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Research Article

Optimal Ordering Policy of a Risk-Averse Retailer Subject to Inventory Inaccuracy

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Inventory inaccuracy refers to the discrepancy between the actual inventory and the recorded inventory information. Inventory inaccuracy is prevalent in retail stores. It may result in a higher inventory level or poor customer service. Earlier studies of inventory inaccuracy have traditionally assumed risk-neutral retailers whose objective is to maximize expected profits. We investigate a risk-averse retailer within a newsvendor framework. The risk aversion attitude is measured by conditional-value-at-risk (CVaR). We consider inventory inaccuracy stemming both from permanent shrinkage and temporary shrinkage. Two scenarios of reducing inventory shrinkage are presented. In the first scenario, the retailer conducts physical inventory audits to identify the discrepancy. In the second scenario, the retailer deploys an automatic tracking technology, radiofrequency identification (RFID), to reduce inventory shrinkage. With the CVaR criterion, we propose optimal policies for the two scenarios. We show monotonicity between the retailer's ordering policy and his risk aversion degree. A numerical analysis provides managerial insights for risk-averse retailers considering investing in RFID technology.

1. Introduction

Inventory inaccuracy occurs when the inventory shown in the information system does not match that of the physically available inventory [1]. Inventory inaccuracy is prevalent in retail stores. For example, DeHoratius and Raman [2] found that up to 65% of inventory records at a leading retailer were inaccurate. This issue is mainly due to four sources: (i) permanent shrinkage, including theft, spoilage, and damage of products; (ii) temporary shrinkage, when a fraction of products are misplaced; (iii) transaction errors at the checkout counter; (iv) random yield of an unreliable supplier [1]. Due to these factors, the inventory system fails to order stock on time or carries more stock than necessary [3]. DeHoratius and Raman [2] reported that inaccuracies in the inventory system could reduce retailers' profit by 10% due to lost sales and extra inventory holding costs.

Some studies have quantified the impact of inventory inaccuracy on retailers' ordering policies (e.g., [4–6]). Most of these articles postulated risk-neutral decision makers whose objective was to maximize expected profits, and they ignored the variance of the profit. However, inaccurate inventory in the information system increases the possibility of lower profits. Some recent empirical studies [7, 8] found that decision makers usually show a risk-averse attitude in reality. Moreover, risk-averse managers are sensitive to lower profits and make decisions based on conservative objectives. Motivated by these investigations, we specifically incorporate a risk aversion attitude into the analysis of inventory inaccuracy.

Our goal is to investigate the impact of a risk aversion attitude on the retailer's ordering policy subject to inventory inaccuracy. According to Atali et al. [3], inventory inaccuracy can be reduced in two ways: conducting physical inventory audits and deploying an automatic inventory tracking technology, such as radiofrequency identification (RFID). After audits, the retailer can reconcile the inventory record with the physically available inventory, as well as return misplaced

items to their original correct locations. The emergence of RFID technology offers another possible solution to the inventory inaccuracy problem. Dai and Tseng [9] pointed out that RFID helps to reduce inventory shrinkage in two ways: visibility and prevention. With respect to visibility, RFID enables virtually full inventory transparency at the stock-keeping-unit level and transmits accurate inventory information in a timely manner. In relation to prevention, RFID, together with other supporting technologies, such as closed circuit television systems (to prevent theft) and smart shelves (to prevent misplacement), reduces inventory shrinkage. In a timely manner, RFID makes more products available for sale. In this paper, we explicitly model and compare the two scenarios under risk aversion.

We model risk aversion by adopting the conditional-value-at-risk (CVaR) decision criterion. CVaR, as a viable risk measure criterion, has gained much attention in the portfolio optimization literature. CVaR measures the risk of possible losses in portfolio selections. For details about CVaR, we refer the reader to Rockafellar and Uryasev [10, 11]. Studies applying CVaR to the newsvendor problem are quite limited. Chen et al. [12] adopted CVaR to address combined price and order quantity decisions of a risk-averse newsvendor. Caliskan-Demirag et al. [13] considered a supply chain rebates contract problem and modeled the retailer's risk aversion with CVaR. Cheng et al. [14] analyzed bilevel newsvendor models with a CVaR objective. Gotoh and Takano [15] studied different objectives based on the CVaR criterion and showed that the CVaR minimization problem in a multiproduct setting can be represented as a linear program. Our paper contributes to this area by analyzing the discrepancy between the actual inventory and the recorded inventory information, which is quite a realistic problem in retail stores.

