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Research Article

Pessimistic Portfolio Choice with One Safe and One Risky Asset and Right Monotone Probability Difference Order

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As is well known, a first-order dominant deterioration in risk does not necessarily cause a risk-averse investor to reduce his holdings of that deteriorated asset under the expected utility framework, even in the simplest portfolio setting with one safe asset and one risky asset. The purpose of this paper is to derive conditions on shifts in the distribution of the risky asset under which the counterintuitive conclusion above can be overthrown under the rank-dependent expected utility framework, a more general and prominent alternative of the expected utility. Two new criterions of changes in risk, named the monotone probability difference (MPD) and the right monotone probability difference (RMPD) order, are proposed, which is a particular case of the first stochastic dominance. The relationship among MPD, RMPD, and the other two important stochastic orders, monotone likelihood ratio (MLR) and monotone probability ratio (MPR), is examined. A desired comparative statics result is obtained when a shift in the distribution of the risky asset satisfies the RMPD criterion.

1. Introduction

In recent years, a great number of empirical lines of evidence suggest that two types of psychological biases play a key role in people's decision-making. They are the pessimism and the optimism, characterized by people's attitude toward favorable and unfavorable events. Intuitively, a pessimist is defined as one who always underestimates the probabilities of favorable events and overestimates the probabilities of unfavorable events, while an optimist always revises up the probabilities of favorable events and revises down the probabilities of unfavorable events [1]. However, as the most important normative decision model under risk, the expected utility (EU) theory fails to identify the two types of behavioral biases. On the contrary, as a more general and promising alternative model of EU, the rank-dependent expected utility (RDEU) introduced originally by Quiggin [2] can characterize these view biases well [3]. In this paper, we will adopt RDEU as an alternative tool to rededicate ourselves to the comparative statics analysis of the classic portfolio problem with one safe asset and one risky asset.

Besides those reasons mentioned above, our choice of RDEU is also motivated by other considerations. Firstly, the prediction of the EU theory is often inconsistent with some observed behaviors. One of the most notable examples is the Allais paradox. But this inconsistency can be interpreted well by RDEU [2, 3]. Second, RDEU can characterize broader behaviors than just risk-averse or risk seeking by integrating the sensitivity of probability into people's risk attitude (e.g., risk seeking with decreasing marginal utility, which is at odd with EU). The model can also capture decision makers' some psychological biases (such as greediness, pessimism) [3]. Finally, RDEU preserves many useful properties and results of the EU theory and nests EU as its a special case [2, 3]. These are the main motivations for us to choose the RDEU model.

In the theory of portfolio selection, an important and often considered question is how a change in risk sways the optimal proportion invested in a risky asset by a risk-averse economic agent who maximizes his expected utility. A plausible intuition should suggest that the risk-averse agent will reduce his investments in the risky asset, when the shift in the distribution of the risky asset makes him worse off, such as in the sense of the first- or second-order stochastic dominance (FSD or SSD). Contrary to the intuition, the deteriorations of a FSD and a SSD in distribution do not always cause all risk-averse agents to reduce their optimal investments in the deteriorated risky asset, which have already been observed in the literature since 1970s. For example, Rothschild and Stiglitz [4] observed that a SSD shift is neither necessary nor sufficient for all risk-averse individuals to adjust their investments in a risky asset in the same direction even in the simple portfolio setting with only one risky asset and one safe asset. Similar claim was also made by Fishburn and Porter [5] for a shift of FSD.

In order to obtain unambiguous comparative static effects, there have been two different strategies in the literature. The first approach is to limit utility functions of decision makers to derive the desired comparative static effects [4–6]. The second one is to impose restrictions on changes in risk that induce all risk-averse agents to adjust their optimal investments in a risky asset in the same direction, which has been widely adopted in the existing literature [7–13]. In addition, many researchers choose to combine the two methods.

Since the two surprising observations in [4, 5], more attention has been paid to looking for proper subsets of SSD in order to obtain unambiguous results. However, there are few studies on determining conditions to overcome the negative conclusion in [5]. The two authors demonstrated that an improvement of FSD in the distribution of a risky asset will induce each risk-averse agent to increase his holding of the improved risky asset if the coefficient of relative risk-aversion never exceeds unity plus the coefficient of absolute risk-averse [5]. Differing from [5], Landsberger and Meilijson [10] focused on finding out proper subsets of FSD so that unambiguous comparative static results could be obtained. They proved that for the simple portfolio problem, a change in risk in the sense of the monotone likelihood ratio (MLR) order will cause all individuals with nondecreasing utility functions to adjust their optimal investments in the risky asset toward the same direction. The MLR order is also widely applied in the literature on testing statistical hypotheses and plays a key role in recent advance in the technology of monotone comparative static analysis under uncertainty [14]. Eeckhoudt and Gollier [11] proposed the monotone probability ratio criterion (MPR) which lies between the MLR order and the FSD order and extended the conclusion in [10] to the larger subset of FSD.

All these papers mentioned above rely on an assumption that decision makers act on the principle of maximizing their expected utility. When we reconsider the simple portfolio problem under RDEU framework, the situation becomes more complicated. On the one hand, the usefulness of each decision-making model severely rests on its ability of making comparative static analysis. This has been pointed out by several authors, for example, Quiggin [15] argued: "... *this increase in scope is of little use if generalized models are unable to, generate sharp comparative static results like those that have made EU theory such a powerful tool of analysis*." Regrettably, in contrast to the enormous literature on the comparative static analysis under the EU framework, there are few studies on this issue from the perspective of the RDEU theory. On the other hand, Quiggin [15] developed a novel approach called "corresponding principle" to investigate the comparative statics problem for the RDEU model. The RDEU functional is regarded as the expected utility with respect to a transformed probability distribution in this approach. This insightful interpretation allows for direct extensions of some comparative static results from EU to RDEU. However, the negative result in [5] for EU is still prevalent for RDEU, so there is a natural question whether the MLR or the MPR shifts will result in unambiguous comparative static effects for all risk-averse and pessimistic RDEU decision makers. In fact, it is easy to show that even one distribution dominates another in the sense of the MLR or the MPR orders and these two transformed distributions via a concave distortion function generally fail to preserve the MLR or the MPR orders, indicating that the nice "corresponding principle" loses its power. Motivated by these considerations, we try to find out a subset of FSD to arrive at a desired comparative static analysis for the more general RDEU preference.

