Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 594340, 11 pages
http://dx.doi.org/10.1155/2013/594340

Research Article

Delay-Dependent Finite-Time and L_2-Gain Analysis for Switched Systems with Time-Varying Delay

Qiongfen Yang

School of Mathematics and Computer Science, Mianyang Normal University, Mianyang, Sichuan 621000, China

Correspondence should be addressed to Qiongfen Yang; myyqf@163.com

Received 10 October 2013; Accepted 23 November 2013

Academic Editor: Zhengguang Wu

For the switched systems, switching behavior always affects the finite-time stability (FTS) property, which was neglected by most previous studies. This paper is mainly concerned with the problem of delay-dependent finite-time and L_2-gain analysis for switched systems with time-varying delay. Several less conservative sufficient conditions related to finite-time stability and boundness of switched system with time-varying delays are proposed; the system trajectory stays within a special bound with the information of switching signal. At last, a numerical example is also given to illustrate the efficiency of the developed method.

1. Introduction

During the past decades, the systems with time delays have received much attention since they often fall in various practical systems, for instance, neural networks, networked control systems, engineering systems, biology, economics, and other fields. However, time delay is always frequently the major cause of oscillation and instability; the stability for systems with time delays has been devoted to a lot of effort. Nowadays, the stability of time delay is parted into two classes: delay-independent stability criteria and delay-dependent ones. It should be pointed out that delay-independent criteria turn out to be more conservative, especially for the small-size delays. The problem of delay-dependent stability analysis for time-delay systems has received considerable attention, and lots of important results have also been reported [1–12].

In recent years, there has been an increasing interest in analysis of hybrid and switched systems due to their importance both in theory and its applications. As an important class of hybrid systems, switched linear systems comprise a collection of linear subsystems described by differential equations as well as a switching law to specify the switching during these subsystems. A switched system is a type of hybrid system with a combination of the discrete and continuous dynamical systems. These systems are recognized as models for appearance which cannot be denoted by exclusively continuous or discrete processes. Recently, based on the Lyapunov functions and other analysis tools, the corresponding stability and stabilization for both linear and nonlinear and switched systems have been investigated and many good results have been obtained; for a recent survey on this issue and related matters, one can also refer to [13–26].

Most of the existing results have concentrated on Lyapunov-based asymptotic stability for the switched systems; the behavior of that is above the infinite time interval. However, in practical engineering, there exists the bad transient characteristics; the system is asymptotically stable without unusable. For the systems which work in a short time interval, for instance, control system, missile system, and communication network system, which mainly care about the behavior over a finite time interval. To deal with this problem, in 1961, Dorato presented the idea of finite-time stability in [27]. Over the past few years, many study efforts have been dedicated to the finite-time stability (FTS) of switched systems due to its wide applications. Comparing with the classical Lyapunov stability, which currently is the focus on a large and growing interdisciplinary area of research. To study the transient behavior of systems, FTS concerns the stability of a system over a finite interval of time and plays an important role. It is important to emphasize the disconnection between classical Lyapunov stability and finite-time stability. The problem about finite-time stabilization has been widely learned in the literature [28–44]. It is worth pointing out that there is a difference between finite-time stability and Lyapunov asymptotic stability, and they are also independent of each other.

Recently, some papers are found to be related to finite-time stability for switched systems. For example, based on the technique of average dwell time, the problem of finite-time boundedness for the switched linear system with time delays was investigated in [36]. The issue of finite-time stability and H_∞ stabilization for switched nonlinear discrete-time systems was developed in [37]. The problem of finite-time quantized H_∞ control problem for a class of discrete-time switched time-delay systems with time-varying exogenous disturbances was also discussed in [38]. But, to the best of the authors' knowledge, the finite-time L_2-gain problems for switched systems have not been completely studied. The previous research has some conservatism of stability criterion; it is natural to look for an alternative way to reduce the conservatism of corresponding stability criteria. This idea motivates to our study.

The main contribution of this paper is that we present a novel approach for finite-time stability of the given switched system. Moreover, several sufficient conditions ensuring the finite-time stability and boundness are proposed with different information we know about the switching signal. It shows the less conservative results when more information about the switching signal is available. By selecting the appropriate Lyapunov-Krasovskii functional, the sufficient conditions are obtained to guarantee finite-time stability of the systems and the closed-loop system trajectory stays in a special bound. The finite-time stability criteria can also be dealt with in the form of linear matrix inequalities and average dwell time. Finally, a numerical example is given to illustrate the effectiveness of the developed techniques.

2. Preliminaries

In this paper, the following switched systems are described as follows:

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-\tau(t)) + G_{\sigma(t)}\omega(t),\\ z(t) &= C_{\sigma(t)}x(t) + D_{\sigma(t)}\omega(t), \quad t \geq 0,\\ \widetilde{x}(t) &= \phi(t), \quad t \in [-h, 0],\end{aligned} \tag{1}$$

where $x(t) \in \mathscr{R}^n$ is the state vector of the system, $z(t) \in \mathscr{R}^m$ is the controlled output, and $\omega(t) \in L_2^q[0,\infty]$ is the external disturbance vector and satisfies the constraint:

$$\int_0^T \omega^{\top}(t)\,\omega(t) \leq d, \quad d \geq 0. \tag{2}$$

$A_{\sigma(t)}$, $B_{\sigma(t)}$, $G_{\sigma(t)}$, $C_{\sigma(t)}$, and $D_{\sigma(t)}$ are the constant real matrices with appropriate dimension. $\tau(t)$ represents the mode-dependent time-varying state delay in the switched system and satisfies the following conditions:

$$0 < \tau(t) \leq h < \infty, \tag{3}$$

$$\dot{\tau}(t) \leq \tau, \tag{4}$$

where h is the upper bound of the time-varying delay $\tau(t)$ and τ is the variation rate of the time-varying delay $\tau(t)$. $\phi(t)$ is the differentiable vector-valued initial function on $[-h,0]$. $\sigma(t) : [0,\infty) \to \mathscr{N} = \{1,2,\ldots,N\}$ is the right continuous piecewise constant switching signal to be designed.

Corresponding to the switching signal $\sigma(t)$, we get the follwing switching sequence:

$$\sum = \{x_0; (i_0, t_0), \ldots, (i_k, t_k), \ldots, \mid i_k \in \mathscr{N}, k = 0, 1, \ldots\}, \tag{5}$$

where t_0 is the initial time when $t_k \in [t_k, t_{k+1})$, $x(t_0)$ is the initial state and i_kth subsystem is active. Therefore, the trajectory $x(t)$ of the switched system (1) is called the trajectory of the i_kth subsystem. As assumed before, we get rid of Zeno behavior for all types related to switching signal. Throughout this paper, we assumed that the state of the switched system (1) does not jump at the switching instants; that is, the trajectory $x(t)$ is continuous everywhere.

Firstly, we will give the definitions and lemmas about switched system (1), which plays an major role in the derivation of our results.

Definition 1. Switch system (1) is said to be finite-time bounded with respect to (c_1, c_2, T, S, d), if condition (6) holds:

$$\begin{aligned}&\max_{-\tau \leq t_0 \leq 0} \{x^{\top}(t_0) S x(t_0), \dot{x}^{\top}(t_0) S \dot{x}(t_0)\}\\ &\quad \leq c_1 \Longrightarrow x^{\top}(t) S x(t) < c_2,\\ &\forall t \in [0,T], \quad \forall \omega(t) : \int_0^T \omega^{\top}(s)\,\omega(s)\,ds \leq d,\end{aligned} \tag{6}$$

where $c_2 > c_1 \geq 0$ and $S > 0$.

