{i 2}^{T} P_{i 2} U_{i 2}\right) \leq \operatorname{Tr}\left(U_{i 2}^{T} \Phi_{i 2} U_{i 2}\right), \\
& \frac{1}{1-g_{i}} \int_{-\eta_{i}(0)}^{0} \dot{\phi}_{i}^{T}(s) \dot{\phi}_{i}(s) d s=L_{i 1}, \\
& f_{i 0} \int_{-f_{i 0}}^{0}\left(s+f_{i 0}\right) \dot{\phi}_{i}^{T}(s) \dot{\phi}_{i}(s) d s=L_{i 2}, \\
& \frac{1}{1-l_{i}} \sum_{j=1, j \neq i}^{N} \int_{-\tau_{i j}(0)}^{0} \phi_{j}^{T}(s) W_{i j} \phi_{j}(s) d s=L_{i 3} .
\end{align*}
$$

Therefore, it follows from (11) that

$$
\begin{align*}
J_{u} \leq & \sum_{i=1}^{N}\left[\alpha_{i}+\operatorname{Tr}\left(U_{i 1}^{T} \Phi_{i 1} U_{i 1}+U_{i 2}^{T} \Phi_{i 2} U_{i 2}\right)\right]  \tag{36}\\
& +\sum_{i=1}^{N}\left(L_{i 1}+L_{i 2}+L_{i 3}\right)
\end{align*}
$$

The minimization of the right hand of inequality (36) implies the minimization of the guaranteed cost $J_{u}$ for unperturbed system (9). This completes the proof.

### 3.2. Nonfragile Guaranteed Cost Control for Uncertain Neutral Interconnected Systems

Theorem 8. Consider uncertain neutral interconnected systems (1) with (2), (3), and (4). If there exist positive numbers $\varepsilon_{i 1}>0, \varepsilon_{i 2}>0$, and $\varepsilon_{i 3}>0$, some symmetric positive definite matrices $Q_{i k}(k=0,1,2), \bar{W}_{j i}, W_{i j}$, and matrix $X_{i}$ such that the following inequalities hold:

$$
\begin{gather*}
\overline{\bar{\Gamma}}_{i}=\left[\begin{array}{cc}
\Gamma_{i} & \bar{\Gamma}_{i} \\
* & \widetilde{\Gamma}_{i}
\end{array}\right]<0,  \tag{37}\\
{\left[\begin{array}{ccc}
-I_{n_{i}}+\varepsilon_{i 3} D_{i \eta_{i}}^{T} D_{i \eta_{i}} & A_{i \eta_{i}}^{T} & 0 \\
* & -I_{n_{i}} & C_{i} \\
* & * & -\varepsilon_{i 3} I_{n_{i}}
\end{array}\right]<0,} \tag{38}
\end{gather*}
$$

then control (4) with $K_{i}=X_{i} Q_{i 0}^{-1}$ is the decentralized nonfragile guaranteed cost control of uncertain neutral interconnected
systems (1) with the guaranteed cost in (11), where $\bar{\Gamma}_{i}=$ $\left[\begin{array}{ll}\varepsilon_{i 2} \Pi_{i 1} & \Pi_{i 2}\end{array}\right]$,

