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Research Article

On the Stability of (α, β, γ)-Derivations and Lie C^*-Algebra Homomorphisms on Lie C^*-Algebras: A Fixed Points Method

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We investigate new generalized Hyers-Ulam stability results for (α, β, γ)-derivations and Lie C^*-algebra homomorphisms on Lie C^*-algebras associated with the additive functional equation: $\sum_{i=1}^{n} f(nx_i + \sum_{j=1, j\neq i}^{n} x_j) + f(\sum_{i=1}^{n} x_i) = 2f(\sum_{i=1}^{n} nx_i)$.

1. Introduction

The theory of finite dimensional complex Lie algebras is an important part of Lie theory. It has several applications in physics and connections with other parts of mathematics. With an increasing amount of theory and applications concerning Lie algebras of various dimensions, it is becoming necessary to ascertain which tools are applicable for handling them. The miscellaneous characteristics of Lie algebras constitute such tools and have also found applications: Casimir operators [1], derived, lower central and upper central sequences, and the Lie algebra of derivations, radicals, nilradicals, ideals, subalgebras [2, 3], and recently megaideals [4]. These characteristics are particularly crucial when considering possible affinities among Lie algebras. Physically motivated relations between two Lie algebras, namely, contractions and deformations, have been extensively studied [5]. Moreover, in modern industry, various analytical approaches for solving mathematical equations are widely applied in analysis of problems in packaging engineering, and so mathematical modeling and computation methods by using mathematical equations play an important role in application of packaging engineering. From now, we wish to note that mathematical equations for stability properties in this paper can have applications to Packaging Engineering.

A C^*-algebra $\mathscr{A}$ endowed with the Lie product $[x, y] = (xy - yx)/2$ on $\mathscr{A}$ is called a *Lie* C^**-algebra*. A $\mathbb{C}$-linear mapping $\delta : \mathscr{A} \to \mathscr{A}$ is called a (α, β, γ)*-derivation* of $\mathscr{A}$ if there exist α, β, and $\gamma \in C$ such that $\alpha\delta[x, y] = \beta[\delta(x), y] + \gamma[x, \delta(y)]$ for all $x, y \in \mathscr{A}$ [6]. Let $\mathscr{A}$ and $\mathscr{B}$ be Lie C^*-algebras. A $\mathbb{C}$-linear mapping $H : \mathscr{A} \to \mathscr{B}$ is called a *Lie* C^**-algebra homomorphism* if $H([x, y]) = [H(x), H(y)]$ for all $x, y \in \mathscr{A}$.

The stability problem of functional equations was originated from a question of Ulam [7, 8] concerning the stability of group homomorphisms as follows.

> *Let $(G_1, *)$ be a group and $(G_2, \diamond, d)$ be a metric group with the metric $d(\cdot, \cdot)$. Given $\epsilon > 0$, does there exist a $\delta(\epsilon) > 0$ such that if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x * y), h(x) \diamond h(y)) < \epsilon$ for all $x, y \in G_1$, then there is a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$?*

If the answer is affirmative, we would say that the equation of a homomorphism $H(x * y) = H(x) \diamond H(y)$ is stable (see also [9–14]). Cădariu and Radu [15] applied the fixed point method to investigation of the stability of a functional equation. In 2008, Novotný and Hrivnák [6] investigated generalizing the concept of Lie derivations via certain complex parameters, obtained various Lie, and established the structure and properties of (α, β, γ)-derivations of Lie algebras. Recently, the generalized Hyers-Ulam stability of problems on C^*-algebras associated with functional equations has been investigated by using a fixed point method (see [16–20]). The following fixed point theorem will play an important role in proving our main theorem.

Theorem 1 (see [21]). *Suppose that we are given a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \to \Omega$ with Lipschitz constant L. Then for each given $x \in \Omega$, either $d(T^n x, T^{n+1}x) = \infty$ for all nonnegative integers $n \geq 0$ or there exists a natural number n_o such that*

(1) *$d(T^n x, T^{n+1}x) < \infty$ for all $n \geq n_o$;*

(2) *the sequence $\{T^n x\}$ is convergent to a fixed point $y*$ of T;*

(3) *$y*$ is the unique fixed point of T in the set $\Lambda = \{y \in \Omega : d(T^{n_o}x, y) < \infty\}$;*

