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Research Article

Asynchronous H_∞ Dynamic Output Feedback Control of Switched Time-Delay Systems with Sensor Nonlinearity and Missing Measurements

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The H_∞ dynamic output feedback control problem for a class of discrete-time switched time-delay systems under asynchronous switching is investigated in this paper. Sensor nonlinearity and missing measurements are considered when collecting output knowledge of the system. Firstly, when there exists asynchronous switching between the switching modes and the candidate controllers, new results on the regional stability and l_2 gain analysis for the underlying system are given by allowing the Lyapunov-like function (LLF) to increase with a random probability. Then, a mean square stabilizing output feedback controller and a switching law subject to average dwell time (ADT) are obtained with a given disturbance attenuation level. Moreover, the mean square domain of attraction could be estimated by a convex combination of a set of ellipsoids, the number of which depends on the number of switching modes. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

The past few decades have witnessed an ever increasing research interest in the control problems that are fundamental to the switched systems. These families of systems have great practical potential in many fields, such as power systems [1] and networked control systems [2]. A discrete-time switched system can be analytically expressed by a finite number of operation modes with switching signals between them. As a significant fact, the switching law depicts the transition between possible system behavior patterns; therefore, the average dwell time (ADT) approach has become one of the most effective methods to deal with the analysis and synthesis of switched system (see [3–5] and references therein). The main advantage of ADT switching is the fact that the stabilizability and other performance requirements could be achieved by regulating the switching rate, that is, to enlarge or reduce the number of switches over a finite time interval not less than a fixed value [6].

It is worth mentioning that an interesting research topic named "asynchronous switching" draws much attention in recent years. It usually means that the switching of controllers has a lag to the switching of system modes, because in many practical applications, it inevitably takes some time to identify the currently operating mode of the switched system and applies a matched controller. The stability, stabilizability, and filtering problem for such an asynchronous mechanism have been well studied by allowing the Lyapunov-like function (LLF) to increase during the unmatched period between the switching mode and the controller [7, 8]. Such an unmatched time depends on the identification of the switching mode and the scheduling of the candidate controller and the length of it may be time-varying according to different running environment. In most cases, the maximum value of the unmatched time must be known *a priori* [9–11]. Another effective method is to divide the active time of switching mode and controller into matched and mismatched intervals and corresponding controller gains could be obtained by solving coupled matrix inequality constraints [12–14]. The obtained results could soon be extended to the asynchronous filtering problems [15, 16]. Burgeoning research works in related areas such as finite-time control or state estimation, system with switching mechanism and time delay, and system with mismatched uncertainties could be reviewed in [17–22].

In most existing literatures concerning asynchronous switching, it has been implicitly assumed that the sensors can always provide unlimited amplitude signal and therefore ignore the possible effect of the sensor nonlinearity such as [23], although sensor nonlinearities are widely exist [24–26]. Moreover, the asynchronous controller design approaches for switched systems rely on the ideal hypothesis of perfect measurements [27]. However, in terms of engineering application, such hypothesis does not always hold. For example, due to the missing measurements [28] or incomplete measurements [29], the signal will be strongly influenced or even only contain noise, which indicate that real signal is jeopardized or missed. There is one more point that we want to touch on the fact that, in the abovementioned relevant references, the unmatched period between switching mode and controller is assumed known *a priori*. However, in almost all types of asynchronous mechanism, such unmatched period could be vague and random in the running time of different operating mode. Therefore, rather than having a large complexity to measure or estimate all the unmatched period or the largest one, it is significant and necessary to further develop more general asynchronous control strategy by allowing the LLF to increase with a random probability during the unmatched period.

In this technical note, we aim to investigate the asynchronous H_∞ dynamic output feedback controller design problem for a class of time-delay switched system with both sensor nonlinearity and missing measurements. Note that the addressed system model is quite comprehensive to cover asynchronous switching, time delay, sensor nonlinearity, missing measurements, and H_∞ performance requirement, hence reflecting the reality closely. The main contributions of this study are trifold: (1) a new system model for time-delay switched system is established to take both sensor nonlinearity and missing measurements into account; (2) new results on the regional stability and l_2 gain analysis for the underlying system are given by employing a binary switching sequence to depict the evolution of the LLF; (3) a mode-dependent ellipsoid constraint, which represented by a convex combination of a set of ellipsoids, is developed to deal with sensor nonlinearity for considering the time-delay within the switching dynamics.

Notation. Notations in this paper are fairly standard. The superscript "T" stands for matrices transport. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote n dimensional Euclidean space and set of all $n \times m$ matrices, respectively. The brief notation $A \geq B$ or $A > B$ (where matrices A and B are symmetric) means that $A - B$ is positive semidefinite or positive definite, respectively. In symmetric matrices, we use $*$ as an ellipsis for the symmetric terms above or below the diagonal; I and 0 denote identity matrix and zero matrix with appropriate dimensions, respectively.

2. Preliminaries and Problem Formulations

Consider the following discrete-time switched system:

$$\begin{aligned} x_{k+1} &= A(r_k)x_k + A^d(r_k)x_{k-d} + B^w(r_k)w_k + B_1^u(r_k)u_k, \\ z_k &= C^z(r_k)x_k + C^d(r_k)x_{k-d} + D^w(r_k)w_k + B_2^u(r_k)u_k, \end{aligned} \tag{1}$$

and m sensors with both saturation and missing measurements:

$$\begin{aligned} y_k^f &= \alpha_k^f \sigma\left((C(r_k))^f x_k\right) + \left(1-\alpha_k^f\right)\beta_k^f (C(r_k))^f x_k \\ &\quad + (D(r_k))^f v_k^f, \quad f = 1,2,\ldots,m, \end{aligned} \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $z_k \in \mathbb{R}^r$ is the controlled output vector, and $y_k^f \in \mathbb{R}$ is the fth sensor observations. Notations $w_k \in l_2([0,\infty),\mathbb{R}^p)$ and $v_k^f \in l_2([0,\infty),\mathbb{R})$ represent the exogenous disturbance and measurement noise, respectively.

The switching signal r_k is a piecewise constant function of time, which takes values in a finite integer set $\mathbb{S} = \{1,2,\ldots,s\}$, and $s > 1$ is the number of the switching modes. The time sequence $k_0 < k_1 < \cdots < k_l < k_{l+1} < \cdots$ represents every switching instant and time k_0 is also denoted by initial time. When $k \in [k_l, k_{l+1})$, the r_{k_l}th mode is active and therefore the trajectory x_k of system (1) is the trajectory under the r_{k_l}th mode. The jumps of the state at the mode switching instants are not considered here.

For each possible value of $r_k = i$, we denote $A(r_k) = A_i$, $A^d(r_k) = A_i^d$, $B^w(r_k) = B_i^w$, $B_1^u(r_k) = B_{1i}^u$, $C^z(r_k) = C_i^z$, $C^d(r_k) = C_i^d$, $D^w(r_k) = D_i^w$, $B_2^u(r_k) = B_{2i}^u$, $(C(r_k))^f = C_i^f$, and $(D(r_k))^f = D_i^f$ for simplicity.

