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Research Article

H_∞ Filter Design for Large-Scale Systems with Missing Measurements

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This paper is concerned with H_∞ filter design problem for large-scale systems with missing measurements. The occurrence of missing measurements is assumed to be a Bernoulli distributed sequence with known probability. The new full-dimensional filter is designed to make the filter error system exponentially mean-square stable and achieve a prescribed H_∞ performance. Sufficient conditions are derived in terms of linear matrix inequality (LMI) for the existence of the filter, and the parameters of filter are obtained by solving the LMI. Finally, the numerical simulation results illustrate the effectiveness of the proposed scheme.

1. Introduction

In many practical applications, due to the limitations imposed by the network, missing measurements often occur due to the network link transmission errors, network congestion, and so forth. Currently, the research of filter and controller design for systems with missing measurements has attracted more attention [1–7]. In [1], the robust control problem with missing measurements was investigated, where the missing measurements were described by a binary switch sequence satisfied conditional probability distribution. The similar model was employed in [2–4], where the filtering problem was investigated in [2, 3], and the distributed state estimation problem was studied in [4]. In [5], the quantized H_∞ control problem is investigated for a class of nonlinear stochastic time-delay network-based systems with probabilistic data missing. In [6], the filtering problem with packet loss was considered using Markov chains to describe probabilistic losses. The problem of robust H_∞ filtering for discrete-time switched systems with missing measurements under asynchronous switching is considered in [7].

Most of the existing research is focused on general linear or nonlinear discrete system. However, many actual systems are large-scale systems which are composed of interconnected subsystems. Although ideas of decentralized control of large-scale systems have attracted much attention in the literature during the past two decades, the research about large-scale systems with missing measurements is seldom. In [8], a decentralized H_∞ controller design for a class of large-scale systems with missing measurements is considered. In [9], a state feedback H_∞ controller is designed for a class of linear discrete-time large-scale system with both measurement data and control data missing simultaneously.

In this paper, H_∞ filter is considered for a class of large-scale systems with missing measurements. We apply Bernoulli distributed sequence to describe the occurrence of missing measurements, and the linear discrete-time large-scale system is modeled as interconnection of N subsystems with missing measurements. Then, we design a new decentralized filter. Sufficient conditions are derived in terms of linear matrix inequality (LMI) which is easy to be solved by using MATLAB LMI Toolbox for the decentralized stabilization of this class of large-scale system.

2. Problem Formulation

Consider the linear discrete-time large-scale system comprising N subsystems Ξ_i, $i = 1, 2, \ldots, N$; the dynamics of the ith subsystem is described by

$$\Xi_i : \begin{cases} x_i(k+1) = A_i x_i(k) + B_i w_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) \\ z_i(k) = C_i x_i(k) + D_i w_i(k) \\ y_i(k) = x_i(k), \end{cases} \tag{1}$$

where $x_i(k) \in R^{n_i}$ is the state vector of the ith subsystem at time k, $y_i(k) \in R^{q_i}$ is the measurement output, $z_i(k) \in R^{p_i}$ is the controlled output, $w_i(k) \in R^{r_i}$ is the disturbance vector belonging to $l_2[0,\infty)$, A_i, B_i, C_i, and D_i are known real constant matrices with appropriate dimensions, and $G_{ij} \in R^{n_i \times n_j}$ is the interconnection matrix of the subsystem of j and i.

The measurement with missing data can be characterized by

$$\overline{x}_i(k) = \alpha_i(k) x_i(k), \tag{2}$$

where $\overline{x}_i(k) \in R^{m_i}$ is the actual measured state, the stochastic variable $\alpha_i(k) \in R$ is a Bernoulli distributed white noise sequence taking the values of 0 and 1 with certain probability

$$\begin{aligned} \operatorname{prob}\{\alpha_i(k) = 1\} &= E\{\alpha_i(k)\} := \overline{\alpha}_i, \\ \operatorname{prob}\{\alpha_i(k) = 0\} &= 1 - E\{\alpha_i(k)\} := 1 - \overline{\alpha}_i, \end{aligned} \tag{3}$$

and $0 < \overline{\alpha}_i < 1$ is a known positive constant.

