Research Article

Stability Analysis of State Saturation 2D Discrete Time-Delay Systems Based on F-M Model

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The problem of stability analysis is investigated for a class of state saturation two-dimensional (2D) discrete time-delay systems described by the Fornasini-Marchesini (F-M) model. The delay is allowed to be a bounded time-varying function. By constructing the delay-dependent 2D discrete Lyapunov functional and introducing a nonnegative scalar β, a sufficient condition is proposed to guarantee the global asymptotic stability of the addressed systems. Subsequently, the criterion is converted into the linear matrix inequalities (LMIs) which can be easily tested by using the standard numerical software. Finally, two numerical examples are given to show the effectiveness of the proposed stability criterion.

1. Introduction

Over the past decades, the two-dimensional (2D) systems have received considerable attention due to their extensive applications [1]. The 2D discrete-time systems have been successfully applied to many practical areas such as signal processing, linear image processing, multidimensional digital filtering, seismographic data processing, water stream heating and thermal processes. To mention a few, the problem of linear image processing has been studied in [2] for 2D discrete systems based on the Roesser model (R model). Aiming at the design of the digital filter, the 2D discrete systems based on the Fornasini-Marchesini model (F-M model) have been discussed in [3]. Recently, the modeling, calculation, and stability of 2D discrete systems have been widely studied and a large amount of results on these topics have been published; see, for example, [4–9].

Saturation, as a common and typical nonlinear constraint for practical control systems, is often encountered in various industrial systems. The phenomena of saturations if not properly handled will inevitably affect the implementations of the designed control schemes and may lead to the occurrence of the zero-input limit cycles [10–13]. Dynamical systems with saturation nonlinearities appear commonly in networks and 2D digital filter [14, 15]. When designing the 2D digital filter by using fixed point arithmetic, saturations are introduced due to the overflow and quantization. Recently, many important results on stability analysis of 2D discrete F-M systems with state saturation have been reported in recent literature; see, for example, [16–21]. To be specific, by using the Lyapunov method and employing the property of matrix norm, some sufficient conditions have been presented in [16, 17] to guarantee the global asymptotic stability of the related systems. In [18], an internally stable condition has been given for the design of 2D filters. By introducing a nonnegative scalar, in [19, 20], the criteria based on linear matrix inequality (LMI) have been proposed to ensure the global asymptotic stability of F-M model. Recently, sufficient conditions have been derived in [21] to guarantee the global asymptotic stability of the addressed F-M systems with state saturation nonlinearities.

As well known, the time delays occur in many practical engineering systems [22–24]. The occurrence of the time delays would yield the instability of the controlled systems in some cases. So far, a great number of results have been reported; see, for example, [25–29]. In particular, considerable research attention has been devoted to the problems of stability analysis of 2D discrete time-delay systems. To be specific, by constructing an appropriate Lyapunov function and a scalar β, sufficient conditions have been proposed

in [26] for state saturation 2D discrete time-delay systems based on the R model. In [27], the state estimation problem has been investigated for 2D complex networks with randomly occurring nonlinearities and probabilistic sensor delays. Accordingly, some sufficient conditions have been established such that the resulted estimation error dynamics are globally asymptotically stable in the mean square sense. To be specific, the problems of global asymptotic stability have been given in [30, 31] for 2D discrete F-M systems with state saturation and constant time delays. However, to the best of author's knowledge, there has not been much work undertaken on the global asymptotic stability for state saturation 2D discrete time-varying delay systems based on F-M model.

In this paper, we aim to investigate the problem of global asymptotic stability for 2D discrete F-M systems with state saturation and time delays. Here, the delays are assumed to be time varying with known lower and upper bounds. By constructing a delay-dependent 2D discrete Lyapunov functional and introducing a nonnegative scalar β based on a row diagonally dominant matrix, a sufficient condition is proposed to guarantee the global asymptotic stability of the addressed systems. Subsequently, the problem of global asymptotic stability is converted into the problem of feasibility by solving the LMIs. Finally, two numerical examples are given to show the effectiveness of the proposed criterion. The main contribution of this paper lies in that new stability criterion is given for state saturation 2D discrete F-M systems with time-varying delays.

Notations. The fundamental notations used in the paper are given as follows: I_p and 0 denote the identity matrix and zero matrix with appropriate dimensions. For a matrix A, A^T stands for transpose of the matrix A. $A > 0$ means that A is positive definite symmetric matrix.