The standard saturation function with appropriate dimensions $\psi : \mathbb{R} \to \mathbb{R}$, which is defined as $\psi(x) = \text{sign}(x)\min\{1,|x|\}$, is employed to describe the sensor nonlinearity in this study. The notation "sign" denotes a signum function. Here we have slightly abused the notation by using $\psi(\cdot)$ to denote both the scalar valued and the vector valued saturation functions.

To deal with the asynchronous switching between switching mode and corresponding controller, we let $[k_l, k_{l+1})$, $\forall l \in \mathbb{N}$, denote the active time interval of some subsystems, while $[k_l, k_l+\tau)$ and $[k_l+\tau, k_{l+1})$ represent the unmatched time and the matched time, respectively. The LLF is assumed to increase during $[k_l, k_l+\tau)$ and decrease during $[k_l+\tau, k_{l+1})$. Because $[k_l, k_l+\tau)$ and $[k_l+\tau, k_{l+1})$ are randomly dispersed intervals, we assume they obey Bernoulli distribution.

The stochastic variables $\alpha_k^f \in \mathbb{R}$ and $\beta_k^f \in \mathbb{R}$ in sensor model (2) are Bernoulli distributed white sequences taking values with

$$\begin{aligned} \text{Prob}\left\{\alpha_k^f = 1\right\} &= \mu_f, \\ \text{Prob}\left\{\alpha_k^f = 0\right\} &= 1-\mu_f, \\ \text{Prob}\left\{\beta_k^f = 1\right\} &= \vartheta_f, \end{aligned}$$

$$\operatorname{Prob}\left\{\beta_k^f = 0\right\} = 1 - \vartheta_f, \quad 1 \le f \le m. \tag{3}$$

It can be recalled from [28] that, in sensor model (2), if $\alpha_k^f = 1$, it means that the sensor f subjects to saturation only; if $\alpha_k^f = 0$ and $\beta_k^f = 1$, it implies that the sensor f works normally; if $\alpha_k^f = 0$ and $\beta_k^f = 0$, the sensor f detects noise only.

Assumption 1. We use stochastic variable θ_k to characterize the randomly dispersed intervals $[k_l, k_l + \tau)$ and $[k_l + \tau, k_{l+1})$, which satisfy

$$\begin{aligned} \operatorname{Prob}\left\{\theta_k = 1\right\} &= \theta, \\ \operatorname{Prob}\left\{\theta_k = 0\right\} &= 1 - \theta. \end{aligned} \tag{4}$$

If $\theta_k = 1$, it means that the system mode and controller are matched and the LLF is decreased during the interval $[k_l + \tau, k_{l+1})$; if $\theta_k = 0$, it implies that the system mode and controller are unmatched and the LLF is increased during the interval $[k_l, k_l + \tau)$.

In this study, a class of switching signals with ADT switching is designed when the controller is obtained. Thus, the definition of ADT is recalled.

Definition 2 (see [6] (average dwell time)). For a switching signal r_k and any time $k > k_0$, let $N_r(k_0, k)$ be the switching numbers of r_k over the finite interval $[k_0, k)$. If, for any given $N_0 > 0$ and $\tau_a > 0$, we have $N_r(k_0, k) \le N_0 + (k - k_0)/\tau_a$, then τ_a and N_0 are called average dwell time and chatter bound, respectively. As commonly used in the references, we choose $N_0 = 0$.

For notational brevity, we rewrite sensor model (2) as

$$\overline{y}_k = \varphi_{\alpha k}\psi\left(\overline{C}_i x_k\right) + \left(I - \varphi_{\alpha k}\right)\varphi_{\beta k}\overline{C}_i x_k + \overline{D}_i \overline{v}_k, \tag{5}$$

where

$$\begin{aligned} \overline{y}_k &= \begin{bmatrix} y_k^1 & y_k^2 & \cdots & y_k^m \end{bmatrix}^{\mathrm{T}}, \\ \overline{v}_k &= \begin{bmatrix} v_k^1 & v_k^2 & \cdots & v_k^m \end{bmatrix}^{\mathrm{T}}, \\ \varphi_{\alpha k} &= \operatorname{diag}\left\{\alpha_k^1, \alpha_k^2, \ldots, \alpha_k^m\right\}, \\ \varphi_{\beta k} &= \operatorname{diag}\left\{\beta_k^1, \beta_k^2, \ldots, \beta_k^m\right\}, \\ \overline{C}_i &= \begin{bmatrix} (C_i^1)^{\mathrm{T}} & \left(C_i^2\right)^{\mathrm{T}} & \cdots & (C_i^m)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ \overline{D}_i &= \operatorname{diag}\left\{D_i^1, D_i^2, \ldots, D_i^m\right\}. \end{aligned} \tag{6}$$

Moreover, we set

$$\begin{aligned} \varphi_\mu &= \operatorname{diag}\left\{\mu_1, \mu_2, \ldots, \mu_m\right\}, \\ \varphi_\vartheta &= \operatorname{diag}\left\{\vartheta_1, \vartheta_2, \ldots, \vartheta_m\right\}. \end{aligned} \tag{7}$$

In this note, we are interested in designing the following dynamic output feedback controller:

$$\begin{aligned} x_{k+1}^c &= A^c\left(r_k\right)x_k^c + B^c\left(r_k\right)\overline{y}_k, \\ u_k &= C^c\left(r_k\right)x_k^c, \end{aligned} \tag{8}$$

where matrices $A^c(r_k) = A_i^c$, $B^c(r_k) = B_i^c$ and $C^c(r_k) = C_i^c$ are controller gains to be determined. By introducing new vectors $\eta_k = [x_k^{\mathrm{T}} \quad (x_k^c)^{\mathrm{T}}]^{\mathrm{T}}$ and $\overline{w}_k = [w_k^{\mathrm{T}} \quad \overline{v}_k^{\mathrm{T}}]^{\mathrm{T}}$, the resulting closed-loop systems under output feedback controller (8) become

$$\begin{aligned} \eta_{k+1} &= \overline{A}_{ij}\eta_k + \overline{A}_i^d\eta_{k-d} + \overline{J}_j\psi\left(\overline{C}_i G\eta_k\right) + \overline{B}_{ij}\overline{w}_k \\ &\quad + \sum_{f=1}^m\left(\varphi_{\alpha k}^f - \mu_f\right)\overline{K}_j\psi\left(\overline{C}_i G\eta_k\right) \\ &\quad + \sum_{f=1}^m\left(\left(1 - \varphi_{\alpha k}^f\right)\varphi_{\beta k}^f - \left(1 - \varphi_\mu\right)\varphi_\vartheta\right)\overline{K}_j\overline{C}_i G\eta_k, \\ z_k &= \overline{C}_{ij}^z\eta_k + \overline{C}_i^d\eta_{k-d} + \overline{D}_i^w\overline{w}_k, \\ &\qquad \forall k \in \left[k_l, k_l + \tau\right), \end{aligned} \tag{9}$$