In order to observe the states of the system (1), we consider the following filter of order n described by

$$\begin{aligned} \widehat{x}_i(k+1) &= A_i \widehat{x}_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} \widehat{x}_j(k) + K_i\left(\overline{\alpha}_i \widehat{x}_i(k) - \overline{x}_i(k)\right), \\ \widehat{z}_i(k) &= C_i \widehat{x}_i(k), \\ \widehat{y}_i(k) &= \widehat{x}_i(k), \end{aligned} \tag{4}$$

where $\widehat{x}_i(k) \in R^{n_i}$ is the state estimate of system (1) and K_i is the observer gain to be determined later.

Define the state estimation error by

$$\varepsilon_i(k) = x_i(k) - \widehat{x}_i(k), \tag{5}$$

and the filter error output is denoted by

$$e_i(k) = z_i(k) - \widehat{z}_i(k). \tag{6}$$

Then it follows from (1), (2), and (4) that

$$\begin{aligned} \varepsilon_i(k+1) = {} & \left(A_i + \overline{\alpha}_i K_i\right)\varepsilon_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) \\ & + \left(\alpha_i(k) - \overline{\alpha}_i\right) K_i x_i(k) + B_i w_i(k), \end{aligned} \tag{7}$$

$$e_i(k) = z_i(k) - \widehat{z}_i(k) = C_i \varepsilon_i(k) + D_i w_i(k). \tag{8}$$

Since it contains stochastic quantities $\alpha_i(k)$, the filter error system (8) is actually a stochastic parameter system. Then we use the following definition.

Definition 1 (see [10]). The filter error system (8) is said to be exponentially mean-square asymptotically stable if with $w(k) = 0$, there exist constants $\kappa > 0$ and $0 < \tau < 1$, such that

$$E\left\{\|e(k)\|^2\right\} < \kappa\tau^k E\left\{\|e(0)\|^2\right\}, \quad \forall e(k) \neq 0, \tag{9}$$

where $e(k) = \left[e_1^T(k) \ \cdots \ e_N^T(k)\right]^T$ and $w(k) = \left[w_1^T(k) \ \cdots \ w_N^T(k)\right]^T$.

With this definition, our objective is to design the full-order filter of form (4), such that

(1) the filter error system (8) is exponentially mean-square asymptotically stable with $w(k) = 0$;

(2) under zero-initial condition, the filter error $e(k)$ satisfies

$$\sum_{k=0}^{\infty} E\left\{\|e(k)\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} E\left\{\|w(k)\|^2\right\}, \tag{10}$$

where γ is a given positive constant.

3. Main Results

For investigating the stability conditions of the filter error system (8), the following lemma is needed.

Lemma 2 (see [10]). *Let $V(\eta(k))$ be a Lyapunov functional. If there exist constants $\lambda \geq 0$, $\mu > 0$, $\nu > 0$, and $0 < \psi < 1$ such that*

$$\begin{aligned} \mu\|\eta(k)\|^2 \leq V(\eta(k)) \leq \nu\|\eta(k)\|^2, \\ E\{V(\eta(k+1) \mid \eta(k))\} - V(\eta(k)) \leq \lambda - \psi V(\eta(k)), \end{aligned} \tag{11}$$

then the sequence $\eta(k)$ satisfies

$$E\left\{\left\|\eta(k)^2\right\|\right\} \leq \frac{\nu}{\mu}\|\eta(0)\|^2(1-\psi)^k + \frac{\lambda}{\mu\psi}. \tag{12}$$

The main results are concluded into the following theorems.