2. Problem Formulation and Preliminaries

In this paper, we consider the following 2D discrete F-M system with state saturation and time-varying delays:

$$x(k+1, l+1) = f(y(k,l)),$$

$$y(k,l) = [y_1(k,l), y_2(k,l), \ldots, y_n(k,l)]^T$$

$$= A \begin{bmatrix} x(k, l+1) \\ x(k+1, l) \end{bmatrix} + A_d \begin{bmatrix} x(k-d_h(k), l+1) \\ x(k+1, l-d_v(l)) \end{bmatrix}$$

$$= A_{11}x(k, l+1) + A_{12}x(k+1, l)$$

$$+ A_{21}x(k-d_h(k), l+1)$$

$$+ A_{22}x(k+1, l-d_v(l)),$$

where $x(k,l)$ is the n-dimensional state vector and (k,l) is the 2-dimensional time variable along the vertical and horizontal

directions satisfying $k \geq 0$, $l \geq 0$. $A = [A_{11}, A_{12}] \in R^{n \times 2n}$, $A_d = [A_{21}, A_{22}] \in R^{n \times 2n}$, $d_h(k)$ and $d_v(l)$ are time-varying delays along the vertical and horizontal directions, respectively. We assume $d_h(k)$ and $d_v(l)$ satisfying:

$$h_m \leq d_h(k) \leq h_M, \qquad v_m \leq d_v(l) \leq v_M, \tag{2}$$

where h_m and h_M denote the lower and upper bounds of the delay $d_h(k)$ along the horizontal directions, v_m and v_M denote the lower and upper bounds of the delay $d_v(l)$ along the vertical directions. Throughout this paper, the superscript T to any vector (or matrix) stands for the transpose of that vector (or matrix).

The saturation nonlinearity $f(\cdot)$ is given by

$$f(y(k,l)) = [f_1(y_1(k,l)), f_2(y_2(k,l)), \ldots, f_n(y_n(k,l))]^T, \tag{3}$$

with

$$f_i(y_i(k,l)) = \begin{cases} 1, & y_i(k,l) > 1, \\ y_i(k,l), & |y_i(k,l)| \leq 1, \\ -1, & y_i(k,l) < -1, \end{cases} \quad (i = 1, 2, \ldots, n). \tag{4}$$

The system (1) has finite initial conditions $x(k, 0)$ and $x(0, l)$ with fixed yet nonzero values for $-h_M \leq k \leq K$ and $-v_M \leq l \leq L$, and

$$x(k, 0) = 0, \quad k \geq K; \qquad x(0, l) = 0, \quad l \geq L, \tag{5}$$

where K and L are positive integers.

To proceed, we introduce the following definitions which will be used in the subsequent derivations.

Definition 1. The origin $x = 0$ of the 2D discrete time-delay system is said to be stable (in the sense of Lyapunov) if for every $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$ such that

$$\|x(k,l)\| < \varepsilon, \tag{6}$$

for all $k \geq 0$, $l \geq 0$, whenever $\|x(k, 0)\| < \delta$ $(-h_M \leq k \leq K)$ and $\|x(0, l)\| < \delta$ $(-v_M \leq l \leq L)$, where K and L are positive integers and $\|\cdot\|$ denotes any of the equivalent norms on R^n.

Definition 2 (see [21]). The origin $x = 0$ of the 2D system (1) is said to be globally asymptotically stable if the following conditions are simultaneously satisfied:

(1) system (1) is stable;

(2) every solution of system (1) tends to the origin as $k + l \to \infty$; that is,

$$\lim_{k \to \infty \text{ or } l \to \infty} x(k,l) = \lim_{k+l \to \infty} x(k,l) = 0. \tag{7}$$

Definition 3 (see [32]). The matrix M is said to be row diagonally dominant if

$$\left| m_{ii} \right| \geq \sum_{j=1, j \neq i}^{n} \left| m_{ij} \right|, \quad i \in [1, n]. \tag{8}$$

Definition 4. For a 2D function $V(k, l) = V^h(k, l) + V^v(k, l)$, denote its difference $\Delta V(k, l)$ as

$$\Delta V(k, l) = \Delta V^h(k, l) + \Delta V^v(k, l), \tag{9}$$

where

$$\Delta V^h(k, l) = V^h(k + 1, l + 1) - V^h(k, l + 1),$$
$$\Delta V^v(k, l) = V^v(k + 1, l + 1) - V^v(k + 1, l). \tag{10}$$

3. Main Results

In this section, we aim to investigate the problem of stability analysis for the 2D discrete F-M system with state saturation and time-varying delays. By resorting to the LMI technique, a sufficient condition is given to ensure the global asymptotic stability of the addressed system.