(4) *$d(y, y*) \leq (1/(1-L))d(y, Ty)$ for all $y \in \Lambda$.*

Let k be a fixed positive integer. We recall that a mapping $\rho : A \to B$ having a domain A and a codomain $(B, \leq)$ that are both closed under addition is called a *contractively subadditive mapping* if there exists a constant L with $0 < L < 1$ such that $\rho(x+y) \leq L(\rho(x) + \rho(y))$ and an *expansively superadditive mapping* if there exists a constant L with $0 < L < 1$ such that $\rho(x+y) \geq (1/L)(\rho(x)+\rho(y))$ for all $x, y \in A$. A mapping $\rho : A \to B$ is called a *homogeneous of degree k* if $\rho(\lambda x) = \lambda^k \rho(x)$ for all $x, y \in A$. Also, if there exists a constant L with $0 < L < 1$ such that a mapping $\rho : A^n \to B$ satisfies $\rho(x_1, \ldots, \lambda^\ell x, \ldots, x_n) \leq \lambda^\ell L\rho(x_1, \ldots, x, \ldots, x_n)$ for all $x, y \in A$ and positive integer $\lambda > 1$, then we say that ρ is a *n-contractively subhomogeneous mapping* if $\ell = 1$, and ρ is an *expansively superhomogeneous mapping* if $\ell = -1$.

Now, we consider a mapping $f : X \to Y$ satisfying the following functional equation [22]:

$$\sum_{i=1}^{n} f\left(nx_i + \sum_{j=1, j\neq i}^{n} x_j\right) + f\left(\sum_{i=1}^{n} x_i\right) = 2f\left(\sum_{i=1}^{n} nx_i\right) \tag{1}$$

for all $x_i, x_j \in X$, where $n \in \mathbb{Z}^+$ is a fixed integer with $n \geq 2$. Note that the mapping $f(x) = ax$ is a solution of the functional equation (1). Eskandni [22] investigated the Hyers-Ulam-Rassias stability of the functional equation (1) in quasi-Banach spaces using the direct method.

In this paper, using some strategies from [15, 18, 22], we investigate new generalized Hyers-Ulam stability for (α, β, γ)-derivations and Lie C^*-algebra homomorphisms on Lie C^*-algebras associated with the functional equation (1) via the fixed point method.

Throughout this paper, let $\mathscr{A}$ and $\mathscr{B}$ be Lie C^*-algebras and $T^1_{1/n_o} = \{e^{i\theta} : 0 \leq \theta \leq 2\pi/n_o\}$. For any mapping $f : \mathscr{A} \to \mathscr{B}$, we define

$$\begin{aligned} \mathscr{D}_\mu f(x_1, \ldots, x_n) = {} & \sum_{i=1}^{n} f\left(n\mu x_i + \sum_{j=1, j\neq i}^{n} \mu x_j\right) \\ & + f\left(\sum_{i=1}^{n} \mu x_i\right) - 2f\left(\sum_{i=1}^{n} n\mu x_i\right), \\ \mathscr{D}_{\alpha,\beta,\gamma} f(x, y) = {} & \alpha f[x, y] - \beta[f(x), y] - \gamma[x, f(y)] \end{aligned} \tag{2}$$

for all $x_1, \ldots, x_n$, $x, y \in X$ $(n \geq 2)$, $\mu \in T^1_{1/n_o}$, and some $\alpha, \beta, \gamma \in \mathbb{C}$.

2. Main Results

In this section, we point out the stability of the functional equation (1) on Lie C^*-algebras using a fixed point method. Let us recall that a mapping $d : X^2 \to [0, \infty)$ is called a *generalized metric* on a nonempty set X if (i) $d(x, y) = 0$ if and only if $x = y$, (ii) $d(x, y) = d(y, x)$, and (iii) $d(x, z) \leq d(x, y) + d(y, z)$ for all x, y, and $z \in X$. We need the following lemma to prove the main result in this paper.