Definition 2. For $\gamma > 0$, $d > 0$, $T > 0$, $\eta > 0$, $\Lambda > 0$, and $c_2 > c_1 > 0$, system (3) is said to be finite-time stable with a weighted L_2 performance γ with respect to (c_1, c_2, T, S, d), if the following condition holds:

$$\int_0^T \left[\eta s - \ln \frac{\lambda_1 c_2}{\Lambda c_1 + d\gamma^2 (1/\eta)(1-e^{\eta T})}\right] z^{\top}(s) z(s)\, ds \le \gamma^2 e^{-\eta T} \int_0^T \omega^{\top}(s)\omega(s)\, ds, \tag{7}$$

and under zero initial condition, it holds for all nonzero ω: $\int_0^T \omega^{\top}(s)\omega(s)ds \le d$.

Definition 3 (see [21]). For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ during (T_1, T_2). If $N_\sigma(T_1, T_2) \le N_0 + (T_2 - T_1)/\tau_a$ holds for $N_0 \ge 0$ and $\tau_a > 0$, then N_0 and τ_a are called chattering bound and average dwell time, respectively. Here we assume $N_0 = 0$ for simplicity as supported by reference.

Lemma 4 (see [9]). *Let $f_i : \mathscr{R}^m \to \mathscr{R}$ $(i = 1, 2, \ldots, N)$ have positive values in an open subset $\mathscr{D}$ of $\mathscr{R}^m$. Then, the reciprocally convex combination of f_i over $\mathscr{D}$ satisfiesz*

$$\min_{\{\beta_i | \beta_i > 0, \sum_i \beta_i = 1\}} \sum_i \frac{1}{\beta_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \ne j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : \mathscr{R}^m \longrightarrow \mathscr{R}, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0 \right\}. \tag{8}$$

Lemma 5 (see [11]). *For any constant matrix $Z \in \mathscr{R}^{n\times n}$, $Z = Z^{\top} > 0$, scalars $h > 0$, such that the following integrations are well defined; then*

$$\begin{aligned} -h\int_{t-h}^{t} x^{\top}(s) Z x(s)\, ds &\le -\left[\int_{t-h}^{t} x(s)\, ds\right]^{\top} Z \left[\int_{t-h}^{t} x(s)\, ds\right], \\ -\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t} x^{\top}(s) Z x(s)\, ds\, d\theta &\le -\left[\int_{-h}^{0}\int_{t+\theta}^{t} x(s)\, ds\, d\theta\right]^{\top} Z \left[\int_{-h}^{0}\int_{t+\theta}^{t} x(s)\, ds\, d\theta\right]. \end{aligned} \tag{9}$$

Lemma 6 (*Schur's complement*). *Given constant matrices X, Y, Z, where $X = X^{\top}$ and $0 < Y = Y^{\top}$, then $X + Z^{\top}Y^{-1}Z < 0$ if and only if*

$$\begin{bmatrix} X & Z^{\top} \\ * & -Y \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -Y & Z \\ * & X \end{bmatrix} < 0. \tag{10}$$

3. Finite-Time Boundedness Analysis

Theorem 7. *System (3) is said to be finite-time stability with respect to (c_1, c_2, R, d, T) if there exist symmetric positive matrices P_i, Q_{si} $s = 1, 2$, X_{ki} $(k = 1, 2)$, Y_i and matrices M_i, N_i, V_i, scalars $\alpha \ge 0$, $\mu \ge 1$, $\lambda_l > 0$ $(l = 1, 2, \ldots, 8)$, $d > 0$, $h > 0$, $\Lambda > 0$, $\tau > 0$, $r(\tau)$; such that $\forall i, j \in \mathscr{N}$, we have the following linear matrix inequalities:*

$$\Xi_{1i} = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & W_i & A_i^{\top}V_i^{\top} & hA_i^{\top}M_i^{\top} + Y_i + Y_i^{\top} & Y_i + Y_i^{\top} & P_iG_i \\ * & \Xi_{22i} & -W_i + \dfrac{X_{2i}}{h} & B_i^{\top}V_i^{\top} & hB_i^{\top}M_i^{\top} & 0 & 0 \\ * & * & -Q_{1i} - \dfrac{X_{2i}}{h} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & -hM_i^{\top} & 0 & V_iG_i \\ * & * & * & * & -hX_{1i} - Y_i - Y_i^{\top} & 0 & hM_iG_i \\ * & * & * & * & * & -Y_i - Y_i^{\top} & 0 \\ * & * & * & * & * & * & -\eta H_i \end{bmatrix} < 0,$$

$$\Xi_{2i} = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & W_i & A_i^{\top}V_i^{\top} & Y_i + Y_i^{\top} & hA_i^{\top}N_i^{\top} + Y_i + Y_i^{\top} & P_iG_i \\ * & \Xi_{22i} & -W_i + \dfrac{X_{2i}}{h} & B_i^{\top}V_i^{\top} & 0 & hB_i^{\top}N_i^{\top} & 0 \\ * & * & -Q_{1i} - \dfrac{X_{2i}}{h} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & -hN_i^{\top} & V_iG_i \\ * & * & * & * & -Y_i - Y_i^{\top} & 0 & 0 \\ * & * & * & * & * & -hX_{1i} - Y_i - Y_i^{\top} & hN_iG_i \\ * & * & * & * & * & * & -\eta H_i \end{bmatrix} < 0, \tag{11}$$

$$\begin{gathered} P_i < \mu P_j, \quad Q_{1i} < \mu Q_{1j}, \quad Q_{2i} < \mu Q_{2j}, \\ X_{1i} < \mu X_{1j}, \quad X_{2i} < \mu X_{2j}, \quad Y_i < \mu Y_j, \end{gathered} \tag{12}$$

$$\lambda_1 c_2 e^{-\eta T} > \Lambda c_1 + d\lambda_8 \left(1 - e^{-\eta T}\right). \tag{13}$$

with the average dwell time of the switching signal σ satisfying

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln(\lambda_1 c_2) - \ln\left[\Lambda c_1 + d\lambda_8 \left(1 - e^{-\eta T}\right)\right] - \eta T}, \tag{14}$$

where

$$\begin{aligned}
\Xi_{11i} &= \delta P_i + P_i A_i + A_i^{\top} P_i + e^{\delta h}\left(Q_{1i} + Q_{2i}\right) \\
&\quad + hX_{1i} - \frac{X_{2i}}{h} - 2Y_i - 2Y_{2i}^{\top}, \\
\Xi_{12i} &= P_i B_i + \frac{X_{2i}}{h} - W_i, \\
\Xi_{22i} &= r(\tau) Q_{1i} - \frac{X_{2i}}{h} + W_i + W_i^{\top} - \frac{X_{2i}^{\top}}{h}, \\
\Xi_{44i} &= hX_{2i} + \frac{e^{\delta h} - \delta h - 1}{\delta^2} Y_i - V_i - V_i^{\top}, \\
r(\tau) &= \begin{cases} -(1-\tau) e^{\delta h}, & \text{if } \tau > 1; \\ -(1-\tau), & \text{if } \tau \le 1. \end{cases}
\end{aligned} \tag{15}$$