This paper adds a new contribution to this type of the literature on the comparative static problem. We present two new stochastic orders called the "monotone probability difference" and "right monotone probability difference" orders, which are based on the concept of monotone difference between two cumulative distribution functions (CDFs). It is worthy of noticing that similar strategies have also been adopted by several authors. For example, Landsberger and Meilijson [16] show the equivalence between Quiggin's mean-preserve monotone spread (MPMS) and the monotone difference between quantile functions of two random variables, which provided the monotone spread increasing in risk with a nice new characterization. Papers [14, 17] developed a novel approach to make comparative static analysis called "monotone comparative statics," which is different from the traditional methods based on the implicit function theorem or revealed preference argument. For this technology, monotone difference between two functions plays also an extra important role [14, 17]. Therefore, our stochastic orders are based on a solid foundation. We examine several relationship among MLP, MPR, here MPD, and RMPD and derive a determined comparative statics result for the RDEU type of decision maker under a RMPD shift in distribution with less restrictions in utility function and probability distortion function.

Finally, we review two relevant works by Quiggin [15, 18] briefly on the comparative static analysis under RDEU. Quiggin [15] made first attempt to derive comparative static effects for the RDEU model and proposed the "corresponding principle." He characterized some categories of distortion function in order to reach determined comparative static effects about some special transformations in random variables by "the correspondence principle". These transformations contain the MPMS criterion (essentially, this means that the final random variable is equal in distribution to the initial one plus a "noise" which is comonotone with it) and a degenerated distribution transformed into another randomization but maintaining its mean invariant. Quiggin [18] extended that idea and those results further by introducing generalized distortion function, which only requires the function being a map from unit interval to unit interval in the real set R. Moreover, the author presented two new concepts of risk-aversion, namely, aversion to monotone spreads decreasing in wealth and aversion to monotone spreads increasing in risk. The two concepts are closely related to the monotone spreads dominance proposed in his earlier articles. Under combination of the two stochastic shifts mentioned above and DARA condition of utility function, Quiggin obtained some desirable comparative static results that can be used in a general economic model. In the two papers, Quiggin's main contribution was to demonstrate that a MPMS shift in risk will lead to an unambiguous comparative static results in both the EU and the RDEU frameworks. In addition, the author also pointed out that the MPMS order is a subset of the SSD order, while so is the FSD order. But the relationship between the FSD and the MPMS orders is still obscure.

The paper is organized as follows. In Section 2, we review the definitions of MLR and MPR briefly and introduce our MPD and RMPD orders. Then, the relationship among them is examined in detail, and the RDEU functional is also recalled. The comparative static theorem in simple portfolio setting, our main result, is stated and proved in Section 3. Section 4 gives some remarks about MPD and RMPD. Section 5 concludes this paper.

2. Definition, Basic Model, and Rank-Dependent Expected Utility

2.1. Right Monotone Probability Difference Order. In this part, we firstly review three well-known stochastic orders in the economic and financial literature. Then, we introduce two new stochastic orders and examine the relationship among these stochastic orders. Consider any pair of random variables (X_1, X_2) with cumulative distribution functions (CDFs) $(F_1(x), F_2(x))$ whose supports are in an interval $[a, b]$. To avoid entanglement in technological details, we assume that all random variables are continuous with bounded supports in $[a, b]$ without losing the generality of our conclusions.

Definition 1 (first-order stochastic dominance). $X_1 \succcurlyeq_{\mathrm{FSD}} X_2$ if their associated CDFs, $F_1(x)$ and $F_2(x)$, satisfy that $F_2(x) \geqslant F_1(x)$ for all x in $[a, b]$.

The following definitions of the MLR and the MPR orders are given by Eeckhoudt and Gollier [11], where the definition of MLR is slightly different from the original one given by [10].

Definition 2 (monotone likelihood ratio). $X_1 \succcurlyeq_{\mathrm{MLR}} X_2$ if there exist a scalar c in $[a, b]$ and a nonnegative nonincreasing function $h : [c, b] \to [0, \infty)$ such that $F_1(x) = 0$ for any $x < c$ and $F_2(x) = F_2(c) + \int_c^x h(s)dF_1(s)$ for any $x \geqslant c$.

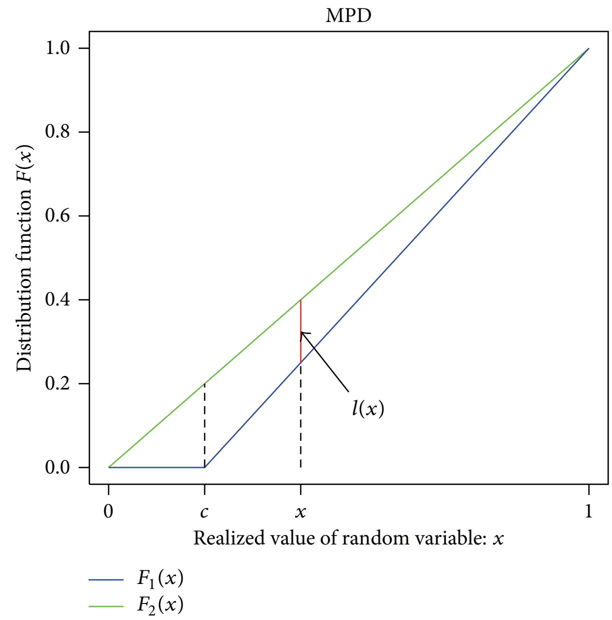

Figure 1: $X_1 \succcurlyeq_{\mathrm{MPD}} X_2$.

Definition 3 (monotone probability ratio). $X_1 \succcurlyeq_{\mathrm{MPR}} X_2$ if there exist a scalar c in $[a, b]$ and a nonnegative nonincreasing function $H : [c, b] \to [1, \infty)$ such that $F_1(x) = 0$ for any $x < c$ and $F_2(x) = H(x)F_1(x)$ for any $x \geqslant c$.

Obviously, the MLR order is defined by a ratio between the probability density functions (PDFs) of the two random variables, while MPR is a ratio between their CDFs. We adopt a different way to define two new stochastic orders by virtue of difference between the two CDFs. Our criteria are defined as follows.

Definition 4 (monotone probability difference). $X_1 \succcurlyeq_{\mathrm{MPD}} X_2$ if there exist a scalar c in $[a, b]$ and a nonnegative nonincreasing function $l : [c, b] \to [0, 1]$ such that $F_1(x) = 0$ for any $x < c$ and $F_2(x) = F_1(x) + l(x)$ for any $x \geqslant c$.

Definition 5 (right monotone probability difference). $X_1 \succcurlyeq^{d}_{\mathrm{RMPD}} X_2$ if $X_1 \succcurlyeq_{\mathrm{FSD}} X_2$, and there are two scalars c and d satisfying $a \leqslant c < d < b$ and a nonnegative nonincreasing function $L : [d, b] \to [0, 1]$ such that $F_1(x) = 0$ for any $x < c$ and $F_2(x) = F_1(x) + L(x)$ for any $x \geqslant d$.

