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Research Article

State Tracking of MRAC Systems in the Presence of Controller Temporary Failure Based on a Switching Method

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The state tracking problem for a class of model reference adaptive control (MRAC) systems in the presence of controller temporary failures is studied. Due to the controller temporary failure, the considered system is viewed as an error switched system. The properties of Lyapunov function candidates without switching are described. Then the notion of global practical stability of switched systems is presented, and sufficient conditions for global practical stability of the error system under the restrictions of controller failure frequency and unavailability rate are provided. An example is presented to demonstrate the feasibility and effectiveness of the proposed method.

1. Introduction

There are often parameter, structural, and environment uncertainties in practical systems [1–4]. Model reference adaptive control (MRAC) has been used as an important control approach for such uncertain systems [5, 6]. Closed-loop signal boundedness and asymptotic tracking can be ensured by a state feedback controller and adaptive laws in MRAC systems [7].

On the other hand, controller temporary failures are often encountered in real control systems due to various environment factors during operation. Some motivations of studying controller failures are summarized in [8]. The reasons can be roughly classified into two categories: passive and positive ones. A typical passive reason is that the signals are not transmitted perfectly or the controller itself is not available for some reasons. For instance, the packet dropout phenomenon in networked control systems leads to controller failure, which is inevitable because of unreliable transmission paths. In contrast to passive reasons, a typical positive reason is that the controller is purposefully suspended from time to time for an economic or system life consideration [9]. Apparently, controller failures may lead to severe performance deterioration of systems. Especially, for adaptive control systems, the controller failure may cause the tracking error divergence due to the uncertainties of systems. Therefore, it is both theoretically and practically important to develop some new techniques to deal with the case of controller temporary failure of adaptive systems.

Recently, there are rapidly growing interests in switched systems and switching control in the control community [10–13]. In the study of stability of switched systems, one of the effective tools is the average dwell time approach [14–16]. Based on this approach, exponential stability is guaranteed if the unavailability rate of the controller is smaller than a specified constant and the average time interval between controller failures is large enough [9, 17, 18]. In [8], this result was further extended to symmetric linear time-invariant system. The concept of controller failure frequency was first introduced in [9], and the cases of controller temporary failure for a class of time-varying delay systems were analyzed in [19]. Interestingly, a nonswitched MRAC system in the presence of controller temporary failure can be viewed as a switched system with a switching signal depending on the time interval between controller failures. Thus, theories and methods of switched systems may be applicable to the study of the state tracking problem for nonswitched MRAC systems with controller temporary failure. However, this issue has been rarely explored so far.

In this paper, we study the state tracking problem for MRAC systems in the presence of controller temporary failure. As in [8], the controller temporary failure means that the controller itself is not available or the controller signals are not transmitted perfectly within a certain time interval. Furthermore, we assume that the parameter estimation is "frozen" in the instant of the controller temporary failure until the controller works normally. There are two main issues to be addressed in this paper. One is to describe a tradition MRAC system in the presence of controller temporary failure as an error switched system with two subsystems: the normal error subsystem which stands for the case without controller failure and the unstable error subsystem which describes the case of controller failure. The other issue is the stability analysis for the error switched system. To address the second issue, we analyze the properties of Lyapunov function candidates without switching and introduce the notions of global practical stability, failure frequency, and unavailability rate.

The results in this paper have three features. First of all, MRAC systems in the presence of controller temporary failure are first considered. Secondly, the state tracking problem is studied from a switched system point of view. Finally, the global practical stability criterion is given for the considered system under the condition of controller failure frequency and unavailability rate.

The organization of the paper is as follows. The state tracking problem in the presence of controller failure is formulated in Section 2. In Section 3, we present an error switched system. Section 4 gives three lemmas and the main result. An example is given to illustrate the effectiveness of the proposed method in Section 5. Finally, the conclusions are presented in Section 6.

The notation is standard. Consider the following:

$\lambda_{\max}(A)$ ($\lambda_{\min}(A)$): the largest (smallest) eigenvalue of matrix A;

$\|A\| = \sqrt{\lambda_{\max}(AA^T)}$: the norm of matrix A;

$\|x\| = \sqrt{(\sum_{i=1}^{n} |x_i|^2)}$: the norm of a vector $x = (x_1, x_2, \ldots, x_n)^T \in R^n$;

$\mathrm{tr}[A]$: the trace of a square matrix A.

2. Problem Statement

Consider a system

$$\dot{x}(t) = \Theta f(x) + Bu(t), \tag{1}$$

where $B \in R^{n\times n}$ is input matrix, $x(t) \in R^n$ is the state, $u(t) \in R^n$ is the control input, $\Theta \in R^{n\times p}$ is an uncertain constant parameter matrix with a bounded δ_Θ, that is, $\|\Theta\| \le \delta_\Theta$, and $f(x) \in R^p$ is a vector which can be described as $f(x) = Fx + g(x)$, where $F \in R^{p\times n}$ is a constant parameter matrix and $\|g(x)\| \le M$ for some $M \ge 0$.

The classical state tracking problem is to design a controller such that the state $x(t)$ of the system (1) tracks a given reference state $x_m(t)$ generated from the reference model system

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \tag{2}$$

where $A_m \in R^{n\times n}$ is a constant Hurwitz matrix, $B_m \in R^{n\times n}$ is a constant input matrix, and $r(t) \in R^n$ is a bounded and piecewise continuous reference input.

Suppose that there exist matrices R, T, and W^* such that the following matching equations are satisfied:

$$A_m = BR, \qquad B_m = BT, \qquad BW^* = \Theta, \tag{3}$$

where W^* is an unknown matrix due to uncertain constant parameter matrix Θ.