Lemma 2 (see [22]). *Let X, Y be linear spaces and $n \geq 2$ a fixed positive integer. A mapping $f : X \to Y$ satisfies the functional equation (1) if and only if f is additive.*

Theorem 3. *Assume that there exist a contractively subadditive mapping $\phi : \mathscr{A}^n \to [0, \infty)$ and a 2-contractively subhomogeneous mapping $\psi : \mathscr{A}^2 \to [0, \infty)$ with a constant $0 < L < 1$ such that a mapping $f : \mathscr{A} \to \mathscr{A}$ satisfies*

$$\left\|\mathscr{D}_\mu f(x_1, \ldots x_n)\right\| \leq \phi(x_1, \ldots, x_n), \tag{3}$$

$$\left\|\mathscr{D}_{\alpha,\beta,\gamma} f(x, y)\right\| \leq \psi(x, y) \tag{4}$$

for all $x_1, \ldots, x_n$, $x, y \in \mathscr{A}$, $\mu \in T^1_{1/n_o}$, and some α, β, and $\gamma \in \mathbb{C}$. Then there exists a unique (α, β, γ)-derivation $\delta : \mathscr{A} \to \mathscr{A}$ which satisfies (1) and the inequality

$$\|f(x) - \delta(x)\| \leq \frac{1}{n(1-L)}\phi(x, 0, \ldots, 0) \tag{5}$$

for all $x \in \mathscr{A}$.

Proof. Let Ω be a set of all mappings from $\mathscr{A}$ into $\mathscr{A}$, and introduce a generalized metric on Ω as follows:

$$\begin{aligned} d(g, h) = \inf \{ & C \in \mathbb{R}^+ : \|g(x) - h(x)\| \\ & \leq C\phi(x, 0, \ldots, 0)\ \forall x \in \mathscr{A}\}. \end{aligned} \tag{6}$$

Then (Ω, d) is a generalized complete metric space. Now, we consider the mapping $T : \Omega \to \Omega$ defined by

$$Tg(x) = \frac{1}{n}g(nx) \tag{7}$$

for all $g \in \Omega$, $x \in \mathscr{A}$, and $n \in \mathbb{Z}^+$. Let $g, h \in \Omega$ and $C \in (0, \infty)$ be an arbitrary constant with $d(g, h) \leq C$. Then we have $\|g(x) - h(x)\| \leq C\phi(x, 0, \ldots, 0)$ and

$$\|Tg(x) - Th(x)\| \leq \frac{C}{n}\phi(nx, 0, \ldots, 0) \leq LC\phi(x, 0, \ldots, 0) \tag{8}$$

for all $x \in \mathscr{A}$, which means that $d(Tg, Th) \leq Ld(g, h)$ for all $g, h \in \Omega$. Thus T is a strictly contractive self-mapping on Ω with the Lipschitz constant L.

Letting $x_1 = x$, $x_2 = x_3 = \cdots = x_n = 0$, and $\mu = 1$ in (3), we have

$$\|f(nx) - nf(x)\| \leq \phi(x, 0, \ldots, 0), \tag{9}$$

which gives

$$\left\| f(x) - \frac{1}{n} f(nx) \right\| \le \frac{1}{n} \phi(x, 0, \ldots, 0) \tag{10}$$

for all $x \in \mathscr{A}$ and $n \in \mathbb{Z}^+$ with $n \ge 2$. Then we have $d(Tf, f) \le 1/n < \infty$. From Theorem 1, there is a mapping δ which is a unique fixed point of T in the set $\Omega_1 = \{g \in \Omega : d(f, g) < \infty\}$ such that

$$\delta(x) = \lim_{m \to \infty} \frac{1}{n^m} f(n^m x) \tag{11}$$

for all $x \in \mathscr{A}$, since $\lim_{m \to \infty} d(T^m f, \delta) = 0$. Again by Theorem 1, we have

$$d(f, \delta) \le \frac{1}{1 - L} d(Tf, f) \le \frac{1}{n(1 - L)} \tag{12}$$

for all $x \in \mathscr{A}$. Thus, inequality (5) holds.