Theorem 3. *Given $0 < \overline{\alpha}_i < 1$ and $w(k) = 0$, the filter error system (8) is exponentially mean-square asymptotically stable if there exist positive definite matrices $P_{1i} = P_{1i}^T$, $P_{2i} = P_{2i}^T$ and gain matrix K_i, $i = 1, 2, \ldots, N$, satisfying*

$$\begin{bmatrix} -P_1 & 0 & Q_1^T + \overline{\alpha}K & 0 & 0 \\ 0 & -P_2 & 0 & Q_1^T & K \\ Q_1 + \overline{\alpha}K & 0 & -P_1^{-1} & 0 & 0 \\ 0 & Q_1 & 0 & -P_2^{-1} & 0 \\ 0 & K & 0 & 0 & -\beta^2 P_1^{-1} \end{bmatrix} < 0, \tag{13}$$

where

$$Q_1 = \begin{bmatrix} A_1 & G_{12} & \cdots & G_{1N} \\ G_{21} & A_2 & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & A_N \end{bmatrix},$$

$$P_1 = \operatorname{diag}\{P_{11}, P_{12}, \ldots, P_{1N}\},$$

$$P_2 = \operatorname{diag}\{P_{21}, P_{22}, \ldots, P_{2N}\},$$

$$K = \operatorname{diag}\{K_1, K_2, \ldots, K_N\}, \qquad \beta = \operatorname{diag}\{\beta_1, \beta_2, \ldots, \beta_N\},$$

$$\beta_i = \left((1-\overline{\alpha}_i)\overline{\alpha}_i\right)^{1/2}, \quad i = 1, 2, \ldots, N,$$

$$\overline{\alpha} = \operatorname{diag}\{\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_N\}. \tag{14}$$

Proof. When $w(k) = 0$, define the Lyapunov functional

$$V(k) = \sum_{i=1}^{N} \varepsilon_i^T(k) P_{1i} \varepsilon_i(k) + \sum_{i=1}^{N} x_i^T(k) P_{2i} x_i(k), \tag{15}$$

where $P_{1i} = P_{1i}^T$, $P_{2i} = P_{2i}^T$ are positive definite matrices. It follows from (8) that

$$\begin{aligned} &V(k+1) - V(k) \\ &= \sum_{i=1}^{N} \Bigg(\left(A_i + \overline{\alpha}_i K_i\right) \varepsilon_i(k) \\ &\qquad + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) + \left(\alpha_i(k) - \overline{\alpha}_i\right) K_i x_i(k) \Bigg)^T P_{1i} \\ &\quad \times \Bigg(\left(A_i + \overline{\alpha}_i K_i\right) \varepsilon_i(k) \\ &\qquad + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) + \left(\alpha_i(k) - \overline{\alpha}_i\right) K_i x_i(k) \Bigg) \\ &\quad + \sum_{i=1}^{N} \left(A_i x_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) \right)^T P_{2i} \\ &\quad \times \left(A_i x_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) \right) \\ &\quad - \sum_{i=1}^{N} \varepsilon_i^T(k) P_{1i} \varepsilon_i(k) - \sum_{i=1}^{N} x_i^T(k) P_{2i} x_i(k). \end{aligned} \tag{16}$$

Noting that $E\{\alpha_i(k) - \overline{\alpha}_i\} = 0$, $E\{(\alpha_i(k) - \overline{\alpha}_i)^2\} = (1-\overline{\alpha}_i)\overline{\alpha}_i \triangleq \beta_i^2$, we have