Among the few studies that have dealt analytically with the inventory inaccuracy problem, Rekik et al. [4] considered a risk-neutral retailer subject to misplacement errors. They showed the impact of misplacement errors on ordering decisions and explored the benefits of obtaining information about these errors. Rekik et al. [5] analyzed a retailer's ordering policy in a multiperiod setting, where the records were inaccurate due to theft. They investigated the contribution of perfect RFID technology to improve the inventory system. Within a newsvendor framework, Fan et al. [6] also considered a retailer subject to permanent inventory shrinkage. Their work incorporated a penalty cost for stockout, and they investigated the incentive of retailers to adopt RFID technology. Our study differs from these articles in that we analyze inventory inaccuracy stemming from both permanent shrinkage and temporary shrinkage, rather than assuming a single source. Our study also focuses on investigating the effect of the retailer's risk attitude on his ordering policy.

The contribution of this paper is threefold: (i) applying CVaR to optimize retailers' inventory policies subject to inaccurate inventory, (ii) investigating the impact of different types of inventory shrinkage on retailers' decisions, and (iii) providing managerial insights for risk-averse retailers to invest in RFID technology. The rest of this paper is organized as follows. Section 2 discusses two scenarios for reducing inventory inaccuracy and presents a simple description of the CVaR criterion. Section 3 analyzes the risk-averse retailer's optimal ordering policy in Scenario 1. Section 4 investigates the retailer's optimal ordering policy in Scenario 2 and specifies the conditions under which investing in RFID is profitable. Section 5 conducts a numerical analysis to provide some insights for managers to deal with the inventory inaccuracy problem. Finally, Section 6 concludes the paper.

2. A Modeling Framework for Risk-Averse Retailers Subject to Inventory Shrinkage

2.1. Problem Description. We consider a single-product, single-period newsvendor framework, where the retailer orders a seasonal product from the supplier at unit cost c and sells the products to customers at price p. Demand D is a random variable, and the retailer's decision variable is the order quantity Q. The retailer receives the order quantity Q at the beginning of the season. Due to inventory shrinkage, the quantity available for sale is less than Q. The whole unsold quantity (items on shelves and misplaced items) is salvaged at price s at the end of the period. Within this framework, we present two scenarios for managing the inventory inaccuracy problem: conducting physical inventory audits and deploying RFID technology.

The notations used throughout this paper are summarized as follows:

p: Selling price per item

c: Purchasing cost per item

s: Salvage price per item

t: RFID tag cost per item

D: Random demand

$f(D)$: Probability density function (PDF) of D

$F(D)$: Cumulative distribution function (CDF) of D

α: Ratio of permanent inventory shrinkage

β: Ratio of temporary inventory shrinkage

γ_i: Ratio between the available quantity for sale and the total physical quantity in Scenario i $(i = 1, 2)$

b: Improvement rate in permanent shrinkage following application of RFID

d: Improvement rate in temporary shrinkage following application of RFID

η: Risk factor that reflects the degree of risk aversion

π_i^{η}: The retailer's performance in Scenario i $(i = 1, 2)$

Q_i: Order quantity in Scenario i $(i = 1, 2)$ (decision variable).

In this study, we make the following assumptions.

Assumption 1. We assume instantaneous shrinkage at the beginning of the period for analytical tractability. The items that are stolen or damaged are no longer available to customers, whereas the misplaced items can be found at the end of the period. We set $s < c < p$ to avoid trivial cases.

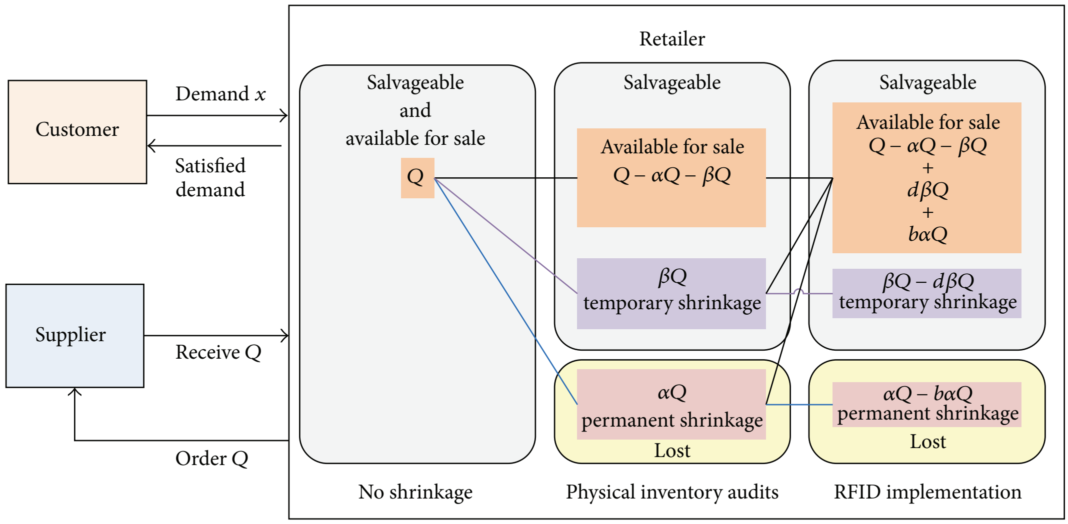

FIGURE 1: Scenario 1 versus Scenario 2.