It follows from (3), (11), and the contractively subadditive mapping of ϕ that

$$\begin{aligned} \left\| \mathscr{D}_\mu \delta(x_1, \ldots, x_n) \right\| &\le \lim_{m \to \infty} \frac{1}{n^m} \left\| \mathscr{D}_\mu f(n^m x_1, \ldots, n^m x_n) \right\| \\ &\le \lim_{m \to \infty} L^m \phi(x_1, \ldots, x_n) = 0, \end{aligned} \tag{13}$$

which gives $\mathscr{D}_\mu \delta(x_1, \ldots, x_n) = 0$ for all $x_1, \ldots, x_n \in \mathscr{A}$ and $\mu \in T^1_{1/n_o}$. If we put $\mu = 1$ in $\mathscr{D}_\mu \delta(x_1, \ldots, x_n) = 0$; then δ satisfies the functional equation (1), and so δ is additive by Lemma 2. Also, we let $x_1 = x$ and $x_2 = \cdots = x_n = 0$, then $\delta(\mu x) = \mu \delta(x)$. By the same reasoning as that of the proof of Theorem 2.1 of [23], the mapping δ is $\mathbb{C}$-linear. So, it follows from the 2-contractively subhomogeneous of ψ, (4), and (11) that

$$\begin{aligned} \left\| \mathscr{D}_{\alpha,\beta,\gamma} \delta(x, y) \right\| &\le \lim_{m \to \infty} \frac{1}{n^{2m}} \psi(n^m x, n^m y) \\ &\le \lim_{m \to \infty} L^{2m} \psi(x, y) = 0 \end{aligned} \tag{14}$$

for all $x, y \in \mathscr{A}$ and some α, β, and $\gamma \in \mathbb{C}$. Then we have

$$\alpha \delta[x, y] = \beta[\delta(x), y] + \gamma[x, \delta(y)] \tag{15}$$

for all $x, y \in \mathscr{A}$ and some α, β, and $\gamma \in \mathbb{C}$. Thus, δ is a unique Lie (α, β, γ)-derivation on Lie C^*-algebra $\mathscr{A}$ satisfying (5). This completes the proof. □

Corollary 4. *Let $0 < r < 1$, $0 < s < 2$, and θ be nonnegative real numbers. Suppose that a mapping $f : \mathscr{A} \to \mathscr{A}$ satisfies*

$$\begin{aligned} \left\| \mathscr{D}_\mu f(x_1, \ldots x_n) \right\| &\le \theta \sum_{i=1}^{n} \|x_i\|^r, \\ \left\| \mathscr{D}_{\alpha,\beta,\gamma} f(x, y) \right\| &\le \theta \left(\|x\|^s + \|y\|^s \right) \end{aligned} \tag{16}$$

for all $x_1, \ldots, x_n$, $x, y \in \mathscr{A}$, $\mu \in T^1_{1/n_o}$, and some α, β, and $\gamma \in \mathbb{C}$. Then there exists a unique (α, β, γ)-derivation $\delta : \mathscr{A} \to \mathscr{A}$ such that

$$\left\| f(x) - \delta(x) \right\| \le \frac{\theta}{n - n^r} \|x\|^r, \tag{17}$$

for all $x \in \mathscr{A}$ and $n \in \mathbb{Z}^+$ with $n \ge 2$.

Proof. The proof follows from Theorem 3 by taking $\phi(x_1, \ldots, x_n) = \theta \sum_{i=1}^{n} \|x_i\|^r$, $\psi(x, y) = \theta(\|x\|^s + \|y\|^s)$ for all $x_1, \ldots, x_n$, $x, y \in \mathscr{A}$. Then we can choose $L = n^{r-1}$, and we obtain the desired result. This completes the proof. □

Theorem 5. *Assume that there exist an expansively superadditive mapping $\phi : \mathscr{A}^n \to [0, \infty)$ and a 2-expansively superhomogeneous mapping $\psi : \mathscr{A}^2 \to [0, \infty)$ with a constant $0 < L < 1$ such that a mapping $f : \mathscr{A} \to \mathscr{A}$ satisfies* (3) *and* (4)*. Then there exists a unique (α, β, γ)-derivation $\delta : \mathscr{A} \to \mathscr{A}$ which satisfies* (1) *and the inequality*

$$\left\| f(x) - \delta(x) \right\| \le \frac{L}{n(1 - L)} \phi(x, 0, \ldots, 0) \tag{18}$$

for all $x \in \mathscr{A}$.