Define the tracking error $e(t) = x_m(t) - x(t)$. To solve the state tracking problem, we use the controller structure [20]

$$u(t) = \psi\omega, \tag{4}$$

where $\psi = \begin{bmatrix} R & \widehat{W}(t) & T \end{bmatrix}$ and $\omega = \begin{bmatrix} x^T \\ -f^T(x) \\ r^T \end{bmatrix}$, $\widehat{W}(t)$ is the estimate of unknown matrix W^*, and $\|\widehat{W}(t)\| \le \delta_{\widehat{W}}$.

Apply a parameter projection adaptive law

$$\dot{\widehat{W}}(t) = -\Gamma B^T P_1 e(t) f^T(x) + F_s, \tag{5}$$

where $\Gamma = \mathrm{diag}\{\Gamma_1, \Gamma_2, \ldots, \Gamma_i, \ldots, \Gamma_M\}$ with positive constants Γ_i, P_1 is a symmetric positive definite matrix satisfying $A_m^T P_1 + P_1 A_m < 0$, and F_s is a vector satisfying

$$F_s = \begin{cases} 0, & \text{if } \left\|\widehat{W}(t)\right\| \le \delta_{\widehat{W}}, \\ \Gamma B^T P_1 e(t) f^T(t), & \text{if } \left\|\widehat{W}(t)\right\| > \delta_{\widehat{W}}. \end{cases} \tag{6}$$

Then, the closed-loop system is

$$\dot{x}(t) = A_m x(t) + B_m r(t) + B\widetilde{W}(t) f(x), \tag{7}$$

where $\widetilde{W}(t) = W^* - \widehat{W}(t)$.

From [5, 20, 21], $x(t)$ converges asymptotically to $x_m(t)$ under the controller (4) and the adaptive law (5), that is, $\lim_{t\to\infty} e(t) = 0$.

We now consider the case of controller temporary failure depicted in Figure 1. Controller failures occur when the controller (a) itself is not available or when the signals are not transmitted perfectly on the route (b). Suppose that the time interval of the controller failures is not more than a specified constant T_s, which means the designed controller can be recovered within a finite time interval [19]. Also, the failed controller implies the complete breakdown of the controller ($u(t) = 0$) in its failure time interval [8]. Hence, the system (1) with the controller temporary failure is dominated by the following piecewise differential equations:

$$\dot{x}(t) = \begin{cases} \Theta f(t) + Bu(t), & \text{when the controller works,} \\ \Theta f(t), & \text{when the controller fails.} \end{cases} \tag{8}$$

We introduce the following definitions which will play key roles in deriving our main results.

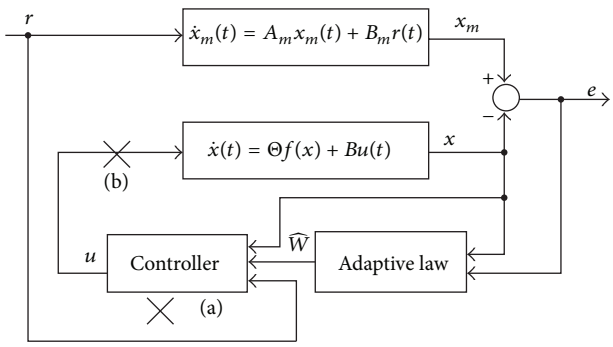

Figure 1: Controller failures occur in MRAC systems.

Definition 1 (see [8]). For any $T_2 > T_1$, denote $T^+(T_1, T_2)$ as the total time interval of controller failure during (T_1, T_2), and call the ratio $T^+(T_1, T_2)/(T_2 - T_1)$ the unavailability rate of the controller in the system.

Definition 2 (see [9]). For any $T_2 > T_1 \geq 0$, let $N_f(T_1, T_2)$ denote the number of control failure in the time interval (T_1, T_2). $F_f(T_1, T_2) = N_f(T_1, T_2)/(T_2 - T_1)$ is referred to as the controller failure frequency in the time interval (T_1, T_2).

In this paper, our objective is to develop conditions under which $x(t)$ tracks $x_m(t)$ subject to controller temporary failure.

3. Error Switched System

From the closed-loop system (7), we have a normal error system

$$\dot{e}(t) = A_m e(t) - B\widetilde{W}(t) f(x). \tag{9}$$

When controller fails, we obtain an unstable error system

$$\dot{e}(t) = A_m e(t) + A_m x(t) + B_m r - \Theta f(x). \tag{10}$$

In this condition, we choose the adaptive law

$$\dot{\widehat{W}}(t) = 0. \tag{11}$$

Remark 3. When controller fails, because the adaptive parameter $\widehat{W}(t)$ has no influence on the tracking error $e(t)$, it is proper that the parameter estimation $\widehat{W}(t)$ is "frozen" in the instants of the controller temporary failure until the controller works normally.

Based on (5), (9), (10), and (11), $e(t)$ is governed by the following error switched system:

$$\dot{e}(t) = A_m e(t) + \Phi_\sigma(t), \tag{12}$$

where $\sigma(t) : [0, +\infty) \to M = \{1, 2\}$, $\Phi_1 = -B\widetilde{W}(t) f(x)$, and $\Phi_2 = A_m x(t) B_m r - \Theta f(x)$.

Meanwhile, we have a switching adaptive law of the following form:

$$\dot{\widehat{W}}(t) = \left(-\Gamma B^T P_1 e(t) f^T(x) + F_s\right)\Psi_\sigma, \tag{13}$$

where $\Psi_1 = I_{p\times p}$ and $\Psi_2 = 0_{p\times p}$.

When $\sigma = 1$, the normal error subsystem is active, which corresponds to the case of no controller failure; when $\sigma = 2$, the unstable error subsystem is active, which denotes that the controller fails.

Therefore, the problem of state tracking in the presence of controller temporary failure can be handled by means of analyzing the stability of the error switched system (12) with the switching adaptive law (13).

To analyze the stability of the error switched system (12), we introduce the following definition.