$$
\begin{align*}
& \Pi_{i 1}=\left[\begin{array}{c}
\bar{\Gamma}_{11}^{i} \\
0 \\
0 \\
\bar{\Gamma}_{41}^{i} \\
0 \\
0
\end{array}\right], \quad \Pi_{i 2}=\left[\begin{array}{c}
\bar{\Gamma}_{12}^{i} \\
\bar{\Gamma}_{22}^{i} \\
0 \\
0 \\
0 \\
\bar{\Gamma}_{62}^{i}
\end{array}\right], \\
& \bar{\Gamma}_{22}^{i}=\left[\begin{array}{cccccc}
0 & 0 & Q_{i 0} D_{i \sigma_{i}}^{T} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{i \eta_{i}}^{T} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -X_{i}^{T} D_{i \delta_{i}}^{T} & 0
\end{array}\right], \\
& \bar{\Gamma}_{11}^{i}=\left[\begin{array}{llllll}
C_{i} & C_{i} & C_{i} & C_{i} & C_{i} & C_{i}
\end{array}\right], \\
& \bar{\Gamma}_{12}^{i}=\left[\begin{array}{llllll}
Q_{i 0} D_{i 1}^{T} & -X_{i}^{T} D_{i 2}^{T} & 0 & 0 & 0 & 0
\end{array}\right], \\
& \bar{\Gamma}_{41}^{i}=\left[\begin{array}{cccccc}
C_{i} & C_{i} & C_{i} & C_{i} & C_{i} & C_{i} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
E_{i}^{1 / 2} C_{i} & 0 & 0 & 0 & 0 & 0 \\
0 & E_{i}^{1 / 2} C_{i} & 0 & 0 & 0 & E_{i}^{1 / 2} C_{i} \\
0 & 0 & E_{i}^{1 / 2} C_{i} & 0 & 0 & 0 \\
0 & 0 & 0 & E_{i}^{1 / 2} C_{i} & 0 & 0 \\
0 & 0 & 0 & 0 & E_{i}^{1 / 2} C_{i} & 0
\end{array}\right], \\
& \bar{\Gamma}_{62}^{i}=\left[\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & \varepsilon_{i 1} M_{i}^{T} D_{i 2}^{T} \\
0 & 0 & 0 & 0 & \varepsilon_{i 1} M_{i}^{T} D_{i \delta_{i}}^{T} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right], \\
& \widetilde{\Gamma}_{i}=\operatorname{diag}\{\underbrace{-\varepsilon_{i 2} I_{n_{i}}, \ldots,-\varepsilon_{i 2} I_{n_{i}}}_{12}\} . \tag{39}
\end{align*}
$$

Proof. From condition (10) with unperturbed neutral interconnected systems (9), one can obtain the corresponding condition to stabilize uncertain neutral interconnected systems (1) as follows:

$$
\begin{equation*}
\Sigma_{i}=\Gamma_{i}+\Pi_{i 1} R_{i 2} \Pi_{i 2}^{T}+\Pi_{i 2} R_{i 2}^{T} \Pi_{i 1}^{T}<0, \tag{40}
\end{equation*}
$$

where $R_{i 2}=\operatorname{diag}\left\{F_{i}(t), F_{i}(t), F_{i}(t), F_{i}(t), F_{i}(t), F_{i}(t)\right\}$.
By Lemma 1 and Schur complement formula, the condition $\overline{\bar{\Gamma}}_{i}<0$ in (37) is equivalent to $\Sigma_{i}<0$ in (40). For the same reason, (38) is equivalent to

$$
\begin{equation*}
\left[A_{i \eta_{i}}+\Delta A_{i \eta_{i}}(t)\right]^{T}\left[A_{i \eta_{i}}+\Delta A_{i \eta_{i}}(t)\right]<I_{n_{i}} . \tag{41}
\end{equation*}
$$

This implies that uncertain neutral interconnected systems (1) are Lipschitz in the term $\dot{x}_{i}\left(t-\eta_{i}(t)\right)$ with Lipschitz constant less than 1 [8]. By the same derivation of Theorem 3, one can complete this proof.

The decentralized nonfragile guaranteed cost control (4) and the minimization of the guaranteed cost $J_{u}$ for uncertain
neutral interconnected systems (1) are determined by the following theorem.

Theorem 9. Consider uncertain neutral interconnected systems (1) with (2), (3), (4), and cost function (6). If the following optimization problem:

$$
\begin{equation*}
\min \sum_{i=1}^{N}\left[\alpha_{i}+\operatorname{Tr}\left(U_{i 1}^{T} \Phi_{i 1} U_{i 1}+U_{i 2}^{T} \Phi_{i 2} U_{i 2}\right)\right] \tag{42}
\end{equation*}
$$

is subject to LMI (37) with Algorithm 6, (38), and (31) has a solution set $\left(\alpha_{i}>0, \varepsilon_{i 1}>0, \varepsilon_{i 2}>0, \varepsilon_{i 3}>0, Q_{i k}>\right.$ $\left.0(k=0,1,2), \Phi_{i 1}>0, \Phi_{i 2}>0, X_{i}\right)$, then control (4) with $K_{i}=X_{i} Q_{i 0}^{-1}$ is the decentralized nonfragile guaranteed cost control for uncertain neutral interconnected systems (1) with the minimization $J^{*}$ of the guaranteed cost $J_{u}$ in (32).

Remark 10. Reference [18] develops a scheme of counteracting the interconnections to simplify the problem, which may add conservatism in some cases. Compared with the approach of treating the interconnections in [18], we utilize an approach of magnifying the terms associated interconnections; for details, one can see the derivation of inequality (16). To some extent, it may reduce the conservatism of the results derived in the paper.