Consider

$$\Lambda = \lambda_2 + he^{\delta h}\left(\lambda_3 + \lambda_4\right) + h^2 e^{\delta h}\left(\lambda_5 + \lambda_6\right) + \frac{1}{2}\lambda_7 h^3 e^{\delta h}, \tag{16}$$

$$\lambda_1 = \min_{i \in \mathcal{N}} \left\{\lambda_{\min}\left(\overline{P}_i\right)\right\}, \qquad \lambda_2 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(\overline{P}_i\right)\right\}, \tag{17}$$

$$\lambda_3 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(\overline{Q}_{1i}\right)\right\}, \qquad \lambda_4 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(\overline{Q}_{2i}\right)\right\}, \tag{18}$$

$$\lambda_5 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(\overline{X}_{1i}\right)\right\}, \qquad \lambda_6 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(\overline{X}_{2i}\right)\right\}, \tag{19}$$

$$\lambda_7 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(\overline{Y}_i\right)\right\}, \qquad \lambda_8 = \max_{i \in \mathcal{N}} \left\{\lambda_{\max}\left(H_i\right)\right\}. \tag{20}$$

Proof. Define $x_t(s) = x(t+s)$. We consider the following Lyapunov-Krasovskii functional:

$$\begin{aligned}
V_{\sigma(t)}\left(x_t, t\right) &= \sum_{l=1}^{4} V_{l\sigma(t)}\left(x_t, t\right), \\
V_{1\sigma(t)}\left(x_t, t\right) &= x^{\top}(t) e^{\delta t} P_{\sigma(t)} x(t), \\
V_{2\sigma(t)}\left(x_t, t\right) &= \int_{t-h}^{t} e^{\delta(s+h)} x^{\top}(s) Q_{1\sigma(t)} x(s)\, ds \\
&\quad + \int_{t-\tau(t)}^{t} e^{\delta(s+h)} x^{\top}(s) Q_{2\sigma(t)} x(s)\, ds, \\
V_{3\sigma(t)}\left(x_t, t\right) &= \int_{-h}^{0}\int_{t+\theta}^{t} e^{\delta(s-\theta)} x^{\top}(s) X_{1\sigma(t)} x(s)\, ds\, d\theta \\
&\quad + \int_{-h}^{0}\int_{t+\theta}^{t} e^{\delta(s-\theta)} \dot{x}^{\top}(s) X_{2\sigma(t)} \dot{x}(s)\, ds\, d\theta, \\
V_{4\sigma(t)}\left(x_t, t\right) &= \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\upsilon}^{t} e^{\delta(s-\theta)} \dot{x}^{\top}(s) Y_{\sigma(t)} \dot{x}(s)\, ds\, d\upsilon\, d\theta.
\end{aligned} \tag{21}$$

Taking the time derivative of $V_{\sigma(t)}(x_t, t)$ along the trajectory of the system (1), one has

$$\begin{aligned}
\dot{V}_{1i}\left(x_t, t\right) &= x^{\top}(t) e^{\delta t}\left(\delta P_i + P_i A_i + A_i^{\top} P_i\right) x(t) \\
&\quad + 2e^{\delta t} x^{\top}(t) P_i B_i x(t - \tau(t)) \\
&\quad + 2e^{\delta t} x^{\top}(t) P_i G_i \omega(t), \\
\dot{V}_{2i}\left(x_t, t\right) &= e^{\delta t} x^{\top}(t) e^{\delta h}\left(Q_{1i} + Q_{2i}\right) x(t) \\
&\quad - e^{\delta t} x^{\top}(t-h) Q_{1i} x(t-h) \\
&\quad - (1 - \dot{\tau}(t)) e^{\delta t} x^{\top}(t - \tau(t)) Q_{2i} x(t - \tau(t)).
\end{aligned} \tag{22}$$

Since

$$0 \le \tau(t) \le h, \tag{23}$$

we define

$$r(\tau) = \begin{cases} -(1-\tau) e^{\delta h}, & \text{if } \tau > 1; \\ -(1-\tau), & \text{if } \tau \le 1. \end{cases} \tag{24}$$

Then,

$$\begin{aligned}
\dot{V}_{2i}\left(x_t, t\right) &= e^{\delta t} x^{\top}(t) e^{\delta h}\left(Q_{1i} + Q_{2i}\right) x(t) \\
&\quad - e^{\delta t} x^{\top}(t-h) Q_{1i} x(t-h) \\
&\quad + e^{\delta t} x^{\top}(t - \tau(t)) r(\tau) Q_{2i} x(t - \tau(t)), \\
\dot{V}_{3i}\left(x_t, t\right) &= e^{\delta t} x^{\top}(t) hX_{1i} x(t) + e^{\delta t} \dot{x}^{\top}(t) hX_{2i} \dot{x}(t) \\
&\quad - \int_{t-h}^{t} e^{\delta(s+h)} x^{\top}(s) X_{1i} x(s)\, ds \\
&\quad - \int_{t-h}^{t} e^{\delta(s+h)} \dot{x}^{\top}(s) X_{2i} \dot{x}(s)\, ds \\
&\le e^{\delta t} x^{\top}(t) hX_{1i} x(t) + e^{\delta t} \dot{x}^{\top}(t) hX_{2i} \dot{x}(t) \\
&\quad - e^{\delta t} \int_{t-\tau(t)}^{t} x^{\top}(s) X_{1i} x(s)\, ds \\
&\quad - e^{\delta t} \int_{t-h}^{t-\tau(t)} x^{\top}(s) X_{1i} x(s)\, ds \\
&\quad - e^{\delta t} \int_{t-h}^{t} \dot{x}^{\top}(s) X_{2i} \dot{x}(s)\, ds.
\end{aligned} \tag{25}$$

By Lemma 5, one can obtain

$$\begin{aligned}
-\int_{t-\tau(t)}^{t} x^{\top}(s) X_{1i} x(s)\, ds &\le -\tau(t) U_1^{\top} X_{1i} U_1, \\
-\int_{t-h}^{t-\tau(t)} x^{\top}(s) X_{1i} x(s)\, ds &\le -(h - \tau(t)) U_2^{\top} X_{1i} U_2,
\end{aligned} \tag{26}$$

where

$$U_1 = \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} x(s)\, ds, \quad U_2 = \frac{1}{h-\tau(t)} \int_{t-h}^{t-\tau(t)} x(s)\, ds,$$
$$\lim_{\tau(t)\to 0} \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} x(s)\, ds = x(t),$$
$$\lim_{\tau(t)\to h} \frac{1}{h-\tau(t)} \int_{t-h}^{t-\tau(t)} x(s)\, ds = x(t-h). \tag{27}$$

From the Leibniz-Newton formula, the following equation is true for any matrices M_i, N_i, and V_i with appropriate dimensions:

$$\begin{aligned}&\left(2\tau(t) U_1^{\top} M_i + 2(h-\tau(t)) U_2^{\top} N_i + 2\dot{x}^{\top}(t) V_i\right)\\ &\quad\times\left[-\dot{x}(t) + A_i x(t) + B_i x(t-\tau(t)) + G_i \omega(t)\right] = 0.\end{aligned} \tag{28}$$

