

## Editorial

# Mathematical Methods Applied to the Celestial Mechanics of Artificial Satellites 2013

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Studies related to celestial mechanics have been made for a long time. The observation of the sky showed the stars, in particular for some of them, called “Planets” by the Greeks, that have a particular motion against the other stars. Those observations challenged the minds of those times and speculations appeared to try to explain those motions. This was the beginning of the astronomy science and opened a particular field of study, focused on understanding the motion of the planets, moons, asteroids, comets and other celestial bodies. Several of the most important names in history are related to this field, like Ptolomeus, Nicolaus Copernicus, Galileo Galilei, Johannes Kepler, Isaac Newton, Albert Einstein and others, who helped to find the physical laws that govern those motions.

More recently, the last few decades brought the combination of this science with more practical applications in the engineering field. This combination created the term “Astrodynamics,” which is usually connected to the study of the motion of space vehicles made to explore the regions of the space around the Earth and beyond. Of course, the physical laws are the same for celestial bodies or spacecraft; it is just a question of emphasizing one or other aspects of the knowledge.

Although very recent in terms of history, this new combination of science and engineering has already made several important contributions to science. From the first artificial

satellite launched from the Earth by the former Soviet Union in 1957, the Sputnik, many important steps were made. The man landed on the Moon in 1969; the two Voyager spacecraft discovered many new moons around the giant planets Jupiter, Saturn, and Uranus in the 70’s and 80’s, also finding many aspects of the planets themselves that would not be found by telescopes based on the Earth.

The combination of theory and practical observations also showed the existence of planetary systems around other stars, the so called “exoplanets.” Soon “exomoons” around those “exoplanets” will be found.

Going back to our mother Earth, people’s life has changed dramatically for better after the space research activities started. Instantaneous communications by telephone, television, and so forth are mostly based on satellites travelling around the Earth in strategic orbits specified by methods used in astrodynamics. Driving a car in a new place is no longer an adventure, after the popularization of small and cheap devices that receive information from a constellation of satellites, like the GPS, and calculate its position in a map. The discoveries of resources on Earth may also benefit from images taken from space. Very important activities, like weather forecast, which contributes in agricultural activities and in the prediction of natural disaster, also rely on satellites.

This special issue is the third one of this journal dedicated to problems related to space research. It covers topics related

to mission design from every perspective, like orbit selection, attitude and orbit propagation, control of spacecraft, and related topics. This issue gives special attention to applications on those fields, in order to be complementary to publications that are more focused on theoretical developments. It has a total of twenty three papers, which are briefly described below.

Five of them are concerned with the problems of optimal maneuvers. One of them is “*Searching for orbits with minimum fuel consumption for station-keeping maneuvers: An application to lunisolar perturbations*,” by A. Prado. It develops a criterion to map orbits with respect to the perturbations received by the satellite. The method is based on the integral of the perturbing forces over the time. This index measures the variation of velocity transferred to the spacecraft by the perturbations. It is a characteristic of the orbit and the dynamical system considered and it does not depend on the engine or station-keeping techniques used. The results show the effects of the perturbations due to the Sun and the Moon in spacecraft around the Earth.

Then, the paper “*Station keeping of constellations using multiobjective strategies*,” by E. Rocco et al., studies how to perform station keeping maneuvers in constellations. The idea is to make those maneuvers under two conditions: low fuel consumption and a time constraint. These conditions are applied to all the members of the constellation, so a strategy is searched to consider the global optimization of those variables. This is a multiobjective problem. To find a solution, strategies that minimize the fuel consumption considering the constraints on the duration of the maneuver are used. Several examples are shown to exemplify the technique.

The next paper of that set is “*Minimum fuel low-thrust transfers for satellites using a Permanent magnet hall thruster*,” by T. Oliveira et al., that studies the problem of the orbital maneuvers required to accomplish the goals of a space mission. In particular, the situation where it is necessary to minimize the fuel consumption is solved, in order to increase the lifetime of the satellite. The paper has two particular goals: to develop an algorithm that can find optimal trajectories with continuous thrust that can be used in several types of missions and taking into account different constraints at the same time and to study the performance of the propulsion devices under development at the Universidade de Brasília to perform orbital maneuvers. It includes a study of the consequence of the errors of the thrust level of these pieces of equipment.

Then comes the paper by G. Zhang et al., “*Tangent orbital rendezvous using linear relative motion with  $J_2$  perturbations*,” that discusses the short-range coplanar problem for elliptic orbits. Three cases are analyzed with the first, the second, or both impulses being tangent. For a given initial point, the first two problems can be transformed into finding all roots of a single variable function about the transfer time, which can be done by the secant method. The bitangent rendezvous problem requires the same solutions obtained by considering the initial coasting time. A numerical example for two coplanar elliptic orbits is given to verify the efficiency of these proposed techniques. However, since this method is based on the linear relative

motion equations, it is only suitable for the short-range case between two spacecraft.