## 4. Illustrative Examples

In this section, some examples are presented to show the validity of the control approach and the advantages of the schemes of dealing with the interconnections.

Example 1. To illustrate the design method of the decentralized nonfragile guaranteed cost control and the optimization approach of the guaranteed cost for uncertain neutral interconnected system, consider uncertain neutral interconnected systems (1) composed of two third-order subsystems:

$$
\begin{gathered}
A_{1}=\left[\begin{array}{ccc}
1.1221 & 70.1414 & -5.1247 \\
4.1437 & -1.1203 & 3.1243 \\
2.4589 & -0.5671 & -2.2548
\end{array}\right], \\
A_{1 \sigma_{1}}=\left[\begin{array}{ccc}
-0.0321 & 0.0012 & -0.0123 \\
0.1325 & -0.0321 & -0.0246 \\
0.0348 & 0.0023 & 0.0236
\end{array}\right], \\
A_{1 \eta_{1}}=\left[\begin{array}{ccc}
0.2236 & -0.2011 & -0.0321 \\
0.2134 & 0.0271 & -0.1282 \\
0.0123 & 0.5621 & -0.0124
\end{array}\right], \\
B_{1}=\left[\begin{array}{cc}
-2.1231 & -4.0126 \\
-1.1245 & 3.4725 \\
0.1243 & -9.3417
\end{array}\right],
\end{gathered}
$$

$$
B_{1 \delta_{1}}=\left[\begin{array}{cc}
0.1012 & -0.0219 \\
0.1427 & -0.0537 \\
-0.0531 & 0.05324
\end{array}\right]
$$

$$
A_{12}=\left[\begin{array}{ccc}
-0.0898 & 0.0161 & -0.0682 \\
-0.0359 & 0.0205 & -0.0542 \\
-0.0205 & 0.0176 & 0.0814
\end{array}\right]
$$

$$
C_{1}=\left[\begin{array}{ccc}
0.0680 & -0.0655 & 0.0283 \\
-0.0086 & -0.0381 & 0.0889 \\
0.0422 & 0.0088 & 0.0366
\end{array}\right]
$$

$$
D_{11}=\left[\begin{array}{ccc}
-0.0051 & 0.0429 & 0.0464 \\
0.0792 & -0.0749 & -0.0321 \\
-0.0579 & 0.0900 & 0.0946
\end{array}\right]
$$

$$
D_{1 \sigma_{1}}=\left[\begin{array}{ccc}
-0.0212 & 0.0481 & -0.0933 \\
-0.0991 & 0.0896 & 0.0941 \\
-0.0347 & -0.0204 & -0.0837
\end{array}\right],
$$

$$
D_{12}=\left[\begin{array}{cc}
0.0928 & -0.0609 \\
-0.0390 & 0.0897 \\
-0.0874 & -0.0064
\end{array}\right]
$$

$$
D_{1 \eta_{1}}=\left[\begin{array}{ccc}
-0.0501 & 0.0860 & -0.0565 \\
-0.0311 & -0.0913 & 0.0185 \\
0.0068 & 0.0621 & -0.0214
\end{array}\right]
$$

$$
M_{1}=\left[\begin{array}{lll}
0.01 & 0.01 & 0.01 \\
0.01 & 0.01 & 0.01
\end{array}\right]
$$

$$
D_{1 \delta_{1}}=\left[\begin{array}{cc}
-0.0175 & 0.0086 \\
0.0172 & 0.0621 \\
0.0142 & -0.0739
\end{array}\right]
$$

$$
N_{1}=\operatorname{diag}\{1,1,1\}
$$

$$
\sigma_{1}(t)=0.1 *(2+\sin (t))
$$

$$
\eta_{1}(t)=0.2 *(1+\cos (t)),
$$

$$
\delta_{1}(t)=0.3 *(1+\sin (t)),
$$

$$
\tau_{12}(t)=0.1 *(1+\cos (t))
$$

$$
A_{2}=\left[\begin{array}{ccc}
8.1906 & 0.4571 & 2.5678 \\
-0.4724 & -4.4540 & 1.4527 \\
0.4561 & -2.4561 & -5.9568
\end{array}\right]
$$

$$
A_{2 \sigma_{2}}=\left[\begin{array}{ccc}
0.0614 & 0.0973 & -0.0627 \\
-0.0819 & -0.0535 & -0.0848 \\
-0.0844 & -0.0895 & 0.0602
\end{array}\right],
$$