$$\begin{aligned} &E\{V(k+1)\} - V(k) \\ &= \sum_{i=1}^{N} \left(\left(A_i + \overline{\alpha}_i K_i\right) \varepsilon_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) \right)^T P_{1i} \\ &\quad \times \left(\left(A_i + \overline{\alpha}_i K_i\right) \varepsilon_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) \right) \\ &\quad + \sum_{i=1}^{N} \left(A_i x_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) \right)^T P_{2i} \\ &\quad \times \left(A_i x_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) \right) \\ &\quad + \sum_{i=1}^{N} \beta_i^2 \left(K_i x_i(k)\right)^T P_{1i} \left(K_i x_i(k)\right) \\ &\quad - \sum_{i=1}^{N} \varepsilon_i^T(k) P_{1i} \varepsilon_i(k) - \sum_{i=1}^{N} x_i^T(k) P_{2i} x_i(k) \\ &= \eta^T(k) \theta_1 \eta(k), \end{aligned} \tag{17}$$

where $x(k) = \left[x_1^T(k) \ \cdots \ x_N^T(k)\right]^T$, $\varepsilon(k) = \left[\varepsilon_1^T(k) \ \cdots \ \varepsilon_N^T(k)\right]^T$, and $\eta(k) = [\varepsilon^T(k), x^T(k)]^T$.

By Schur complement, inequality (13) implies $\theta_1 < 0$. Then we have

$$\begin{aligned} E\{V(k+1)\} - V(k) &= \eta^T(k) \theta_1 \eta(k) \\ &\leq -\lambda_{\min}(-\theta_1) \eta^T(k) \eta(k) \\ &< -\gamma \eta^T(k) \eta(k), \end{aligned} \tag{18}$$

where $0 < \gamma < \min\{\lambda_{\min}(-\theta_1), \sigma\}$, and then from (18), we get

$$\begin{aligned} E\{V(k+1)\} - V(k) &< -\gamma \eta^T(k) \eta(k) \\ &< -\frac{\gamma}{\sigma} V(\eta(k)) = -\psi V(\eta(k)), \end{aligned} \tag{19}$$

where $\psi = \gamma/\sigma \in (0, 1)$.

From Definition 1 and Lemma 2, we can conclude that the filter error system (8) is exponentially mean-square asymptotically stable. This completes the proof. □

In the sequel, we further provide method for solving matrix inequality (13) that is not a linear matrix inequality (LMI).

Theorem 4. *Given $0 < \overline{\alpha}_i < 1$ if there exist positive definite matrices $M_1 = M_1^T$, $M_2 = M_2^T$ and gain matrices N_1, N_2 that satisfy linear matrix inequality*

$$\left[\begin{array}{cc|ccc} -M_1 & 0 & M_1Q_1^T + \overline{\alpha}N_1 & 0 & 0 \\ 0 & -M_2 & 0 & M_2Q_1^T & \beta N_2 \\ \hline Q_1M_1 + \overline{\alpha}N_1 & 0 & -M_1 & 0 & 0 \\ 0 & Q_1M_2 & 0 & -M_2 & 0 \\ 0 & \beta N_2 & 0 & 0 & -M_1 \end{array}\right] < 0 \quad (20)$$

and equation

$$N_1P_1 = N_2P_2 = K, \quad (21)$$

where $M_1 = P_1^{-1}$, $M_2 = P_2^{-1}$, $N_1 = KP_1^{-1}$, and $N_2 = KP_2^{-1}$, then the error system (8) is exponentially mean-square asymptotically stable.

Proof. Through left- and right-multiplying (13) by $\operatorname{diag}\{P_1^{-1}, P_2^{-1}, I, I, \beta I\}$, we have

$$\left[\begin{array}{cc|ccc} -P_1^{-1} & 0 & P_1^{-1}(Q_1 + \overline{\alpha}K)^T & 0 & 0 \\ 0 & -P_2^{-1} & 0 & P_2^{-1}Q_1^T & \beta P_2^{-1}K \\ \hline (Q_1 + \overline{\alpha}K)P_1^{-1} & 0 & -P_1^{-1} & 0 & 0 \\ 0 & Q_1P_2^{-1} & 0 & -P_2^{-1} & 0 \\ 0 & \beta KP_2^{-1} & 0 & 0 & -P_1^{-1} \end{array}\right] < 0. \quad (22)$$

For the definitions of the matrices M_1, M_2, N_1, and N_2, the matrix inequality (13) is equivalent to (20) and (21). This completes the proof. □

By solving the linear matrix inequality (20) and (21), we have M_1, M_2 and N_1, N_2. Moreover, the matrices are given by $P_1 = M_1^{-1}$, $P_2 = M_2^{-1}$, and $K = N_1P_1 = N_2P_2$.