Definition 4 (see [22]). Consider system (12). Given a constant $r^* > 0$, the system (12) is said to be globally practically stable with respect to r^* if there exist a switching law $\sigma(t)$ and a constant $T = T(e(t_0), r^*) \geq 0$ which depends on $e(t_0)$ and r^* such that $e(t; t_0, e(t_0)) \in S(r^*) \triangleq \{e \mid \|e\| \leq r^*\}$ for $t \geq t_0 + T$.

Remark 5. Unlike the ε-practical stability concept [23], the initial error in Definition 4 is not required to be bounded. If the initial error is constrained by $e(t_0) \in B(r^*)$, then the global practical stability, given by Definition 4, degenerates into ε-practical stability [22]. Global practical stability stated here expresses a global version of the existing practical stability concept. Obviously, Definition 4 covers the ε-practical stability as a special case.

4. Main Result

In this section, firstly, we give three lemmas to analyze the properties of Lyapunov function candidates without switching. Secondly, we present a theorem to give some conditions under which the error switched system (12) is globally practically stable. Let

$$z(t) = \left[e^T(t), \widetilde{w}_1^T(t), \ldots \widetilde{w}_n^T(t), \ldots \widetilde{w}_p^T(t)\right]^T \in R^{n+n\times p}, \tag{14}$$

where $\widetilde{w}_i(t) \in R^n$ is the ith column of $\widetilde{W}(t), i = 1, 2, \ldots, p$; that is,

$$\widetilde{W}(t) = \left[\widetilde{w}_1(t), \ldots, \widetilde{w}_p(t)\right] \in R^{n\times p}. \tag{15}$$

According to [21], we have

$$\operatorname{tr}\left[\widetilde{W}^T \Gamma^{-1} F_s\right] \leq 0. \tag{16}$$

Note that the parameter estimates $\widehat{W}$ are bounded; thus there exists a constant $l > 0$ defined as

$$l = \max\left(\operatorname{tr}\left[\widetilde{W}^T \Gamma^{-1} \widetilde{W}\right]\right). \tag{17}$$

Consider the situation of the system (1) without controller failure. A Lyapunov functional candidate of the normal error subsystem of (12) is chosen as

$$V_1(z(t)) = e^T(t) P_1 e(t) + \operatorname{tr}\left[\widetilde{W}^T(t) \Gamma^{-1} \widetilde{W}(t)\right]. \tag{18}$$

Differentiating V_1 along the trajectory of the normal error subsystem of (12) and the adaptive law (13) gives

$$\dot{V}_1(z(t)) = e^T(t)\left(A_m^T P_1 + P_1 A_m\right) e(t) + 2\operatorname{tr}\left[\widetilde{W}^T \Gamma^{-1} F_s\right]. \tag{19}$$

The following lemma gives the estimate of the convergence rate of V_1 along the trajectory of the normal error subsystem of (12).

Lemma 6. *Consider the normal error subsystem of* (12). *For any given* $r_1 > \sqrt{l/\lambda_{\min}(P_1)}$, *denote* $S(r_1) = \{e \mid \|e\| < r_1\}$. *If* $e(t) \in R^n/S(r_1)$, *then the inequality*

$$V(z(t)) \le \exp(-2\lambda_1(t-t_0))V_1(z(t_0)) \tag{20}$$

holds for any λ_1 *satisfying* $0 < \lambda_1 \le (\overline{\lambda_1}/2)(1-(l/r_1^2\lambda_{\min}(P_1)))$.

Proof. From (17) and (18), it is easy to get

$$e^T(t)P_1e(t) \le V_1(z(t)) \le e^T(t)P_1e(t) + l. \tag{21}$$

It is obvious that

$$\lambda_{\min}(P_1)\|e(t)\|^2 \le V_1(z(t)) \le \lambda_{\max}(P_1)\|e(t)\|^2 + l. \tag{22}$$

From (16) and (19), it holds that

$$\dot{V}_1(z(t)) \le e^T\left(A_m^TP_1 + P_1A_m\right)e < 0. \tag{23}$$

Since A_m is Hurwitz matrix, there exists a scalar $\overline{\lambda_1} > 0$ such that $A_m^TP_1 + P_1A_m + \overline{\lambda_1}P_1 < 0$.

With the help of (21)– (23), we have

$$\begin{aligned}
&\dot{V}_1(z(t)) \\
&\quad \le -\overline{\lambda_1}V_1(z(t)) + \overline{\lambda_1}\, l \\
&\quad = -2\lambda_1V_1(z(t)) - \left(\overline{\lambda_1} - 2\lambda_1\right)V_1(z(t)) + \overline{\lambda_1}\, l \\
&\quad \le -2\lambda_1V_1(z(t)) - \lambda_{\min}(P_1)\left(\overline{\lambda_1} - 2\lambda_1\right)\|e(t)\|^2 + \overline{\lambda_1}\, l.
\end{aligned} \tag{24}$$

Given that $r_1 > \sqrt{l/\lambda_{\min}(P_1)}$, when $e(t) \in R^n/S(r_1)$, applying (24) leads to $\dot{V}_1(z(t)) \le -2\lambda_1V_1(z(t))$ that is, $V(z(t)) \le \exp(-2\lambda_1(t-t_0))V_1(z(t_0))$; for any λ_1 satisfying $0 < \lambda_1 \le (\overline{\lambda_1}/2)(1-(l/r_1^2\lambda_{\min}(P_1)))$.

This completes the proof. □

When the controller fails, for the unstable error subsystem of (12), we choose another Lyapunov functional candidate of the following form:

$$V_2(z(t)) = e^T(t)P_2e(t) + tr\left[\widetilde{W}^T(t)\Gamma^{-1}\widetilde{W}(t)\right], \tag{25}$$

where P_2 is a positive definite matrix.