From Lemma 4, it yields

$$\begin{aligned}
&-\int_{t-h}^{t} \dot{x}^{\top}(s) X_{2i} \dot{x}(s)\, ds \\
&\quad= -\int_{t-\tau(t)}^{t} \dot{x}^{\top} X_{2i} \dot{x}(s)\, ds \\
&\qquad- \int_{t-h}^{t-\tau(t)} \dot{x}^{\top} X_{2i} \dot{x}(s)\, ds \\
&\quad\le -\frac{h}{h-\tau(t)} \left[\int_{t-h}^{t-\tau(t)} \dot{x} ds\right]^{\top} \frac{X_{2i}}{h} \left[\int_{t-h}^{t-\tau(t)} \dot{x} ds\right] \\
&\qquad- \frac{h}{\tau(t)} \left[\int_{t-\tau(t)}^{t} \dot{x} ds\right]^{\top} \frac{X_{2i}}{h} \left[\int_{t-\tau(t)}^{t} \dot{x} ds\right] \\
&\quad\le -\begin{bmatrix} x(t) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-h) \end{bmatrix}^{\top} \\
&\qquad\times \begin{bmatrix} \frac{X_{2i}}{h} & W_i \\ * & \frac{X_{2i}}{h} \end{bmatrix} \begin{bmatrix} x(t) - x(t-\tau(t)) \\ x(t-\tau(t)) - x(t-h) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\dot{V}_{4i}(x_t, t) &= e^{\delta t} \dot{x}^{\top}(t) Y_i \dot{x}(t) \int_{-h}^{0} \int_{\theta}^{0} e^{-\delta v} dv\, d\theta \\
&\quad- e^{\delta t} \int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}^{\top}(s) Y_i \dot{x}(s)\, ds\, d\theta \\
&= e^{\delta t} \dot{x}^{\top}(t) \frac{e^{\delta h} - \delta h - 1}{\delta^2} Y_i \dot{x}(t) \\
&\quad- e^{\delta t} \int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}^{\top}(s) Y_i \dot{x}(s)\, ds\, d\theta.
\end{aligned} \tag{29}$$

By using Lemma 5, we have

$$\begin{aligned}
&-\int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}(s) Y_i \dot{x}(s)\, ds\, d\theta \\
&\quad= -\int_{-\tau(t)}^{0} \int_{t+\theta}^{t} \dot{x}(s) Y_i \dot{x}(s)\, ds\, d\theta \\
&\qquad- \int_{-h}^{-\tau(t)} \int_{t+\theta}^{t} \dot{x}(s) Y_i \dot{x}(s)\, ds\, d\theta \\
&\quad\le -\frac{2}{\tau^2(t)} \left[\int_{-\tau(t)}^{0} \int_{t+\theta}^{t} \dot{x}(s)\, ds\, d\theta\right]^{\top} \\
&\qquad\times Y_i \left[\int_{-\tau(t)}^{0} \int_{t+\theta}^{t} \dot{x}(s)\, ds\, d\theta\right] \\
&\qquad- \frac{2}{(h-\tau(t))^2} \left[\int_{-h}^{-\tau(t)} \int_{t+\theta}^{t} \dot{x}(s)\, ds\, d\theta\right]^{\top} \\
&\qquad\times Y_i \left[\int_{-h}^{-\tau(t)} \int_{t+\theta}^{t} \dot{x}(s)\, ds\, d\theta\right] \\
&\quad= -\frac{2}{\tau^2(t)} \left[\tau(t) x(t) - \int_{t-\tau(t)}^{t} x(s)\, ds\right]^{\top} \\
&\qquad\times Y_i \left[\tau(t) x(t) - \int_{t-\tau(t)}^{t} x(s)\, ds\right] \\
&\qquad- \frac{2}{(h-\tau(t))^2} \left[(h-\tau(t) x(t)) - \int_{t-h}^{t-\tau(t)} x(s)\, ds\right]^{\top} \\
&\qquad\times Y_i \left[(h-\tau(t) x(t)) - \int_{t-h}^{t-\tau(t)} x(s)\, ds\right] \\
&\quad= -2\left[x(t) - \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} x(s)\, ds\right]^{\top} \\
&\qquad\times Y_i \left[x(t) - \frac{1}{\tau(t)} \int_{t-\tau(t)}^{t} x(s)\, ds\right] \\
&\qquad- 2\left[x(t) - \frac{1}{h-\tau(t)} \int_{t-h}^{t-\tau(t)} x(s)\, ds\right]^{\top} \\
&\qquad\times Y_i \left[x(t) - \frac{1}{h-\tau(t)} \int_{t-h}^{t-\tau(t)} x(s)\, ds\right] \\
&\quad= -2\left[x(t) - U_1\right]^{\top} Y_i \left[x(t) - U_1\right] \\
&\qquad- 2\left[x(t) - U_2\right]^{\top} Y_i \left[x(t) - U_2\right].
\end{aligned} \tag{30}$$

Therefore, for a given $\eta > 0$ and from (21)–(30), one can obtain that

$$\dot{V}_i(x_t, t) - \eta \omega^{\top} H_i \omega(t) \le e^{\delta t} \xi^{\top}(t) \Xi_i \xi(t), \tag{31}$$

where

$$\xi^{\top}(t)=\left[x^{\top}(t),x^{\top}(t-\tau(t)),x^{\top}(t-h),\dot{x}^{\top}(t),U_1^{\top},U_2^{\top},\omega^{\top}(t)\right].$$

$$\overline{\Xi}_i=\begin{bmatrix}\Xi_{11i} & \Xi_{12i} & W_i & A_i^{\top}V_i^{\top} & \tau(t)A_i^{\top}M_i^{\top}+Y_i+Y_i^{\top} & (h-\tau(t))A_i^{\top}N_i^{\top}+Y_i+Y_i^{\top} & P_iG_i\\ * & \Xi_{22i} & -W_i+\dfrac{X_{2i}}{h} & B_i^{\top}V_i^{\top} & \tau(t)B_i^{\top}M_i^{\top} & (h-\tau(t))B_i^{\top}N_i^{\top} & 0\\ * & * & -Q_{1i}-\dfrac{X_{2i}}{h} & 0 & 0 & 0 & 0\\ * & * & * & \Xi_{44i} & -\tau(t)M_i^{\top} & -(h-\tau(t))N_i^{\top} & V_iG_i\\ * & * & * & * & -\tau(t)X_{1i}-Y_i-Y_i^{\top} & 0 & \tau(t)M_iG_i\\ * & * & * & * & * & -(h-\tau(t))X_{1i}-Y_i-Y_i^{\top} & (h-\tau(t))N_iG_i\\ * & * & * & * & * & * & -\eta H_i\end{bmatrix}. \tag{32}$$

The LMIs (11) lead to $\tau(t)\rightarrow h$ and to $\tau(t)\rightarrow 0$, respectively. It is easy to see that Ξ_{1i} results from $\overline{\Xi}_i\mid\tau(t)=h$ and Ξ_{2i} results from $\overline{\Xi}_i\mid\tau(t)=0$. Thus, we obtain

$$\dot{V}_i(x_t,t)-\eta V_i(x_t,t)<\eta\omega^{\top}(t)H_i\omega(t). \tag{33}$$

Notice that

$$\frac{d}{dt}\left(e^{-\eta t}V_i(x_t,t)\right)<\eta e^{-\eta t}\omega^{\top}(t)H_i\omega(t). \tag{34}$$

Integrating (34) from t_k to t, we can get that

$$V_i(x_t,t)<e^{\eta(t-t_k)}V_i(x_{t_k},t_k)+\eta\int_{t_k}^{t}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds. \tag{35}$$

Note that (12) and $\mu\geq 1$, it yields

$$V_{\sigma(t_k)}(x_{t_k},t_k)\leq\mu V_{\sigma(t_{k-1})}(x_{t_k},t_k). \tag{36}$$

Then, we can easily have

$$\begin{aligned}V_{\sigma(t_{k-1})}(x_{t_k},t_k)&<e^{\eta(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(x_{t_{k-1}},t_{k-1})\\&\quad+\eta\int_{t_{k-1}}^{t_k}e^{\eta(t_k-s)}\omega^{\top}(s)H_i\omega(s)\,ds.\end{aligned} \tag{37}$$