$$\begin{aligned} \eta_{k+1} &= \overline{A}_i\eta_k + \overline{A}_i^d\eta_{k-d} + \overline{J}_i\psi\left(\overline{C}_i G\eta_k\right) + \overline{B}_i\overline{w}_k \\ &\quad + \sum_{f=1}^m\left(\varphi_{\alpha k}^f - \mu_f\right)\overline{K}_i\psi\left(\overline{C}_i G\eta_k\right) \\ &\quad + \sum_{f=1}^m\left(\left(1 - \varphi_{\alpha k}^f\right)\varphi_{\beta k}^f - \left(1 - \varphi_\mu\right)\varphi_\vartheta\right)\overline{K}_i\overline{C}_i G\eta_k, \\ z_k &= \overline{C}_i^z\eta_k + \overline{C}_i^d\eta_{k-d} + \overline{D}_i^w\overline{w}_k \\ &\qquad \forall k \in \left[k_l + \tau, k_{l+1}\right), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \overline{A}_i &= \begin{bmatrix} \widetilde{A}_i \\ \widetilde{R}_i \end{bmatrix} = \begin{bmatrix} A_i & B_{1i}^u C_i^c \\ B_i^c\left(I - \varphi_\mu\right)\varphi_\vartheta\overline{C}_i & A_i^c \end{bmatrix}, \\ \overline{A}_{ij} &= \begin{bmatrix} \widetilde{A}_{ij} \\ \widetilde{R}_{ij} \end{bmatrix} = \begin{bmatrix} A_j & B_{1i}^u C_j^c \\ B_j^c\left(I - \varphi_\mu\right)\varphi_\vartheta\overline{C}_i & A_j^c \end{bmatrix}, \\ \overline{B}_i &= \begin{bmatrix} \overline{B}_{1i} \\ \overline{B}_{2i} \end{bmatrix} = \begin{bmatrix} B_i^w & 0 \\ 0 & B_i^c\overline{D}_i \end{bmatrix}, \\ \overline{B}_{ij} &= \begin{bmatrix} \overline{B}_{1i} \\ \overline{B}_{2ij} \end{bmatrix} = \begin{bmatrix} B_i^w & 0 \\ 0 & B_j^c\overline{D}_i \end{bmatrix}, \\ \overline{J}_i &= \begin{bmatrix} 0 \\ B_i^c\varphi_\mu \end{bmatrix}, \qquad \overline{J}_j = \begin{bmatrix} 0 \\ B_j^c\varphi_\mu \end{bmatrix}, \qquad \overline{K}_i = \begin{bmatrix} 0 \\ B_i^c E_f \end{bmatrix}, \\ \overline{K}_j &= \begin{bmatrix} 0 \\ B_j^c E_f \end{bmatrix}, \qquad E_f = \operatorname{diag}\left\{\overbrace{0, \ldots 0}^{f-1}, 1, \overbrace{0, \ldots 0,}^{m-f}\right\}, \end{aligned}$$

$$G = \begin{bmatrix} I & 0 \end{bmatrix}, \qquad \overline{C}_i^z = \begin{bmatrix} C_i^z & B_{2i}^u C_i^c \end{bmatrix},$$
$$\overline{C}_{ij}^z = \begin{bmatrix} C_i^z & B_{2i}^u C_j^c \end{bmatrix}, \qquad \overline{A}_i^d = \begin{bmatrix} \widetilde{A}_i^d \\ 0 \end{bmatrix} = \begin{bmatrix} A_i^d & 0 \\ 0 & 0 \end{bmatrix},$$
$$\overline{C}_i^d = \begin{bmatrix} C_i^d & 0 \end{bmatrix}, \qquad \overline{D}_i^w = \begin{bmatrix} D_i^w & 0 \end{bmatrix}. \tag{11}$$

In order to deal with the regional mean square stability of closed-loop systems (9)~(10) and the saturation nonlinearity $\psi(\cdot)$, the following preliminaries are given.

Definition 3. Denote $\eta_{k,\eta_0,\overline{w}}$ be the state trajectory of closed-loop systems (9)~(10) starting from the initial value η_0; then the set satisfying

$$\mathbb{Z} = \left\{ \eta_0 \in \mathbb{R}^{2n} : \lim_{k\to\infty} \mathrm{E} \left\| \eta_{k,\eta_0,\overline{w}} \right\|_2^2 = 0 \right\} \tag{12}$$

is said to be the mean square domain of attraction of the origin.

Lemma 4 (see [28]). *Assume the nonlinear function $\psi(\cdot)$ satisfies $[\psi(x) - a_f x][\psi(x) - x] \le 0$ and $|x| \le a_f^{-1}$, where a_f is a positive scalar satisfying $0 < a_f < 1$. Set $\varphi_a = \mathrm{diag}\{a_1, a_2, \ldots, a_m\}$ and define*

$$\mathbb{F}\left(\varphi_a \overline{C}_i G\right) = \left\{ \eta \in \mathbb{R}^{2n} : \left| a_f C_i^f G \eta \right| \le 1 \right\},$$
$$i = 1, 2, \ldots, s, \quad f = 1, 2, \ldots, m. \tag{13}$$

Then, it can be verified that $0 < \varphi_a < I$ and

$$\left[\psi\left(\overline{C}_i G \eta_k\right) - \varphi_a \overline{C}_i G \eta_k\right]^{\mathrm{T}} \left[\psi\left(\overline{C}_i G \eta_k\right) - \overline{C}_i G \eta_k\right] \le 0, \tag{14}$$

for each $\eta \in \mathbb{F}(\varphi_a \overline{C}_i G)$.

Definition 5. Define a switching level set as follows:

$$\Omega\left(P_i, Q\right) = \left\{ \eta \in \mathbb{R}^{2n} : \eta^{\mathrm{T}} P_i \eta + \sum_{l=k-d}^{k-1} \eta_l^{\mathrm{T}} Q \eta_l \le 1 \right\}, \tag{15}$$

where $P_i, Q \in \mathbb{R}^{2n}$ are positive definite matrices.

Remark 6. The level set $\Omega(P_i, Q)$ is graphic and simple in form but could not be directly employed to estimate the domain of attraction $\mathbb{Z}$. This problem will be tackled in the following part of the paper.

The purpose of this study is to design H_∞ dynamic output feedback controller of form (8) subject to ADT switching such that the following two conditions are satisfied.

(i) Closed-loop systems (9)~(10) with $\overline{w}_k = 0$ are regional mean square stabilizable and the switching level set $\Omega(P_i, Q) \subset \mathbb{Z}$.

(ii) Under the zero-initial condition, the controlled output satisfies

$$\|z\|_2^2 < \gamma^2 \|\overline{w}\|_2^2, \tag{16}$$

where

$$\|z\|_2^2 = \left\{ \mathrm{E}\left[\sum_{k=0}^{\infty} z_k^{\mathrm{T}} z_k \right] \right\} < \infty,$$
$$\|\overline{w}\|_2^2 = \sum_{k=0}^{\infty} \overline{w}_k^{\mathrm{T}} \overline{w}_k < \infty \tag{17}$$

for any nonzero $\overline{w}_k$ and a prescribed attenuation level $\gamma > 0$.