Theorem 5. *Given $0 < \overline{\alpha}_i < 1$ if there exist positive definite matrices $P_{1i} = P_{1i}^T$, $P_{2i} = P_{2i}^T$ and gain matrix $K_i, i = 1, 2, \ldots, N$, that satisfy the following linear matrix inequality:*

$$\begin{bmatrix} -P_1 & * & * & * & * & * & * \\ 0 & -P_2 & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * \\ Q_1 + \overline{\alpha}K & 0 & B & -P_1^{-1} & * & * & * \\ 0 & Q_1 & B & 0 & -P_2^{-1} & * & * \\ 0 & K & 0 & 0 & 0 & -\beta^2 P_1^{-1} & * \\ C & 0 & D & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (23)$$

where $C = \operatorname{diag}\{C_1, C_2, \ldots, C_N\}$, $D = \operatorname{diag}\{D_1, D_2, \ldots, D_N\}$, $B = \operatorname{diag}\{B_1, B_2, \ldots, B_N\}$, and Q_1, P_1, P_2, K, $\overline{\alpha}$, β_i, and β are the same as (13), then the filter error system (8) is exponentially mean-square asymptotically stable and achieves the prescribed H_∞ performance.

Proof. When $w(k) = 0$, (23) is equivalent to (13), so the filter error system is exponentially mean-square asymptotically stable.

When $w(k) \neq 0$, define the Lyapunov functional as

$$V(k) = \sum_{i=1}^{N} \varepsilon_i^T(k) P_{1i} \varepsilon_i(k) + \sum_{i=1}^{N} x_i^T(k) P_{2i} x_i(k), \quad (24)$$

then

$$\begin{aligned} &E\{V(k+1)\} - V(k) \\ &\quad + E\left\{e^T(k)e(k)\right\} - \gamma^2 E\left\{w^T(k)w(k)\right\} \\ &\quad = \sum_{i=1}^{N}\left(\left(A_i + \overline{\alpha}_i K_i\right)\varepsilon_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) + B_i w_i(k)\right)^T \\ &\qquad \times P_{1i} \\ &\qquad \times \left(\left(A_i + \overline{\alpha}_i K_i\right)\varepsilon_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}\varepsilon_j(k) + B_i w_i(k)\right) \\ &\qquad + \sum_{i=1}^{N}\left(A_i x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij} x_j(k) + B_i w_i(k)\right)^T P_{2i} \\ &\qquad \times \left(A_i x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij} x_j(k) + B_i w_i(k)\right) \\ &\qquad + \beta_i^2 \sum_{i=1}^{N}\left(K_i x_i(k)\right)^T P_{1i}\left(K_i x_i(k)\right) \\ &\qquad + E\left\{\sum_{i=1}^{N}\left(C_i\varepsilon_i(k) + D_i w_i(k)\right)^T \right. \\ &\qquad\qquad \left. \times \left(C_i\varepsilon_i(k) + D_i w_i(k)\right)\right\} \\ &\qquad - \sum_{i=1}^{N}\varepsilon_i^T(k) P_{1i}\varepsilon_i(k) \\ &\qquad - \sum_{i=1}^{N} x_i^T(k) P_{2i} x_i(k) - \gamma^2 E\left\{w^T(k)w(k)\right\} \\ &\quad = \overline{\eta}^T(k)\theta_2\overline{\eta}(k), \end{aligned} \quad (25)$$

where $\overline{\eta}(k) = [\varepsilon^T(k), x^T(k), w^T(k)]^T$. By the Schur complement, inequality (23) implies $\theta_2 < 0$, and then we have

