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Letter to the Editor

Comment on "An Approximation to Solution of Space and Time Fractional Telegraph Equations by He's Variational Iteration Method"

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The variational iteration method was applied to the time fractional telegraph equation and some variational iteration formulae were suggested in (Sevimlican, 2010). Those formulae are improved by Laplace transform from which the approximate solutions of higher accuracies can be obtained.

Sevimlican [1] considered the application of the variational iteration method [2, 3] to find approximate solutions of space and time fractional telegraph equations. The author suggested the following variational iteration formula for (5.1) in [1]

$$u_{n+1} = u_n + \int_0^x \lambda(s,x)\left(\frac{\partial^\alpha u(s,t)}{\partial s^\alpha} - \frac{\partial^2 u(s,t)}{\partial t^2} - \frac{\partial u(s,t)}{\partial t} - u(s,t)\right) ds, \quad 1<\alpha<2, \tag{1}$$

$$\lambda(s,x) = s - x.$$

However, in this comment, it is pointed out that the identification of the Lagrange multiplier $\lambda(s,x) = s - x$ from (4.1) to (4.9) can be improved.

According to the technique of determination of the Lagrange multipliers [4, 5], firstly, construct a correctional functional as

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^x \lambda(s,x)\left(\frac{\partial^\alpha u_n(s,t)}{\partial s^\alpha} - \frac{\partial^2 u_n(s,t)}{\partial t^2} - \frac{\partial u_n(s,t)}{\partial t} - u_n(s,t)\right) ds, \quad \alpha > 0. \tag{2}$$

Assuming the Lagrange multiplier $\lambda(s,x) = \lambda(x-s)$, take the Laplace transform to both sides of (2)

$$\overline{u}_{n+1}(S,t) = \overline{u}_n(S,t) + L\left[\int_0^x \lambda(s,x)\left(\frac{\partial^\alpha u_n(s,t)}{\partial s^\alpha} - \frac{\partial^2 u_n(s,t)}{\partial t^2} - \frac{\partial u_n(s,t)}{\partial t} - u_n(s,t)\right) ds\right], \tag{3}$$

where $\overline{u}_n(S,t)$ is the Laplace transform of $u_n(x,t)$.

Taking the variation δ with respect to $\overline{u}_n(S,t)$, one can obtain

$$\begin{aligned}\delta\overline{u}_{n+1}(S,t) &= \delta\overline{u}_n(S,t) + \delta L\left[\int_0^x \lambda(s,x)\left(\frac{\partial^\alpha u_n(s,t)}{\partial s^\alpha} - \frac{\partial^2 u_n(s,t)}{\partial t^2} - \frac{\partial u_n(s,t)}{\partial t} - u_n(s,t)\right) ds\right] \\ &= \left(1 + \overline{\lambda}(S) S^\alpha\right)\delta\overline{u}_n(S,t), \\ \overline{\lambda}(S) &= -\frac{1}{S^\alpha}.\end{aligned} \tag{4}$$

Then, the Lagrange multiplier can be determined as

$$\lambda(s,x) = \frac{(-1)^{\alpha}(s-x)^{\alpha-1}}{\Gamma(\alpha)}. \tag{5}$$

Instead $\lambda(s,x) = s - x$ (see (4.9) in [1]).

As a result, the variational iteration formula is obtained as

$$u_{n+1} = u_n + \int_0^x \frac{(-1)^{\alpha}(s-x)^{\alpha-1}}{\Gamma(\alpha)} \left(\frac{\partial^{\alpha} u(s,t)}{\partial s^{\alpha}} - \frac{\partial^2 u(s,t)}{\partial t^2} - \frac{\partial u(s,t)}{\partial t} - u(s,t) \right) ds, \quad 0 < \alpha. \tag{6}$$

The variational iteration formulae (5.10) and (5.17) are not right which also should be corrected, respectively.

Equation (5.10) in [1] should be

$$u_{n+1} = u_n + \int_0^x \frac{(s-x)^{2\alpha-1}}{\Gamma(2\alpha)} \left(\frac{\partial^{2\alpha} u(s,t)}{\partial s^{2\alpha}} - \frac{\partial^2 u(s,t)}{\partial t^2} - \frac{\partial u(s,t)}{\partial t} - u(s,t) + s^2 + t - 1 \right) ds, \quad 0 < \alpha. \tag{7}$$

Equation (5.17) in [1] should be

$$u_{n+1} = u_n + \int_0^t \frac{(s-t)^{2\alpha-1}}{\Gamma(2\alpha)} \left(\frac{\partial^{2\alpha} u(x,s)}{\partial s^{2\alpha}} + \lambda \frac{\partial^2 u(x,s)}{\partial s^2} - \upsilon \frac{\partial u(x,s)}{\partial s} \right) ds, \quad 0 < \alpha. \tag{8}$$

Conclusions

As is well known, the VIM became an efficient analytical tool in nonlinear science since it was proposed and the method was often used in fractional differential equations. This study illustrates the method in fractional calculus can be improved by the Laplace tranform method with which the Lagrange mutipliers can be identified explicitly.

Recently, there are also other new applications of the variation iteration method to various nonlinear problems, that is, fuzzy equations [6, 7] and q-fractional differential equations [8]. Readers are referred to the recent review article [9].

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