In contrast with MLR and MPR, the MPD and the RMPD orders here are more interpretable than them. To see this point more clearly, it is instructive to examine the characteristics of logarithms of the two ratios in Definitions 2 and 3. In this way, we easily conclude that the difference between the logarithms of the two associated PDFs $(f_1(x), f_2(x))$ is nonincreasing in x and so is the difference between the logarithms of the two CDFs. However, the interpretations of log differences are not very apparent. On the contrary, the definitions of MPD and RMPD built directly from the difference between the two CDFs seem to be easier to understand, since it implies that $f_1(x) \geqslant f_2(x)$ for $x \in [c, b]$,

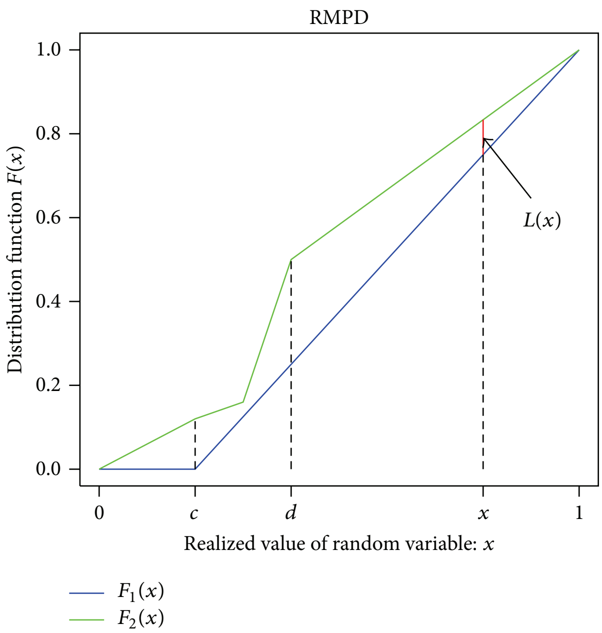

FIGURE 2: $X_1 \succcurlyeq^d_{\text{RMPD}} X_2$.

if $X_1 \succcurlyeq_{\text{MPD}} X_2$; $f_1(x) \geqslant f_2(x)$ for $x \in [d, b]$, if $X_1 \succcurlyeq^d_{\text{RMPD}} X_2$. It should be noticed that the RMPD criterion has less demand than MPD. In fact, taking the same scale c in Definitions 4 and 5, it is obvious that $[d, b] \subset [c, b]$, which implies that Definition 5 only requires the monotonicity of $F_2(x) - F_1(x)$ on the interval $[d, b]$ other than on the larger interval $[c, b]$. Figures 1 and 2 illustrate this distinction between the MPD and the RMPD orders.

The following Proposition 6 goes further to explore some implications of our stochastic orders and to investigate these relationships among MLR, MPR, MPD, and RMPD.

Proposition 6. *(1) $X_1 \succcurlyeq_{MLR} X_2 \Rightarrow X_1 \succcurlyeq_{MPR} X_2$ and $X_1 \succcurlyeq_{MPD} X_2 \Rightarrow X_1 \succcurlyeq_{MPR} X_2$;*

(2) $X_1 \succcurlyeq_{MLR} X_2 \Rightarrow X_1 \succcurlyeq^d_{RMPD} X_2$ and $X_1 \succcurlyeq_{MPD} X_2 \Rightarrow X_1 \succcurlyeq^d_{RMPD} X_2$.

Proof. (1) The first relationship has been shown in [11]. Consider the second relationship. Because of $X_1 \succcurlyeq_{\text{MPD}} X_2$, from the definition of MPD, there is a nonnegative nonincreasing function $l(x)$ such that $F_2(x) = F_1(x) + l(x)$ for any $x \geqslant c$. Using this relation, we obtain following formula

$$\frac{F_2(x)}{F_1(x)} = 1 + \frac{l(x)}{F_1(x)}, \quad \text{when } x \geqslant c. \tag{1}$$

Let $H(x) = 1 + (l(x)/F_1(x))$; by the monotonicity of $l(x)$ and CDF $F_1(x)$, it is obvious that $H(x)$ is a positive nonincreasing function defined on $[c, b]$. Therefore, according to Definition 3, $X_1 \succcurlyeq_{\text{MPR}} X_2$.

(2) Only the first relationship needs to be demonstrated. If $X_1 \succcurlyeq_{\text{MLR}} X_2$, then $F_2(x) = F_2(c) + \int_c^x h(s) dF_1(s)$ for any $x \geqslant c$. Because $X_1 \succcurlyeq_{\text{MLR}} X_2$ implies that their PDFs satisfy the single crossing condition (the single crossing condition is very important for the approach of monotone comparative statics under uncertainty (see [14])) according to the assumption of $h(x)$ being nonnegative nonincreasing, there exists a scalar d in $[c, b]$ such that $h(x) - 1 \leqslant 0$ for all $x \geqslant d$. Let

$$L(x) = F_2(c) + \int_c^x h(s) dF_1(s) - F_1(x), \quad \text{for } x \geqslant d. \tag{2}$$

It is obvious that

$$F_2(x) = F_1(x) + L(x), \quad \text{for } x \geqslant d. \tag{3}$$

As a result, $L(x) \geqslant 0$ since $X_1 \succcurlyeq_{\text{FSD}} X_2$. To differentiate $L(x)$, we have

$$\begin{aligned} L'(x) &= \left(\int_c^x h(s) dF_1(s) \right)' - F_1'(x) \\ &= (h(x) - 1) F_1'(x) \leqslant 0, \quad x \geqslant d. \end{aligned} \tag{4}$$

So $L(x)$ is a nonnegative nonincreasing function on $[d, b]$. According to Definition 5, we can derive $X_1 \succcurlyeq^d_{\text{RMPD}} X_2$. □

The relationship among the various orderings mentioned above can be illustrated clearly from following diagram:

$$\begin{tikzcd} \text{MLR} \arrow[r, Rightarrow] \arrow[dr, dotted] & \text{MPR} \arrow[r, Rightarrow] & \text{FSD} \arrow[d, equal] \\ \text{MPD} \arrow[r, Rightarrow] \arrow[ur, dotted] & \text{RMPD} \arrow[r, Rightarrow] & \text{FSD} \end{tikzcd} \tag{5}$$

It is obvious that the MLR and MPD orders have stricter limitations than the MPR and RMPD orders. According to the definition of MPD, it is convenient to interpret its implication, but this stochastic order is too restrictive to be widely applied. On the contrary, the RMPD order as a larger subset of FSD can capture a number of phenomena of changes in distribution (such as translations of some random variables, MLR shifts), so it has much broader application than the MPD order.

In successive parts, we assume that an investor's preference satisfies so-called law invariant property, namely, he always considers any random variables with the same CDF as indifferent. Thus, these stochastic orders defined above can be described by the random variables or by their CDFs. For instance, $X_1 \succcurlyeq^d_{\text{RMPD}} X_2$ can be denoted as $F_1 \succcurlyeq^d_{\text{RMPD}} F_2$.