3. Main Results

Let us start with tackling the switching level set $\Omega(P_i, Q)$. Given ellipsoid sets

$$\Lambda\left(P_i\right) = \left\{ \eta \in \mathbb{R}^{2n} : \eta^{\mathrm{T}} P_i \eta \le 1 \right\}, \quad i \in \mathbb{S},$$
$$\Lambda(Q) = \left\{ \eta \in \mathbb{R}^{2n} : \eta^{\mathrm{T}} Q \eta \le 1 \right\}, \tag{18}$$

where matrices P_i and Q are positive definite, then we denote

$$\Lambda\left(P_i, Q\right) = \mathrm{Co}\left\{ \Lambda\left(P_1\right), \Lambda\left(P_2\right) \cdots \Lambda\left(P_s\right), \Lambda(Q) \right\}, \tag{19}$$

where Co represents a convex hull. Viewing P_i and Q as vertices of the level set $\Lambda(P_i, Q)$ and by using the property of the polytope [30], (19) immediately yields $\Omega(P_i, Q) \subset \Lambda(P_i, Q)$. Obviously, the estimation of the domain of attraction could be enlarged when we replace the level set $\Omega(P_i, Q)$ by the ellipsoid set $\Lambda(P_i, Q)$. Thus, the domain of attraction could be estimated by ellipsoid (19) only if $\Lambda(P_i, Q) \subset \mathbb{Z}$ is satisfied.

Theorem 7. *Consider systems (9)~(10) with $\overline{w}_k = 0$ and let the controller gain A_i^c, B_i^c, and C_i^c be given. If, for some given constants $\widetilde{\alpha} = 1 - \alpha$ $(0 < \alpha < 1)$, $\widetilde{\beta} = 1 + \beta$ $(\beta \ge 0)$, and $\kappa > 1$, there exist matrices $P_i > 0$ and $Q > 0$ and scalar $\varepsilon > 0$ such that*

$$\Lambda\left(P_i\right) \subset \mathbb{F}\left(\varphi_a \overline{C}_i G\right), \qquad \Lambda(Q) \subset \mathbb{F}\left(\varphi_a \overline{C}_i G\right), \tag{20}$$

$$\Xi_i = \begin{bmatrix} \Xi_{i11} - \varepsilon_1 G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \varphi_a \overline{C}_i G - \widetilde{\alpha} P_i & \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i^d & \Xi_{i13} \\ * & \left(\widetilde{A}_i^d\right)^{\mathrm{T}} P_{1i} \widetilde{A}_i^d - Q & 0 \\ * & * & \Xi_{i33} - \varepsilon I \end{bmatrix} < 0, \tag{21}$$

$$\Xi_{ij} = \begin{bmatrix} \Xi_{ij11} - \varepsilon_1 G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \varphi_a \overline{C}_i G_i - \widetilde{\beta} P_i & \widetilde{A}_{ij}^{\mathrm{T}} P_{1i} \widetilde{A}_i^d & \Xi_{ij13} \\ * & \left(\widetilde{A}_i^d\right)^{\mathrm{T}} P_{1i} \widetilde{A}_i^d - Q & 0 \\ * & * & \Xi_{ij33} - \varepsilon I \end{bmatrix} < 0, \tag{22}$$

$$P_i \le \kappa P_j, \tag{23}$$

then the system is regional mean square stable and ellipsoid $\Lambda(P_i, Q)$ *is contained in the mean square domain of attraction* $\mathbb{Z}$ *for switching signal* r_k *with ADT:*

$$\tau_a > \tau_a^* = -\frac{\ln\kappa}{\ln\left(1-\theta\alpha+(1-\theta)\beta\right)}, \qquad -1 < -\theta\alpha + (1-\theta)\beta < 0, \tag{24}$$

where

$$\begin{aligned}
\Xi_{i11} &= \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i + \widetilde{R}_i^{\mathrm{T}} P_{2i} \widetilde{R}_i + Q \\
&\quad + \sum_{f=1}^{m} \left(\delta_{2f}+\delta_{3f}\right) G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G, \\
\Xi_{i13} &= \widetilde{R}_i^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu + \frac{\varepsilon G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \left(I+\varphi_a\right)}{2}, \\
\Xi_{i33} &= \varphi_\mu \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu + \sum_{f=1}^{m} \left(\delta_{1f}+\delta_{3f}\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{i2} B_i^c E_f, \\
\delta_{1f} &= \mu_f\left(1-\mu_f\right), \\
\delta_{2f} &= \left(1-\mu_f\right)\vartheta_f - \left(1-\mu_f\right)^2 \vartheta_f^2, \\
\delta_{3f} &= \left(1-\mu_f\right)\mu_f\vartheta_f, \\
\Xi_{ij11} &= \widetilde{A}_{ij}^{\mathrm{T}} P_{1i} \widetilde{A}_{ij} + \widetilde{R}_{ij}^{\mathrm{T}} P_{2i} \widetilde{R}_{ij} + Q \\
&\quad + \sum_{f=1}^{m} \left(\delta_{2f}+\delta_{3f}\right) G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G, \\
\Xi_{ij13} &= \widetilde{R}_{ij}^{\mathrm{T}} P_{2i} B_j^c \varphi_\mu + \frac{\varepsilon G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \left(I+\varphi_a\right)}{2}, \\
\Xi_{ij33} &= \varphi_\mu \left(B_i^c\right)^{\mathrm{T}} P_{2j} B_i^c \varphi_\mu + \sum_{f=1}^{m} \left(\delta_{1f}+\delta_{3f}\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{2j} B_i^c E_f.
\end{aligned} \tag{25}$$

Proof. For the convenience of manipulation, we assume the matrices P_i and Q have the diagonal form; that is, $P_i = \operatorname{diag}\{P_{1i}, P_{2i}\}$ and $Q = \operatorname{diag}\{Q_1, Q_2\}$. Condition (20) means that if $\eta_k \in \Lambda(P_i, Q)$, then the nonlinear constraint could always be satisfied. In order to simplify the proof, we only consider closed-loop system (10). The results for system (9) can be obtained in a similar way.

Consider the quadratic LLF:

$$V\left(r_k, k\right) = V_i(k) = \eta_k^{\mathrm{T}} P_i \eta_k + \sum_{l=k-d}^{k-1} \eta_l^{\mathrm{T}} Q \eta_l, \qquad k \in \left[k_l, k_{l+1}\right); \tag{26}$$

then

$$\begin{aligned}
&\mathrm{E}\left\{V_i(k+1)\right\} \\
&= \mathrm{E}\Bigg\{\eta_k^{\mathrm{T}} \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i \eta_k + \eta_k^{\mathrm{T}} \widetilde{R}_i^{\mathrm{T}} P_{2i} \widetilde{R}_i \eta_k + \eta_{k-d}^{\mathrm{T}} \left(\widetilde{A}_i^d\right)^{\mathrm{T}} P_{1i} \widetilde{A}_i^d \eta_{k-d} \\
&\quad + \psi^{\mathrm{T}}\left(\overline{C}_i G \eta_k\right) \varphi_\mu \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu \psi\left(\overline{C}_i G \eta_k\right) \\
&\quad + \sum_{f=1}^{m} \delta_{1f} \psi^{\mathrm{T}}\left(\overline{C}_i G \eta_k\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \psi\left(\overline{C}_i G \eta_k\right) \\
&\quad + \sum_{f=1}^{m} \delta_{2f} \eta_k^{\mathrm{T}} G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G \eta_k \\
&\quad + 2\eta_k^{\mathrm{T}} \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i^d \eta_{k-d} + 2\eta_k^{\mathrm{T}} \widetilde{R}_i^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu \psi\left(\overline{C}_i G \eta_k\right) \\
&\quad - 2\sum_{f=1}^{m} \delta_{3f} \psi^{\mathrm{T}}\left(\overline{C}_i G \eta_k\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G \eta_k \\
&\quad + \eta_k^{\mathrm{T}} Q \eta_k - \eta_{k-d}^{\mathrm{T}} Q \eta_{k-d} + \sum_{l=k-d}^{k-1} \eta_l^{\mathrm{T}} Q \eta_l \Bigg\}.
\end{aligned} \tag{27}$$