Proof. Let Ω and d be as in the proof of Theorem 3. Then (Ω, d) becomes a generalized complete metric space, and we consider the mapping $T : \Omega \to \Omega$ defined by $(Tg)(x) = ng(x/n)$ for all $g \in \Omega$ and $x \in \mathscr{A}$. So, $d(Tg, Th) \le Ld(g, h)$ for all $g, h \in \Omega$. It follows from (9) that

$$\left\| f(x) - nf\left(\frac{x}{n}\right) \right\| \le \frac{L}{n} \phi(x, 0, \ldots, 0) \tag{19}$$

for all $x \in \mathscr{A}$ and $n \in \mathbb{Z}^+$ with $n \ge 2$. Then, $d(Tf, f) \le L/n < \infty$. From Theorem 1, there exists a unique mapping δ which is a unique fixed point of T in the set $\Omega_1 = \{g \in \Omega : d(f, g) < \infty\}$ such that $\delta(x) = \lim_{m \to \infty} n^m f(x/n^m)$ for all $x \in \mathscr{A}$. Thus, we have

$$d(f, \delta) \le \frac{1}{1 - L} d(Tf, f) \le \frac{L}{n(1 - L)}, \tag{20}$$

which implies that (18) holds. The remaining assertion goes through in similar method to the corresponding part of Theorem 3. This completes the proof. □

Corollary 6. *Let $r > 1$, $s > 2$ and θ be nonnegative real numbers. Suppose that a mapping $f : \mathscr{A} \to \mathscr{A}$ satisfies* (16)*. Then there exists a unique (α, β, γ)-derivation $\delta : \mathscr{A} \to \mathscr{A}$ such that*

$$\left\| f(x) - \delta(x) \right\| \le \frac{\theta}{n^r - n} \|x\|^r, \tag{21}$$

for all $x \in \mathscr{A}$ and $n \in \mathbb{Z}^+$ with $n \ge 2$.

Proof. The proof follows from Theorem 5 by taking $\phi(x_1, \ldots, x_n) = \theta \sum_{i=1}^{n} \|x_i\|^r$, $\psi(x, y) = \theta(\|x\|^s + \|y\|^s)$ for all $x_1, \ldots, x_n$, $x, y \in \mathscr{A}$. Then we can choose $L = n^{1-r}$, and we obtain the desired result. This completes the proof. □

Next, we establish another theorem about the stability of the functional equation (1).

Theorem 7. *Assume that there exists a contractively subadditive mapping $\varphi : \mathscr{A}^{n+2} \to [0, \infty)$ with a constant $L < 1$ such that a mapping $f : \mathscr{A} \to \mathscr{A}$ satisfies*

$$\left\| \mathscr{D}_\mu f(x_1, \ldots, x_n) + \mathscr{D}_{\alpha,\beta,\gamma} f(x, y) \right\| \le \varphi(x_1, \ldots, x_n, x, y) \tag{22}$$

for all $x_1, \ldots, x_n$, $x, y \in \mathscr{A}$, $\mu \in T^1_{1/n_o}$, *and some* α, β, *and* $\gamma \in \mathbb{C}$. *Then there exists a unique* (α, β, γ)*-derivation* $\delta : \mathscr{A} \to \mathscr{A}$ *which satisfies* (1) *and the inequality*

$$\|f(x) - \delta(x)\| \le \frac{1}{n(1-L)}\varphi(x, 0, \ldots, 0, 0, 0) \quad (23)$$

for all $x \in \mathscr{A}$.

Proof. Substituting $x_1 = a$, $x_2 = \cdots, x_n = x = y = 0$, and $\mu = 1$ in (22), we obtain

$$\left\|\frac{1}{n}f(na) - f(a)\right\| \le \frac{1}{n}\varphi(a, 0, \ldots, 0, 0, 0) \quad (24)$$

for all $a \in \mathscr{A}$ and a positive integer $n \ge 2$. Let Ω and d be as in the proof of Theorem 3 such that (Ω, d) becomes a generalized complete metric space. Let a mapping $T : \Omega \to \Omega$ defined by $(Tg)(x) = (1/n)g(nx)$ for all $x \in \mathscr{A}$, $g \in \Omega$. Then, we have $d(Tg, Th) \le Ld(g, h)$ for all $g, h \in \Omega$. It follows from (24) that $d(Tf, f) \le 1/n < \infty$. The remaining assertion goes through in a similar way to the corresponding part of Theorem 3. This completes the proof. □

Corollary 8. *Assume that there exists an expansively superadditive mapping* $\varphi : \mathscr{A}^{n+2} \to [0, \infty)$ *with a constant* $L < 1$ *such that a mapping* $f : \mathscr{A} \to \mathscr{A}$ *satisfies* (22). *Then there exists a unique* (α, β, γ)*-derivation* $\delta : \mathscr{A} \to \mathscr{A}$ *which satisfies* (1) *and the inequality*

$$\|f(x) - \delta(x)\| \le \frac{L}{n(1-L)}\varphi(x, 0, \ldots, 0, 0, 0) \quad (25)$$

for all $x \in \mathscr{A}$.

