Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 623757, 6 pages
http://dx.doi.org/10.1155/2013/623757

Research Article

A New Distance Measure and Ranking Method for Generalized Trapezoidal Fuzzy Numbers

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Received 28 January 2013; Accepted 13 April 2013

Academic Editor: Gozde Ulutagay

This study presents an approximate approach for ranking fuzzy numbers based on the centroid point of a fuzzy number and its area. The total approximate is determined by convex combining of fuzzy number's relative and its area that is based on decision maker's optimistic perspectives. The proposed approach is simple in terms of computational efforts and is efficient in ranking a large quantity of fuzzy numbers. A group of examples by Bortolan and Degani (1985) demonstrate the accuracy and applicability of the proposed approach. Finally by this approach, a new measure is introduced between two fuzzy numbers.

1. Introduction

The fuzzy set theory pioneered by Zadeh [1] has been extensively used. Fuzzy numbers or fuzzy subsets of the real line R are applied to represent the imprecise numerical measurements of different alternatives. Therefore, comparing the different alternatives is actually comparing the resulting fuzzy numbers.

Also in many applications of fuzzy set theory to decision making, we need to know the best select from a collection of possible solutions. This selection process may require that we rank or order fuzzy numbers.

Several researchers presented ranking methods [2–7]. Various techniques are applied to compare the fuzzy numbers. Some of the exiting approaches are difficult to understand and have suffered from different plights, for example, the lack of discrimination, producing counterintuitive orderings, and ultimately resulting in inconsistent ordering if a new fuzzy number is added. Also nearly all approaches should acquire membership functions of fuzzy numbers before the ranking is performed; however, this may be infeasible in real applications. Furthermore, accuracy and efficiency should be of priority concern in the ranking process if ranking a large amount of fuzzy numbers. In light of the above discussion, specially in [6], Chen and Lu have proposed an approximate approach for ranking fuzzy numbers, in that they worked with dominance.

Also we studied here the approach is determined by convex combining the centroid point (x_0, y_0) of x_0 and area s of a fuzzy number that performs simple arithmetic operations for the ranking purpose, and it can be applied to rank the combination case of some fuzzy numbers and crisp numbers and the case of discrete fuzzy numbers and it is useful in ranking a large quantity of fuzzy numbers. Comparing the proposed approximate approach with the existing methods using both Bortolan and Degani's examples [3].

The methods of measuring the distance between fuzzy numbers have became important due to the significant applications in diverse fields like remote sensing, data mining, pattern recognition, multivariate data analysis and so on. Several distance measures for precise numbers are well established in the literature. Several researchers focused on computing the distance between fuzzy numbers [2, 3, 5, 7–10]. Here we introduce a new distance between two trapezoidal fuzzy numbers by the new approach proposed for them.

This paper is organized as follows.

In Section 2, the basic concept of fuzzy number operation is brought. Section 3 introduces the ranking approach and presents some comparative examples which demonstrate the accuracy of the proposed approach over the exiting methods.

Table 1: Case studies using Boltolan and Degani's examples in Figure 1.