From the above equality, we have

$$\mathrm{E}\left\{V_i(k+1)\right\} \le \mathrm{E}\left\{\xi_k^{\mathrm{T}} \Xi_i \xi_k + \widetilde{\alpha} \sum_{l=k-d}^{k-1} \eta_l^{\mathrm{T}} Q \eta_l\right\}, \tag{28}$$

where

$$\xi_k = \left[\eta_k^{\mathrm{T}} \quad \eta_{k-d}^{\mathrm{T}} \quad \psi^{\mathrm{T}}\left(\overline{C}_i G \eta_k\right)\right]^{\mathrm{T}},$$
$$\overline{\Xi}_i = \begin{bmatrix} \Xi_{i11} & \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i^d & \widetilde{R}_i^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu \\ * & \left(\widetilde{A}_i^d\right)^{\mathrm{T}} P_{1i} \widetilde{A}_i^d - Q & 0 \\ * & * & \Xi_{i33} \end{bmatrix}, \tag{29}$$

which comes from the fact that

$$\begin{aligned}
&-2\psi^{\mathrm{T}}\left(\overline{C}_i G \eta_k\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G \eta_k \\
&\quad \le \psi^{\mathrm{T}}\left(\overline{C}_i G \eta_k\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \psi\left(\overline{C}_i G \eta_k\right) \\
&\quad + \eta_k^{\mathrm{T}} G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G \eta_k.
\end{aligned} \tag{30}$$

It follows from (14) that

$$\begin{aligned}
&\mathrm{E}\left\{V_i(k+1)\right\} \\
&\le \mathrm{E}\Bigg\{\xi_k^{\mathrm{T}} \overline{\Xi}_i \xi_k - \varepsilon\left[\psi\left(\overline{C}_i G \eta_k\right) - \varphi_a \overline{C}_i G \eta_k\right]^{\mathrm{T}} \\
&\qquad \times \left[\psi\left(\overline{C}_i G \eta_k\right) - \overline{C}_i G \eta_k\right] + \widetilde{\alpha} \sum_{l=k-d}^{k-1} \eta_l^{\mathrm{T}} Q \eta_l \Bigg\}
\end{aligned}$$

$$\leq \mathrm{E}\left\{\xi_k^{\mathrm{T}}\overline{\Xi}_i\xi_k - \varepsilon\psi^{\mathrm{T}}\left(\overline{C}_iG\eta_k\right)\psi\left(\overline{C}_iG\eta_k\right) + \varepsilon\eta_k^{\mathrm{T}}G^{\mathrm{T}}\overline{C}_i^{\mathrm{T}}\psi\left(\overline{C}_iG\eta_k\right) + \varepsilon\eta_k^{\mathrm{T}}G^{\mathrm{T}}\overline{C}_i^{\mathrm{T}}\varphi_a\psi\left(\overline{C}_iG\eta_k\right) - \varepsilon\eta_k^{\mathrm{T}}G^{\mathrm{T}}\overline{C}_i^{\mathrm{T}}\varphi_a\overline{C}_iG\eta_k + \widetilde{\alpha}\sum_{l=k-d}^{k-1}\eta_l^{\mathrm{T}}Q\eta_l\right\} = \mathrm{E}\left\{\xi_k^{\mathrm{T}}\widetilde{\Xi}_i\xi_k + \widetilde{\alpha}\sum_{l=k-d}^{k-1}\eta_l^{\mathrm{T}}Q\eta_l\right\}, \tag{31}$$

where

$$\widetilde{\Xi}_i = \begin{bmatrix} \Xi_{i11} - \varepsilon G^{\mathrm{T}}\overline{C}_i^{\mathrm{T}}\varphi_a\overline{C}_iG & \widetilde{A}_i^{\mathrm{T}}P_{1i}\widetilde{A}_i^d & \Xi_{i13} \\ * & \left(\widetilde{A}_i^d\right)^{\mathrm{T}}P_{1i}\widetilde{A}_i^d - Q & 0 \\ * & * & \Xi_{i33} - \varepsilon I \end{bmatrix}. \tag{32}$$

Because (21) holds, that is, $\Xi_i < 0$, we know that

$$\widetilde{\Xi}_i < \begin{bmatrix} \widetilde{\alpha}P_i & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}. \tag{33}$$

Then, according to (31) and (33), one has

$$\mathrm{E}\left\{V_i(k+1)\right\} \leq \mathrm{E}\left\{\widetilde{\alpha}\eta_k^{\mathrm{T}}P_i\eta_k + \widetilde{\alpha}\sum_{l=k-d}^{k-1}\eta_l^{\mathrm{T}}Q\eta_l\right\} = \widetilde{\alpha}\mathrm{E}\left\{V_i(k)\right\}, \quad \forall k \in \left[k_l+\tau, k_{l+1}\right). \tag{34}$$

Following the same lines of the proof of (34), we can get

$$\mathrm{E}\left\{V_i(k+1)\right\} \leq \mathrm{E}\left\{\widetilde{\beta}\eta_k^{\mathrm{T}}P_i\eta_k + \widetilde{\beta}\sum_{l=k-d}^{k-1}\eta_l^{\mathrm{T}}Q\eta_l\right\} = \widetilde{\beta}\mathrm{E}\left\{V_i(k)\right\}, \quad \forall k \in \left[k_l, k_l+\tau\right). \tag{35}$$

It follows from (4), (34), and (35) that

$$\begin{aligned} \mathrm{E}\left\{\Delta V_i(k)\right\} &= \mathrm{E}\left\{V_i(k+1) - V_i(k)\right\} \\ &= \mathrm{E}\left\{\theta_k\left(-\alpha V_i(k)\right) + \left(1-\theta_k\right)\left(\beta V_i(k)\right)\right\} \\ &= \mathrm{E}\left\{\left(-\theta\alpha - \left(\theta_k - \theta\right)\alpha + (1-\theta)\beta + \left(1-\theta_k\right)\beta - (1-\theta)\beta\right)V_i(k)\right\} \\ &= \left(-\theta\alpha + (1-\theta)\beta\right)\mathrm{E}\left\{V_i(k)\right\}. \end{aligned} \tag{36}$$