Assumption 2. We assume imperfect RFID technology in the retail store. In other words, RFID implementation can only reduce a certain fraction of inventory shrinkage. Many analytical studies assumed that RFID is perfect for the sake of simplicity. However, empirical studies have shown that the read rate of pallet-level RFID tags is only 74–79%, and shrinkage can only be reduced by 47% at the retailer level with RFID deployment [16]. Therefore, we assume imperfect RFID.

As illustrated in Figure 1, we describe the two scenarios as follows.

Scenario 1. The retailer estimates inventory shrinkage by physical inventory audits.

By audits, the retailer uncovers factors that cause inventory inaccuracy and estimates the value of α and β based on historical data. Then, the retailer makes his ordering policy by taking inventory shrinkage into account.

Scenario 2. RFID is implemented to reduce inventory shrinkage.

With the deployment of RFID, part of the inventory shrinkage is eliminated, and this inventory becomes available to meet customers' demands. Specifically, $d\beta Q$ items are prevented from misplacement and available for sale, and $b\alpha Q$ items are prevented from shrinkage and converted into a demand for purchase.

2.2. The Risk Measure Criterion: CVaR. CVaR measures the risk of possible losses in investment decisions. Let $L(x, y)$ be a loss function, which depends on a decision variable, x, and a random variable, y. For a specified confidence level λ, the value-at-risk, which is denoted by $\mathrm{VaR}_\lambda(x)$, is defined as

$$\mathrm{VaR}_\lambda(x) = \inf\{\omega \mid P[L(x, y) \le \omega] \ge \lambda\}, \quad (1)$$

where $P[A]$ defines the probability of Event A.

VaR_λ gives the lowest amount ω such that, with $100\lambda\%$ confidence, the loss will not exceed the given threshold ω. $\mathrm{CVaR}_\lambda(x)$ is defined as the expected value of loss exceeding VaR_λ, which is

$$\begin{aligned}\mathrm{CVaR}_\lambda(x) &= E[L(x, y) \mid L(x, y) \ge \mathrm{VaR}_\lambda(x)] \\ &= \frac{1}{1-\lambda}\int_{L(x,y)\ge \mathrm{VaR}_\lambda(x)} L(x, y)\, g(y)\, dy,\end{aligned} \quad (2)$$

where $g(y)$ is the density function of y.

A method of calculating $\mathrm{CVaR}_\lambda(x)$ was introduced by Rockafellar and Uryasev [11]. They proved that $\mathrm{CVaR}_\lambda(x)$ can be expressed by the following minimization formula:

$$\mathrm{CVaR}_\lambda(x) = \min_\omega G_\lambda(x, \omega), \quad (3)$$

with

$$\begin{gathered} G_\lambda(x, \omega) = \omega + \frac{1}{1-\lambda}\int_{y\in\mathbf{R}^m} [L(x, y) - \omega]^+ g(y)\, dy \\ [t]^+ = \max\{t, 0\}, \qquad [t]^- = \min\{t, 0\}. \end{gathered} \quad (4)$$

Moreover, $G_\lambda(x, \omega)$ is jointly convex in x and ω if $L(x, y)$ is convex w.r.t. x. Thus, CVaR is expressed as follows:

$$\min_x \mathrm{CVaR}_\lambda(x) = \min_{x,\omega} G_\lambda(x, \omega). \quad (5)$$

2.3. CVaR Applied to the Newsvendor Problem. Let Q denote a retailer's order quantity, which is a decision variable, and D denote a customer demand, which is a random variable. $\pi(Q, D)$ denotes the expected profit. In order to apply CvaR to the newsvendor problem, the variables (x, y) are replaced by (Q, D), where the order quantity Q and the demand D replace (x, y) in CvaR.

Let $-\pi(Q, D)$ denote the net loss of the profit. By substituting $-\pi(Q, D)$ for $L(x, y)$ in the above equations, we get

$$\begin{aligned}
\mathrm{CVaR}_\lambda(Q) &= E\left[-\pi(Q, D) \mid -\pi(Q, D) \geq \mathrm{VaR}_\lambda(Q)\right] \\
&= \frac{1}{1-\lambda}\int_{-\pi(Q,D)\geq \mathrm{VaR}_\lambda(Q)} -\pi(Q, D) f(D)\, dD, \\
\min_Q \mathrm{CVaR}_\lambda(Q) & \\
&= \min_{Q,\omega}\left\{\omega + \frac{1}{1-\lambda}\int_0^{+\infty}\left[-\pi(Q, D)-\omega\right]^+ f(D)\, dD\right\}.
\end{aligned} \tag{6}$$

$\mathrm{CVaR}_\lambda(Q)$ measures the expected value of $-\pi(Q, D)$ falling below a certain quantile level. Hence, $\mathrm{CVaR}_\lambda(Q)$ can be used as the decision criterion of the expectation of $\pi(Q, D)$ over a critical level.