	Chang	Kerre	Lee-Lee		Liou-Wang			Jain			Proposed		
			O	G	$\gamma = 1$	$\gamma = .5$	$\gamma = 0$	$K = 1$	$K = 2$	$K = .5$	$\lambda = 1$	$\lambda = .5$	$\lambda = 0$
A_1	.02	.80	0	0	.15	.1	.05	.18	.03	.40	.1	.1	.1
A_2	.18	1	1	1	.95	.9	.85	.90	.84	.95	.9	.5	.1
B_1	.14	.80	0	0	.75	.7	.65	.72	.55	.84	.7	.4	.1
B_2	.18	1	1	1	.95	.9	.85	.90	.84	.95	.9	.5	.1
C_1	.16	.85	0	0	.85	.8	.75	.81	.70	.90	.8	.45	.1
C_2	.09	1	1	1	.98	.95	.93	.95	.92	.97	.95	.5	.5
D_1	.04	.80	0	0	.25	.2	.15	.32	.12	.55	.2	.15	.1
D_2	.08	.80	0	.5	.45	.4	.35	.55	.33	.72	.1	.25	.4
D_3	.14	1	1	1	.75	.7	.65	.89	.80	.94	.1	.4	.7
E_1	0	.89	0	0	.5	.25	0	.09	0	.26	.03	.04	.05
E_2	.12	.85	0	.5	.65	.6	.55	.63	.42	.78	.6	.35	.1
E_3	.10	1	1	1	1	.98	.95	1	1	1	.96	.5	.05
F_1	.40	.96	.67	1	.75	.59	.43	.66	.53	.78	.55	.43	.32
F_2	.34	.89	.32	.32	.75	.59	.43	.69	.51	.81	.61	.47	.32
G_1	.58	.51	.40	.23	.75	.4	.05	.66	.53	.78	.41	.55	.7
G_2	.12	.89	.60	1	.65	.6	.55	.63	.42	.78	.6	.35	.1
H_1	.58	.42	.40	.13	.75	.4	.05	.66	.53	.78	.41	.55	.7
H_2	.14	.95	.70	1	.75	.7	.65	.72	.54	.84	.7	.4	.1
I_1	.46	1	.65	1	.95	.8	.65	.90	.84	.95	.76	.53	.3
I_2	.41	.86	.35	.61	.85	.7	.55	.76	.65	.86	.7	.5	.3
I_3	.38	.76	.20	.18	.75	.6	.45	.66	.54	.78	.63	.46	.3
J_1	.28	1	.65	1	.8	.7	.6	.82	.71	.89	.7	.45	.2
J_2	.37	.91	.34	.60	.8	.65	.5	.82	.71	.89	.63	.46	.3
J_3	.52	.75	.24	.23	.8	.58	.35	.82	.71	.89	.57	.51	.45
K_1	.56	1	.62	1	.85	.63	.4	.90	.82	.94	.62	.53	.45
K_2	.33	.85	.38	.62	.7	.55	.4	.69	.56	.80	.56	.43	.3
K_3	.20	.75	.24	.21	.6	.5	.4	.64	.45	.77	.5	.35	.2
L_1	.29	.91	.50	1	.65	.5	.35	.73	.60	.83	.5	.4	.3
L_2	.10	.91	.50	1	.55	.5	.45	.67	.48	.80	.5	.3	.1

A new measure between fuzzy numbers is defined in Section 4. Concluding remarks are finally made in Section 5.