If we denote $\varpi = -\theta\alpha + (1-\theta)\beta$, then, from the above, we can get

$$\mathrm{E}\left\{V\left(r_k, k\right)\right\} \leq \varpi^{k-k_l}\mathrm{E}\left\{V\left(r_k, k_l\right)\right\}. \tag{37}$$

Note that

$$\begin{aligned} \mathrm{E}\left\{V\left(r_{k_l}, k_l\right)\right\} &= \mathrm{E}\left\{x_{k_l}^{\mathrm{T}}P\left(r_{k_l}\right)x_{k_l} + \sum_{l=k_l-d}^{k_l-1}\eta_l^{\mathrm{T}}Q\eta_l\right\} \\ &\leq \mathrm{E}\left\{\kappa x_{k_l}^{\mathrm{T}}P\left(r_{k_{l-1}}\right)x_{k_l} + \sum_{l=k_l-d}^{k_l-1}\eta_l^{\mathrm{T}}Q\eta_l\right\}; \\ \mathrm{E}\left\{V\left(r_{k_{l-1}}, k_l\right)\right\} &\leq \varpi^{k_l-k_{l-1}}\mathrm{E}\left\{V\left(r_{k_{l-1}}, k_{l-1}\right)\right\}, \end{aligned} \tag{38}$$

then, according to (23) and (26), one has

$$\begin{aligned} &\mathrm{E}\left\{V\left(r_{k_l}, k_l\right)\right\} \\ &\leq \kappa\mathrm{E}\left\{V\left(r_{k_{l-1}}, k_l\right)\right\} - \kappa\sum_{l=k_l-d}^{k_l-1}\eta_l^{\mathrm{T}}Q\eta_l + \sum_{l=k_l-d}^{k_l-1}\eta_l^{\mathrm{T}}Q\eta_l \\ &\leq \varpi^{k_l-k_{l-1}}\kappa\mathrm{E}\left\{V\left(r_{k_{l-1}}, k_{l-1}\right)\right\} + (1-\kappa)\mathrm{E}\left\{\sum_{l=k_l-d}^{k_l-1}\eta_l^{\mathrm{T}}Q\eta_l\right\} \\ &\leq \varpi^{k_l-k_{l-1}}\kappa\mathrm{E}\left\{V\left(r_{k_{l-1}}, k_{l-1}\right)\right\}. \end{aligned} \tag{39}$$

Thus,

$$\begin{aligned} \mathrm{E}\left\{V\left(r_k, k\right)\right\} &\leq \varpi^{k-k_{l-1}}\kappa\mathrm{E}\left\{V\left(r_{k_{l-1}}, k_{l-1}\right)\right\} \\ &\leq \varpi^{k-k_{l-2}}\kappa^2\mathrm{E}\left\{V\left(r_{k_{l-2}}, k_{l-2}\right)\right\} \\ &\leq \cdots \leq \varpi^{k-k_0}\kappa^{k-k_0/\tau_a}\mathrm{E}\left\{V\left(r_{k_0}, k_0\right)\right\} \\ &\leq \left\{\varpi\kappa^{1/\tau_a}\right\}^{k-k_0}\mathrm{E}\left\{V\left(r_{k_0}, k_0\right)\right\}. \end{aligned} \tag{40}$$

If ADT satisfies (24), one has

$$\varpi\kappa^{1/\tau_a} = e^{\ln\{\varpi\kappa^{1/\tau_a}\}} = e^{\ln\varpi + (1/\tau_a)\ln\kappa} < e^0 = 1. \tag{41}$$

Therefore, we conclude that $V(r_k, k)$ exponentially converges to zero as $k \to \infty$ in the mean square sense; then the mean square stability can be deduced. From (19), we know that constraint (20) indicates that $\Lambda(P_i, Q) \subset \mathbb{F}(\varphi_a\overline{C}_iG)$ (see [31] for details). Therefore, for each $\eta_k \in \Omega(P_i, Q) \subset \Lambda(P_i, Q) \subset \mathbb{F}(\varphi_a\overline{C}_iG)$, it follows immediately that $\eta_k \in \mathbb{Z}$ (see [32, 33] for details). Thus, the proof is completed. □

Remark 8. In this work, we focus our study on the asynchronous switching by considering randomly occurring sensor nonlinearity and missing measurements when collecting the output knowledge. The saturation function, which describes the sensor nonlinearity, restricts the system state and controller state in a regional area of the state space. Since $\Lambda(P_i, Q)$ is expressed by a convex combination of $\Lambda(P_i)$ and $\Lambda(Q)$, it has two main advantages. First, the invariant ellipsoid $\Lambda(P_i, Q)$ depends on all switching modes not just a single one; thus it could enlarge the mean square domain of attraction. Second, the possible largest estimation of the mean square domain of attraction could be obtained by employing a similar approach presented in Algorithm 1 in [31].

Remark 9. The mean square domain of attraction could be estimated by $\Lambda(P_i, Q)$, which is constructed by the convex combination of ellipsoid $\Lambda(P_1), \Lambda(P_2), \ldots, \Lambda(P_s), \Lambda(Q)$. It can be easily verified that

$$\Lambda(P_i, Q) = \left\{\eta \in \mathbb{R}^n : \eta^{\mathrm{T}}(\lambda_1 P_1 + \lambda_2 P_2 + \cdots + \lambda_s P_s + \lambda_{s+1} Q)\ \eta \le 1\right\}, \tag{42}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_s, \lambda_{s+1} > 0$ and satisfying $\lambda_1 + \lambda_2 + \cdots + \lambda_s + \lambda_{s+1} = 1$.

Remark 10. Compared with the results presented in [7], the maximal value of the unmatched time interval is not required to be known *a priori* and the probability of the matched and unmatched time interval could be obtained by using a statistical method. If $\theta = 1$, the results of Theorem 7 degenerate to the case that the switching of the system modes and controllers are matched. A simple corollary is omitted here owing to limited space.

Next, we direct our attention to the l_2 gain analysis. Sufficient conditions for both the regional mean square stability and H_∞ performance of the closed-loop systems (9)~(10) are derived in the following theorem.