Substituting $\nu = -\omega$, $\pi^\eta(Q, D) = \mathrm{CVaR}_\lambda(Q)$, and $\eta = 1-\lambda$, we derive the following formula to maximize the expectation of profit over a critical level:

$$\begin{aligned}
\min_Q \mathrm{CVaR}_\lambda(Q) & \\
&= \max_Q \pi^\eta(Q, D) \\
&= \max_{Q,\nu}\left\{\nu + \frac{1}{\eta}\int_0^{+\infty}\left[\pi(Q, D)-\nu\right]^- f(D)\, dD\right\}.
\end{aligned} \tag{7}$$

Note that η denotes the degree of risk aversion. Retailers become more risk-averse as η decreases. Meanwhile, $\eta = 1$ indicates risk neutrality, which can be considered a special case of risk aversion.

3. A Risk-Averse Retailer's Optimal Ordering Policy with Physical Inventory Audits

In this section, we analyze the case of a retail store without RFID deployment (Scenario 1). We explore the optimal ordering policy with the CVaR criterion and investigate the impact of different types of inventory shrinkage on the retailer's decisions.

In Scenario 1, the retailer's profit is given by

$$\begin{aligned}
\text{profit} = {} & p\,\mathrm{Min}(D, \gamma_1 Q_1) + s\left[\gamma_1 Q_1 - \mathrm{Min}(D, \gamma_1 Q_1) + \beta Q_1\right] \\
& - cQ_1, \quad \text{where } \gamma_1 = 1-\alpha-\beta.
\end{aligned} \tag{8}$$

The expected profit function is

$$\begin{aligned}
\pi_1(Q_1, D) = {} & \int_0^{\gamma_1 Q_1}\left(pD + s\left(\gamma_1 Q_1 - D + \beta Q_1\right)\right) dF(D) \\
& + \int_{\gamma_1 Q_1}^{+\infty}\left(p\gamma_1 Q_1 + s\beta Q_1\right) dF(D) - cQ_1.
\end{aligned} \tag{9}$$

Taking the second-order derivative of $\pi_1(Q_1, D)$ w.r.t. Q_1, we get

$$\frac{\partial \pi_1(Q_1, D)}{\partial Q_1} = -\gamma_1^2 (p-s) f(\gamma_1 Q_1). \tag{10}$$

The second-order derivative is always less than zero when $s < p$ (Assumption 1), which implies that $\pi_1(Q_1, D)$ is concave in Q_1. Substituting $\pi_1(Q_1, D)$ for $\pi(Q, D)$ in (7), we obtain the risk-averse retailer's objective function, which is

$$\begin{aligned}
\max_{Q_1} \pi_1^\eta(Q_1, D) & \\
&= \max_{Q_1,\nu}\left\{\nu + \frac{1}{\eta}\int_0^{+\infty}\left[\pi_1(Q_1, D)-\nu\right]^- f(D)\, dD\right\}.
\end{aligned} \tag{11}$$

Proposition 3. *The retailer's optimal ordering policy in Scenario 1 is as follows.*

(1) *The optimal order quantity is*

$$Q_1^* = \begin{cases} \dfrac{1}{\gamma_1}F^{-1}\left[\dfrac{\eta(p\gamma_1 + s\beta - c)}{\gamma_1(p-s)}\right], & \text{if } p\alpha + (p-s)\beta \leq p-c \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

(2) *Under the CVaR criterion, the retailer's performance is maximized, which is*

$$\pi_1^\eta(Q_1^*) = \begin{cases} \dfrac{p-s}{\eta}\displaystyle\int_0^{\gamma_1 Q_1^*} D\, dF(D), & \text{if } p\alpha + (p-s)\beta \leq p-c \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Proof. See Appendix. □

From Proposition 3, we find that $Q_1^* \geq 0$ has a positive relationship with η. Therefore, we can infer that the risk-averse retailer orders less than the risk-neutral retailer. When inventory shrinkage is severe ($p\alpha + (p-s)\beta > p-c$), the retailer stops placing orders. It is worth noting that the stop ordering point is the same for retailers with different degrees of risk aversion.

Proposition 4. *The impact of different types of inventory shrinkage on the risk-averse retailer's ordering policy is as follows:*

(1) $Q_1^* \geq 0$ *is decreasing in* k, *where* $k = \alpha/(\alpha+\beta)$,

(2) $\pi_1^\eta(Q_1^*) \geq 0$ *is decreasing in* k, α, *and* β.