$$
A_{2 \eta_{2}}=\left[\begin{array}{ccc}
0.1147 & -0.0218 & 0.0157 \\
-0.1254 & 0.0282 & -0.0515 \\
0.0919 & -0.0763 & -0.2169
\end{array}\right]
$$

$$
B_{2}=\left[\begin{array}{cc}
9.7954 & -1.3341 \\
-7.5894 & -1.0482 \\
-0.0893 & -0.3494
\end{array}\right],
$$

$$
\begin{gather*}
B_{2 \delta_{2}}=\left[\begin{array}{ccc}
-0.0349 & -0.0189 \\
-0.0854 & -0.0231 \\
0.0312 & 0.0993
\end{array}\right], \\
A_{21}=\left[\begin{array}{ccc}
-0.2898 & 0.3161 & -0.0682 \\
-0.0359 & 0.0205 & -0.0542 \\
-0.0205 & 0.3176 & 0.3814
\end{array}\right], \\
C_{2}=\left[\begin{array}{ccc}
0.0387 & 0.0738 & 0.0668 \\
-0.0441 & 0.0863 & -0.0823 \\
-0.0909 & 0.0750 & 0.0727
\end{array}\right], \\
D_{21}=\left[\begin{array}{ccc}
0.0417 & 0.0333 & 0.0483 \\
0.0805 & -0.0888 & 0.0914 \\
0.0205 & -0.0677 & 0.0988
\end{array}\right], \\
D_{2 \sigma_{2}}=\left[\begin{array}{ccc}
-0.0149 & -0.0275 & -0.0303 \\
-0.0488 & 0.0497 & -0.0892 \\
-0.0614 & 0.0711 & -0.0632
\end{array}\right], \\
D_{22}=\left[\begin{array}{ccc}
-0.0860 & -0.0073 \\
0.0380 & 0.0673 \\
0.0462 & 0.0578
\end{array}\right], \\
D_{2 \eta_{2}}=\left[\begin{array}{ccc}
-0.0103 & -0.0529 & -0.0389 \\
0.0136 & 0.0132 & -0.0028 \\
0.0579 & -0.0099 & -0.0987
\end{array}\right], \\
D_{2 \delta_{2}}=\left[\begin{array}{cc}
0.0566 & 0.0541 \\
-0.0304 & -0.0335 \\
0.0894 & 0.0378
\end{array}\right], \\
M_{2}=M_{1}, \\
N_{2}=N_{1}, \\
\sigma_{2}(t)=0.12 *(1+\cos (t)), \\
\eta_{2}(t)=0.1 *(2+\sin (t)), \\
\tau_{21}(t)=0.2 *(1+\cos (t)),  \tag{43}\\
0.2 *(2+\sin (t)) \\
\hline
\end{gather*},
$$

Let

$$
\begin{gather*}
S_{11}=\operatorname{diag}\{0.3,0.3,0.3\}, \\
S_{12}=\operatorname{diag}\{0.5,0.5\},  \tag{44}\\
S_{21}=\operatorname{diag}\{0.1,0.1,0.1\}, \\
S_{22}=\operatorname{diag}\{1,1\},
\end{gather*}
$$

and give the following initial condition:

$$
\begin{align*}
& \phi_{1}(t)=\left[\begin{array}{lll}
-0.1 e^{2 t} & 0.15 & 0.2 t+0.05
\end{array}\right]^{T}  \tag{45}\\
& \phi_{2}(t)=\left[\begin{array}{lll}
0.2 e^{t} & -0.06+0.2 t & 0.06
\end{array}\right]^{T}
\end{align*}
$$

According to Algorithm 6, $W_{12}$ and $W_{21}$ are chosen as follows:

$$
\begin{align*}
& W_{12}=\operatorname{diag}\{7.7378,7.7123,7.7253\}, \\
& W_{21}=\operatorname{diag}\{6.9514,9.0184,7.2122\} . \tag{46}
\end{align*}
$$



Figure 1: State response of the first open-loop subsystem.

Solving the optimization problem (42) subject to condition LMI (37) with Algorithm 6, (38), and (31), one can obtain

$$
\begin{gather*}
K_{1}=\left[\begin{array}{ccc}
-3.9594 & -1.2404 & -1.2278 \\
-0.6353 & -0.0710 & -0.6896
\end{array}\right], \\
K_{2}=\left[\begin{array}{ccc}
1.0313 & 0.0551 & 0.1222 \\
-4.5397 & 0.3862 & -1.8594
\end{array}\right],  \tag{47}\\
J^{*}=1.3053
\end{gather*}
$$

The simulation results are shown in Figures 1-6 based on the above parameters. From Figures 1 and 2, one can see that the uncertain neutral systems (1) without controller are divergent. From Figures 3 and 4, one can see that the nonlinear neutral systems (1) with control law (4) are indeed well stabilized. The control signals $u_{1}(t)$ and $u_{2}(t)$ are rather smooth in Figures 5 and 6.