Next, we investigate the Lie C^*-algebra homomorphisms on Lie C^*-algebras associated with the functional equation (1). The results in Theorems 9 and 10 are similar to those in [24].

Theorem 9. *Assume that there exist a contractively subadditive mapping* $\phi : \mathscr{A}^n \to [0, \infty)$ *and a 2-contractively subhomogeneous mapping* $\psi : \mathscr{A}^2 \to [0, \infty)$ *with a constant* $0 < L < 1$ *such that a mapping* $f : \mathscr{A} \to \mathscr{B}$ *satisfies* (3) *and*

$$\|f([x, y]) - [f(x), f(y)]\| \le \psi(x, y) \quad (26)$$

for all $x, y \in \mathscr{A}$ *and* $\mu \in T^1_{1/n_o}$. *Then there exists a unique Lie* C^**-algebra homomorphism* $H : \mathscr{A} \to \mathscr{B}$ *satisfying*

$$\|f(x) - H(x)\| \le \frac{1}{n(1-L)}\phi(x, 0, \ldots, 0) \quad (27)$$

for all $x \in \mathscr{A}$.

Proof. By the same method as in Theorem 3, we obtain a $\mathbb{C}$-linear mapping $H : \mathscr{A} \to \mathscr{B}$ satisfying (27). The mapping is given by $H(x) = \lim_{m\to\infty}(1/n^m)f(n^m x)$ for all $x \in \mathscr{A}$. It follows from (25) that

$$\begin{aligned}
&\|H[x, y] - [H(x), H(y)]\| \\
&\quad = \lim_{m\to\infty}\frac{1}{n^{2m}}\left\|f\left(n^{2m}[x, y]\right) - [f(n^m x), f(n^m y)]\right\| \\
&\quad \le \lim_{m\to\infty} L^{2m}\psi(x, y) = 0
\end{aligned} \quad (28)$$

for all $x, y \in \mathscr{A}$. Thus, H is a Lie C^*-algebra homomorphism. This completes the proof. □

Theorem 10. *Assume that there exist an expansively superadditive mapping* $\phi : \mathscr{A}^n \to [0, \infty)$ *and a 2-expansively superhomogeneous mapping* $\psi : \mathscr{A}^2 \to [0, \infty)$ *with a constant* $0 < L < 1$ *such that a mapping* $f : \mathscr{A} \to \mathscr{B}$ *satisfies* (3) *and* (26). *Then there exists a unique Lie* C^**-algebra homomorphism* $H : \mathscr{A} \to \mathscr{B}$ *satisfying*

$$\|f(x) - H(x)\| \le \frac{L}{n(1-L)}\phi(x, 0, \ldots, 0) \quad (29)$$

for all $x \in \mathscr{A}$.

Proof. The proof is similar to the proofs of Theorems 5 and 9. □

Corollary 11. *Let* $r < 1$, $s < 2$, *and* θ *be nonnegative real numbers. Suppose that a mapping* $f : \mathscr{A} \to \mathscr{B}$ *satisfies*

$$\begin{gathered}
\left\|\mathscr{D}_\mu f(x_1, \ldots, x_n)\right\| \le \theta\sum_{i=1}^{n}\|x_i\|^r, \\
\|f([x, y]) - [f(x), f(y)]\| \le \theta\left(\|x\|^s + \|y\|^s\right)
\end{gathered} \quad (30)$$

for all $x_1, \ldots, x_n$, $x, y \in \mathscr{A}$, $\mu \in T^1_{1/n_o}$. *Then there exists a unique Lie* C^**-algebra homomorphism* $H : \mathscr{A} \to \mathscr{B}$ *satisfying*

$$\|f(x) - H(x)\| \le \frac{\theta}{n - n^r}\|x\|^r, \quad (31)$$

for all $x \in \mathscr{A}$ *and* $n \in \mathbb{Z}^+$ *with* $n \ge 2$.

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