Differentiating V_2 along the trajectory of the normal error subsystem of (12) and the adaptive law (13) gives

$$\dot{V}_2(z(t)) = \dot{e}(t)P_2e(t) + e(t)P_2\dot{e}(t). \tag{26}$$

Then, in the following lemma, we estimate the divergence rate of V_2 along the trajectory of the unstable error subsystem of (12).

Lemma 7. *Consider the unstable error subsystem of* (12). *For any given* $r_2 > 0$, *denote* $S(r_2) = \{e \mid \|e\| < r_2\}$. *If* $e(t) \in R^n/S(r_2)$, *then the inequality*

$$V(z(t)) \le \exp(2\lambda_2(t-t_0))V_2(z(t_0)) \tag{27}$$

holds for any λ_2 *satisfying* $0 < \lambda_2 \le (1/2\lambda_{\min}(P_2))((\beta_2/r_2) + \beta_1)$.

Proof. From (17), we have

$$e^T(t)P_2e(t) \le V_2(z(t)) \le e^T(t)P_2e(t) + l. \tag{28}$$

It is obvious that

$$\lambda_{\min}(P_2)\|e(t)\|^2 \le V_2(z(t)) \le \lambda_{\max}(P_2)\|e(t)\|^2 + l. \tag{29}$$

From (10) and (26), we have

$$\begin{aligned}
\dot{V}_2(z(t)) &= e^T(t)\left(A_m^TP_2 + P_2A_m\right)e(t) \\
&\quad + 2e^T(t)P_2\left(A_mx_m + B_mr(t) - \Theta f(x)\right) \\
&= e^T(t)\left(A_m^TP_2 - P_2A_m + 2\Theta F\right)e(t) \\
&\quad + 2e^T(t)P_2\left((A_m - \Theta F)x_m + B_mr(t) - \Theta g(x)\right) \\
&= e^T(t)P_3e(t) + e^T(t)\xi_1 - e^T(t)P_2\Theta g(x),
\end{aligned} \tag{30}$$

where $P_3 \triangleq A_m^TP_2 - P_2A_m + 2\Theta F$ and $\xi_1 \triangleq 2P_2(A_m - \Theta F)x_m + B_mr(t)$.

Since Θ and $r(t)$ are bounded, we have

$$\begin{aligned}
\dot{V}_2(z(t)) &\le \|e(t)\|^2\|P_3\| + \|e(t)\|\|\xi_1\| \\
&\quad + \|e(t)\|\|P_2\|\ \|\Theta\|\|g(x)\| \\
&\le \|e(t)\|^2\|P_3\| + \|e(t)\|\|\xi_1\| \\
&\quad + M\|e(t)\|\|P_2\|\|\Theta\| \\
&\le \|e(t)\|^2\|P_3\| + \|e(t)\|\left(\|\xi_1\| + M\|P_2\|\|\Theta\|\right).
\end{aligned} \tag{31}$$

Then, it holds that

$$\dot{V}_2(z(t)) \le \beta_1\|e(t)\|^2 + \beta_2\|e(t)\|, \tag{32}$$

where $\beta_1 = \|P_3\|$ and $\beta_2 = \|\xi_1\| + M\|P_2\|\|\Theta\|$.

Therefore, for any $r_2 > 0$, when $e(t) \in R^n/S(r_2)$, then the inequality $\dot{V}_2(z(t)) \le 2\lambda_2V_2(z(t))$, that is, $V(z(t)) \le \exp(2\lambda_2(t-t_0))V_2(z(t_0))$, holds for any λ_2 satisfying $0 < \lambda_2 \le (1/2\lambda_{\min}(P_2))((\beta_2/r_2) + \beta_1)$.

This completes the proof. □

Based on Lemmas 6 and 7, we have the following lemma.

Lemma 8. *Consider the subsystems of* (12). *For any* $r_3 > 0$, *if* $e(t) \in R^n/S(r_3)$, *then the inequality*

$$\eta_1\|e(t)\|^2 \le V_i(z(t)) \le \eta_3\|e(t)\|^2, \tag{33}$$

holds for $\eta_1 = \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}$, $\eta_2 = \max\{\lambda_{\max}(P_1), \lambda_{\max}(P_2)\}$, *and* $\eta_3 = \eta_2 + (l/r_3^2)$.

Proof. Form (22) and (29), we have

$$e^T(t)P_ie(t) \le V_i(z(t)) \le e^T(t)P_ie(t) + l. \quad (34)$$

Denote that $\eta_1 = \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}$ and $\eta_2 = \max\{\lambda_{\max}(P_1), \lambda_{\max}(P_2)\}$. Then, we have

$$\eta_1\|e(t)\|^2 \le V_i(z(t)) \le \eta_2\|e(t)\|^2 + l. \quad (35)$$

For any $r_3 > 0$, when $e(t) \in R^n/S(r_3)$, we can find a constant $\eta_3 = \eta_2 + (l/r_3^2)$ such that $\eta_1\|e(t)\|^2 \le V_i(z(t)) \le \eta_3\|e(t)\|^2$.

This completes the proof. □

Furthermore, according to Lemmas 6–8, for any given $r_0' = \max\{r_1, r_2, r_3\}$ and $t > t_0$, if $e(t) \in R^n/S(r_0')$, it is true that

$$V(z(t)) \le \begin{cases} \exp(-2\lambda_1(t-t_0))V_1(z(t_0)) & \text{when the controller works,} \\ \exp(2\lambda_2(t-t_0))V_2(z(t_0)) & \text{when the controller fails.} \end{cases} \quad (36)$$

For (18) and (25), it is obvious that there exists $\mu = \eta_3/\eta_1$ such that

$$V_i(z(t)) \le \mu V_j(z(t)) \quad (i, j = 1, 2). \quad (37)$$

Without loss of generality, for $k = 0, 1, \ldots$, we assume that the controller works during $[t_{2k}, t_{2k+1})$, which means that the first subsystem is active on $[t_{2k}, t_{2k+1})$, while the controller fails during $[t_{2k+1}, t_{2k+2})$, which denotes that the second subsystem is active on $[t_{2k+1}, t_{2k+2})$.