Example 2. To the best of the authors' knowledge, the nonfragile control and optimization for neutral interconnected systems have not been studied. But in order to show the advantages of the schemes of dealing with the interconnections, the authors have to simplify the model of neutral interconnected systems (1) to compare with the existing results.

In contrast to the model of system (13a) in [18], let

$$
\begin{array}{cc}
A_{1 \eta_{1}}=0, & D_{1 \eta_{1}}=0 \\
M_{1}=0, & N_{1}=0 \\
\sigma_{1}(t)=0.3, & \eta_{1}(t)=0
\end{array}
$$



Figure 2: State response of the second open-loop subsystem.


Figure 3: State response of the first closed-loop subsystem.

$$
\begin{gather*}
\delta_{1}(t)=0.6, \quad \tau_{12}(t)=0 \\
A_{2 \eta_{2}}=0, \quad D_{2 \eta_{2}}=0 \\
M_{2}=0, \quad N_{2}=0, \quad \sigma_{2}(t)=0.24 \\
\eta_{2}(t)=0, \quad \delta_{2}(t)=0.3, \quad \tau_{21}(t)=0 \tag{48}
\end{gather*}
$$

and other parameters be the same as Example 1.


Figure 4: State response of the second closed-loop subsystem.


Figure 5: Control signal of the first subsystem.

Solving the optimization problem (42) subject to condition LMI (37) with Algorithm 6 and (31), the minimization of the guaranteed cost $J_{u}$ is given by

$$
\begin{equation*}
J^{*}=0.9429 \tag{49}
\end{equation*}
$$

Since system (13a) in [18] is a nonlinear large-scale system, the authors choose the nonlinear vector function $g_{i j}=W_{i j} x_{j}$ satisfying the assumptions in [18] and $G_{i j}=\operatorname{diag}\{1,1,1\}$, $D_{i j}=E_{i j}=0$. By Theorem 4.2 in [18], one can obtain the minimization $J^{*}$ as follows:

$$
\begin{equation*}
J^{*}=2.2627 \tag{50}
\end{equation*}
$$



Figure 6: Control signal of the second subsystem.

Remark 11. It is clear from Example 2 that the minimization of the guaranteed cost provided by Theorem 9 in this paper is less than that of [18]. Viewing from this point, the results derived in this paper have the less conservatism.

## 5. Conclusion

The nonfragile guaranteed cost control and optimization are complex and challenging for uncertain interconnected systems of neutral type. In this paper, the sufficient conditions for the existence of the decentralized nonfragile guaranteed cost control for unperturbed and uncertain neutral interconnected systems are derived, which are presented in terms of coupled nonlinear inequalities. A novel algorithm is proposed to solve the nonlinear problems of coupled inequalities (10). Also, a good optimization scheme is introduced to solve the nonconvex problem of the guaranteed cost. Two numerical examples with the corresponding simulation results and the comparison results have elucidated the validity of the present control approach and the advantages of the schemes of dealing with the interconnections over the existing results in the literature.

## Acknowledgments

This research is supported by Natural Science Foundation of China (no. 61104106), Science Foundation of Department of Education of Liaoning Province (no. L2012422), and Start-up Fund for Doctors of Shenyang University (no. 1120212340).