Theorem 5. *The origin $x = 0$ of 2D discrete system (1) is globally asymptotically stable if there exist matrices P, Q, and M, where $P = P_1 + P_2$ and $Q = Q_1 + Q_2$ with $P_1 = [p_{ij}^1] \in R^{n \times n}$, $P_2 = [p_{ij}^2] \in R^{n \times n}$, $Q_1 = [q_{ij}^1] \in R^{n \times n}$, and $Q_2 = [q_{ij}^2] \in R^{n \times n}$ being symmetric positive definite matrices and $M = [m_{ij}] \in R^{n \times n}$ is row diagonally dominant with nonnegative diagonal elements, satisfying the following matrix inequality:*

$$\begin{bmatrix} \Pi_1 & M^T A_{11} & M^T A_{12} & M^T A_{21} & M^T A_{22} \\ * & \Pi_2 & 0 & 0 & 0 \\ * & * & \Pi_3 & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix} < 0, \tag{11}$$

with

$$\Pi_1 = P_1 + P_2 - \left(M + M^T \right),$$
$$\Pi_2 = Q_1 - P_1 + \left(h_M - h_m \right) Q_1, \tag{12}$$
$$\Pi_3 = Q_2 - P_2 + \left(v_M - v_m \right) Q_2.$$

Proof. In order to prove the global asymptotic stability of the 2D discrete F-M system (1), we construct the following Lyapunov functional as in [26]:

$$V(k, l) = \sum_{i=1}^{3} V_i(k, l), \tag{13}$$

where

$$V_1(k, l) = V_1^h(k, l) + V_1^v(k, l)$$
$$= x^T(k, l) P_1 x(k, l) + x^T(k, l) P_2 x(k, l),$$

$$V_2(k, l) = V_2^h(k, l) + V_2^v(k, l)$$
$$= \sum_{i=k-d_h(k)}^{k-1} x^T(i, l) Q_1 x(i, l)$$
$$+ \sum_{j=l-d_v(l)}^{l-1} x^T(k, j) Q_2 x(k, j), \tag{14}$$

$$V_3(k, l) = V_3^h(k, l) + V_3^v(k, l)$$
$$= \sum_{j=-h_M+1}^{-h_m} \sum_{i=k+j}^{k-1} x^T(i, l) Q_1 x(i, l)$$
$$+ \sum_{i=-v_M+1}^{-v_m} \sum_{j=l+i}^{l-1} x^T(k, j) Q_2 x(k, j)$$

with $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, and $Q_2 > 0$ being matrices to be determined.

According to Definition 4, the corresponding difference of $\Delta V(k, l)$ along the trajectory of system (1) can be calculated as follows:

$$\Delta V(k, l) = \sum_{i=1}^{3} \Delta V_i(k, l), \tag{15}$$

with

$$\Delta V_1(k, l) = V_1(k + 1, l + 1) - x^T(k, l + 1) P_1 x(k, l + 1)$$
$$- x^T(k + 1, l) P_2 x(k + 1, l),$$

$$\Delta V_2(k, l) = \sum_{i=k+1-d_h(k+1)}^{k} x^T(i, l + 1) Q_1 x(i, l + 1)$$

$$- \sum_{i=k-d_h(k)}^{k-1} x^T(i, l+1) Q_1 x(i, l+1)$$

$$+ \sum_{j=l+1-d_v(l+1)}^{l} x^T(k+1, j) Q_2 x(k+1, j)$$

$$- \sum_{j=l-d_v(l)}^{l-1} x^T(k+1, j) Q_2 x(k+1, j),$$

$$\Delta V_3(k,l) = \sum_{j=-h_M+1}^{-h_m} \sum_{i=k+j+1}^{k} x^T(i, l+1) Q_1 x(i, l+1)$$

$$- \sum_{j=-h_M+1}^{-h_m} \sum_{i=k+j}^{k-1} x^T(i, l+1) Q_1 x(i, l+1)$$

$$+ \sum_{i=-v_M+1}^{-v_m} \sum_{j=l+i+1}^{l} x^T(k+1, j) Q_2 x(k+1, j)$$

$$- \sum_{i=-v_M+1}^{-v_m} \sum_{j=l+i}^{l-1} x^T(k+1, j) Q_2 x(k+1, j). \tag{16}$$