When $e(t_{2k}) \in S(r_1)$, from (22), we have $e(t_{2k+1}) \in S(r_1')$ for any given $r_1' \ge \sqrt{(r_1^2\lambda_{\max}(P_1) + l)/\lambda_{\min}(P_1)}$. It is obvious that (36) and (37) still hold for $r_0 \ge \max\{r_1', r_0'\}$.

Remark 9. Because of the uncertainties of the systems, the properties of the Lyapunov function candidates are restricted outside the ball with the radius r_0 as described as in Lemmas 6–8.

Now, we are in the position to give the main result of this paper.

Theorem 10. *Consider the error switched system* (12) *with the adaptive control law* (13). *For* $k = 0, 1, \ldots$, *if the switching law* $\sigma(t)$ *satisfies the following two conditions:*

Condition 1

$$\frac{T^+(t_{2k}, t)}{(t - t_{2k})} \le \frac{(\lambda_1 - \lambda^*)}{(\lambda_1 + \lambda_2)} \quad (38)$$

holds for some scalar $\lambda^* \in (0, \lambda_1)$,

Condition 2

$$F_f(t_{2k}, t) \le F_f^* = \frac{\lambda}{\ln\mu} \quad (39)$$

holds for some scalar $\lambda \in (0, \lambda^*)$, *then, the error switched system* (12) *is globally practically stable.*

Proof. For any given $r_0 \ge \max\{r_1', r_0'\}$, denote

$$r^* = \exp[\|\Theta\|\,\|F\|(T_s)] \cdot r_0 + \frac{(\kappa + M\|\Theta\|)[\exp(\|\Theta\|\,\|F\|(T_s)) - 1]}{\|\Theta\|\,\|F\|}, \quad (40)$$

where $\kappa \ge \|(A_m - \Theta F)x_m(t) + B_m r(t)\|$ for some $\kappa > 0$. When the initial error $e(t_0) \in R^n$, we will show that there exists a constant $T = T(e(t_0), r^*) \ge 0$, concerned with $e(t_0)$ and r^* such that $e(t; t_0, e(t_0)) \in S(r^*)$ for $t \ge t_0 + T$ under the switching law $\sigma(t)$ satisfying Conditions 1-2. To this end, we will prove the theorem in three cases.

(a) For $e(t_{2k}) \in S(r_0)$, we will show that $e(t_{2k+2}) \in S(r^*)$.

(b) For $e(t_{2k}) \in R^n/S(r_0)$, we will prove that there exists $T_{2k} = T_{2k}(e(t_{2k}), r_0) \ge 0$ such that $\|e(t_{2k} + T_{2k})\| = r_0$ for $e(t) \in R^n/S(r_0)$ and $t \in [t_{2k}, T_{2k})$.

(c) When the initial error $e(t_0) \in R^n$, we will show that there exists a constant $T = T(e(t_0), r^*) \ge 0$ such that $e(t; t_0, e(t_0)) \in S(r^*)$ for $t \ge t_0 + T$ under the switching law $\sigma(t)$ satisfying Conditions 1 and 2.

We first prove (a). Consider $e(t_{2k}) \in S(r_0)$. Because of the asymptotical stability of the normal subsystem of (12), it is obvious that $e(t_{2k+1}) \in S(r_0)$. When $t \in [t_{2k+1}, t_{2k+2})$, the second subsystem is active. From (10), we have

$$\dot{e}(t) = \Theta F e(t) + (A_m - \Theta F)x_m + B_m r - \Theta g(x); \quad (41)$$

then, the trajectory of the error switched system (12) satisfies

$$\begin{aligned} e(t_{2k+2}) &= \exp[\Theta F(t_{2k+2} - t_{2k+1})]e(t_{2k+1}) \\ &\quad + \int_{t_{2k+1}}^{t_{2k+2}} \{\exp[\Theta F(t_{2k+2} - t)] \\ &\qquad \times [(A_m - \Theta F)x_m(t) + B_m r(t) - \Theta g(x(t))]\}\,dt. \end{aligned} \quad (42)$$

With the help of $\|e^A\| \le e^{\|A\|}$, it is obvious that

$$\begin{aligned} &\|e(t_{2k+2})\| \\ &\le \exp[\|\Theta\|\,\|F\| \cdot (t_{2k+2} - t_{2k+1})]\,\|e(t_{2k+1})\| \\ &\quad + \int_{t_{2k+1}}^{t_{2k+2}} \{\exp[\|\Theta\|\,\|F\| \cdot (t_{2k+2} - \tau)] \\ &\qquad \times [\|(A_m - \Theta F)x_m(\tau) + B_m r(\tau)\| \\ &\qquad + \|\Theta\|\,\|g(x(\tau))\|]\}\,d\tau \end{aligned}$$

$$\begin{aligned}
&\le \exp\left[\|\Theta\|\,\|F\|\left(t_{2k+2}-t_{2k+1}\right)\right]\left\|e\left(t_{2k+1}\right)\right\| \\
&\quad+\int_{t_{2k+1}}^{t_{2k+2}}\left\{\left(\Lambda+M\|\Theta\|\right)\exp\left[\|\Theta\|\,\|F\|\left(t_{2k+2}-\tau\right)\right]\right\}d\tau \\
&=\exp\left[\|\Theta\|\,\|F\|\left(t_{2k+2}-t_{2k+1}\right)\right]\left\|e\left(t_{2k+1}\right)\right\| \\
&\quad+\frac{\left(\Lambda+M\|\Theta\|\right)\left[\exp\left(\|\Theta\|\,\|F\|\left(t_{2k+2}-t_{2k+1}\right)\right)-1\right]}{\|\Theta\|\,\|F\|}.
\end{aligned} \tag{43}$$