## References

[1] V. Kolmanovskii and A. Myshkis, Introduction to the Theory and Applications of Functional Differential Equations, Kluwer Academic, Dordrecht, The Netherlands, 1999.
[2] S. Zhou and L. Zhou, "Improved exponential stability criteria and stabilisation of T-S model-based neutral systems," IET Control Theory \& Applications, vol. 4, no. 12, pp. 2993-3002, 2010.
[3] L. Huang and X. Mao, "Delay-dependent exponential stability of neutral stochastic delay systems," IEEE Transactions on Automatic Control, vol. 54, no. 1, pp. 147-152, 2009.
[4] A. Alif, M. Darouach, and M. Boutayeb, "Design of robust $H_{\infty}$ reduced-order unknown-input filter for a class of uncertain linear neutral systems," IEEE Transactions on Automatic Control, vol. 55, no. 1, pp. 6-19, 2010.
[5] H. R. Karimi, "Robust delay-dependent $H_{\infty}$ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters," IEEE Transactions on Circuits and Systems I, vol. 58, no. 8, pp. 1910-1923, 2011.
[6] W. H. Chen, W. X. Zheng, and Y. Shen, "Delay-dependent stochastic stability and $H_{\infty}$-control of uncertain neutral stochastic systems with time delay," IEEE Transactions on Automatic Control, vol. 54, no. 7, pp. 1660-1667, 2009.
[7] J. Yang, W. Luo, G. Li, and S. Zhong, "Reliable guaranteed cost control for uncertain fuzzy neutral systems," Nonlinear Analysis: Hybrid Systems, vol. 4, no. 4, pp. 644-658, 2010.
[8] C. H. Lien, "Non-fragile guaranteed cost control for uncertain neutral dynamic systems with time-varying delays in state and control input," Chaos, Solitons and Fractals, vol. 31, no. 4, pp. 889-899, 2007.
[9] C. H. Lien and K. W. Yu, "Non-fragile $H_{\infty}$ control for uncertain neutral systems with time-varying delays via the LMI optimization approach," IEEE Transactions on Systems, Man, Cybernetics B, vol. 37, no. 2, pp. 493-499, 2007.
[10] S. Y. Xu, J. Lam, J. L. Wang, and G. H. Yang, "Non-fragile positive real control for uncertain linear neutral delay systems," Systems \& Control Letters, vol. 52, no. 1, pp. 59-74, 2004.
[11] S. I. Niculescu, Delay Effects on Stability: A Robust Control Approach, vol. 269 of Lecture Notes in Control and Information Sciences, Springer, Berlin, Germany, 2001.
[12] R. K. Brayton, "Bifurcation of periodic solutions in a nonlinear difference-differential equations of neutral type," Quarterly of Applied Mathematics, vol. 24, pp. 215-224, 1966.
[13] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, vol. 191 of Mathematics in Science and Engineering, Academic Press, Boston, Mass, USA, 1993.
[14] T.-J. Tarn, T. Yang, X. Zeng, and C. Guo, "Periodic output feedback stabilization of neutral systems," IEEE Transactions on Automatic Control, vol. 41, no. 4, pp. 511-521, 1996.
[15] G. Chen, Z. Xiang, and M. S. Mahmoud, "Stability and $H_{\infty}$ performance analysis of switched stochastic neutral systems," Circuits, Systems, and Signal Processing, vol. 32, no. 1, pp. 387400, 2013.
[16] Z. Xiang, Y. N. Sun, and M. S. Mahmoud, "Robust finitetime $H_{\infty}$ control for a class of uncertain switched neutral systems," Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 4, pp. 1766-1778, 2012.
[17] Z. Xiang, Y. N. Sun, and Q. Chen, "Robust reliable stabilization of uncertain switched neutral systems with delayed switching," Applied Mathematics and Computation, vol. 217, no. 23, pp. 9835-9844, 2011.
[18] H. Mukaidani, "An LMI approach to decentralized guaranteed cost control for a class of uncertain nonlinear large-scale delay systems," Journal of Mathematical Analysis and Applications, vol. 300, no. 1, pp. 17-29, 2004.
[19] M. S. Mahmoud and Y. Xia, "A generalized approach to stabilization of linear interconnected time-delay systems," Asian Journal of Control, vol. 14, no. 6, pp. 1539-1552, 2012.
[20] G. Pujol, J. Rodellar, J. Rossell, and F. Pozo, "Decentralised reliable guaranteed cost control of uncertain systems: an LMI design," IET Control Theory and Applications, vol. 1, no. 3, pp. 779-785, 2007.
[21] D. Zhao, Q. L. Zhang, H. L. Hu, and C. Y. Zhao, "Nonfragile guaranteed cost control for uncertain neutral large-scale interconnected systems," Journal of Systems Engineering and Electronics, vol. 21, no. 4, pp. 635-642, 2010.
[22] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, LMI Control Toolbox User's Guide, The Math Works, Natick, Mass, USA, 1995.


Advances in Operations Research $-$


The Scientific World Journal


Advances in
Decision Sciences
= -


## Hindawi

Submit your manuscripts at
http://www.hindawi.com


Mathematical Problems in Engineering


Journal of Function Spaces
$\underline{=}$



International Journal of Differential Equations 5