Theorem 11. *Consider systems (9)~(10) with $\overline{w}_k = 0$ and let the controller gain A_i^c, B_i^c, and C_i^c be given. If, for some given constants $\widetilde{\alpha} = 1-\alpha$ $(0 < \alpha < 1)$, $\widetilde{\beta} = 1+\beta$ $(\beta \ge 0)$, and $\kappa > 1$, there exist matrices $P_i > 0$ and $Q > 0$ and scalar $\varepsilon > 0$ such that*

$$\Lambda(P_i) \subset \mathbb{F}\left(\varphi_a \overline{C}_i G\right), \qquad \Lambda(Q) \subset \mathbb{F}\left(\varphi_a \overline{C}_i G\right), \tag{43}$$

$$\Theta_i = \begin{bmatrix} \Theta_{i11} & \Theta_{i12} & \Theta_{i13} & \Theta_{i14} \\ * & \Theta_{i22} & 0 & \Theta_{i24} \\ * & * & \Theta_{i33} - \varepsilon I & \Theta_{i34} \\ * & * & * & \Theta_{i44} \end{bmatrix} < 0, \tag{44}$$

$$\Theta_{ij} = \begin{bmatrix} \Theta_{ij11} & \Theta_{ij12} & \Theta_{ij13} & \Theta_{ij14} \\ * & \Theta_{i22} & 0 & \Theta_{i24} \\ * & * & \Theta_{ij33} - \varepsilon I & \Theta_{ij34} \\ * & * & * & \Theta_{ij44} \end{bmatrix} < 0, \tag{45}$$

$$P_i \le \kappa P_j, \tag{46}$$

then the system is regional mean square stable with a given disturbance attenuation level $\gamma > 0$ and ellipsoid $\Lambda(P_i, Q)$ is contained in the mean square domain of attraction $\mathbb{Z}$ for switching signal r_k with ADT:

$$\tau_a > \tau_a^* = -\frac{\ln \kappa}{\ln(1 - \theta\alpha + (1-\theta)\beta)}, \qquad -1 < -\theta\alpha + (1-\theta)\beta < 0, \tag{47}$$

where

$$\begin{aligned}
\Theta_{i11} &= \widehat{\Theta}_{i11} - \varepsilon G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \varphi_a \overline{C}_i G - \widetilde{\alpha} P_i, \\
\widehat{\Theta}_{i11} &= \kappa\widetilde{\alpha} \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i + \kappa\widetilde{\alpha} \widetilde{R}_i^{\mathrm{T}} P_{2i} \widetilde{R}_i + \widetilde{\alpha} Q \\
&\quad + \kappa\widetilde{\alpha} \sum_{f=1}^{m} \left(\delta_{2f} + \delta_{3f}\right) G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f \overline{C}_i G \\
&\quad + \left(\overline{C}_i^z\right)^{\mathrm{T}} \overline{C}_i^z, \\
\Theta_{i12} &= \kappa\widetilde{\alpha} \widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i^d + \left(\overline{C}_i^z\right)^{\mathrm{T}} \overline{C}_i^d, \\
\Theta_{i13} &= \kappa\widetilde{\alpha} \widetilde{R}_i^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu + \frac{\varepsilon G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} (I + \varphi_a)}{2}, \\
\Theta_{i14} &= \kappa\widetilde{\alpha} \widetilde{A}_i^{\mathrm{T}} P_{1i} \overline{B}_{1i} + \kappa\widetilde{\alpha} \widetilde{R}_i^{\mathrm{T}} P_{2i} \overline{B}_{2i} + \left(\overline{C}_i^z\right)^{\mathrm{T}} \overline{D}_i^w, \\
\Theta_{i22} &= \kappa\widetilde{\alpha} \left(\widetilde{A}_i^d\right)^{\mathrm{T}} P_{1i} \widetilde{A}_i^d - \widetilde{\alpha} Q + \left(\overline{C}_i^d\right)^{\mathrm{T}} \overline{C}_i^d, \\
\Theta_{i24} &= \kappa\widetilde{\alpha} \left(\widetilde{A}_i^d\right)^{\mathrm{T}} P_{1i} \overline{B}_{1i} + \left(\overline{C}_i^d\right)^{\mathrm{T}} \overline{D}_i^w, \\
\Theta_{i33} &= \kappa\widetilde{\alpha} \varphi_\mu \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c \varphi_\mu \\
&\quad + \kappa\widetilde{\alpha} \sum_{f=1}^{m} \left(\delta_{1f} + \delta_{3f}\right) E_f \left(B_i^c\right)^{\mathrm{T}} P_{2i} B_i^c E_f, \\
\Theta_{i34} &= \kappa\widetilde{\alpha} \varphi_\mu \left(B_i^c\right)^{\mathrm{T}} P_{2i} \overline{B}_{2i}, \\
\Theta_{i44} &= -\gamma^2 I + \kappa\widetilde{\alpha} \overline{B}_i^{\mathrm{T}} P_i \overline{B}_i + \left(\overline{D}_i^w\right)^{\mathrm{T}} \overline{D}_i^w, \\
\Theta_{ij11} &= \widehat{\Theta}_{ij11} - \varepsilon G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \varphi_a \overline{C}_i G - \widetilde{\beta} P_i, \\
\widehat{\Theta}_{ij11} &= \kappa\widetilde{\alpha} \widetilde{A}_{ij}^{\mathrm{T}} P_{1i} \widetilde{A}_{ij} + \kappa\widetilde{\alpha} \widetilde{R}_{ij}^{\mathrm{T}} P_{2i} \widetilde{R}_{ij} + \widetilde{\alpha} Q \\
&\quad + \kappa\widetilde{\alpha} \sum_{f=1}^{m} \left(\delta_{2f} + \delta_{3f}\right) G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} E_f \left(B_j^c\right)^{\mathrm{T}} P_{2i} B_j^c E_f \overline{C}_i G \\
&\quad + \left(\overline{C}_{ij}^z\right)^{\mathrm{T}} \overline{C}_{ij}^z, \\
\Theta_{ij12} &= \kappa\widetilde{\alpha} \widetilde{A}_{ij}^{\mathrm{T}} P_{1i} \widetilde{A}_i^d + \left(\overline{C}_{ij}^z\right)^{\mathrm{T}} \overline{C}_i^d, \\
\Theta_{ij13} &= \kappa\widetilde{\alpha} \widetilde{R}_{ij}^{\mathrm{T}} P_{2i} B_{ij}^c \varphi_\mu + \frac{\varepsilon G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} (I + \varphi_a)}{2}, \\
\Theta_{ij14} &= \kappa\widetilde{\alpha} \widetilde{A}_{ij}^{\mathrm{T}} P_{1i} \overline{B}_{1i} + \kappa\widetilde{\alpha} \widetilde{R}_{ij}^{\mathrm{T}} P_{2i} \overline{B}_{2ij} + \left(\overline{C}_{ij}^z\right)^{\mathrm{T}} \overline{D}_i^w, \\
\Theta_{ij33} &= \kappa\widetilde{\alpha} \varphi_\mu \left(B_j^c\right)^{\mathrm{T}} P_{2i} B_j^c \varphi_\mu \\
&\quad + \kappa\widetilde{\alpha} \sum_{f=1}^{m} \left(\delta_{1f} + \delta_{3f}\right) E_f \left(B_j^c\right)^{\mathrm{T}} P_{2i} B_j^c E_f, \\
\Theta_{ij34} &= \kappa\widetilde{\alpha} \varphi_\mu \left(B_j^c\right)^{\mathrm{T}} P_{2i} \overline{B}_{2ij}, \\
\Theta_{ij44} &= -\gamma^2 I + \kappa\widetilde{\alpha} \overline{B}_{ij}^{\mathrm{T}} P_i \overline{B}_{ij} + \left(\overline{D}_i^w\right)^{\mathrm{T}} \overline{D}_i^w.
\end{aligned} \tag{48}$$

Proof. In order to simplify the proof, we only consider closed-loop system (10). The results of system (9) can be obtained in a similar way. The LLF is chosen as the same to the one in the proof of Theorem 7.