2.2. Model and Rank-Dependent Expected Utility. Let us consider a standard portfolio problem with a safe asset and a risky asset. An investor plans to allocate his positive initial wealth w_0 between the two assets. Suppose that the safe asset has a risk-free net return r and the risky one has a random net return X with its support contained in $[a, b]$ and the associated CDF $F(x)$. The investor allocates α units in the risky asset and remaining $w_0 - \alpha$ in the safe asset. We here rule out the short-selling situation in order to avoid discussing the case where the problem has not finite solution. Therefore, we focus on $\alpha \geqslant 0$, where $\alpha = 0$ indicates that the investor only holds the safe asset. So the investor's end-of-period wealth can be expressed as $W = \alpha(1 + X) + (w_0 - \alpha)(1 + r)$

or $W = \alpha(X - r) + w_0(1 + r)$, and the term $X - r$ is the excess return. Furthermore, we also assume that there is no arbitrage opportunity in the finance market. As a result, both the probabilities $P(X > r)$ and $P(X < r)$ are strict positive; that is, $P(X > r) > 0$ and $P(X < r) > 0$.

The investor chooses the optimal level of α in order to maximize his utility function. Instead of EU which has been used in most of the published literature on the comparative static problem, we assume that the investor is an RDEU type of decision maker. That is, he acts in accordance with the principle of maximizing his RDEU functional. The problem can be formalized as follows:

$$\max_{\alpha} \mathrm{RU}(\alpha, F(x)) = E_g[u(W)] = \int_a^b u[\alpha(x - r) + w_0(1 + r)]\, dg(F(x)), \tag{6}$$

where the von Neumann-Morgenstern utility function $u(x)$ is always assumed to be continuous and increasing and the distortion function $g : [0, 1] \rightarrow [0, 1]$ is a continuous and increasing function with $g(0) = 0$ and $g(1) = 1$. Clearly, a RDEU investor's attitude toward risk is described by the combined properties of $u(x)$ and $g(x)$. For the distortion function $g(x)$, what is the meaning of it all? According to the observations in paper [19], we can provide $g(x)$ with an interesting interpretation. The author pointed out that there exists a difference between the "objective given" probability and the probability "perceived" by the investor that is used to value his decision [19]. Typically, the "perceived" probability depends on many different factors, such as the investor's characteristics and the decision-making setting which he is facing. In the RDEU functional given in (6), $F(x)$ can be interpreted as the "objective given" CDF, while $g(F(x))$ can be considered as the "perceived" one. The different functions $g_i(x)$ (i belonging to some index set I) can represent the distortion functions of different decision makers as well as an individual's distortion functions in different situations.

It is easy to illustrate that the model identifies EU and DU (dual utility) as its specific cases. In fact, when $g(x)$ is the identity function, formula (6) becomes

$$\max_{\alpha} \mathrm{EU}(\alpha, F(x)) = E[u(W)] = \int_a^b u[\alpha(x - r) + w_0(1 + r)]\, dF(x). \tag{7}$$

When $u(x)$ is the identity function, we can rewrite the formula as follows:

$$\max_{\alpha} \mathrm{DU}(\alpha, F(x)) = E_g[W] = \int_a^b [\alpha(x - r) + w_0(1 + r)]\, dg(F(x)). \tag{8}$$

In (8), the symbol $E_g[W]$ denotes the expectation of the random wealth W about the transformed distribution $g(F(x))$. It is obvious that DU is dual to EU theory since it distorts the probabilities of random events other than transforming the outcomes resulting from those events. Therefore, RDEU is also called the generalized expected utility by several authors [20, 21].

Besides these hypotheses above, we also suppose that both $u(x)$ and $g(x)$ are concave. This property about $u(x)$ represents individual attitude toward outcomes, which is usually interpreted as decreasing marginal utility. The concavity of $g(x)$ indicates two different meanings. First, the mixture of the concavities of $u(x)$ and $g(x)$ fully characterizes an investor's behavior that is averse to increasing in risk in the sense of SSD, called strong risk-averse agent in the published literature [20]. Second, it can also capture an investor's "*pessimistic*" bias [21]. As mentioned in the introduction, a pessimist always attaches a greater probability to an unfavorable event and a lesser probability to a favorable event. A great number of empirical observations have ascertained that the pessimistic bias pervades everywhere [1, 21]. This paper focuses on the effects of the first-order shifts in the distribution of the risky asset on the optimal levels of α selected by any strong risk-averse and pessimistic agents. In addition, we also assume that these two functions, $u(x)$ and $g(x)$, are twice continuously differentiable. It is convenient to discuss the problem of portfolio selection with this assumption.

3. The Comparative Statics Analysis

In this section, we offer a comparative static results under the framework of RDEU theory. The effects of both the outcome sensitivity and the probability sensitivity are considered. At first, we study the properties of the RDEU functional from its first two derivatives:

$$\frac{\partial \mathrm{RU}(\alpha, F(x))}{\partial \alpha} = \int_a^b u'[\alpha(x - r) + w_0(1 + r)] \times (x - r)\, dg(F(x)), \tag{9}$$

$$\frac{\partial^2 \mathrm{RU}(\alpha, F(x))}{\partial \alpha^2} = \int_a^b u''[\alpha(x - r) + w_0(1 + r)] \times (x - r)^2 dg(F(x)). \tag{10}$$

From (10), we know that the concavity of $u(x)$ ensures the concavity of the RDEU functional in the controllable variable α. Consequently, this guarantees that the portfolio problem (6) has unique optimal solution in $[0, w_0]$. If the solution lies in the interior of $[0, w_0]$, the optimal level of α can be derived from the first-order condition that

$$\frac{\partial \mathrm{RU}(\alpha, F(x))}{\partial \alpha} = 0. \tag{11}$$

But if the maximizing problem (6) has a corner solution, namely, $\alpha^* = 0$ or $\alpha^* = w_0$, then the associated first-order conditions can be expressed as follows, respectively:

$$\frac{\partial \mathrm{RU}(0, F(x))}{\partial \alpha} \leqslant 0, \qquad \frac{\partial \mathrm{RU}(w_0, F(x))}{\partial \alpha} \geqslant 0. \tag{12}$$

As observed by Quiggin [15, 18], the function $g(F(x))$ is still a CDF, corresponding to a "perceived" probability measure denoted as P_g; that is, $P_g(X \leqslant x) = g \circ P(X \leqslant x)$. Although the nonarbitrage assumption guarantees that $P(X > r) > 0$ and $P(X < r) > 0$, it is still possible that the investor's "perceived" probability satisfies $P_g(X > r) = 0$, if he is enough pessimistic. In this case, the RDEU investor puts all wealth w_0 on the safe asset, which can be seen from the formula (9) and the condition (12). Of course, this is very uninteresting for the problem which we will discuss. Therefore, we assume that the probability perceived by the investor always satisfies that $P_g(X > r) > 0$ and $P_g(X < r) > 0$.