Another paper of this topic is “*Application of the Kalman filter to estimate the state of an aerobraking maneuver*,” by W. G. dos Santos et al., that study an application of the Kalman filter used to estimate the position and velocity of a spacecraft that is performing an aerobraking maneuver in the atmosphere of the Earth. Two examples are used: the maneuver made by the Hiten spacecraft and an aerobraking of a satellite that is in a low Earth orbit. The model considers a reference trajectory and a trajectory perturbed by external forces and nonidealities of the sensors and actuators. A PID controller and propulsive jets are used to maneuver the spacecraft. The results show comparisons using and not using the Kalman filter, showing the importance of its application.

Another topic covered in this special issue is related to the determination, simulation, and propagation of orbits of spacecraft. There are five papers related to this topic. One of them is “*Onboard and real-time artificial satellite orbit determination using GPS*,” by A. P. M. Chiaradia et al., which introduces an algorithm for orbit determination applying the extended Kalman Filter method. An analysis is performed to ascertain an adequacy of the modeling complexity versus accuracy. The minimum set to estimate states to reach the level of accuracy of tens of meters is found to have at least the position, velocity, and user clock offset components. The dynamical model is assessed through several tests, covering the force model, the numerical integration scheme and the step-size control, using simplified variational equations. The algorithm is tested with real-time data from a satellite with a GPS receiver on board. The satellite orbit is estimated using the algorithm developed with good accuracy and minimum computational cost. In this procedure, the position and velocity errors obtained along one day vary from 15 to 20 m and from 0.014 to 0.018 m/s, respectively, with standard deviations from 6 to 10 m and from 0.006 to 0.008 m/s, respectively.

Another paper of this topic is “*Precise analytical computation of frozen-eccentricity, low Earth orbits in a tesseral potential*,” by M. Lara et al., that considers the problem of finding frozen orbits using a higher degree geopotential, which takes into account the short-period effects of the tesseral harmonics. This approach can describe the behavior of elliptic frozen orbits in conditions where a model based on the  $J_2$ - $J_3$  terms is not good enough. Using this model it is possible to find low Earth orbits that show smaller long-period effects in long-term propagations than those obtained when using the zonal model design.

Also in this topic there is the paper “*Dynamics of artificial satellites around Europa*,” by J. P. dos S. Carvalho et al., that searched for stable orbits around Europa. The orbital elements of an artificial satellite in orbit around Europa are analyzed by developing the disturbing potential in a power series of the eccentricity and inclination of the satellite. This satellite is under the gravitational effects of Europa, with its gravitational coefficients  $J_2$  and  $J_3$  and Jupiter. The Lagrange planetary equations are numerically integrated and the results show that in the range of semimajor axis between 2000 km and 3000 km several polar orbits can survive for

about 300 days. A particular orbit with semimajor axis equals to 2341 km presents smaller variations in its orbital elements and may be important for future missions. They also found that the critical inclination for this particular orbit is  $46.1^\circ$  and  $138.4^\circ$  for the prograde and the retrograde cases, respectively.

Next in this topic is the paper by R. C. Domingos et al. entitled “*A study of double and single second order averaged models to evaluate third body perturbation considering elliptic orbits for the perturbing body*,” where the equations for the variations of the Keplerian elements of the orbit of the spacecraft perturbed by a third body are developed using a single-averaged approach over the motion of the spacecraft, considering an elliptic orbit for the disturbing body. A comparison is made of the results obtained by this approach and the more used double-averaged technique, as well as with the full elliptic restricted three-body problem. Different initial eccentricities for the perturbed body are considered, since the measurement of the effects of this variable is one of the goals of this paper. The results show the impact of this parameter as well as the differences between both models compared to the full elliptic restricted three-body problem. Regions below, near, and above the well-known critical angle of the third-body perturbations are considered, as well as different altitudes for the orbit of the spacecraft. The results point out that short oscillations in the inclination and eccentricity for initial inclinations below the critical value, increase fast in amplitude around the critical value and then the typical behavior of having the inclinations remaining constant for a long time and then returning very fast to the initial value, increase fast again to its original values. The eccentricity shows an opposite behavior and increases when the inclination decreases and vice versa. The results also showed that circular orbits exist, but frozen orbits do not exist under this model. The circular orbits, in general, do not keep the inclinations constant, as happened in the double averaged model. The increase of the semimajor axis causes an increase of the perturbation and this fact accelerates the dynamics of the system. In general, both second order averaged models have results that are much closer to each other than closer to the full model. It means that both averaged models tend to give similar errors when compared to the full model.