Thus, from (35)–(37), it yields

$$\begin{aligned}V_{\sigma(t)}(x_t,t)&\leq e^{\eta(t-t_k)}V_{\sigma(t_k)}(x_{t_k},t_k)\\&\quad+\eta\int_{t_k}^{t}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\leq\mu e^{\eta(t-t_k)}V_{\sigma(t_{k-1})}(x_{t_k},t_k)\\&\quad+\eta\int_{t_k}^{t}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\leq\mu e^{\eta(t-t_{k-1})}V_{\sigma(t_{k-1})}(x_{t_{k-1}},t_{k-1})\\&\quad+\eta\mu\int_{t_{k-1}}^{t_k}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\quad+\eta\int_{t_k}^{t}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\leq\mu^2e^{\eta(t-t_{k-2})}V_{\sigma(t_{k-2})}(x_{t_{k-2}},t_{k-2})\\&\quad+\eta\mu^2\int_{t_{k-2}}^{t_{k-1}}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\quad+\eta\mu\int_{t_{k-1}}^{t_k}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\quad+\eta\int_{t_k}^{t}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\leq\cdots\leq\mu^{N_\sigma(0,t)}e^{\eta t}V_{\sigma(0)}(x_0,0)\\&\quad+\eta\mu^{N_\sigma(0,t)}\int_0^{t_1}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\quad+\eta\mu^{N_\sigma(t_1,t)}\int_{t_1}^{t_2}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&\quad+\cdots+\eta\int_{t_k}^{t}e^{\eta(t-s)}\omega^{\top}(s)H_i\omega(s)\,ds\\&=\mu^{N_\sigma(0,t)}e^{\eta t}V_{\sigma(0)}(x_0,0)\end{aligned}$$

$$+\eta\int_0^t e^{\eta(t-s)}\mu^{N_\sigma(s,t)}\omega^{\top}(s)H_i\omega(s)\,ds$$
$$\le \mu^{N_\sigma(0,t)}e^{\eta T}V_{\sigma(0)}(x_0,0)$$
$$+\eta\mu^{N_\sigma(0,t)}d\lambda_{\max}(H_i)e^{\alpha T}\int_0^t e^{-\eta s}ds$$
$$\le \mu^{N_\sigma(0,T)}e^{\eta T}$$
$$\times\left\{V_{\sigma(0)}(x_0,0)+d\lambda_{\max}(H_i)\eta\int_0^T e^{-\eta s}ds\right\}$$
$$\le \mu^{T/\tau_a}e^{\eta T}\left\{V_{\sigma(0)}(x_0,0)+d\lambda_{\max}(H_i)\left(1-e^{-\eta T}\right)\right\}$$
$$\le \mu^{N_\sigma(0,T)}e^{\eta T}$$
$$\times\left\{V_{\sigma(0)}(x_0,0)+d\lambda_{\max}(H_i)\eta\int_0^T e^{-\eta s}ds\right\}$$
$$=\mu^{T/\tau_a}e^{\eta T}\left\{V_{\sigma(0)}(x_0,0)+d\lambda_8\left(1-e^{-\eta T}\right)\right\}. \tag{38}$$

Define $\overline{P}_i=R^{-1/2}P_iR^{-1/2}$, $\overline{Q}_{si}=R^{-1/2}Q_{si}R^{-1/2}$ $(s=1,2)$, $\overline{X}_{li}=R^{-1/2}X_{li}R^{-1/2}$ $(l=1,2)$, $\overline{Y}_i=R^{-1/2}Y_iR^{-1/2}$.

Note that

$$V_{\sigma(0)}(x_0,0)$$
$$=\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{P}_i\right)x^{\top}(0)Rx(0)$$
$$+\left(\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{Q}_{1i}\right)+\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{Q}_{2i}\right)\right)e^{\delta h}$$
$$\times\int_{-h}^0 e^{\delta s}x^{\top}(s)Rx(s)\,ds$$
$$+\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{X}_{1i}\right)e^{\delta h}\int_{-h}^0\int_\theta^0 e^{-\delta\theta}x^{\top}(s)Rx(s)\,ds\,d\theta$$
$$+\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{X}_{2i}\right)e^{\delta h}\int_{-h}^0\int_\theta^0 e^{-\delta\theta}\dot{x}^{\top}(s)R\dot{x}(s)\,ds\,d\theta$$
$$+\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{X}_{2i}\right)e^{\delta h}$$
$$\times\int_{-h}^0\int_\theta^0\int_v^0 e^{-\delta\theta}\dot{x}^{\top}(s)R\dot{x}(s)\,ds\,d\theta\,dv$$
$$\le\left\{\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{P}_i\right)\right.$$
$$+he^{\delta h}\left(\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{Q}_{1i}\right)+\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{Q}_{2i}\right)\right)$$
$$+h^2e^{\delta h}\left(\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{X}_{1i}\right)+\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{X}_{2i}\right)\right)$$
$$\left.+\frac{1}{2}h^3e^{\delta h}\max_{i\in\mathcal{N}}\lambda_{\max}\left(\overline{Y}_i\right)\right\}$$
$$\times\sup_{-h\le s\le 0}\left\{x^{\top}(s)Rx(s),\dot{x}^{\top}(s)R\dot{x}(s)\right\}$$
$$\le\left(\lambda_2+he^{\delta h}(\lambda_3+\lambda_4)\right.$$
$$\left.+h^2e^{\delta h}(\lambda_5+\lambda_6)+\frac{1}{2}\lambda_7h^3e^{\delta h}\right)$$
$$\times\sup_{-h\le s\le 0}\left\{x^{\top}(s)Rx(s),\dot{x}^{\top}(s)R\dot{x}(s)\right\}$$
$$\le\left(\lambda_2+he^{\delta h}(\lambda_3+\lambda_4)+h^2e^{\delta h}(\lambda_5+\lambda_6)\right.$$
$$\left.+\frac{1}{2}\lambda_7h^3e^{\delta h}\right)c_1=\Lambda c_1. \tag{39}$$

Thus,

$$\begin{aligned}V_{\sigma(t)}(x_t,t)&\le\mu^{T/\tau_a}e^{\alpha T}\left\{\Lambda c_1+d\lambda_8\left(1-e^{-\eta T}\right)\right\}\\&=e^{(\alpha+\ln\mu/\tau_a)T}\left\{\Lambda c_1+d\lambda_8\left(1-e^{-\eta T}\right)\right\}.\end{aligned} \tag{40}$$

On the other hand,

$$V_{\sigma(t)}(x_t,t)\ge\lambda_{\min}\left(\overline{P}_i\right)x^{\top}(t)Rx(t)=\lambda_1x^{\top}(t)Rx(t). \tag{41}$$

From (40) and (41), one obtains

$$x^{\top}(t)Rx(t)\le\frac{\Lambda c_1+d\lambda_8\left(1-e^{-\eta T}\right)}{\lambda_1}e^{(\eta+\ln\mu/\tau_a)T}. \tag{42}$$

When $\mu=1$, which is the trivial case, from (13), $x^{\top}(t)Rx(t)<c_2e^{-\eta T}e^{\eta T}=c_2$. When $\mu\ge1$, from (13), $\ln(\lambda_1c_2)-\ln[\Lambda c_1+d\lambda_8(1-e^{-\eta T})]-\alpha T>0$, we have

$$\begin{aligned}\frac{T}{\tau_a}&<\frac{\ln(\lambda_1c_2)-\ln\left[\Lambda c_1+d\lambda_8\left(1-e^{-\eta T}\right)\right]-\eta T}{\ln\mu}\\&=\frac{\ln\left(\lambda_1c_2e^{-\eta T}/\left(\Lambda c_1+d\lambda_8\left(1-e^{-\eta T}\right)\right)\right)}{\ln\mu}.\end{aligned} \tag{43}$$

Substituting (43) into (42) yields

$$x^{\top}(t)Rx(t)<c_2. \tag{44}$$

The proof is completed. □

Remark 8. It should be mentioned that to reduce the conservatism, $-\int_{-\tau(t)}^0\int_{t+\theta}^t\dot{x}^{\top}(s)Z_i\dot{x}dsd\theta$ and $-\int_{-h}^{-\tau(t)}\int_{t+\theta}^t\dot{x}^{\top}(s)Z_i\dot{x}dsd\theta$ are bounded with $-2[x(t)-U_1]^{\top}Z_i[x(t)-U_1]$ and $-2[x(t)-U_2]^{\top}Z_i[x(t)-U_2]$, respectively.