$$\begin{aligned} &E\{V(k+1)\} - V(k) + E\left\{e^T(k)e(k)\right\} \\ &\qquad - \gamma^2 E\left\{w^T(k)w(k)\right\} < 0, \\ &\sum_{k=0}^{\infty} E\left\{e^T(k)e(k)\right\} < \gamma^2 \sum_{k=0}^{\infty} E\left\{w^T(k)w(k)\right\} \\ &\qquad + E\{V(0)\} - E\{V(\infty)\}. \end{aligned} \quad (26)$$

Since the system is exponentially mean-square asymptotically stable, it is straightforward to see that

$$\sum_{k=0}^{\infty} E\left\{\|e(k)\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} E\left\{\|w(k)\|^2\right\}, \tag{27}$$

under the zero-initial condition. This completes the proof. □

In the sequel, we further present how to convert the matrix inequality (23) into an LMI with matrix equality constraint.

Theorem 6. *Given $0 < \overline{\alpha}_i < 1$ if there exist positive definite matrices $M_1^T = M_1$, $M_2^T = M_2$ and gain matrices N_1, N_2 that satisfy the following linear matrix inequality*

$$\begin{bmatrix} -M_1 & * & * & * & * & * & * \\ 0 & -M_2 & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * \\ Q_1 M_1 + \overline{\alpha} N_1 & 0 & B & -M_1 & * & * & * \\ 0 & Q_1 M_2 & B & 0 & -M_2 & * & * \\ 0 & \beta N_2 & 0 & 0 & 0 & -M_1 & * \\ C M_1 & 0 & D & 0 & 0 & 0 & -I \end{bmatrix} < 0,$$
$$K = N_1 P_1 = N_2 P_2, \tag{28}$$

then the filter error system (8) *is exponentially mean-square asymptotically stable and achieves the prescribed H_∞ performance.*

Proof. Through left- and right-multiplying (23) by $\operatorname{diag}\{P_1^{-1}, P_2^{-1}, I, I, I, \beta I, I\} > 0$, we have

$$\begin{bmatrix} -P_1^{-1} & * & * & * & * & * & * \\ 0 & -P_2^{-1} & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * \\ (Q_1 + \alpha K) P_1^{-1} & 0 & B & -P_1^{-1} & * & * & * \\ 0 & Q_1 P_2^{-1} & B & 0 & -P_2^{-1} & * & * \\ 0 & \beta K P_2^{-1} & 0 & 0 & 0 & -P_1^{-1} & * \\ C P_1^{-1} & 0 & D & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{29}$$

Similar to the proof of Theorem 4, we define $M_1 = P_1^{-1}$, $M_2 = P_2^{-1}$, $N_1 = KP_1^{-1}$, and $N_2 = KP_2^{-1}$. Then the matrix inequality (29) is equivalent to (23). From Theorem 5, we can conclude that the filter error system (8) is exponentially mean-square asymptotically stable and achieves the prescribed H_∞ performance. The proof is completed. □

4. Numerical Simulations

Consider a linear discrete-time large-scale system which consists of two interconnected subsystems:

$$\begin{aligned} x_1(k+1) &= \begin{bmatrix} -1 & 3 \\ 0 & -0.1 \end{bmatrix} x_1(k) + \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \end{bmatrix} x_2(k) \\ &\quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w_1(k) \\ &= A_1 x_1(k) + G_{12} x_2(k) + E_1 w_1(k), \\ z_1 &= \begin{bmatrix} 1 & 0 \\ -1 & 0.8 \end{bmatrix} x_1(k) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} w_1(k) \\ &= C_1 x_1(k) + D_1(k) w_1(k), \\ y_1(k) &= x_1(k), \\ x_2(k+1) &= \begin{bmatrix} -1 & 1 \\ 0 & -0.2 \end{bmatrix} x_2(k) + \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.2 \end{bmatrix} x_1(k) \\ &\quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w_2(k) \\ &= A_2 x_2(k) + G_{21} x_1(k) + E_2 w_2(k), \\ z_2 &= \begin{bmatrix} 1 & 0 \\ 1 & -0.8 \end{bmatrix} x_2(k) + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} w_2(k) \\ &= C_2 x_2(k) + D_2(k) w_2(k), \\ y_2(k) &= x_2(k). \end{aligned} \tag{30}$$