Proof. Equation (12) can be transformed to

$$\begin{aligned}
Q_1^* &= \frac{1}{\gamma_1}F^{-1}\left[\eta\left(1 - \frac{h}{\gamma_1(h+u)} - \frac{k(1-\gamma_1)s}{\gamma_1(h+u)}\right)\right], \\
&\qquad \text{where } h = c-s,\ u = p-c,
\end{aligned} \tag{14}$$

where h is the holding cost, and u is the underage cost. Differentiating Q_1^* w.r.t. k, it is obvious that Q_1^* decreases in

k. We can also explain this monotonicity by distinguishing temporary shrinkage from permanent shrinkage. Note that misplaced items can be found and sold at a salvage price at the end of the period, whereas the items that are stolen or damaged are permanently lost. As a consequence, the retailer orders less with an increasing ratio of permanent shrinkage.

Since $\gamma_1^* Q_1^*$ decreases in k, α, and β (12), from (13), we deduce that $\pi_1^\eta(Q_1^*) \geq 0$ decreases in k, α, and β. □

Let $\pi_0^\eta(Q_0^*)$ denote the retailer's performance without inventory inaccuracy. With $\alpha = \beta = 0$, it is straightforward that $\pi_0^\eta(Q_0^*) \geq \pi_1^\eta(Q_1^*)$. Thus, we explore whether the deployment of RFID can cover the loss due to inventory inaccuracy in the next section.

4. A Risk-Averse Retailer's Optimal Ordering Policy with RFID Implementation

Despite the potential advantages of RFID, currently RFID is not extensively used in retail stores because of its relatively high costs compared to physical inventory audits. Besides the fixed costs related to the purchase and implementation of the necessary infrastructure, the substantial cost of RFID tags seems to prohibit widespread application of item-level RFID. This section investigates the risk-averse retailer's optimal ordering policy with RFID implementation and evaluates whether RFID implementation is cost-effective compared to the nontechnical approach. We only consider the additional tag cost t incurred by each item. The fixed infrastructure cost of RFID is beyond the scope of our model.

The model with RFID is equivalent to that of Scenario 1 with $c' = c+t, \alpha' = (1-b)\alpha, \beta' = (1-d)\beta$, and $\gamma_2 = 1-\alpha'-\beta'$. Thus, the retailer's optimal order quantity in Scenario 2 can be expressed as follows:

$$Q_2^* = \begin{cases} \dfrac{1}{\gamma_2} F^{-1}\left[\dfrac{\eta\left(p\gamma_2 + s(1-d)\beta - (c+t)\right)}{\gamma_2(p-s)}\right] & \text{if } p(1-b)\alpha + (p-s)(1-d)\beta \leq p-(c+t), \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Under the CVaR criterion, the retailer's performance is maximized, which is

$$\pi_2^\eta(Q_2^*) = \begin{cases} \dfrac{p-s}{\eta} \displaystyle\int_0^{\gamma_2 Q_2^*} D\, dF(D) & \text{if } p(1-b)\alpha + (p-s)(1-d)\beta \leq p-(c+t), \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Similar to Proposition 3, from Equation (15), we can infer that the risk-averse retailer also orders less than the risk-neutral retailer with RFID implementation.

Let $\Delta\pi^\eta$ denote the benefit of RFID deployment compared with the traditional physical counting approach, where $\Delta\pi^\eta = \pi_2^\eta(Q_2^*) - \pi_1^\eta(Q_1^*)$. $\Delta\pi^\eta \geq 0$ indicates that it is more economical to deploy RFID technology in retail stores.

Proposition 5. *The impact of the retailer's risk aversion degree on his benefit from RFID deployment is that $\Delta\pi^\eta$ increases in η if $\Delta\pi^\eta \geq 0$. Otherwise, $\Delta\pi^\eta$ decreases in η.*

Proof. We discuss the following three cases based on the retailer's different ordering policies.

(1) $\pi_2^\eta(Q_2^*) > 0, \pi_1^\eta(Q_1^*) = 0$.

In this case, $\Delta\pi^\eta = \pi_2^\eta(Q_2^*) > 0$. Taking the first-order derivative of $\Delta\pi^\eta$ w.r.t. η, we get

$$\frac{\partial \Delta\pi^\eta}{\partial \eta} = \frac{p-s}{\eta^2} \int_0^{\gamma_2 Q_2^*} F(D)\, dD > 0. \tag{17}$$

(2) $\pi_2^\eta(Q_2^*) = 0, \pi_1^\eta(Q_1^*) > 0$.

We have $\Delta\pi^\eta = -\pi_1^\eta(Q_1^*) < 0$, and $\partial\Delta\pi^\eta/\partial\eta = -((p-s)/\eta^2)\int_0^{\gamma_1 Q_1^*} F(D)dD < 0$.

(3) $\pi_2^\eta(Q_2^*) > 0, \pi_1^\eta(Q_1^*) > 0$.