Remark 9. One can clearly see that criteria given in Theorem 7 are delay-dependent. It is well known that the delay-dependent criteria are less conservative than delay-independent criteria when the delay is small or belongs to a given interval. Interval time-varying delay is a time delay that varies in an interval in which the lower bound is not restricted to zero.

Remark 10. In Theorem 7, the free weight matrix technology is employed, which reduces the conservativeness of the stability condition.

4. Finite-Time Weighted L_2-Gain Analysis

Theorem 11. *System (3) is finite-time bounded with respect to (c_1, c_2, R, d, T) if there exist symmetric positive matrices P_i, Q_{si} $s = 1, 2$, X_{ki} $(k = 1, 2)$, Y_i and matrices M_i, N_i, V_i, scalars $\alpha \geq 0$, $\mu \geq 1$, $\lambda_l > 0$ $(l = 1, 2, \ldots, 7)$, $d > 0$, $h > 0$, $\Lambda > 0$, $\gamma > 0$, $\tau > 0$, $r(\tau)$, such that $\forall i, j \in \mathcal{N}$; we have the following linear matrix inequalities:*

$$\Sigma_{1i} = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & W_i & A_i^{\top} V_i^{\top} & hA_i^{\top} M_i^{\top} + Y_i + Y_i^{\top} & Y_i + Y_i^{\top} & P_i G_i & C_i^{\top} \\ * & \Xi_{22i} & -W_i + \dfrac{X_{2i}}{h} & B_i^{\top} V_i^{\top} & hB_i^{\top} M_i^{\top} & 0 & 0 & 0 \\ * & * & -Q_{1i} - \dfrac{X_{2i}}{h} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & -hM_i^{\top} & 0 & V_i G_i & 0 \\ * & * & * & * & -hX_{1i} - Y_i - Y_i^{\top} & 0 & hM_i G_i & 0 \\ * & * & * & * & * & -Y_i - Y_i^{\top} & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & D_i^{\top} \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0,$$

$$\Sigma_{2i} = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & W_i & A_i^{\top} V_i^{\top} & Y_i + Y_i^{\top} & hA_i^{\top} N_i^{\top} + Y_i + Y_i^{\top} & P_i G_i & C_i^{\top} \\ * & \Xi_{22i} & -W_i + \dfrac{X_{2i}}{h} & B_i^{\top} V_i^{\top} & 0 & hB_i^{\top} N_i^{\top} & 0 & 0 \\ * & * & -Q_{1i} - \dfrac{X_{2i}}{h} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & -hN_i^{\top} & V_i G_i & 0 \\ * & * & * & * & -Y_i - Y_i^{\top} & 0 & 0 & 0 \\ * & * & * & * & * & -hX_{1i} - Y_i - Y_i^{\top} & hN_i G_i & 0 \\ * & * & * & * & * & * & -\gamma^2 I & D_i^{\top} \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{45}$$

$$\begin{gathered} P_i < \mu P_j, \quad Q_{1i} < \mu Q_{1j}, \quad Q_{2i} < \mu Q_{2j}, \\ X_{1i} < \mu X_{1j}, \quad X_{2i} < \mu X_{2j}, \quad Y_i < \mu Y_j, \\ \lambda_1 c_2 e^{-\eta T} > \Lambda c_1 + d\gamma^2 \frac{1}{\eta} \left(1 - e^{-\eta T}\right), \end{gathered} \tag{46}$$

with the average dwell time of the switching signal σ satisfying

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln(\lambda_1 c_2) - \ln\left[\Lambda c_1 + d\gamma^2 (1/\eta)\left(1 - e^{-\eta T}\right)\right] - \eta T}. \tag{47}$$

Proof. Choose the same Lyapunov-Krasovskii functional as in Theorem 7; after some mathematical manipulation and Schur complement, we can get

$$\dot{V}_{\sigma(t)}(x_t, t) + e^{\top}(t) e(t) - \gamma^2 \omega^{\top}(t) \omega(t) = \xi^{\top}(t) \Sigma_{si} \xi(t), \quad (s = 1, 2). \tag{48}$$

Define

$$J(t) = e^{\top}(t) e(t) - \gamma^2 \omega^{\top}(t) \omega(t). \tag{49}$$

From (45), we obtain

$$\dot{V}_{\sigma(t)}(x_t, t) - \eta V_{\sigma(t)}(x_t, t) + J(t) < 0. \tag{50}$$

When $t \in [t_k, t_{k+1}]$, where t_k is the kth switching instant,

$$V_{\sigma(t)}(x_t, t) < e^{\eta(t - t_k)} V_{\sigma(t_k)}\left(x_{t_k}, t_k\right) - \int_{t_k}^{t} e^{\eta(t-s)} J(s)\, ds. \tag{51}$$

Notice that $x(t_k) = x(t_k^-)$; then one obtains

$$V_{\sigma(t_k)}(x(t_k), t_k) \leq \mu V_{\sigma(t_k^-)}(x(t_k), t_k). \tag{52}$$

For any $t \in [0, T]$, one has

$$\begin{aligned} V_{\sigma(t)}(x_t, t) &\leq e^{\eta(t-t_k)} V_{\sigma(t_k)}\left(x_{t_k}, t_k\right) + \eta \int_{t_k}^{t} e^{\eta(t-s)} J(s)\, ds \\ &\leq \mu e^{\eta(t - t_{k-1})} V_{\sigma(t_{k-1})}\left(x_{t_{k-1}}, t_{k-1}\right) \\ &\quad + \eta\mu \int_{t_{k-1}}^{t_k} e^{\eta(t-s)} J(s)\, ds + \eta \int_{t_k}^{t} e^{\eta(t-s)} J(s)\, ds \\ &\quad + \eta\mu \int_{t_{k-1}}^{t_k} e^{\eta(t-s)} \omega^{\top}(s) H_i \omega(s)\, ds \\ &\quad + \eta \int_{t_k}^{t} e^{\eta(t-s)} J(s)\, ds \end{aligned}$$

$$\begin{aligned}
&\leq \cdots \leq \mu^{N_\sigma(0,t)} e^{\eta t} V_{\sigma(0)}(x_0, 0) \\
&\quad + \eta \mu^{N_\sigma(0,t)} \int_0^{t_1} e^{\eta(t-s)} J(s)\, ds \\
&\quad + \eta \mu^{N_\sigma(t_1,t)} \int_{t_1}^{t_2} e^{\eta(t-s)} J(s)\, ds \\
&\quad + \cdots + \eta \int_{t_k}^{t} e^{\eta(t-s)} J(s)\, ds \\
&\leq \mu^{N_\sigma(0,T)} e^{\eta T} V_{\sigma(0)}(x_0, 0) \\
&\quad + \eta \int_0^T e^{\eta(T-s)} \mu^{N_\sigma(s,T)} J(s)\, ds.
\end{aligned} \tag{53}$$