2. Preliminaries

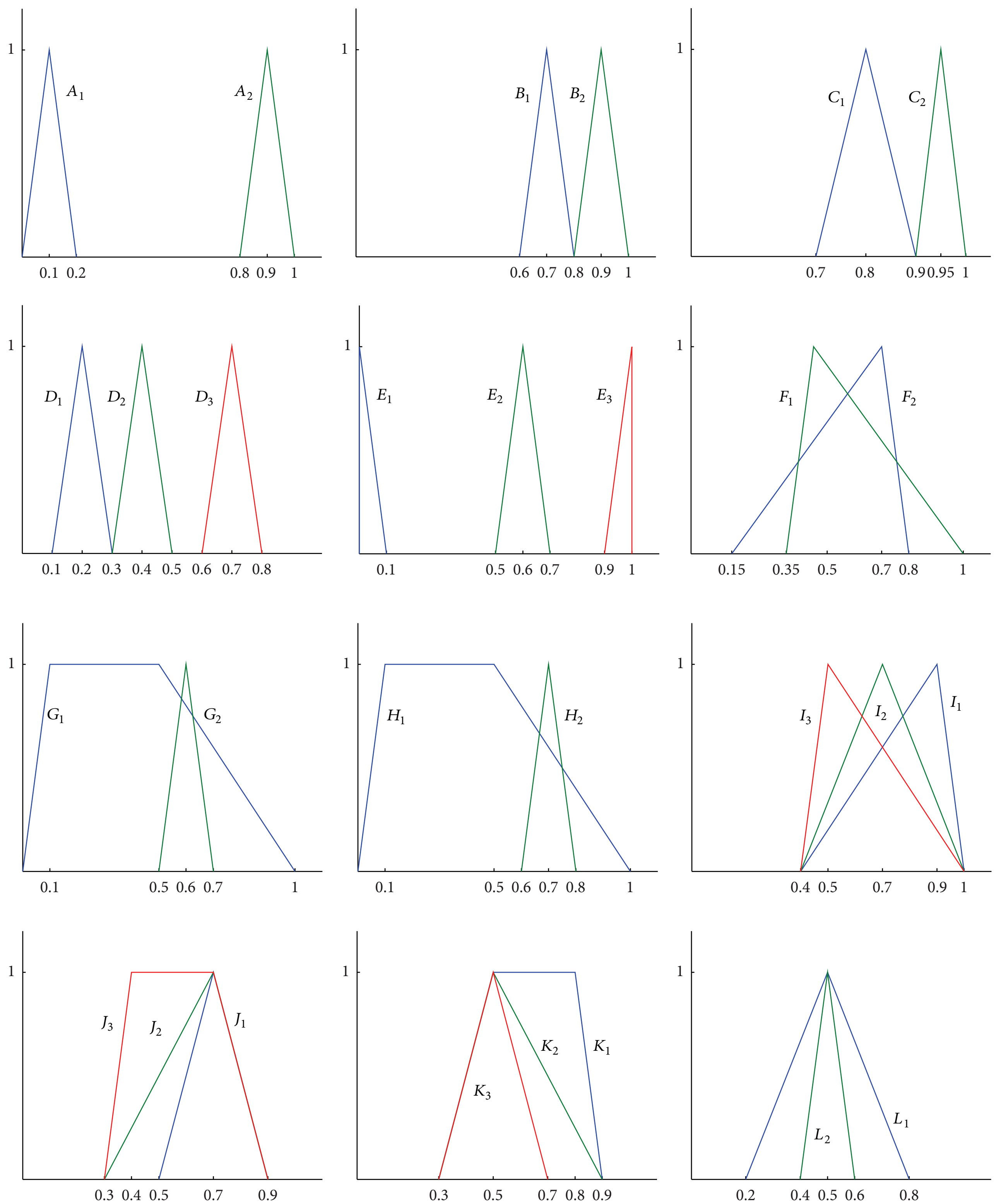
A fuzzy number is a convex fuzzy subset of real line R and is completely defined by its membership function. Let A be a fuzzy number, whose membership function $A(x)$ can generally be defined as

$$A(x) = \begin{cases} A_L(x), & \text{for } a \le x < b \\ w, & \text{for } b \le x < c \\ A_R(x), & \text{for } c < x \le d \\ 0, & \text{Otherwise,} \end{cases} \tag{1}$$

where $0 < w \le 1$ is a constant, $A_L : [a, b) \to [0, w]$ is monotonic increasing continuous from the right function, and $A_R : (c, d] \to [0, w]$ is monotonic decreasing continuous from the left function. If $w = 1$, then A is a normal fuzzy number; otherwise, it is said to be a nonnormal fuzzy number. If the membership function $A(x)$ is piecewise linear and continuous, then A is referred to as a trapezoidal fuzzy number and is usually denoted by $A = (a, b, c, d; w)$ or $A = (a, b, c, d)$ if $w = 1$. In this case, $A_L(x) = (w/(b-a))(x-a)$, $a \le x < b$, and $A_R(x) = (w/(c-d))(x-d)$, $c \le x < d$. In particular, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $A = (a, b, d; w)$ or $A = (a, b, d)$ if $w = 1$ [11].