Let $\alpha(u, g, F)$ denote the solution of the maximizing problem (6), that is,

$$\alpha(u, g, F) = \arg\max_{\alpha} \mathrm{RU}(\alpha, F(x)). \tag{13}$$

Obviously, the scalar depends upon the three factors: $u(x)$, $g(x)$, and $F(x)$.

We review a famous result in financial economics before analysis. The result makes an important difference between the classic EU and the RDEU here. Now, let α_E denote an EU investor's optimal investment and α_R indicate a RDEU investor's optimal investment.

Lemma 7. *For a risk-averse EU type of investor and the simple portfolio problem with one safe asset and one risky asset,* (i) $\alpha_E = 0$ *if and only if* $E[X] \leqslant r$; (ii) $\alpha_E > 0$ *if and only if* $E[X] > r$.

The proof can refer to Zhang and Zhao [22]. But this result here does not consider the case of short-selling. A similar proposition for RDEU investors is proposed as follows.

Proposition 8. *For a RDEU type of investor with concave $u(x)$ and $g(x)$ under the simple portfolio setting with one safe asset and one risky asset,* (i) $\alpha_R = 0$ *if and only if* $E_g[X] \leqslant r$; (ii) $\alpha_R > 0$ *if and only if* $E_g[X] > r$.

In terms of (11) and (12), we can easily derive the conclusion by utilizing the same method as the proof of Lemma 7. Although the two results are similar in form, they have different meanings and essence. For a risk-averse EU investor, Lemma 7 tell us that whether he holds the risky asset depends merely on the signal of the excess return $E[X] - r$, regardless of degree of his risk-averse which can be measured by the coefficient of absolute risk-aversion derived from the utility function $u(x)$. For a RDEU type of investor, however, Proposition 8 shows that he will invest a positive quantity in the risky asset only if its expected return about the "perceived" probability distribution $g(F(x))$ exceeds the risk-free return. Consequently, a RDEU investor's decision depends on his attitude toward risk, the decision setting as well as some individual psychological biases, since all these factors commonly influence on shape of the distortion function $g(x)$. Even if there is a positive excess return under the objective probability distribution $F(x)$, a risk-averse and pessimistic RDEU investor may not hold the risky asset. The fact has already been observed by Yaari [23]. To see it more clearly, we observe a specific example with four discrete random variables.

In the example above, we suppose that the risk-free return is 0.04. The expected returns of four different risky assets are $E[X_1] = 0.04$, $E[X_2] = 0.05$, $E[X_3] = 0.07$, and $E[X_4] = 0.10$, respectively. We use the concave function $g(x) = x^{\beta}$ as the distortion function, where the parameter β can indicate whether an investor is a pessimist or an optimist. Specifically, the investor is pessimistic if $0 < \beta < 1$ or optimistic if $\beta > 1$. While $\beta = 1$ denotes that the investor is an EU decision maker. The lower the value β is, the more pessimistic the investor is when $\beta < 1$. From Table 1, we find out that an optimist always adjusts $E[X_i]$ upward to the "distorted" expectations $E_g[X_i]$, while a pessimist always readjusts the four expectations downward. In addition, with increase in the pessimistic degree, the "distorted" expectations tend to decrease gradually. In the extreme pessimistic situation, the "distorted" expectations $E_g[X_i]$ go down to the smallest realized value of each risky asset, which shows that an extreme pessimistic investor's decision depends on the relationship between the risk-free return and the smallest value of the risky asset.

Though we cannot assert whether an RDEU investor with concave $u(x)$ and $g(x)$ will hold the risky asset when $E[X] > r$, it is easy to arrive at the following affirmative result.

Corollary 9. *For each RDEU type of investor with concave $u(x)$ and $g(x)$ under the simple portfolio setting, he always decides to put all wealth only in the safe asset, when $E[X] \leqslant r$.*

This is obvious, since the inequality $g(F(x)) \geqslant F(x)$ will result in $F(x)$ dominating $g(F(x))$ in the sense of FSD when $g(x)$ is concave, which implies that $E_g[X] \leqslant E[X] \leqslant r$.

Given the initial random return X, suppose that it undergoes a shift in distribution from $F(x)$ to $G(x)$ with its support contained in $[a, b]$. The shift induces the RDEU investor with the utility function $u(x)$ and the distortion function $g(x)$ to adjust his optimal investment from $\alpha(u, g, F)$ to $\alpha(u, g, G)$. Now, the classic comparative static problem can formally be described as what are the conditions satisfied by the two CDFs $F(x)$ and $G(x)$, which enable all risk-averse and pessimistic RDEU investors to adjust their optimal levels of α in the same direction.

Before our main result, Theorem 11, is stated, we firstly demonstrate a key theorem whose conclusion is similar to the condition provided by Gollier [12], which has also been generalized to a more general situation without requiring the monotone payoff function in risk by Hau [24]. However, there exist several important distinctions: first, the function $T(x, \alpha, F)$ in Gollier [12] did not consider the distortion function $g(x)$; second, our conclusion is similar to his necessary and sufficient condition on the determined comparative static result, which is a joint condition on the shifts in distribution and the economic model represented by $z(X, \alpha)$, and he did also not discuss the implication of the changes in distribution. But we go a step further to explore the implication of the changes in distribution behind the condition and to take account of the effects of an individual's pessimistic bias on his optimal choice. For a given $\overline{x}$ in $[a, b]$ and the CDF $F(x)$,

Table 1: Example of distorted excess return with distortion function $g(x) = x^{\beta}$.

Asset	Probability				r	$E[X]$	$E_g[X]$					
	0.25	0.25	0.25	0.25			$x^{1.01}$	$x^{0.95}$	$x^{0.90}$	$x^{0.80}$	$x^{0.10}$	$x^{0.01}$
X_1	0.01	0.03	0.04	0.08	0.04	0.04	0.040	0.039	0.038	0.036	0.014	0.011
X_2	0.01	0.04	0.05	0.10	0.04	0.05	0.050	0.049	0.047	0.045	0.017	0.011
X_3	0.03	0.04	0.06	0.15	0.04	0.07	0.070	0.069	0.067	0.092	0.035	0.031
X_4	0.05	0.07	0.08	0.20	0.04	0.10	0.100	0.098	0.096	0.092	0.057	0.051

Let X denote a lottery $(x_1, p_1; x_2, p_2; x_3, p_3; x_4, p_4)$, where the $E_g[X]$ can be calculated via the formula: $E_g[X] = \sum_{i=1}^{4} x_i w(p_i)$, where $w(p_i) = [g(P_i) - g(P_{i-1})]$, P_i being the cumulative probability and $P_0 = 0$.

let $\mathscr{G}(F, \overline{x})$ indicate the set of some CDFs, $\{G \mid F \succcurlyeq^{d}_{\mathrm{RMPD}} G, c \leqslant d \leqslant \overline{x}\}$. That is, the set contains all CDFs which are dominated by the $F(x)$ in the sense of RMPD with the corresponding thresholds d located at the left side of $\overline{x}$ and their supports contained in $[a, b]$.