In this case, $\Delta\pi^\eta = ((p-s)/\eta^2)\int_{\gamma_1 Q_1^*}^{\gamma_2 Q_2^*} D\,dF(D)$, and $\partial\Delta\pi^\eta/\partial\eta = ((p-s)/\eta^2)\int_{\gamma_1 Q_1^*}^{\gamma_2 Q_2^*} F(D)dD$.

If $\gamma_2 Q_2^* > \gamma_1 Q_1^*$, it is obvious that $\partial\Delta\pi^\eta/\partial\eta > 0$, where $\Delta\pi^\eta > 0$.

Otherwise, we have $\Delta\pi^\eta < 0$ and $\partial\Delta\pi^\eta/\partial\eta < 0$.

In conclusion, $\partial\Delta\pi^\eta/\partial\eta < 0$ when $\Delta\pi^\eta < 0$, and $\partial\Delta\pi^\eta/\partial\eta > 0$ when $\Delta\pi^\eta > 0$. □

Proposition 5 indicates that RFID implementation is not always cost-effective compared to physical audits. If RFID implementation is more profitable than physical audits, risk-neutral retailers obtain greater profits than risk-averse retailers. Otherwise, risk-averse retailers are more profitable.

In practice, it is crucial for retailers to know when it is more profitable to deploy RFID technology. Thus, we specify

the conditions under which RFID implementation is more profitable.

Proposition 6. *The critical RFID tag price t_c such that $\pi_2^\eta(Q_2^*) = \pi_1^\eta(Q_1^*)$ is given by*

$$t_c = \frac{b\alpha(c - s\beta) + d\beta(c - s(1-\alpha))}{\gamma_1}. \quad (18)$$

It is only profitable for retailers to adopt RFID when the actual RFID tag price t is cheaper than t_c. Hence, the higher the critical price is, the more likely that RFID is deployed in retail stores. The influence of $b(d)$ on the critical RFID tag price is intuitively expected: the more improvements RFID can provide, the higher RFID tag price retailers are willing to pay.

5. Numerical Analysis

In this section, we conduct a numerical analysis to provide managerial insights for the risk-averse retailer in terms of his ordering policy affected by inventory shrinkage and risk aversion, as well as his investing decisions in RFID technology.

We assume the demand is uniformly distributed in $[0, D_{\max}]$ and take the values of the parameters as $D_{\max} = 30$, $p = 10$, $c = 4$, and$s = 2$.

5.1. Impact of Different Types of Inventory Shrinkage on the Retailer's Profit. Figure 2 shows the change of the retailer's profit with permanent shrinkage and temporary shrinkage. Note that the x-axis represents t the change of $\alpha(\beta)$ when $\beta(\alpha)$ is fixed to 0.1. Figure 2 indicates that the retailer's profit is more sensitive to permanent shrinkage. This is because, unlike the misplaced items, the items that are stolen or damaged cannot be sold at a salvage price. In other words, permanent shrinkage incurs a higher loss to the retailer.

This finding highlights the importance of distinguishing between permanent shrinkage and temporary shrinkage for retailers to deal with the inventory inaccuracy problem. For example, when retailers deploy RFID technology, they are advised to invest in additional antitheft applications, such as closed circuit television systems and motion detectors, rather than antimisplacement applications, such as smart shelves. In other words, the retailer should balance the store's investments in different applications and choose the right RFID application that generates the biggest benefits. However, when there is no salvage value, the two types of shrinkage have the same monetary impact (Figure 2). Thus, distinguishing the type of shrinkage is not necessary.

5.2. Analysis of the Benefit of RFID Implementation. With emerging RFID applications in different fields, various types of RFID tags are available in the market. RFID tags can cost as little as 50 cents or as much as 50 dollars depending on the type of tag, its efficiency, and its applications. We consider different priced RFID tags by setting $t = rc$. Figure 3 shows that the benefit of RFID changes with the retailer's risk aversion degree and the RFID tag price. When RFID is cost-effective, the benefit of RFID has a negative relationship with the retailer's risk aversion degree. Hence, risk-neutral retailers have a higher tendency to adopt RFID than risk-averse retailers. However, when the tag price is relatively high with respect to the product price, the retailer may encounter a loss. In this situation, risk-neutral retailers lose more profits than risk-averse retailers. This finding is consistent with the definition of risk aversion, which states that risk-averse retailers are willing to give up some profits for avoidance of possible losses.

Figure 4 shows that the retailer's tolerable RFID tag price goes up with the price of the product. Thus, we can infer that RFID is practicable for high-price products (such as jewelry, watches, and digital products). Moreover, when inventory shrinkage becomes severe, the retailer tolerates a higher tag price.