Definition 1 (see [11]). Consider a general trapezoidal fuzzy number $A = [a, b, c, d; w]$, whose membership function is defined as

$$A(x) = \begin{cases} w\dfrac{x-a}{b-a}, & \text{for } a \le x < b \\ w, & \text{for } b \le x < c \\ w\dfrac{d-x}{d-c}, & \text{for } c < x \le d \\ 0, & \text{Otherwise,} \end{cases} \tag{2}$$



1
A_1
A_2
0.1 0.2 0.8 0.9 1
B_1
B_2
0.6 0.7 0.8 0.9 1
C_1
C_2
0.7 0.8 0.90.95 1
D_1
D_2
D_3
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
E_1
E_2
E_3
0.1 0.5 0.6 0.7 0.9 1
F_1
F_2
0.15 0.35 0.5 0.7 0.8 1
G_1
G_2
0.1 0.5 0.6 0.7 1
H_1
H_2
0.1 0.5 0.6 0.7 0.8 1
I_3
I_2
I_1
0.4 0.5 0.7 0.9 1
J_3
J_2
J_1
0.3 0.4 0.5 0.7 0.9
K_3
K_2
K_1
0.3 0.5 0.7 0.8 0.9
L_1
L_2
0.2 0.4 0.5 0.6 0.8

Figure 1

Table 2: Comparison of other methods for the examples in Figure 1.

Chang	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_3 \prec E_2$	$F_1 \succ F_2$
Kerre	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \approx D_2 \prec D_3$	$E_2 \prec E_1 \prec E_3$	$F_1 \succ F_2$
Lee-Lee						
O	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \approx D_2 \prec D_3$	$E_1 \approx E_2 \prec E_3$	$F_1 \succ F_2$
G	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \succ F_2$
Liou-Wang						
$\gamma = 1$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \approx F_2$
$\gamma = .5$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \approx F_2$
$\gamma = 0$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \approx F_2$
Jain						
$k = 1$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \succ F_2$
$k = 2$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \succ F_2$
$k = .5$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \approx F_2$
Proposed						
$\lambda = 1$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \succ F_2$
$\lambda = .5$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \prec D_2 \prec D_3$	$E_1 \prec E_2 \prec E_3$	$F_1 \succ F_2$
$\lambda = 0$	$A_1 \prec A_2$	$B_1 \prec B_2$	$C_1 \prec C_2$	$D_1 \approx D_2 \approx D_3$	$E_1 \approx E_3 \prec E_2$	$F_1 \approx F_2$

where $0 < w \leq 1$. The centroid point (x_0, y_0) of A has been determined by

$$x_0 = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)}\right],$$
$$y_0 = \frac{w}{3}\left[1 + \frac{c - d}{(d + c) - (a + b)}\right]. \tag{3}$$

Definition 2. Consider a general trapezoidal fuzzy number $A = [a, b, c, d; w]$; the area of A determined by

$$s = \frac{w}{2}\left((d + c) - (a + b)\right). \tag{4}$$

3. The Ranking Method

Let A be a trapezoidal generalized fuzzy number. We define the particular convex combination of the fuzzy number's relative locations on the x-axis, x_0, and the fuzzy number's area, s, in Definitions 1 and 2, respectively, as follows

$$D_\lambda(A) = \lambda x_0 + (1 - \lambda) s, \tag{5}$$

where the index $\lambda \in [0, 1]$.

The following choices are related to find approximate approach for rankings fuzzy numbers based on decision maker's opinions.

A decision maker is able to denote his preference for the fuzzy number's relative locations on the x-axis, x, or the fuzzy number's area, s, with ordering suitable index λ in obtaining approximate approach for ranking fuzzy numbers. If the decision maker has a preference for x_0 then λ must be chosen in $(1/2, 1]$. Also if the area s is a choice then λ must be chosen in $[0, 1/2)$.

It is clear that for $\lambda = 0$, the choice is just s and for $\lambda = 1$ is x_0. If $\lambda = 1/2$ then both x_0 and s have the same effects.