Choose the disturbance input $w_1(k) = w_2(k) = 0.01 \left[\begin{smallmatrix} \sin(100k) \\ \sin(100k) \end{smallmatrix}\right]$. The initial state values of original system and its observer are $x_1(0) = \left[\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right]$, $x_2(0) = \left[\begin{smallmatrix} 2 \\ 2 \end{smallmatrix}\right]$ and $\hat{x}_1(0) = \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$, $x_2(0) = \left[\begin{smallmatrix} -1 \\ 0 \end{smallmatrix}\right]$, respectively. Suppose that stochastic sequence obeys Bernoulli distribution with probability $E\{\alpha_1(k) \mid \alpha_1(k) = 1\} = E\{\alpha_2(k) \mid \alpha_2(k) = 1\} = 0.996$, given $\gamma = 1$. For simplicity, let $M = P_1^{-1} = P_2^{-1}$ and $N = KP_1^{-1} = KP_2^{-1}$. We can obtain the following parameters in Theorem 6 by using the MATLAB YALMIP Toolbox:

$$M_1 = \begin{bmatrix} 1.3549 & 0.4014 \\ 0.4014 & 0.3819 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 1.1058 & 0.7881 \\ 0.7881 & 1.6449 \end{bmatrix},$$
$$N_1 = \begin{bmatrix} 0.1755 & -0.3232 \\ -0.3232 & 0.0041 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} 0.2464 & -0.3847 \\ -0.3847 & 0.3580 \end{bmatrix}. \tag{31}$$

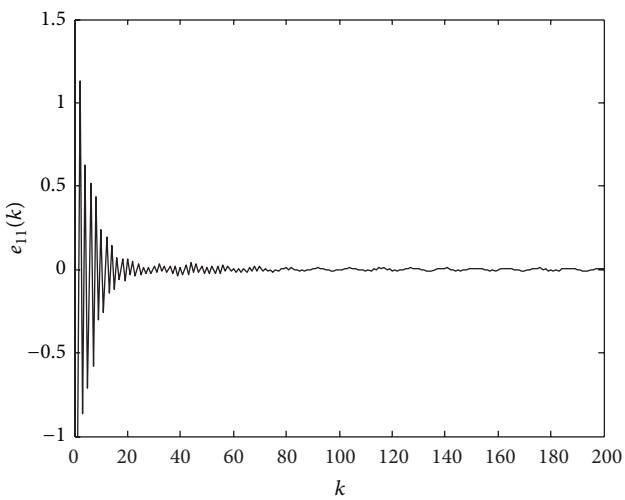

Figure 1: Filter error $e_{11}(k)$.

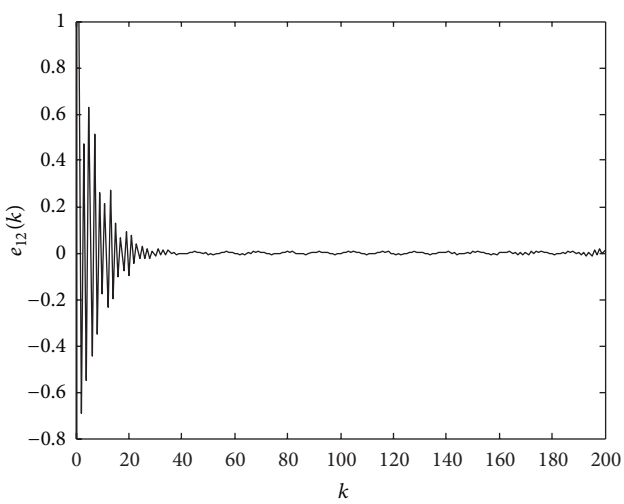

Figure 2: Filter error $e_{12}(k)$.