Because $T_s=\max_k\{t_{2k+2}-t_{2k+1}\}$, we have

$$\begin{aligned}
\left\|e\left(t_{2k+2}\right)\right\| &\le \left\|e\left(t_{2k+1}+T_s\right)\right\| \\
&=\exp\left[\|\Theta\|\,\|F\|\left(T_s\right)\right]\left\|e\left(t_{2k+1}\right)\right\| \\
&\quad+\frac{\left(\kappa+M\|\Theta\|\right)\left[\exp\left(\|\Theta\|\,\|F\|\left(T_s\right)\right)-1\right]}{\|\Theta\|\,\|F\|}.
\end{aligned} \tag{44}$$

Note that r_0 is the maximum value of $\|e(t_{2k+1})\|$; thus we have

$$\begin{aligned}
\left\|e\left(t_{2k+2}\right)\right\| &\le r^* \\
&=\exp\left[\|\Theta\|\,\|F\|\left(T_s\right)\right]\ r_0 \\
&\quad+\frac{\left(\kappa+M\|\Theta\|\right)\left[\exp\left(\|\Theta\|\,\|F\|\left(T_s\right)\right)-1\right]}{\|\Theta\|\,\|F\|}.
\end{aligned} \tag{45}$$

Because of $r^*>r_0$, we have $e(t_{2k+2})\in S(r^*)$ when $e(t_{2k})\in S(r_0)$.

Then, we prove (b). For $e(t_{2k})\in R^n/S(r_0)$, obviously, we only need to consider the trajectory $e(t)$ being totally outside $S(r_0)$. We discuss two cases for $t\ge t_{2k}$. One is $t\in[t_{2j},t_{2j+1})$, and the other is $t\in[t_{2j+1},t_{2j+2})$, where $j=k,\ k+1,\ldots$.

Thus, from Lemmas 6 and 7, it is true that

$$\begin{aligned}
&V\left(z\left(\tau\right)\right) \\
&\le\begin{cases}\exp\left(-2\lambda_1\left(\tau-t_{2k}\right)\right)V_1\left(z\left(t_{2k}\right)\right), & \text{if } t_{2k}\le\tau<t_{2k+1}\\ \exp\left(2\lambda_2\left(\tau-t_{2k+1}\right)\right)V_2\left(z\left(t_{2k+1}\right)\right), & \text{if } t_{2k+1}\le\tau<t_{2k+2}.\end{cases}
\end{aligned} \tag{46}$$

If $t\in[t_{2j},t_{2j+1})$, according to (37) and (46), it holds that

$$\begin{aligned}
V\left(z\left(t\right)\right) &\le \exp\left(-2\lambda_1\left(t-t_{2j}\right)\right)V_1\left(z\left(t_{2j}\right)\right) \\
&\le \exp\left(-2\lambda_1\left(t-t_{2j}\right)\right)\mu V_2\left(z\left(t_{2j}\right)\right) \\
&\le \mu\exp\left(-2\lambda_1\left(t-t_{2j}\right)\right)\exp\left(2\lambda_2\left(t_{2j}-t_{2j-1}\right)\right) \\
&\quad\times V_2\left(z\left(t_{2j-1}\right)\right) \\
&\le \mu^2\exp\left(-2\lambda_1\left(t-t_{2j}\right)\right)\exp\left(2\lambda_2\left(t_{2j}-t_{2j-1}\right)\right) \\
&\quad\times V_1\left(z\left(t_{2j-1}\right)\right) \\
&\le \mu^2\exp\left(-2\lambda_1\left(t-t_{2j}\right)\right)\exp\left(2\lambda_2\left(t_{2j}-t_{2j-1}\right)\right) \\
&\quad\times\exp\left(-2\lambda_1\left(t_{2j-1}-t_{2j-2}\right)\right)V_1\left(z\left(t_{2j-2}\right)\right) \\
&\le\cdots\le \mu^{2^{(j-k)}}\exp\left(-2\lambda_1\left(t-t_{2k}-T^+\left(t_{2k},t\right)\right)\right) \\
&\quad\times\exp\left(2\lambda_2T^+\left(t_{2k},t\right)\right)V_1\left(z\left(t_{2k}\right)\right).
\end{aligned} \tag{47}$$

If $t\in[t_{2j+1},t_{2j+2})$, again from (37) and (46), we have

$$\begin{aligned}
V\left(z\left(t\right)\right) &\le \exp\left(2\lambda_2\left(t-t_{2j+1}\right)\right)V_2\left(z\left(t_{2j+1}\right)\right) \\
&\le \exp\left(2\lambda_2\left(t-t_{2j+1}\right)\right)\mu V_1\left(z\left(t_{2j+1}\right)\right) \\
&\le \mu\exp\left(2\lambda_2\left(t-t_{2j+1}\right)\right)\exp\left(-2\lambda_1\left(t_{2j+1}-t_{2j}\right)\right) \\
&\quad\times V_1\left(z\left(t_{2j}\right)\right) \\
&\le \mu^2\exp\left(2\lambda_2\left(t-t_{2j+1}\right)\right)\exp\left(-2\lambda_1\left(t_{2j+1}-t_{2j}\right)\right) \\
&\quad\times V_2\left(z\left(t_{2j}\right)\right) \\
&\le \mu^2\exp\left(2\lambda_2\left(t-t_{2j+1}\right)\right)\exp\left(-2\lambda_1\left(t_{2j+1}-t_{2j}\right)\right) \\
&\quad\times\exp\left(2\lambda_2\left(t_{2j}-t_{2j-1}\right)\right)V_2\left(z\left(t_{2j-1}\right)\right) \\
&\le\cdots\le \mu^{2^{(j-k)+1}}\exp\left(-2\lambda_1\left(t-t_{2k}-T^+\left(t_{2k},t\right)\right)\right) \\
&\quad\times\exp\left(2\lambda_2T^+\left(t_{2k},t\right)\right)V_1\left(z\left(t_{2k}\right)\right).
\end{aligned} \tag{48}$$