Another paper of this category is “*Averaging tesseral effects: closed form relegation versus expansions of elliptic motion*,” by M. Lara et al. It shows that the terms of the geopotential dependent on the longitude have short-period effects in the orbit propagation of artificial satellites that needs to be taken into account, if accurate analytical and semianalytical algorithms are required. The actual methods neglect the tesseral effects for subsynchronous orbits that are related to the eccentricity of the orbit, which is a point that limits its applications to a certain range of values for the eccentricity. The paper shows comparisons with averaging procedures and a balance of advantages and disadvantages of the different approaches.

After that comes the papers about attitude of the satellites. Three papers cover this topic. The first one is the paper “*Attitude dynamics and stability of a simple solar Photon thruster*,” by A. D. Guerman et al., that develops a model for the attitude dynamics of a nonideal simple solar photon

Thruster (SSPT). It also analyses the motion of a sail craft with respect to its center of mass. The analytical equations for the torque and the force caused by the solar radiation are obtained for the situation where there is a misalignment of the SSPT axis with the direction of the Sun. After that the study of the dynamics and stability properties of a sail craft is made. The results show that an ideally reflecting sail is unstable, while for a sail craft with nonideal collector the symmetry axis is stable with respect to the direction of the Sun. A large variety of system parameters are used. The motion around the symmetry axis is always unstable and an active stabilizer is necessary.

After that comes the paper by H. K. Kuga and V. Carrara, “*Attitude determination with magnetometers and accelerometers to use in satellite simulator*,” that presents the implementation and tests of a fully self-contained algorithm for the determination of attitude using magnetometer and an accelerometer units, for application on a satellite attitude simulator based on frictionless air bearing tables. In order to allow that the measurements are used with accuracy, a calibration method is implemented which proves to be an essential part of the procedure to achieve better attitude determination accuracy. The calibration procedure does not need the attitude knowledge, so that it can be applied without any special approach or environment. Magnetometers were the most biased sensor, as expected, and major source of attitude errors (62,5%). Accelerometer biases contributed only with 37,5% of the weight in attitude determination algorithm. For the stepwise real-time attitude determination the QUEST algorithm is used, which yields quick response with reduced computer resources. The algorithms are tested and qualified with actual data collected on the streets under controlled situation. The GPS receiver is used to obtain positional information. The results show that the implemented process of attitude determination reaches the requirements for real-time operation with enough accuracy to a regular standby or emergency mode operations.

The other paper in this topic deals with microsatellites, that take a very important place in space missions, such as position location, Earth observation, atmospheric data collection, space science, and communications. The paper “*Design of microsatellite attitude control system using mixed  $H_2/H_\infty$  method via LMI optimization*,” by E. R. Pinheiro and L. C. G. Souza, presents a model for a microsatellite subjected to external disturbances taking into account the uncertainties in the moment of inertia. A control system design is performed using the robust control called mixed  $H_2/H_\infty$  method. The simulations have shown that the  $H_\infty$  controller has presented better robustness and performance than the  $H_2$  controller, with respect to uncertainty due to the inertia moment variation and due to external disturbances. However, in all simulations, the  $H_\infty$  controller signal was larger than the  $H_2$  controller, which can cause larger overshoot and can saturate the actuator. As a conclusion, the way to achieve robustness stability and good performance was to design the controller using the mixed  $H_2/H_\infty$  control, because in this procedure one can choose an adequate value for the tuning parameter, so it is possible to have robust control with low control signal.

Three papers discuss problems related to close approach maneuvers. One of them is “*The study of the asymmetric multiple encounters problem and its application to obtain Jupiter gravity assisted maneuvers*,” by D. P. S. dos Santos et al., that presents a study of interplanetary mission trajectory. The main goal of this paper is to find trajectories which reduce the fuel consumption for interplanetary missions. They analyzed the transfer of a spacecraft which leaves and returns to the same body (multiple encounters problem). This spacecraft can also, during its trajectory, perform a swing-by. The system is formed by the spacecraft and two primary bodies. The numerical results show that the velocity and the energy variation of the spacecraft increase as a function of the eccentricity of the primary bodies.

The second one is the paper “*A new celestial navigation method for spacecraft on a gravity assist trajectory*,” by N. Xiaolin et al., that proposes a solution for the problem of autonomous celestial navigation for a spacecraft performing a gravity assist maneuver. The method combines the direct calculation method and the filter method, using an interacting multiple model unscented Kalman filter (IMMUKF). Experimental results show that this method can give a better navigation performance and a higher reliability when compared to the traditional direct calculation and the filter methods. This method helps to deal with a problem that exists in the autonomous celestial navigation of spacecraft during a gravity assist, which is the low accuracy of the trajectory model that may causes a large estimation error and, sometimes, filter divergence.