Noting (13)–(15), it can be derived that

$$\Delta V_1(k,l) = x^T(k+1, l+1) P_1 x(k+1, l+1)$$

$$+ x^T(k+1, l+1) P_2 x(k+1, l+1)$$

$$- x^T(k, l+1) P_1 x(k, l+1)$$

$$- x^T(k+1, l) P_2 x(k+1, l),$$

$$\Delta V_2(k,l) = x^T(k, l+1) Q_1 x(k, l+1)$$

$$- x^T(k-d_h(k), l+1) Q_1 x(k-d_h(k), l+1)$$

$$+ \sum_{i=k+1-d_h(k+1)}^{k-1} x^T(i, l+1) Q_1 x(i, l+1)$$

$$- \sum_{i=k+1-d_h(k)}^{k-1} x^T(i, l+1) Q_1 x(i, l+1)$$

$$+ x^T(k+1, l) Q_2 x(k+1, l)$$

$$- x^T(k+1, l-d_v(l)) Q_2 x(k+1, l-d_v(l))$$

$$+ \sum_{j=l+1-d_v(l+1)}^{l-1} x^T(k+1, j) Q_2 x(k+1, j)$$

$$- \sum_{j=l+1-d_v(l)}^{l-1} x^T(k+1, j) Q_2 x(k+1, j)$$

$$\leq \sum_{i=k+1-h_M}^{k-1} x^T(i, l+1) Q_1 x(i, l+1)$$

$$- \sum_{i=k+1-h_m}^{k-1} x^T(i, l+1) Q_1 x(i, l+1)$$

$$+ \sum_{j=l+1-v_M}^{l-1} x^T(k+1, j) Q_2 x(k+1, j)$$

$$- \sum_{j=l+1-v_m}^{l-1} x^T(k+1, j) Q_2 x(k+1, j)$$

$$+ x^T(k, l+1) Q_1 x(k, l+1)$$

$$- x^T(k-d_h(k), l+1) Q_1 x(k-d_h(k), l+1)$$

$$+ x^T(k+1, l) Q_2 x(k+1, l)$$

$$- x^T(k+1, l-d_v(l)) Q_2 x(k+1, l-d_v(l))$$

$$= \sum_{i=k+1-h_M}^{k-h_m} x^T(i, l+1) Q_1 x(i, l+1)$$

$$+ \sum_{j=l+1-v_M}^{l-v_m} x^T(k+1, j) Q_2 x(k+1, j)$$

$$+ x^T(k, l+1) Q_1 x(k, l+1)$$

$$- x^T(k-d_h(k), l+1) Q_1 x(k-d_h(k), l+1)$$

$$+ x^T(k+1, l) Q_2 x(k+1, l)$$

$$- x^T(k+1, l-d_v(l)) Q_2 x(k+1, l-d_v(l))$$

$$\Delta V_3(k,l) = (h_M - h_m) x^T(k, l+1) Q_1 x(k, l+1)$$

$$- \sum_{i=k+1-h_M}^{k-h_m} x^T(i, l+1) Q_1 x(i, l+1)$$

$$+ (v_M - v_m) x^T(k+1, l) Q_2 x(k+1, l)$$

$$- \sum_{j=l+1-v_M}^{l-v_m} x^T(k+1, j) Q_2 x(k+1, j). \tag{17}$$

Denoting

$$\varphi(k,l) = [f(y(k,l)), x(k, l+1), x(k+1, l),$$
$$x(k-d_h(k), l+1), x(k+1, l-d_v(l))]^T, \tag{18}$$

we have

$$\Delta V(k,l) \leq \varphi^T(k,l) \Theta \varphi(k,l), \tag{19}$$

where

$$\Theta = \begin{bmatrix} P_1 + P_2 & 0 & 0 & 0 & 0 \\ * & (h_M - h_m + 1)Q_1 - P_1 & 0 & 0 & 0 \\ * & * & (v_M - v_m + 1)Q_2 - P_2 & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix}. \quad (20)$$