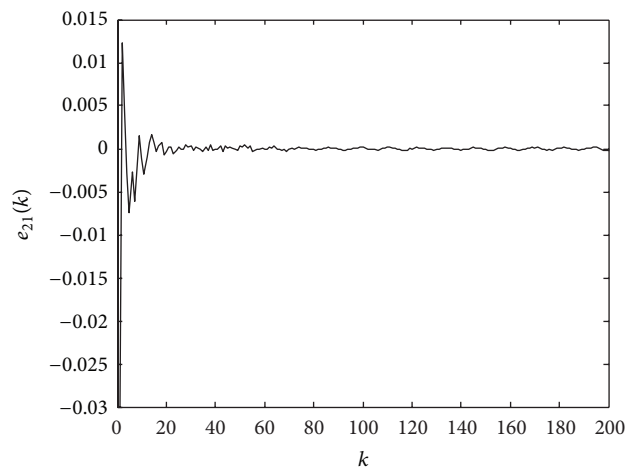

Figure 3: Filter error $e_{21}(k)$.

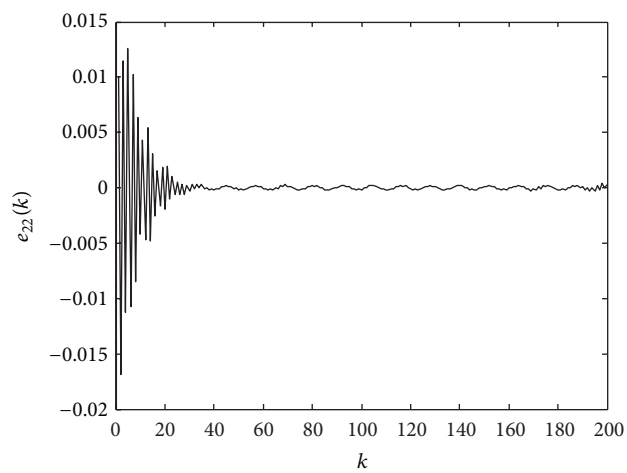

Figure 4: Filter error $e_{22}(k)$.

The Lyapunov function solution matrices and observer parameter are given by

$$\begin{aligned} P_{11} = P_{21} = M_1^{-1} &= \begin{bmatrix} 1.0718 & -1.1264 \\ -1.1264 & 3.8021 \end{bmatrix}, \\ P_{12} = P_{22} = M_2^{-1} &= \begin{bmatrix} 1.3732 & -0.6579 \\ -0.6579 & 0.9231 \end{bmatrix}, \\ K_1 = N_1 P_1 &= \begin{bmatrix} 0.5521 & -1.4264 \\ -0.3510 & 0.3789 \end{bmatrix}, \\ K_2 = N_2 P_2 &= \begin{bmatrix} 0.5919 & -0.5172 \\ -0.7637 & 0.5835 \end{bmatrix}. \end{aligned} \tag{32}$$

The simulation results are shown in Figures 1, 2, 3, and 4.

It can be verified that $\sum_{k=0}^{\infty} E\{\|e(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty}\{\|w(k)\|^2\}$ and the filter error system satisfies the prescribed H_∞ performance.

5. Conclusion

In this paper, the H_∞ filter for a class of linear discrete-time large-scale system has been designed, where the measurements are probably missing. The missing probability is assumed to obey Bernoulli distribution. By employing the Lyapunov stability theory combined with stochastic analysis method, a filter is designed to reconstruct the states of original system such that the filter error system is exponentially stable in the sense of mean square and achieves the prescribed H_∞ performance.

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