Under zero initial condition, we have

$$\int_0^T e^{-\eta s} \mu^{N_\sigma(s,T)} J(s)\, ds < 0, \tag{54}$$

which implies that

$$\begin{aligned}
&\int_0^T e^{-\alpha s} \mu^{N_\sigma(s,T)} z^\top(s) z(s)\, ds \\
&\quad < \int_0^T \mu^{N_\sigma(s,T)} \gamma^2 \omega^\top(s) \omega(s)\, ds.
\end{aligned} \tag{55}$$

Multiplying both sides of (55) by $\mu^{-N_\sigma(0,T)}$ yields

$$\begin{aligned}
&\int_0^T e^{-\alpha s} \mu^{-N_\sigma(0,s)} z^\top(s) z(s)\, ds \\
&\quad < \int_0^T \mu^{-N_\sigma(0,s)} \gamma^2 \omega^\top(s) \omega(s)\, ds.
\end{aligned} \tag{56}$$

It is easy to deduce from (47) that

$$\begin{aligned}
N_\sigma(0,s) &\leq \frac{s}{\tau_a} \\
&\leq \frac{\ln\left(\lambda_1 c_2 / \left(\Lambda c_1 + d\gamma^2 (1/\eta)\left(1 - e^{\eta T}\right)\right)\right) - \eta s}{\ln \mu}.
\end{aligned} \tag{57}$$

Since $\mu \geq 1$, we have

$$\begin{aligned}
&\int_0^T \mu^{\ln((\eta s - \ln(\lambda_1 c_2/(\Lambda c_1 + d\gamma^2(1/\eta)(1-e^{\eta T}))))/\ln\mu)} \\
&\quad \times z^\top(s) z(s)\, ds \\
&\leq \int_0^T \mu^{-N_\sigma(0,s)} e^{-\eta s} z^\top(s) z(s)\, ds \\
&\leq \int_0^T \mu^{-N_\sigma(0,s)} e^{-\eta s} \gamma^2 \omega^\top(s) \omega(s)\, ds \\
&\leq e^{-\eta T} \int_0^T \gamma^2 \omega^\top(s) \omega(s)\, ds.
\end{aligned} \tag{58}$$

Therefore, we can obtain

$$\begin{aligned}
&\int_0^T \left[\eta s - \ln \frac{\lambda_1 c_2}{\Lambda c_1 + d\gamma^2 (1/\eta)\left(1 - e^{\eta T}\right)}\right] z^\top(s) z(s)\, ds \\
&\quad \leq \gamma^2 e^{-\eta T} \int_0^T \omega^\top(s) \omega(s)\, ds.
\end{aligned} \tag{59}$$

According to Definition 2, we know that Theorem 11 holds. This completes the proof. □

Remark 12. It can be seen that Theorem 11 presents a more general result than [33–42] which contain the decay η; such a condition is derived by advantage of the simultaneous usage of the Lyapunov functional and the state variable transformation.

Remark 13. In many real applications, the minimum value of $\gamma^2_{\min}$ is of interest. In Theorem 11, with a fixed μ and η, $\gamma_{\min}$ can be obtained through following procedure:

$$\begin{aligned}
&\min \ \gamma^2 \\
&\text{s.t.} \ \ \text{LMIs } ((37)\text{–}(39)).
\end{aligned} \tag{60}$$

5. Illustrative Example

Example 1. Consider the system (1) with the following data:

$$\begin{aligned}
&A_1 = \begin{bmatrix} -1.7 & 1.7 & 0 \\ 1.3 & -1 & 0.7 \\ 0.7 & 1 & -0.6 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0.7 & 0 & -0.6 \\ 1.7 & 0 & -1.7 \end{bmatrix}, \\
&B_1 = \begin{bmatrix} 1.5 & -1.7 & 0.1 \\ -1.3 & 1 & -0.3 \\ -0.7 & 1 & 0.6 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -1 & 0 & 0.1 \\ 1.3 & -0.1 & 0.6 \\ 1.5 & 0.1 & 1.8 \end{bmatrix}, \\
&G_1 = G_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad x(t) = \begin{bmatrix} 0.7 \\ 0 \\ 0 \end{bmatrix}, \\
&\omega(t) = \begin{bmatrix} 0.03\sin(t) \\ 0.02\cos(2t) \\ 0.015(\sin(t+1) + \cos(t-2)) \end{bmatrix}, \quad t \in [-\tau, 0].
\end{aligned} \tag{61}$$

The values of c_1, c_2, T, d and matrix R are given as follows:

$$\begin{aligned}
&c_1 = 0.5, \qquad T = 10, \qquad R = I, \\
&d = 0.01, \qquad \delta = 0.042, \qquad \eta = 0.075.
\end{aligned} \tag{62}$$

Through Theorem 7, when $c_2 = 50$, we can see that the admissible maximum h computed by [36] is 0.2, and the maximal value of τ in this paper is 0.262. It is obvious that our method shows less conservatism result than that in [36]. Moreover, when $\tau = 0.2$, the optimal bound with minimum value of $c_{2\min}$ relies on the parameter η. One can obtain clearly that Theorem 7 in our paper can indeed provide much smaller admissible $c_{2\min}$ than the stability criteria in [36] which shows the less conservative result in this paper.

6. Conclusions

In this paper, we have examined the problems of delay-dependent finite-time and L_2-gain analysis for switched systems with time-varying delay by allowing new Lyapunov-Krasovskii functional and average dwell time. A numerical example has also been given to demonstrate the effectiveness of the proposed approach. It is possible to extend the main results obtained in this paper to real systems which contain some practical constrains, for example, the failures of controllers. Details will be reported in our follow-up work in the future.

Acknowledgment

This work was supported by the Fundamental Research Funds for the Central Universities (103.1.2E022050205).

References

[1] Z. Wu, H. Su, and J. Chu, "Delay-dependent H_∞ filtering for singular Markovian jump time-delay systems," *Signal Processing*, vol. 90, no. 6, pp. 1815–1824, 2010.

[2] Z. Wu, P. Shi, H. Su, and J. Chu, "Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled-data," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 1796–1806, 2013.

[3] H. Shen, S. Xu, J. Lu, and J. Zhou, "Passivity-based control for uncertain stochastic jumping systems with mode-dependent round-trip time delays," *Journal of the Franklin Institute*, vol. 349, no. 5, pp. 1665–1680, 2012.

[4] H. Shen, S. Xu, J. Zhou, and J. Lu, "Fuzzy H_∞ filtering for nonlinear Markovian jump neutral systems," *International Journal of Systems Science*, vol. 42, no. 5, pp. 767–780, 2011.

[5] Z. Wu, H. Su, and J. Chu, "H_∞ filtering for singular Markovian jump systems with time delay," *International Journal of Robust and Nonlinear Control*, vol. 20, no. 8, pp. 939–957, 2010.

[6] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Passivity analysis for discrete-time stochastic markovian jump neural networks with mixed time delays," *IEEE Transactions on Neural Networks*, vol. 22, no. 10, pp. 1566–1575, 2011.

[7] H. R. Karimi, "Robust delay-dependent H_∞ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters," *IEEE Transactions on Circuits and Systems I*, vol. 58, no. 8, pp. 1910–1923, 2011.

[8] H. R. Karimi, "A sliding mode approach to H_∞ synchronization of master-slave time-delay systems with Markovian jumping parameters and nonlinear uncertainties," *Journal of the Franklin Institute*, vol. 349, no. 4, pp. 1480–1496, 2012.