By Definition 2, we know $N_f(t_{2k},t)=2(j-k)$ for $t\in[t_{2j},t_{2j+1})$ and $N_f(t_{2k},t)=2(j-k)+1$ for $t\in[t_{2j+1},t_{2j+2})$. Thus, for any $t\in[t_{2j+1},t_{2(j+1)})$, from (47) and (48), we can obtain

$$\begin{aligned}
V\left(z\left(t\right)\right) &\le \mu^{N_f(t_{2k},t)}\exp\left(-2\lambda_1\left(t-t_{2k}-T^+\left(t_{2k},t\right)\right)\right) \\
&\quad\times\exp\left(2\lambda_2T^+\left(t_{2k},t\right)\right)V_1\left(z\left(t_{2k}\right)\right).
\end{aligned} \tag{49}$$

With the help of Lemma 8, it holds that

$$\begin{aligned}
\left\|e\left(z\left(t\right)\right)\right\| &\le \sqrt{\frac{\eta_2}{\eta_1}}\ \exp\left(N_f\left(t_{2k},t\right)\ln\mu\right) \\
&\quad\times\exp\left(-\lambda_1\left(t-t_{2k}-T^+\left(t_{2k},t\right)\right)\right) \\
&\quad\times\exp\left(\lambda_2T^+\left(t_{2k},t\right)\right)\left\|e\left(z\left(t_{2k}\right)\right)\right\|.
\end{aligned} \tag{50}$$

Applying Condition 1 gives

$$\begin{aligned}
&\exp\left(-\lambda_1\left(t-t_{2k}-T^+\left(t_{2k},t\right)\right)+\lambda_2T^+\left(t_{2k},t\right)\right) \\
&\quad\le\exp\left(-\lambda^*\left(t-t_{2k}\right)\right).
\end{aligned} \tag{51}$$

From Condition 2 and Definition 1, we have

$$\exp\left(N_f\left(t_{2k},t\right)\ln\mu\right)\le\exp\left(\lambda\left(t-t_{2k}\right)\right). \tag{52}$$

Using (49), (51), and (52) results in

$$\begin{aligned}&\left\|e\left(z\left(t;t_{2k},e\left(t_{2k}\right)\right)\right)\right\|\\&\quad\leq\exp\left(-\left(\lambda^{*}-\lambda\right)\left(t-t_{2k}\right)\right)\left\|e\left(z\left(t_{2k}\right)\right)\right\|.\end{aligned}\tag{53}$$

Therefore, when $e(t_{2k}) \in R^n/S(r_0)$, there exists $T_{2k} = T_{2k}(e(t_{2k}), r_0) \geq 0$ such that $\|e(t_{2k} + T_{2k})\| = r_0$ under the switching law $\sigma(t)$ satisfying Conditions 1-2. Obviously, $\{T_{2k}\}$ is a decreasing sequence, and thus $T_0 = \max_k\{T_{2k}\}$.

Finally, we prove (c). If $e(t_0) \in R^n/S(r^*)$, by applying (a), (b), and $r^* > r_0$, there exists a positive constant $T = T(e(t_0), r^*) < T_0$ such that $e(t; t_0, e(t_0)) \in S(r^*)$ for $t \geq t_0 + T$. If $e(t_0) \in S(r^*)$, the result remains true with $T = 0$.

This completes the proof. □

Remark 11. When the initial error $e(t_0) \in S(r^*)$, the error switched system is ε-practical stability [23].

Remark 12. The error switched system (12) is globally practically stable if the controller fails only for a short time interval and with a low frequency of occurrence.

5. Example

In this section, we present an example to demonstrate the effectiveness of the proposed method in this paper.

Consider the system (1) with

$$\begin{aligned}\Theta&=\begin{bmatrix}1&3\\1&2\end{bmatrix},\qquad F=\begin{bmatrix}1&0\\0&1\end{bmatrix},\\g(x)&=\begin{bmatrix}\sin x_1\\\cos x_2\end{bmatrix},\qquad B=\begin{bmatrix}1&0\\0&2\end{bmatrix}.\end{aligned}\tag{54}$$

The reference state x_m is generated by the reference model (2) with $A_m = \left[\begin{smallmatrix}-2&0\\0&-3\end{smallmatrix}\right]$, $B_m = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$, and the reference input is $r = 1$.

Choose $R = \left[\begin{smallmatrix}-2&0\\0&-1.5\end{smallmatrix}\right]$, $T = \left[\begin{smallmatrix}1\\0.5\end{smallmatrix}\right]$, $P_1 = P_2 = \left[\begin{smallmatrix}0.4449&0\\0&0.2862\end{smallmatrix}\right]$, and $\Gamma = \left[\begin{smallmatrix}20&0\\0&20\end{smallmatrix}\right]$. We have $r^* = 6.8147$ when $r_0 = 5$. Then, according to (38) and (39), we obtain $T^{+}(t_{2k}, t)/(t - t_{2k}) \leq 0.0302$ and $F_f(t_{2k}, t) \leq F_f^* = 0.548$. The switching signal is chosen as

$$\sigma(t)=\begin{cases}1, & t\in\Pi,\\2, & t\in\dfrac{[0,+\infty)}{\Pi},\end{cases}\tag{55}$$

where $\Pi = [15k, 15k + 1.95) \cup [15k + 2, 15k + 4) \cup [15k + 4.05, 15k + 6.35) \cup [15k + 6.4, 15k + 8.85) \cup [15k + 8.9, 15k + 11.55) \cup [15k + 11.6, 15k + 14.55) \cup [15k + 14.6, 15k + 15)$, $k = 0, 1, 2, \ldots$, which is described in Figure 2. It is easy to verify that $\sigma(t)$ satisfies Conditions 1-2 of Theorem 10.