Now, construct the following parameter β:

$$\beta = 2\sum_{i=1}^{n} \left[y_i(k,l) - f_i(y_i(k,l)) \right]$$

$$\times \left[m_{ii} f_i(y_i(k,l)) + \sum_{j=1, j \neq i}^{n} m_{ij} f_j(y_j(k,l)) \right]$$

$$= y^T(k,l) Mf(y(k,l)) + f^T(y(k,l)) M^T y(k,l)$$

$$- f^T(y(k,l)) (M + M^T) f(y(k,l)). \quad (21)$$

Let

$$\alpha_i = \left[y_i(k,l) - f_i(y_i(k,l)) \right]$$

$$\times \left[m_{ii} f_i(y_i(k,l)) + \sum_{j=1, j \neq i}^{n} m_{ij} f_j(y_j(k,l)) \right]. \quad (22)$$

The saturation region can be divided into two regions, and we have

$$f_i(y_i(k,l)) = 1,$$

$$y_i(k,l) - f_i(y_i(k,l)) > 0, \text{ in Region 1,}$$

$$f_i(y_i(k,l)) = -1, \quad (23)$$

$$y_i(k,l) - f_i(y_i(k,l)) < 0, \text{ in Region 2.}$$

Thus, we obtain

$$\alpha_i = \begin{cases} (y_i(k,l) - 1)\left(m_{ii} + \sum_{j=1, j \neq i}^{n} m_{ij} f_j(y_j(k,l)) \right), \\ \qquad\qquad\qquad\qquad \text{in Region 1,} \\ (y_i(k,l) + 1)\left(-m_{ii} + \sum_{j=1, j \neq i}^{n} m_{ij} f_j(y_j(k,l)) \right), \\ \qquad\qquad\qquad\qquad \text{in Region 2.} \end{cases} \quad (24)$$

Subsequently, due to the property of row diagonally dominant matrix M, we have $\alpha_i \geq 0$. Furthermore, note that β is the sum of nonnegative scalars, and hence $\beta \geq 0$. Therefore,

$$\Delta V \leq \varphi^T(k,l) W\varphi(k,l) - \beta, \quad (25)$$

where

$$W = \begin{bmatrix} \Pi_1 & M^T A_{11} & M^T A_{12} & M^T A_{21} & M^T A_{22} \\ * & \Pi_2 & 0 & 0 & 0 \\ * & * & \Pi_3 & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix}. \quad (26)$$

If $W < 0$, then $\Delta V(k,l) < 0$; that is

$$V^h(k+1, l+1) + V^v(k+1, l+1)$$

$$\leq V^h(k, l+1) + V^v(k+1, l). \quad (27)$$

Hence, for any nonnegative integer $d \geq \max\{K, L\}$, we have

$$\sum_{k+l=d+1} V(k,l) = V(1,d) + V(2, d-1) + \cdots + V(d,1)$$

$$\leq V^h(0,d) + V^v(1, d-1) + V^h(1, d-1)$$

$$+ V^v(2, d-2) + \cdots + V^h(d-1, 1)$$

$$+ V^v(d, 0)$$

$$= \sum_{k+l=d} V(k,l). \quad (28)$$

It is clear that the sum of the Lyapunov functional is a decreasing function along the state trajectories of system (1). Then, noting the initial condition $x(0,d) = x(d,0) = 0$, we have

$$\lim_{k \to \infty \text{ or } l \to \infty} x(k,l) = \lim_{k+l \to \infty} x(k,l) = 0. \quad (29)$$

Summarizing the above discussions, it can be shown that (11) is a sufficient condition which ensures the global asymptotic stability of system (1). This completes the proof of Theorem 5. □

As special cases, if there is no time delay in system (1), that is, $A_{21} = 0$ and $A_{22} = 0$, or the time delays are constant, then we can have the following corollaries.

Corollary 6. *Consider the system* (1) *without time delay. If there exist matrices P and M, with $P = P_1 + P_2$, $P_1 = [p_{ij}^1] \in R^{n \times n}$, and $P_2 = [p_{ij}^2] \in R^{n \times n}$ being symmetric positive definite matrices and $M = [m_{ij}] \in R^{n \times n}$ is row diagonally dominant*

with nonnegative diagonal elements, satisfying the following matrix inequality:

$$\begin{bmatrix} P - \left(M + M^T \right) & M^T A_{11} & M^T A_{12} \\ * & -P_1 & 0 \\ * & 0 & -P_2 \end{bmatrix} < 0 \quad (30)$$

then the origin $x = 0$ of 2D discrete system (1) without time delay is globally asymptotically stable.