Theorem 10. *For any $G \in \mathscr{G}(F, r)$ where r is the risk-free return, then for any x in $[a, b]$, the inequality, that is, $T(x, G, g) \leqslant T(x, F, g)$, always holds for all concave distortion functions $g(x)$, where $T(x, F, g) = \int_a^x (s - r) dg(F(s))$.*

Proof. For an arbitrary given $G \in \mathscr{G}(F, r)$, when $a \leqslant x \leqslant r$, by integrating the formula $T(x, G, g) = \int_a^x (s - r) dg(G(s))$ by parts, we have

$$\begin{aligned} T(x, G, g) &= g(G(s))(s-r)\big|_a^x - \int_a^x g(G(s))\, ds \\ &= g(G(x))(x-r) - \int_a^x g(G(s))\, ds \\ &\leqslant g(F(x))(x-r) - \int_a^x g(F(s))\, ds \\ &= T(x, F, g). \end{aligned} \tag{14}$$

The inequality is due to the assumptions $x - r \leqslant 0$ and $G \in \mathscr{G}(F, r)$, which implies that $G(x) \geqslant F(x)$ for any x in $[a, b]$, and $g(G(x)) \geqslant g(F(x))$.

When $r < x \leqslant b$, integrating $T(x, G, g)$ by parts again yields the following inequality:

$$\begin{aligned} T(x, G, g) &= \int_a^x (s-r)\, dg(G(s)) \\ &= \int_a^r (s-r)\, dg(G(s)) \\ &\quad + \int_r^x (s-r)\, dg(G(s)) \\ &= \int_a^r (s-r)\, dg(G(s)) \\ &\quad + g(G(s))(s-r)\big|_r^x - \int_r^x g(G(s))\, ds \\ &= \int_a^r (s-r)\, dg(G(s)) \\ &\quad + g(G(x))(x-r) - \int_r^x g(G(s))\, ds \\ &= \int_a^r (s-r)\, dg(G(s)) \\ &\quad + g(G(x)) \int_r^x ds - \int_r^x g(G(s))\, ds \\ &= \int_a^r (s-r)\, dg(G(s)) \\ &\quad + \int_r^x [g(G(x)) - g(G(s))]\, ds \\ &\leqslant \int_a^r (s-r)\, dg(F(s)) \\ &\quad + \int_r^x [g(G(x)) - g(G(s))]\, ds. \end{aligned} \tag{15}$$

The last inequity derives from the first step, namely, $T(r, G, g) \leqslant T(r, F, g)$. To arrive at the conclusion, we just need to demonstrate the following relationship:

$$g(G(x)) - g(G(s)) \leqslant g(F(x)) - g(F(s)), \quad \text{for } r < s \leqslant x \leqslant b, \tag{16}$$

which is equivalent to showing

$$g(G(x)) - g(F(x)) \leqslant g(G(s)) - g(F(s)), \quad \text{for } r < s \leqslant x \leqslant b. \tag{17}$$

To derive the inequality, let $K(x) = g(G(x)) - g(F(x))$, the statement is equivalent to prove that $K(x)$ is nonincreasing in x over $[r, b]$. To differentiate $K(x)$, we obtain

$$K'(x) = g'(G(x))\, G'(x) - g'(F(x))\, F'(x) \leqslant 0. \tag{18}$$

In fact, by the assumption $G \in \mathscr{G}(F, r)$ and the definition of RMPD, the inequality $G'(x) \leqslant F'(x)$ holds when $r < x \leqslant b$. In addition, $g'(G(x)) \leqslant g'(F(x))$ which results from concavity of $g(x)$ and $G(x) \geqslant F(x)$ for any $x \in [a, b]$. Thus, the inequality $K'(x) \leqslant 0$ always holds when $x \in [r, b]$, which implies that $K(x)$ is a nonincreasing function over $[r, b]$.

The inequality (17) is always true under these given conditions and then when $x \in (r,b]$, we have the following inequality:

$$\begin{aligned} T(x,G,g) &\leqslant \int_a^r (s-r)\,dg(F(s)) \\ &\quad + \int_r^x [g(G(x)) - g(G(s))]\,ds \\ &\leqslant \int_a^r (s-r)\,dg(F(s)) \\ &\quad + \int_r^x [g(F(x)) - g(F(s))]\,ds \\ &= T(x,F,g). \end{aligned} \tag{19}$$

In summary, $\forall x \in [a,b]$ and the inequality $T(x,G,g) \leqslant T(x,F,g)$ always holds for all concave distortion functions $g(x)$ only if $G \in \mathcal{G}(F,r)$. □

In this part, since we only consider the effects of changes in risk, we simplify the RDEU investor's optimal selection $\alpha(u,g,F)$ as α_F. Let $\mathbb{G}$ indicate the set of all concave distortion functions $g(x)$ which satisfies $g \circ P(X > r) > 0$ and $g \circ P(X < r) > 0$. Assume that the random return X satisfies $E[X] > r$; we define two sets of distortion functions as follows: $\mathbb{G}_1 = \{g \mid E_g[X] \leqslant r, g \in \mathbb{G}\}$ and $\mathbb{G}_2 = \{g \mid E_g[X] > r, g \in \mathbb{G}\}$. Obviously, there exists a relationship that $\mathbb{G} = \mathbb{G}_1 \cup \mathbb{G}_2$.

Theorem 11. *For each RDEU investor with concave $u(x)$ and $g(x)$ who selects α_F under the initial distribution $F(x)$, choosing α_G after a change in distribution from $F(x)$ to $G(x)$, (1) if $g \in \mathbb{G}_1$, then even $F \succcurlyeq_{FSD} G$, the investor will reject to hold the risky asset; that is, $\alpha_G = \alpha_F = 0$; (2) if $g \in \mathbb{G}_2$, then $G \in \mathcal{G}(F,r)$ implies $\alpha_G \leqslant \alpha_F$.*

Proof. Conclusion (1) being clear, we only need to prove conclusion (2). Because of $g \in \mathbb{G}_2$, we know from Proposition 8 that the investor has a positive holding of the risky asset. When $\alpha_F = w_0$, the conclusion is trivial. Thus, we only focus on the nontrivial case $\alpha_F < w_0$. Using the result and the notation in Theorem 10, we have

$$\begin{aligned} \frac{\partial RU(\alpha,G)}{\partial \alpha} &= \int_a^b u'[\alpha(x-r) + w_0(1+r)] \\ &\quad \times (x-r)\,dg(G(x)) \\ &= \int_a^b u'[\alpha(x-r) + w_0(1+r)]\,dT(x,G,g). \end{aligned} \tag{20}$$