Now a decision maker can rank a pair of fuzzy numbers, A and B, using $D_\lambda(A)$ and $D_\lambda(B)$ based on the following rules:

(i) $D_\lambda(A) < D_\lambda(B)$, then $A \prec B$,

(ii) $D_\lambda(A) = D_\lambda(B)$, then $A \approx B$,

(iii) $D_\lambda(A) > D_\lambda(B)$, then $A \succ B$.

We can see calculation results of the proposed method in Table 1 for the twelve-pair sets in Figure 1 that were used in [4].

Comparisons of the proposed method to the exiting methods for the twelve-pair sets in Figure 1 are given in Tables 2 and 3.

4. A New Measure between Two Trapezoidal Generalized Fuzzy Numbers

Let A and B be two arbitrary trapezoidal fuzzy numbers with x_0 and s defined by (1) and (2), respectively. The distance between A and B is defined as

$$d(A, B) = D_\lambda(A) - D_\lambda(B). \tag{6}$$

A decision maker can order a pair of trapezoidal fuzzy numbers, A and B, too using $d(A, B)$ based on the following rules:

(i) $d(A, B) > 0$, then $A \succ B$,

(ii) $d(A, B) = 0$, then $A \approx B$,

(iii) $d(A, B) < 0$, then $A \prec B$.

5. Some Properties

Some valuable properties are described in the following, which are useful in ranking a large quantity of generalized

Table 3: Comparison of other methods for the examples in Figure 1.

Chang	$G_1 \succ G_2$	$H_1 \succ H_2$	$I_3 \prec I_2 \prec I_1$	$J_1 \prec J_2 \prec J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \succ L_2$
Kerre	$G_1 \prec G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_3 \prec J_2 \prec J_1$	$K_3 \prec K_2 \prec K_1$	$L_1 \approx L_2$
Lee-Lee						
O	$G_1 \prec G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_3 \prec J_2 \prec J_1$	$K_3 \prec K_2 \prec K_1$	$L_1 \approx L_2$
G	$G_1 \prec G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_3 \prec J_2 \prec J_1$	$K_3 \prec K_2 \prec K_1$	$L_1 \approx L_2$
Liou-Wang						
$\gamma = 1$	$G_1 \prec G_2$	$H_1 \approx H_2$	$I_3 \prec I_2 \prec I_1$	$J_1 \approx J_2 \approx J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \prec L_2$
$\gamma = .5$	$G_1 \prec G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_3 \prec J_2 \prec J_1$	$K_3 \prec K_2 \prec K_1$	$L_1 \approx L_2$
$\gamma = 0$	$G_1 \prec G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_3 \prec J_2 \prec J_1$	$K_3 \approx K_2 \approx K_1$	$L_1 \prec L_2$
Jain						
$k = 1$	$G_1 \succ G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_1 \approx J_2 \approx J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \succ L_2$
$k = 2$	$G_1 \succ G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_1 \approx J_2 \approx J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \succ L_2$
$k = .5$	$G_1 \approx G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_1 \approx J_2 \approx J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \succ L_2$
Proposed						
$\lambda = 1$	$G_1 \prec G_2$	$H_1 \prec H_2$	$I_3 \prec I_2 \prec I_1$	$J_3 \prec J_2 \prec J_1$	$K_3 \prec K_2 \prec K_1$	$L_1 \approx L_2$
$\lambda = .5$	$G_1 \succ G_2$	$H_1 \succ H_2$	$I_3 \prec I_2 \prec I_1$	$J_1 \prec J_2 \prec J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \succ L_2$
$\lambda = 0$	$G_1 \succ G_2$	$H_1 \succ H_2$	$I_1 \approx I_2 \approx I_3$	$J_1 \prec J_2 \prec J_3$	$K_3 \prec K_2 \prec K_1$	$L_1 \succ L_2$

trapezoidal fuzzy numbers simultaneously. Assume that there are three different generalized trapezoidal fuzzy numbers, A, B, and C, to be ranked:

(i) $d(A, A) = 0$

(ii) $d(A, B) = -d(B, A)$,

(iii) if $d(A, B) > 0$, and $d(B, C) > 0$ then $d(A, C) > 0$,

(iv) more than two fuzzy numbers can be ranked by comparing with the benchmark fuzzy number. let A be the benchmark, $d(A, B) = a$, and $d(A, C) = b$. By using the previous two properties, obviously $d(A, B) = d(A, C) - d(B, C) = b - a$. Therefore, if $b > a$, then $d(B, C) > 0$; that is, $B \succ C$,

(v) the ranking if more than two fuzzy numbers has the robustness property, that is,

$$\text{if } d(A, B) < \varepsilon, \quad \text{then } |d(A, C) - d(B, C)| < \varepsilon. \tag{7}$$

This equation suggests that the dominance difference between one fuzzy number and the other two fuzzy numbers is insignificant, if the two fuzzy numbers are close to each other. These properties hold since $d(A, C) - d(B, C) = d(A, B)$.

6. Conclusion

Ranking fuzzy numbers is a critical task in a fuzzy decision making process. Particularly, when ranking a large quantity of fuzzy numbers and only limited information about them can be obtained, an effective, efficient, and accurate ranking method becomes necessary. The proposed ranking approach only considers convex combining of the centroid point (x_0, y_0) of x_0 and area s of a fuzzy number with a decision maker's optimistic perspectives, that is, actually the measurement of the degree of difference between two fuzzy numbers. This ranking is compared with some exiting ranking methods.

Here too, a new distance measure has been introduced for computing crisp distances for fuzzy numbers.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and Computation*, vol. 8, pp. 338–353, 1965.

[2] S. Abbasbandy and S. Hajighasemi, "A fuzzy distance between two fuzzy numbers," in *Information Processing and Management of Uncertainty in Knowledge-Based Systems*, vol. 81 of *Applications Communications in Computer and Information Science*, pp. 376–382, 2010.

[3] S. Abbasbandy and T. Hajjari, "A new approach for ranking of trapezoidal fuzzy numbers," *Computers & Mathematics with Applications*, vol. 57, no. 3, pp. 413–419, 2009.

[4] G. Bortolan and R. Degani, "A review of some methods for ranking fuzzy subsets," *Fuzzy Sets and Systems*, vol. 15, no. 1, pp. 1–19, 1985.

[5] C. Chakraborty and D. Chakraborty, "A theoretical development on a fuzzy distance measure for fuzzy numbers," *Mathematical and Computer Modelling*, vol. 43, no. 3-4, pp. 254–261, 2006.

[6] L.-H. Chen and H.-W. Lu, "An approximate approach for ranking fuzzy numbers based on left and right dominance," *Computers & Mathematics with Applications*, vol. 41, no. 12, pp. 1589–1602, 2001.

[7] C.-H. Cheng, "A new approach for ranking fuzzy numbers by distance method," *Fuzzy Sets and Systems*, vol. 95, no. 3, pp. 307–317, 1998.

[8] P. Grzegorzewski, "Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric," *Fuzzy Sets and Systems*, vol. 148, no. 2, pp. 319–328, 2004.

[9] L. Tran and L. Duckstein, "Comparison of fuzzy numbers using a fuzzy distance measure," *Fuzzy Sets and Systems*, vol. 130, no. 3, pp. 331–341, 2002.

[10] W. Voxman, "Some remarks on distances between fuzzy numbers," *Fuzzy Sets and Systems*, vol. 100, pp. 353–365, 1998.

[11] Y.-M. Wang, J.-B. Yang, D.-L. Xu, and K.-S. Chin, "On the centroids of fuzzy numbers," *Fuzzy Sets and Systems*, vol. 157, no. 7, pp. 919–926, 2006.