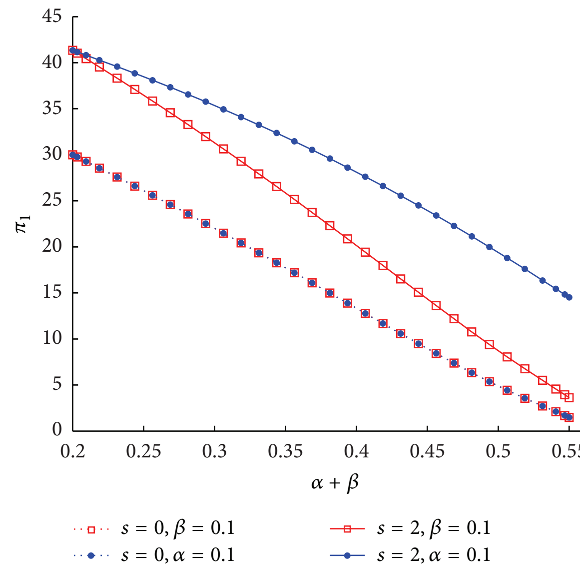

Figure 2: Impact of α and β on π for different values of s.

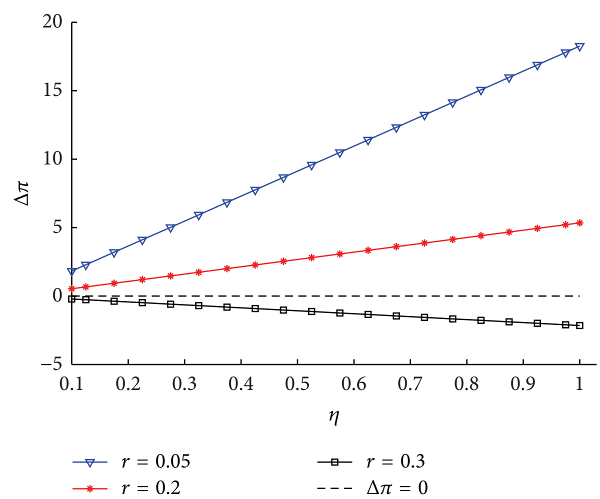

Figure 3: Variation of $\Delta\pi$ with η for different values of r.

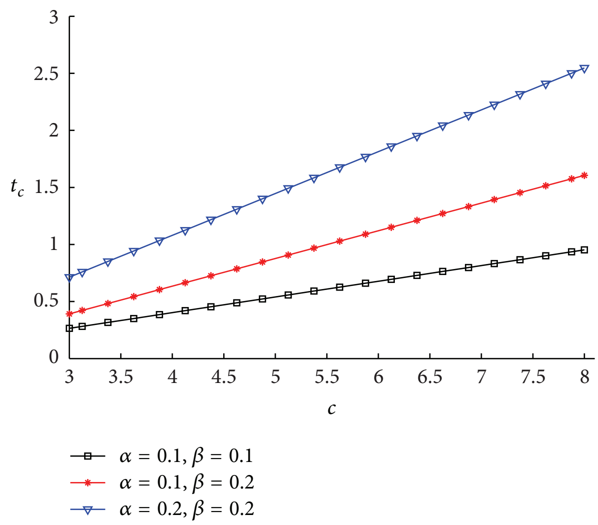

FIGURE 4: Variation of t_c with c for different values of α and β.

6. Conclusion

In this paper, we utilized CVaR to explore a risk-averse retailer's optimal ordering policy subject to inventory shrinkage and investigated his incentive to invest in RFID technology. We contribute to the literature by analyzing two sources of inventory shrinkage and highlighting the impact of inventory shrinkage on retailers' ordering policies. We compared two scenarios (non-RFID and RFID) for reducing inventory inaccuracy and specified the conditions under which RFID implementation is cost-effective. We showed that risk-neutral retailers have a higher tendency to adopt RFID than risk-averse retailers. By considering imperfect RFID and distinguishing its ability to yield improvements with different types of shrinkage, we identified benefits of RFID closer to those experienced in practice. We also provided some managerial insights, such that permanent shrinkage incurs a higher loss to retailers than temporary shrinkage, and RFID implementation is practicable for high-price products.

Appendix

Proof of Proposition 3. Let $g(v, Q_1)$ denote a jointly concave function of (v, Q_1), where

$$g(v, Q_1) = v + \frac{1}{\eta} E\left[\left[\pi(Q_1, D) - v\right]^-\right]. \tag{A.1}$$

Substituting (9) into the above equation, we get

$$\begin{aligned} g(v, Q_1) &= v + \frac{1}{\eta}\int_0^{\gamma_1 Q_1} \left[pD + s(\gamma_1 Q_1 - D + \beta Q_1) - cQ_1 - v\right]^- dF(D) \\ &\quad + \frac{1}{\eta}\int_{\gamma_1 Q_1}^{+\infty} \left[p\gamma_1 Q_1 + s\beta Q_1 - cQ_1 - v\right]^- dF(D). \end{aligned} \tag{A.2}$$

The joint optimal solution (v^*, Q_1^*) should satisfy the first-order conditions $\partial g(v, Q_1)/\partial v = 0$ and $\partial g(v, Q_1)/\partial Q_1 = 0$ simultaneously.