Integrating the last formula by parts yields the following inequality:

$$\begin{aligned} \left.\frac{\partial \mathrm{RU}(\alpha,G)}{\partial \alpha}\right|_{\alpha=\alpha_F} &= \int_a^b u'(w)(x-r)\,dg(G(x)) \\ &= \int_a^b u'(w)\,dT(x,G,g) \\ &= T(x,G,g)u'(w)\Big|_a^b \\ &\quad - \int_a^b \alpha_F u''(w)\,T(x,G,g)\,dx \\ &= T(b,G,g)\,u'[\alpha_F(b-r) + w_0(1+r)] \\ &\quad - \int_a^b \alpha_F u''(w)\,T(x,G,g)\,dx \\ &\leqslant T(b,F,g)\,u'[\alpha_F(b-r) + w_0(1+r)] \\ &\quad - \int_a^b \alpha_F u''(w)\,T(x,F,g)\,dx \\ &= \left.\frac{\partial \mathrm{RU}(\alpha,F)}{\partial \alpha}\right|_{\alpha=\alpha_F} = 0. \end{aligned} \tag{21}$$

The inequality results from our assumptions that $u'(x) > 0$ and $u''(x) \leqslant 0$, and the conclusion $T(x,G,g) \leqslant T(x,F,g)$ in Theorem 10. Thus, the concavity of the RDEU functional $\mathrm{RU}(\alpha,G)$ implies $\alpha_G \leqslant \alpha_F$. □

From Theorem 11, we can see that for all strong risk-averse and pessimistic RDEU investors facing the risky asset X with a positive excess return, some are the "plunging" invertors who only hold the safe asset, but others are the "diversifiers" who hold the two assets. When a change in risk makes them worse in the sense of RMPD, some "diversifiers" may become the "plungers," even the changed risky asset satisfying $E[Y] > r$. However, each risk-averse EU investor still preserves to adopt the diversified investment strategy when the risky asset is changed to Y with $E[Y] > r$. To derive desirable comparative statics result, we require that the change in distribution must satisfy $X \succcurlyeq^{d}_{\mathrm{RMPD}} Y$ with the thresholds d located at left-side of the risk-free return r. Now, let us explore the implication of the MPD and the RMPD orders further. Supposing that the PDFs of the two random variables (X,Y) are $f(x)$ and $p(x)$, respectively, by Definition 5, we can conclude that $f(x) \leqslant p(x)$ always holds when $x \geqslant d$. If $G(x) \in \mathcal{G}(F,r)$, then $d \leqslant r$. As a result, Theorem 11 indicates that if $p(x)$ is derived from $f(x)$ by transferring some probability weights from the right-side part of $f(x)$ which exceeds d to the left-side part of $f(x)$ which is less than d, then each RDEU investor will adjust his optimal investment on the risky asset to a lower level. However, the statement does not apply to the general FSD changes.

To observe the proof process of Theorem 11, we easily see that the conclusion is also true for all risk-averse EU types of investors and we only consider the nontrivial situation with $E[X] > r$.

Corollary 12. *For each risk-averse EU investor who selects α_F under $F(x)$ and turns to choose α_G after a change in distribution from $F(x)$ to $G(x)$, he will always adjust his investment on the risky asset downward; that is, $\alpha_G \leqslant \alpha_F$, if $G(x) \in \mathcal{G}(F,r)$.*

It should be noticed that there is another proof of Theorem 11 by Quiggin's "corresponding principle." According to "corresponding principle," it suffices to show that a RMPD deterioration of the risky return will lead all risk-averse EU investors to reduce their optimal investments on the risky asset (i.e., Corollary 12), and the RMPD dominating relationship between the original distribution and the changed one will be preserved by a concave distortion function $g(x)$. The first step can be easily completed by using the same method as Theorems 10 and 11 without considering the distortion function $g(x)$. The second claim is also obvious. In fact, if two distributions satisfy that $F \succcurlyeq^{d}_{\mathrm{RMPD}} G$, then $G(x) \geqslant F(x)$ for all $x \in [a,b]$ and there is a scalar $d > c$ such that $L(x) = G(x) - F(x)$ being nonincreasing when $x \in [d,b]$ by Definition 5. For an increasing and concave distortion function $g(x)$, it is obvious that $g(G(x)) \geqslant g(F(x))$ for all $x \in [a,b]$ and $g(F(x)) = 0$ for $x < c$ because of the assumptions $g(0) = 0$ and $F(x) = 0$ for $x < c$. Furthermore, the function $K(x) = g(G(x)) - g(F(x))$ is also nonincreasing function over $[d,b]$ from the proof of Theorem 10; Therefore, $g(F(x)) \succcurlyeq^{d}_{\mathrm{RMPD}} g(G(x))$ by Definition 5. That is, the RMPD (or MPD) stochastic dominance relationship between $F(x)$ and $G(x)$ can be maintained well under a concave distortion of distribution function, therefore, the "corresponding principle" can be used.

4. Some Remarks

Several issues need a further discussion before we conclude this paper. Firstly, we should reconsider the relationship among MLR, MPR, here MPD, and RMPD. Essentially, the MLR order is equivalent to the difference between the logarithms of two PDFs being nonincreasing in x. In comparison with MLR, the MPR order is equal to applying the criterion of monotonic log-difference between two functions to two CDFs other than two PDFs. To see it more clearly, we construct two functions $T_{\mathrm{MLR}}(x)$ and $T_{\mathrm{MPR}}(x)$ as follows:

$$\begin{aligned} T_{\mathrm{MLR}}(x) &= \ln f_2(x) - \ln f_1(x), \quad x \in [c,b], \\ T_{\mathrm{MPR}}(x) &= \ln F_2(x) - \ln F_1(x), \quad x \in [c,b]. \end{aligned} \tag{22}$$

Then the MLR and the MPR orders can be formally defined as

$$\begin{aligned} &F_1 \succcurlyeq_{\mathrm{MLR}} F_2 \\ &\quad\Longleftrightarrow T_{\mathrm{MLR}}(x) \text{ is a nonincreasing function in } x, \\ &F_1 \succcurlyeq_{\mathrm{MPR}} F_2 \\ &\quad\Longleftrightarrow T_{\mathrm{MPR}}(x) \text{ is a nonincreasing function in } x. \end{aligned} \tag{23}$$

Contrary to MLR and MPR, let $l(x) = F_2(x) - F_1(x)$, $x \in [c,b]$; the MPD order is defined as

$$\begin{aligned} &F_1 \succcurlyeq_{\mathrm{MPD}} F_2 \\ &\quad\Longleftrightarrow l(x) \text{ is a nonincreasing function in } x. \end{aligned} \tag{24}$$

From (23) and (24), we see that all of the MLR, the MPR, and the MPD orders are defined by the nonincreasing property of the corresponding functions, $T_{\mathrm{MLR}}(x)$, $T_{\mathrm{MPR}}(x)$, and $D(x)$. For any concave distortion function $g(x)$, suppose that $T_{\mathrm{MPD}}(x) = g(F_2(x)) - g(F_1(x))$ and then it is easy to prove

$$\begin{aligned} &F_1 \succcurlyeq_{\mathrm{MPD}} F_2 \\ &\quad\Longrightarrow T_{\mathrm{MPD}}(x) \text{ is a nonincreasing function in } x. \end{aligned} \tag{25}$$