Corollary 7. *Consider the constant time delay $d_h(k) = d_h$, $d_v(l) = d_v$. If there exist matrices P, Q, and M, where $P = P_1 + P_2$ and $Q = Q_1 + Q_2$, with $P_1 = [p_{ij}^1] \in R^{n \times n}$, $P_2 = [p_{ij}^2] \in R^{n \times n}$, $Q_1 = [q_{ij}^1] \in R^{n \times n}$, and $Q_2 = [q_{ij}^2] \in R^{n \times n}$ being symmetric positive definite matrices and $M = [m_{ij}] \in R^{n \times n}$ is row diagonally dominant with nonnegative diagonal elements, satisfying the following matrix inequality:*

$$\begin{bmatrix} P_1 + P_2 - \left(M + M^T \right) & M^T A_{11} & M^T A_{12} & M^T A_{21} & M^T A_{22} \\ * & Q_1 - P_1 & 0 & 0 & 0 \\ * & * & Q_2 - P_2 & 0 & 0 \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_2 \end{bmatrix} < 0, \quad (31)$$

then the origin $x = 0$ of 2D discrete system (1) with constant time delay is globally asymptotically stable.

Remark 8. Note that the problem of global asymptotic stability has been investigated in [33] for state saturation 2D discrete system without time delay. Accordingly, a stability criterion has been derived in [33] but a positive diagonally matrix is needed to be searched. It can be easily seen that the stability condition in [33] is a special case of (30) in Corollary 6. Therefore, the stability condition proposed in this paper is less conservative than the one in [33]. Meanwhile, note that a stability condition has been proposed in [21] where an unknown matrix G must be given firstly. The values of G must take specific values. Compared with the results in [21], we only need to find matrix M. It concludes that the stability condition in [21] is more conservative than our result.

Remark 9. It is worth mentioning that the delay-fractioning approach has been employed in [34, 35] to reduce the conservativeness of the time delay. It has been shown that the developed approach performs well when dealing with the time delay compared with other methods. Accordingly, some effective SMC/SMO schemes based on the delay-fractioning idea have been proposed for discrete time-delay nonlinear stochastic systems with randomly occurring incomplete information. Motivated by the results in [34, 35], we are now researching into the stability criterion based on the delay-fractioning approach for the state saturation 2D discrete time-delay systems. The corresponding results will appear in the near future. Moreover, note that the recursive filters have been designed in [36–38] for time-varying networked non-linear systems with missing measurements. It is also interesting to consider the analysis and synthesis of 2D discrete nonlinear systems with state saturations and missing measurements.

Remark 10. It is worth noting that the conditions in Theorem 5 are not strict LMI due to the fact that the matrix $M = [m_{ij}] \in R^{n \times n}$ is row diagonally dominant

with nonnegative diagonal elements. Hence, the results of Theorem 5 and Corollary 6 are not convex which lead to the computational difficulties. In the following, an alternative approach is developed to deal with the nonconvex problem.

Let e_l be n-dimensional column vectors in which the lth element is 1 and other elements are 0. Let Y_l be the set of n-dimensional column vectors in which the lth element is -1 and other elements are either 1 or -1 and $Y_{ls} \in Y_l$ ($s \in [1, 2^{n-1}]$). Then, the condition where matrix $M = [m_{ij}] \in R^{n \times n}$ is row diagonally dominant and the diagonal is composed of nonnegative elements can be equivalently converted into the following LMIs:

$$e_l^T M Y_{ls} < 0, \quad l = 1, 2, \ldots, m + n; \; s \in \left[1, 2^{m+n-1} \right]. \quad (32)$$

Together with (11) and (32), we can see that the proposed stability condition is a convex one and then can be easily solved by using the standard numerical software.

4. Numerical Examples

In this section, two numerical examples are given here to illustrate the effectiveness of the main results.