We consider three cases of $g(v, Q_1)$:

(1) $v \le (s\gamma_1 + s\beta - c)Q_1$.

In this case, $[pD + s(\gamma_1 Q_1 - D + \beta Q_1) - cQ_1 - v]^- = 0$, and $[p\gamma_1 Q_1 + s\beta Q_1 - cQ_1 - v]^- = 0$. Then, we get $g(v, Q_1) = v$, and $\partial g(v, Q_1)/\partial v = 1 > 0$, and thus no solution exists.

(2) $v > (p\gamma_1 + s\beta - c)Q_1$.

In this case, we get

$$\begin{aligned} g(v, Q_1) &= v + \frac{1}{\eta}\int_0^{\gamma_1 Q_1} (pD + s(\gamma_1 Q_1 - D + \beta Q_1) - cQ_1 - v)\, dF(D) \\ &\quad + \frac{1}{\eta}\int_{\gamma_1 Q_1}^{+\infty} (p\gamma_1 Q_1 + s\beta Q_1 - cQ_1 - v)\, dF(D), \end{aligned} \tag{A.3}$$

and $\partial g(v, Q_1)/\partial v = 1 - (1/\eta)(\int_0^{\gamma_1 Q_1} dF(D) + \int_{\gamma_1 Q_1}^{+\infty} dF(D)) = 1 - 1/\eta \le 0$. Thus, no solution exists.

(3) $(s\gamma_1 + s\beta - c)Q_1 < v \le (p\gamma_1 + s\beta - c)Q_1$.

In this case, $[p\gamma_1 Q_1 + s\beta Q_1 - cQ_1 - v]^- = 0$. We also have $[pD + s(\gamma_1 Q_1 - D + \beta Q_1) - cQ_1 - v]^- = 0$ when $D > v - (s\gamma_1 + s\beta - c)Q_1/(p - s)$.

Thus, we get $g(v, Q_1) = v + (1/\eta)\int_0^{(v - (s\gamma_1 + s\beta - c)Q_1)/(p-s)} (pD + s(\gamma_1 Q_1 - D + \beta Q_1) - cQ_1 - v)dF(D)$, and $\partial g(v, Q_1)/\partial v = 1 - (1/\eta)F((v - (s\gamma_1 + s\beta - c)Q_1)/(p - s))$.

Note that $(\partial g(v, Q_1)/\partial v)|_{v=(s\gamma_1 + s\beta - c)Q_1} = 1 > 0$ when v approaches $(s\gamma_1 + s\beta - c)Q_1$ from the right, and $(\partial g(v, Q_1)/\partial v)|_{v=(p\gamma_1 + s\beta - c)Q_1} = 1 - (1/\eta)F(\gamma_1 Q_1)$ when v approaches $(p\gamma_1 + s\beta - c)Q_1$ from the left. Thus, it is straightforward that $v^* = (p\gamma_1 + s\beta - c)Q_1$ if $Q_1 \le F^{-1}(\eta)/\gamma_1$. Otherwise, v^* can be obtained from $1 - (1/\eta)F((v - (s\gamma_1 + s\beta - c)Q_1)/(p - s)) = 0$.

Based on this observation, we discuss the following situations.

(1) $Q_1 > (1/\gamma_1)F^{-1}(\eta)$.

In this situation, $v^* = (p - s)F^{-1}(\eta) + (s\gamma_1 + s\beta - c)$. Substituting v^* into $g(v, Q_1)$ and differentiating $g(v^*, Q_1)$ w.r.t. Q_1, we get $\partial g(v^*, Q_1)/\partial Q_1 = s\gamma_1 + s\beta - c < 0$, and thus no solution exists.

(2) $Q_1 \le (1/\gamma_1)F^{-1}(\eta)$.

In this situation, $v^* = (p\gamma_1 + s\beta - c)Q_1$. Substituting v^* into $g(v, Q_1)$ and differentiating $g(v^*, Q_1)$ w.r.t. Q_1,we get $\partial g(v^*, Q_1)/\partial Q_1 = p\gamma_1 + s\beta - c - ((p - s)/\eta)F(\gamma_1 Q_1)$, where $Q_1^* = (1/\gamma_1)F^{-1}(\eta(p\gamma_1 + s\beta - c)/\gamma_1(p - s))$ satisfies the first-order condition.

Therefore, the optimal solution is $v^* = (p\gamma_1 + s\beta - c)Q_1^*$ and $Q_1^* = (1/\gamma_1)F^{-1}(\eta(p\gamma_1 + s\beta - c)/\gamma_1(p - s))$. Substituting v^* and Q_1^* into (11), we get $\pi_1^\eta(Q_1^*)$. □

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