Therefore, the MPD order here places greater demands on the two distributions than do both the MLR and the MPR orders. This is partly due to the fact that our comparative statics result is required to hold for all strong risk-averse and pessimistic RDEU investors and all risk-averse EU investors. The MPD order is in accordance with some plausible intuitions, but it is too restrictive for applications. Therefore, we generalize the concept to the weaker RMPD order, which partially preserves the intuitive appealing of MPD and can be applied to a broader scope. We use the RMPD order to derive the unambiguous comparative analysis effects for all RDEU investors. The RMPD criterion requires that there exists a threshold d such that the function $l(x)$ is monotone nonincreasing when $x \geqslant d$. This property implies that the probability mass of X_2 is less than or equal to that of X_1 in the right side of the threshold d. Our main result also depends on the threshold d which must be less than or equal to the risk-free return r. It is worthy of noticing that the RMPD criterion does not place additional restriction to the left-side part of the two distributions except maintaining $F_2(x) \geqslant F_1(x)$. On the contrary, the classic MLR order demands that the PDFs of X_1 and X_2 must meet the single-crossing property, which is very demanding for the two PDFs. The result in [10] did not rest on the location of the crossing point, but the MLR order places higher demands on both the left and right tails of the two PDFs. As a result, there are some tradeoffs of advantages among the MLR, the MPR, and here the RMPD orders.

As for the pessimism and the optimism, there are several different definitions in the existing literature. Quiggin [21] used the property $g(x) > x$ of distortion function to characterize the weak pessimism, while we here define the concept of pessimism by concavity of $g(x)$, which is called strong pessimism [23]. Papers [25, 26] used a pessimistic index $\inf_{0<x<1}[((1-g(x))/(1-x))/(g(x)/x)]$ to describe this type of behavioral bias. Recently, paper [27] adopted NEO-additive capacity to capture the pessimistic and the optimistic biases. However, there exist some gaps among these different definitions and their relationship is still not explicit.

5. Conclusion

This paper proposed the MPD order and its generalized version called RMPD. The relationships among the classic MLR, the MPR, and them were analyzed in detail. Consequently, we obtained an unambiguous comparative static result for all strong risk-averse and pessimistic RDEU investors.

As far as RDEU is concerned, the theories are relatively mature. However, there is a mismatch between considerable knowledge about RDEU and the extent of their applications to relevant problems in finance and economics. The key to apply the RDEU model to a broader scope lies in its ability of making unambiguous comparative static analysis. In this paper, we added new comparative static results to this type of the literature on the RDEU theory. Therefore, we believe that the present research could help us to improve knowledge about applications of the RDEU model. Further studies will concentrate on exploring more implications for this model in practice and using our result to investigate other problems, such as the coinsurance problem and the problem of the optimal leverage of financial intermediaries.

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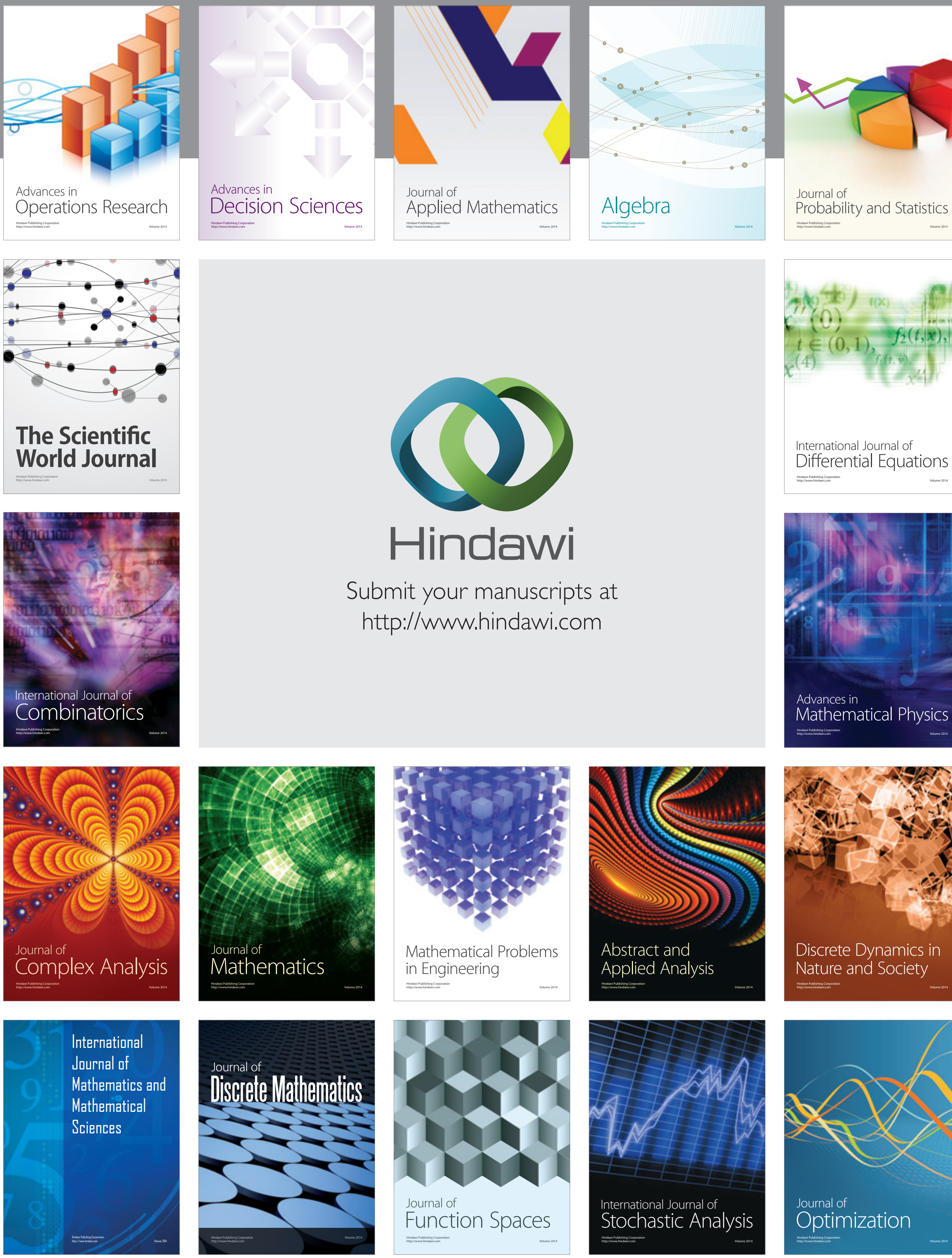

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