The last one of this topic is the paper “*Studying close approaches for a cloud of particles considering atmospheric drag*,” by V. M. Gomes et al., that studies close approach trajectories between a planet and a group of particles. The model includes the atmosphere of the planet and the cloud is created by the passage of the spacecraft by that atmosphere. The equations of motion are given by the circular planar restricted three-body problem and the forces given by the atmospheric drag. The initial conditions are given at the periapsis, which is the point where the explosion occurs. The results show the effects of the inclusion of the atmosphere in the motion of the satellite and the cloud of particles.

The final eight papers are related to specific applications, including inertial measurement. One of this type is the paper “*Discrete model reference adaptive control for gimbal servo-system of control moment gyro with harmonic drive*,” by B. Han et al., that studies the problem of controlling the gimbal angle of the double gimbal control moment gyro (DGCMG). The authors propose the use of a harmonic drive instead of a traditional motor. Then, a model reference adaptive control (MRAC) based on Popov superstability theory is designed, including some numerical experiments to validate the proposal. The authors provide some Matlab simulations showing the effectiveness of their control algorithm.

Then, comes the paper “*Circular orbit target capture using space tether-net system*,” by G. Zhai et al., that considers a space tether-net system to capture and remove space debris. In this paper the authors propose a new flexible TNS (tether net system) which presents several and important differences when compared to the typical space tethers available in

the literature. Therefore, through Lagrangian mechanics, the authors investigate the dynamics of the system for free and nonfree deployment. In some cases, depending on the deployment rate, an in-plan libration is possible and it can be described analytically. The motion for postdeployment is also discussed.

The next one is the paper “*Fault detection and isolation in inertial measurement units based on  $X^2$ -CUSUM and wavelet packet*,” by E. J. Oliveira et al., that presents a fault detection algorithm based on signal processing techniques developed for an inertial measurement unit with minimal redundancy of fiber optic gyros. In this paper, the recursive median filter is applied in order to remove impulses arising from data acquisition process and parity vector operations, improving the fault detection and isolation performance. The algorithm is divided into two blocks: *fault detection* and *fault isolation*. The results show the effectiveness of the proposed method, in which the *fault detection algorithm* is able to indicate low level step bias fault with short days and a high index of correct decision of the *fault isolation algorithm* also with low level step bias fault.

Still in this topic, the paper “*Estimating friction parameters in reaction wheels for attitude control*,” by V. Carrara and H. K. Kuga, considers the accuracy and reliability of pointing systems. Attitude control systems depend on sensors and actuators that satisfy these requirements but are still expensive. The use of reaction wheels for attitude control is usual. The present research presents a characterization of a typical reaction wheel of 0.65 Nms maximum angular momentum storage. Then, they used a friction model that takes into account the Coulomb friction, viscous friction and static friction, according to the Stribeck formulation. The parameters were estimated using a nonlinear batch least squares procedure, based on data raised experimentally. The results show wide agreement with the experimental data, and were also close to a deterministic model, previously obtained for this wheel.

Next, the paper “*Trajectory estimation of aircraft in a double-satellite passive positioning system with the adjoint method*,” by A. Cao et al., constructs a double-satellite passive positioning system using the theory of space geometry. The existence of two observation coordinate and one fundamental coordinate systems is assumed. There is a ray from each of the observation satellites to the aircraft. The adjoint method is used to build a dynamic model to estimate the trajectory of the aircraft during the process of rocket propulsion. The results show that, even for a small number of observations, the trajectory of the aircraft can be estimated.

The paper “*Averaging methods for design of spacecraft hysteresis damper*,” by R. Gama et al., studies the problem of using averaging methods in the problem of the dynamics of the attitude stabilization of a spacecraft. The use of passive gravity-gradient attitude stabilization based on hysteresis rods is analyzed by using discontinuous differential equations. Recently developed averaging techniques for discontinuous system are applied in order to simplify its analysis and to perform the optimization of parameters. The results are compared with numerical optimization.



In the last paper of this topic, it is considered that many methods have been proposed to provide algorithms to transform Cartesian coordinates into Geodetic coordinates. Calculations for artificial satellites in low Earth orbit (LEO) to Geosynchronous Earth orbit (GEO) involve this kind of transformation, Cartesian-to-geodetic coordinates. J. D. Turner and T. Elgohary, in the paper “*A simple perturbation algorithm for inverting the Cartesian to Geodetic transformation*”, present a noniterative series-based solution algorithm which provides a closed-form accurate solution for the Cartesian-to-Geodetic transformation which can be used in a range of LEO-to-GEO applications. A numerical example, used to compare the accuracy of this algorithm at each order of expansion, showed that this algorithm is efficient.

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