When $x_m(t_0) = [0, 0]^T$ and $x(t_0) = [5, 6]^T$, the norm of the tracking error of (12) with and without controller failures is shown in Figures 3 and 4, respectively.

Simulations are carried out for $\|e(t_0)\| = 7.81 \geq r_0$ and $\|e(t_0)\| = 2.236 < r^*$. The results are depicted in Figures 5 and 6.

From Figures 5 and 6, we can conclude that whether $\|e(t_0)\| \geq r_0$ or not, the states of the system (1) with controller temporary failure track the reference signal $x_m(t)$ well under the switching signal $\sigma(t)$, and the tracking error $\|e(t)\|$ is small in the sense of r^*. Simulation illustrates the effectiveness of the proposed method.

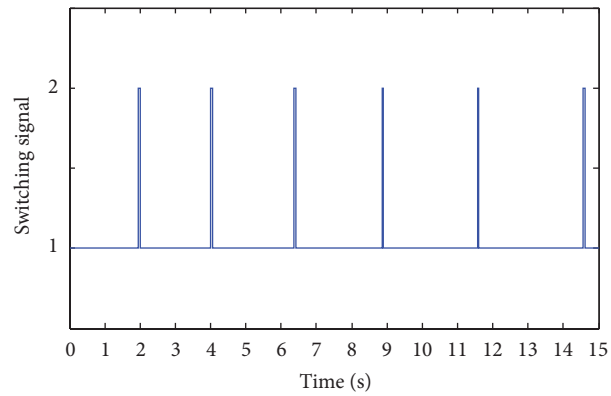

Figure 2: Switching signal.

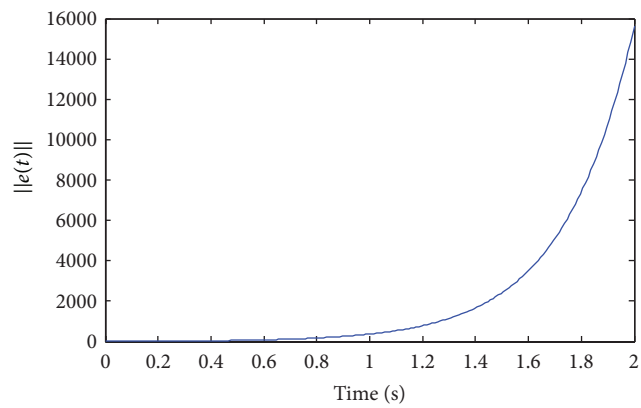

Figure 3: The norm of the tracking error of (12) when controller fails.

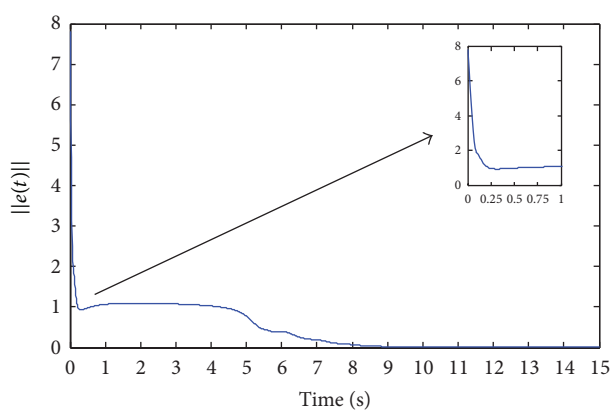

Figure 4: The norm of the tracking error of (12) without controller failure.

6. Conclusion

This paper has considered the state tracking problem for a class of MRAC systems in the presence of controller

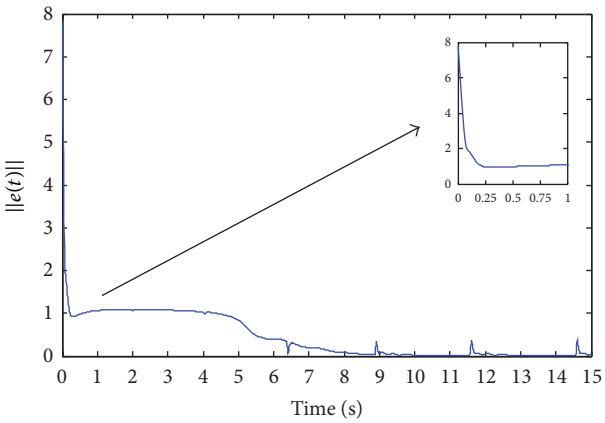

FIGURE 5: The norm of the tracking error of (12) under $\sigma(t)(\|e(t_0)\| = 7.81)$.

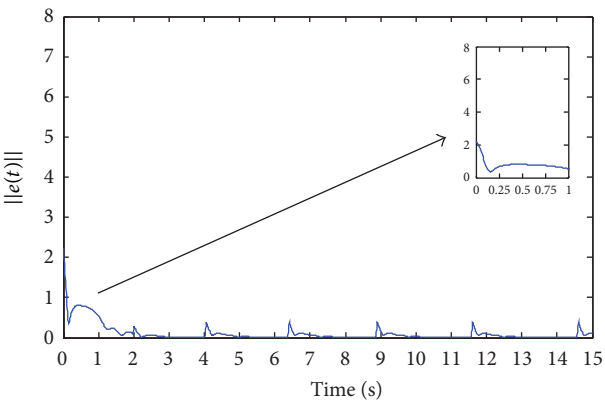

FIGURE 6: The norm of the tracking error of (12) under $\sigma(t)(\|e(t_0)\| = 2.236)$.

temporary failure. A key point is to describe such a system as an error switched system. The properties of Lyapunov function candidates without switching have been given. Then, the global practical stability of the error switched system can be ensured by the proposed scheme, providing that the controller suffers from failures only for a relatively short time interval and with a low frequency of occurrence. It is an interesting topic to extend the results for the output tracking problem of adaptive systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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