[9] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[10] C.-H. Lien, K.-W. Yu, Y.-J. Chung, H.-C. Chang, and J.-D. Chen, "Switching signal design for global exponential stability of uncertain switched nonlinear systems with time-varying delay," *Nonlinear Analysis: Hybrid Systems*, vol. 5, no. 1, pp. 10–19, 2011.

[11] J. Tian and S. Zhong, "Improved delay-dependent stability criteria for neural networks with two additive time-varying delay components," *Neurocomputing*, vol. 77, no. 1, pp. 114–119, 2012.

[12] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Delay-dependent stability analysis for switched neural networks with time-varying delay," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 41, no. 6, pp. 1522–1530, 2011.

[13] D. Zhang, L. Yu, Q. Wang, and C. Ong, "Estimator design for discrete-time switched neural networks with asynchronous switching and time-varying delay," *IEEE Transactions on Neural Networks on Learning Systems*, vol. 23, no. 5, pp. 827–834, 2012.

[14] Z. Xiang, C. Liang, and Q. Chen, "Robust L_2-L_∞ filtering for switched systems under asynchronous switching," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 8, pp. 3303–3318, 2011.

[15] W. Feng, J. Tian, and P. Zhao, "Stability analysis of switched stochastic systems," *Automatica*, vol. 47, no. 1, pp. 148–157, 2011.

[16] D. Xie, L. Wang, F. Hao, and G. Xie, "LMI approach to L_2-gain analysis and control synthesis of uncertain switched systems," *IEE Proceedings: Control Theory and Applications*, vol. 151, no. 1, pp. 21–28, 2004.

[17] C. Yang and W. Zhu, "Stability analysis of impulsive switched systems with time delays," *Mathematical and Computer Modelling*, vol. 50, no. 7-8, pp. 1188–1194, 2009.

[18] V. N. Phat, T. Botmart, and P. Niamsup, "Switching design for exponential stability of a class of nonlinear hybrid time-delay systems," *Nonlinear Analysis: Hybrid Systems*, vol. 3, no. 1, pp. 1–10, 2009.

[19] J. Cheng, H. Zhu, S. Zhong, and Y. Zhang, "Robust stability of switched delay systems with average dwell time under asynchronous switching," *Journal of Applied Mathematics*, vol. 2012, Article ID 956370, 17 pages, 2012.

[20] G. S. Deaecto, J. C. Geromel, and J. Daafouz, "Dynamic output feedback H_∞ control of switched linear systems," *Automatica*, vol. 47, no. 8, pp. 1713–1720, 2011.

[21] H. Li, "New criteria for synchronization stability of continuous complex dynamical networks with non-delayed and delayed coupling," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 2, pp. 1027–1043, 2011.

[22] L. Zhang and P. Shi, "Stability, L_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time," *IEEE Transactions on Automatic Control*, vol. 54, no. 9, pp. 2192–2199, 2009.

[23] B. Niu and J. Zhao, "Stabilization and L_2-gain analysis for a class of cascade switched nonlinear systems: an average dwell-time method," *Nonlinear Analysis: Hybrid Systems*, vol. 5, no. 4, pp. 671–680, 2011.

[24] L. Wu, Z. Feng, and W. X. Zheng, "Exponential stability analysis for delayed neural networks with switching parameters: average dwell time approach," *IEEE Transactions on Neural Networks*, vol. 21, no. 9, pp. 1396–1407, 2010.

[25] W. Xiang and J. Xiao, "H_∞ finite-time control for switched nonlinear discrete-time systems with norm-bounded disturbance," *Journal of the Franklin Institute*, vol. 348, no. 2, pp. 331–352, 2011.

[26] D. Liberzon, *Switching in Systems and Control*, Birkhäuser, Boston, Mass, USA, 2003.

[27] P. Dorato, "Short time stability in linear time-varying systems," in *Proceedings of the IRE International Convention Record Part 4*, pp. 83–87, 1961.

[28] X. Huang, W. Lin, and B. Yang, "Global finite-time stabilization of a class of uncertain nonlinear systems," *Automatica*, vol. 41, no. 5, pp. 881–888, 2005.

[29] C. Qian and J. Li, "Global finite-time stabilization by output feedback for planar systems without observable linearization,"

IEEE Transactions on Automatic Control, vol. 50, no. 6, pp. 885–890, 2005.

[30] Y. Hong, "Finite-time stabilization and stabilizability of a class of controllable systems," *Systems & Control Letters*, vol. 46, no. 4, pp. 231–236, 2002.

[31] S. He and F. Liu, "Finite-time boundedness of uncertain time-delayed neural network with Markovian jumping parameters," *Neurocomputing*, vol. 103, pp. 87–92, 2013.

[32] Z. Xiang, Y.-N. Sun, and M. S. Mahmoud, "Robust finite-time H_∞ control for a class of uncertain switched neutral systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 4, pp. 1766–1778, 2012.

[33] L. Zhu, Y. Shen, and C. Li, "Finite-time control of discrete-time systems with time-varying exogenous disturbance," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 2, pp. 361–370, 2009.

[34] X. Luan, F. Liu, and P. Shi, "Finite-time filtering for non-linear stochastic systems with partially known transition jump rates," *IET Control Theory & Applications*, vol. 4, no. 5, pp. 735–745, 2010.

[35] S. He and F. Liu, "Observer-based finite-time control of time-delayed jump systems," *Applied Mathematics and Computation*, vol. 217, no. 6, pp. 2327–2338, 2010.

[36] X. Lin, H. Du, and S. Li, "Finite-time boundedness and L_2-gain analysis for switched delay systems with norm-bounded disturbance," *Applied Mathematics and Computation*, vol. 217, no. 12, pp. 5982–5993, 2011.

[37] H. Song, L. Yu, D. Zhang, and W.-A. Zhang, "Finite-time H_∞ control for a class of discrete-time switched time-delay systems with quantized feedback," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 12, pp. 4802–4814, 2012.

[38] Z. Zuo, Y. Liu, Y. Wang, and H. Li, "Finite-time stochastic stability and stabization of linear discrete-time Markovian jump systems with partly unknown transition probabilities," *ITE Control Theory and Applications*, vol. 6, no. 10, pp. 1522–1526, 2011.

[39] Y. Zhang, C. Liu, and X. Mu, "Robust finite-time H_∞ control of singular stochastic systems via static output feedback," *Applied Mathematics and Computation*, vol. 218, no. 9, pp. 5629–5640, 2012.

[40] X. Zhang, G. Feng, and Y. Sun, "Finite-time stabilization by state feedback control for a class of time-varying nonlinear systems," *Automatica*, vol. 48, no. 3, pp. 499–504, 2012.

[41] G. Garcia, S. Tarbouriech, and J. Bernussou, "Finite-time stabilization of linear time-varying continuous systems," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 364–369, 2009.

[42] Z. Xiang, C. Qiao, and M. S. Mahmoud, "Finite-time analysis and H_∞ control for switched stochastic systems," *Journal of the Franklin Institute*, vol. 349, no. 3, pp. 915–927, 2012.

[43] J. Cheng, H. Zhu, S. Zhong, Y. Zeng, and X. Dong, "Finite-time H_∞ control for a class of Markovian jump systems with mode-dependent time-varying delays via new Lyapunov functionals," *ISA Transactions*, vol. 52, pp. 768–774, 2013.

[44] J. Cheng, H. Zhu, S. Zhong, Y. Zhang, and Y. Li, "Finite-time H_∞ control for a class of discrete-time Markov jump systems with partly unknown time-varying transition probabilities subject to average dwell time switching," *International Journal of Systems Science*, 2013.