Example 1. Consider the state saturation 2D discrete time-delay system (1) with

$$A_{11} = \begin{bmatrix} 1.678 & 0.2853 & -0.2432 & -0.1246 \\ -0.84 & 0.23 & -0.1 & 1.45 \\ 0.4 & 0.0217 & 0.1785 & 0.1662 \\ 0.0245 & -0.00884 & 0.0321 & -0.2428 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} -0.495 & -0.687 & -0.030 & -0.3013 \\ -0.1630 & -0.4817 & -0.993 & 0.1814 \\ 0.2842 & 0.1790 & -0.5551 & 0.0799 \\ -0.2112 & 0.1887 & 0.0895 & 0.5604 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -9.987 & -0.2 & 0 & 0 \\ 0 & -0.15 & 0 & -0.1 \\ 0 & 0 & -0.78 & 0 \\ -0.23 & 0 & 0 & -0.98 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 5.763 & -0.067 & 0 & 0.04 \\ -2.314 & 0.342 & 0.098 & -5.78 \\ 4.56 & 0.001 & -0.98 & 0.231 \\ 0.23 & -0.76 & 0.043 & 8.6 \end{bmatrix}. \tag{33}$$

The lower and upper bounds are given by $h_M = 3$, $h_m = 1$, $v_M = 4$, and $v_m = 2$.

Solving the inequalities (11) and (32) in the Matlab environment, we can obtain

$$P_1 = \begin{bmatrix} 3.0067 & -0.0360 & -0.0043 & 0.0125 \\ -0.0360 & 3.1420 & 0.1635 & 0.2573 \\ -0.0043 & 0.1635 & 2.9443 & 0.1253 \\ 0.0125 & 0.2573 & 0.1253 & 2.9694 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 2.9848 & -0.0225 & -0.0006 & 0.0463 \\ -0.0225 & 3.1494 & 0.1757 & 0.2382 \\ -0.0006 & 0.1757 & 3.0049 & 0.0990 \\ 0.0463 & 0.2382 & 0.0990 & 2.9051 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.5521 & -0.0048 & -0.0017 & 0.0417 \\ -0.0048 & 0.6168 & 0.0461 & 0.0573 \\ -0.0017 & 0.0461 & 0.5631 & 0.0356 \\ 0.0417 & 0.0573 & 0.0356 & 0.4917 \end{bmatrix}, \tag{34}$$

$$Q_2 = \begin{bmatrix} 0.5676 & -0.0144 & -0.0043 & 0.0175 \\ -0.0144 & 0.6115 & 0.0374 & 0.0709 \\ -0.0043 & 0.0374 & 0.5198 & 0.0545 \\ 0.0175 & 0.0709 & 0.0545 & 0.5376 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.2812 & 0.0241 & -0.0475 & -0.0020 \\ 0.0353 & 0.6877 & 0.0609 & 0.2516 \\ -0.0769 & 0.0654 & 0.7074 & 0.0128 \\ 0.0027 & 0.1886 & -0.0018 & 0.5205 \end{bmatrix}.$$

Then, we can see that there exist the required matrices P_1, P_2, Q_1, Q_2, and M satisfying LMIs (11) and (32). As such, according to Theorem 5, the origin $x = 0$ of system (1) is globally asymptotically stable which confirms the feasibility of the proposed main results.

Example 2. Consider a 2D system described by (1) without time delay. The system parameters are given as

$$A_{11} = \begin{bmatrix} 1.2 & -2.8 \\ 0.1 & 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 0.01 \\ 0 & 0.02 \end{bmatrix}. \tag{35}$$

By using the Matlab LMI Toolbox, it can be easily verified that the following feasible solutions can be obtained

$$P_1 = \begin{bmatrix} 1421.7 & -203.9 \\ -203.9 & 1698.1 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 1321 & 25 \\ 25 & 1167.5 \end{bmatrix},$$

$$M = \begin{bmatrix} 434.848 & -11.3033 \\ -9.1992 & 818.5481 \end{bmatrix}. \tag{36}$$

That is, there exist the required matrices P_1, P_2, and M satisfying LMI (30). According to Corollary 6, the origin of system (1) is globally asymptotically stable which confirms the effectiveness of the presented results. However, it can be tested that the conditions in [33] are infeasible. Therefore, the result in our paper is less conservative than the one in [33].

5. Conclusions

In this paper, we have discussed the problem of global asymptotic stability for 2D discrete F-M systems with state saturation and time-varying delays. By constructing the 2D discrete-time Lyapunov functional, a new stability criterion has been established to ensure that the addressed system is globally asymptotically stable. The proposed stability criterion is in terms of the LMIs which can be easily tested by using the Matlab matrix toolbox. Two numerical examples have been given to demonstrate the feasibility of the proposed stability condition. It is worth mentioning that the construction of the scalar β has taken full effects from time delays with hope to reduce the conservativeness. One of the future research topics would be the extension of the proposed main results to more general state saturation 2D discrete systems as in [39–41] with network-induced phenomena.

Acknowledgments

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