Because (44) holds (i.e., $\Theta_i < 0$) and $\kappa > 1$, we know that

$$\widetilde{\Theta}_i < \begin{bmatrix} \tilde{\alpha} P_i & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix}. \tag{49}$$

Because (49) holds and $0 < \tilde{\alpha} < 1$, we obtain $\Xi_i < 0$ (in (21)). Thus, it can be concluded that $\Xi_i < 0$ if $\Theta_i < 0$. Similarly, we know that $\Xi_{ij} < 0$ if $\Theta_{ij} < 0$. According to Theorem 7, conditions (43), (46), and (47) with $\Xi_i < 0$ and $\Xi_{ij} < 0$ guarantee the mean square stability of systems (9)~(10) and ellipsoid $\Lambda(P_i, Q)$ is contained in the mean square domain of attraction $\mathbb{Z}$.

Next, we pay our attention to the H_∞ performance analysis. Define

$$\begin{aligned}
J_k &= \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l\right)\right\} \\
&= \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \theta_k \tilde{\alpha} V\left(r_{l+1}, l+1\right)\right.\right. \\
&\qquad + \theta_k \tilde{\beta} V\left(r_{l+1}, l+1\right) - \theta_k \tilde{\alpha} V\left(r_l, l\right) \\
&\qquad \left.\left. - \theta_k \tilde{\beta} V\left(r_l, l\right)\right)\right\} \\
&\quad - \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(\theta_k \tilde{\alpha} V\left(r_{l+1}, l+1\right) + \theta_k \tilde{\beta} V\left(r_{l+1}, l+1\right)\right.\right. \\
&\qquad \left.\left. - \theta_k \tilde{\alpha} V\left(r_l, l\right) - \theta_k \tilde{\beta} V\left(r_l, l\right)\right)\right\} \\
&= \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\right.\right. \\
&\qquad \left.\left. \times \left(V\left(r_{l+1}, l+1\right) - V\left(r_l, l\right)\right)\right)\right\} \\
&\quad - \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\left(V\left(r_{l+1}, l+1\right) - V\left(r_l, l\right)\right)\right\} \\
&= \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\right.\right. \\
&\qquad \left.\left. \times \left(V\left(r_{l+1}, l+1\right) - V\left(r_l, l\right)\right)\right)\right\} \\
&\quad - \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\mathrm{E}\left\{V\left(r_k, k\right)\right\} \\
&\le \mathrm{E}\left\{\sum_{l=0}^{k-1}\left(z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\right.\right. \\
&\qquad \left.\left. \times \left(V\left(r_{l+1}, l+1\right) - V\left(r_l, l\right)\right)\right)\right\}.
\end{aligned} \tag{50}$$

Since

$$\begin{aligned}
&\mathrm{E}\left\{z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\right. \\
&\quad \left. \times \left(V\left(r_{l+1}, l+1\right) - V\left(r_l, l\right)\right)\right\} \\
&\quad = z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right) \\
&\qquad \times \left(x_{l+1}^{\mathrm{T}} P\left(r_{l+1}\right) x_{l+1} - x_l^{\mathrm{T}} P\left(r_l\right) x_l + x_l^{\mathrm{T}} Q x_l - x_{l-d}^{\mathrm{T}} Q x_{l-d}\right) \\
&\quad \le z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right) \\
&\qquad \times \left(\kappa x_{l+1}^{\mathrm{T}} P\left(r_l\right) x_{l+1} - x_l^{\mathrm{T}} P\left(r_l\right) x_l + x_l^{\mathrm{T}} Q x_l - x_{l-d}^{\mathrm{T}} Q x_{l-d}\right),
\end{aligned} \tag{51}$$

conditions (44) and (45) imply that

$$\begin{aligned}
&z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \tilde{\alpha}\left(\kappa x_{l+1}^{\mathrm{T}} P\left(r_l\right) x_{l+1} - x_l^{\mathrm{T}} P\left(r_l\right) x_l\right. \\
&\qquad \left. + x_l^{\mathrm{T}} Q x_l - x_{l-d}^{\mathrm{T}} Q x_{l-d}\right) \le 0, \\
&z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \tilde{\beta}\left(\kappa x_{l+1}^{\mathrm{T}} P\left(r_l\right) x_{l+1} - x_l^{\mathrm{T}} P\left(r_l\right) x_l\right. \\
&\qquad \left. + x_l^{\mathrm{T}} Q x_l - x_{l-d}^{\mathrm{T}} Q x_{l-d}\right) \le 0.
\end{aligned} \tag{52}$$

Therefore, with the property of polytope [30], we get

$$\begin{aligned}
&z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right) \\
&\quad \times \left(\kappa x_{l+1}^{\mathrm{T}} P\left(r_l\right) x_{l+1} - x_l^{\mathrm{T}} P\left(r_l\right) x_l + x_l^{\mathrm{T}} Q x_l\right. \\
&\quad \left. - x_{l-d}^{\mathrm{T}} Q x_{l-d}\right) \le 0.
\end{aligned} \tag{53}$$

According to (51) and the above inequality, one further gets

$$\begin{aligned}
&\mathrm{E}\left\{z_l^{\mathrm{T}} z_l - \gamma^2 w_l^{\mathrm{T}} w_l + \left(\theta \tilde{\alpha} + (1-\theta)\tilde{\beta}\right)\right. \\
&\quad \left. \times \left(V\left(r_{l+1}, l+1\right) - V\left(r_l, l\right)\right)\right\} \le 0.
\end{aligned} \tag{54}$$

It follows immediately that $J_k < 0$. Therefore, (16) holds, which means a prescribed H_∞ disturbance attenuation level γ is obtained. Thus, the proof is completed. □

According to the regional H_∞ performance analysis presented in Theorem 11, a solution to the asynchronous H_∞ control for the underlying system with randomly occurring sensor nonlinearity and missing measurements is given in what follows. The dynamic output feedback gain can be obtained by solving a set of linear matrix inequalities (LMIs).

Theorem 12. *Consider switched system (1) and sensors model (2) and let* $\tilde{\alpha} = 1 - \alpha$ $(0 < \alpha < 1)$, $\tilde{\beta} = 1 + \beta$ $(\beta \ge 0)$, $\kappa > 1$, $\gamma > 0$ *be given constants. If there exist matrices* $P_{1i} = P_{1i}^{\mathrm{T}} > 0$, $P_{2i} = P_{2i}^{\mathrm{T}} > 0$, $Q_1 = Q_1^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$, $W_i = W_i^{\mathrm{T}} > 0$, X_i,

Y_i, and C_i^c and scalar a_f, where $i = 1, 2, \ldots, s$, $f = 1, 2, \ldots, m$, such that

$$P_{1i} W_i = I, \tag{55}$$

$$0 < a_f < 1, \tag{56}$$

$$\begin{bmatrix} -P_{1i} & 0 & \left(C_i^f\right)^{\mathrm{T}} a_f \\ * & -P_{2i} & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{57}$$

$$\begin{bmatrix} -Q_1 & 0 & \left(C_i^f\right)^{\mathrm{T}} a_f \\ * & -Q_2 & 0 \\ * & * & -I \end{bmatrix} < 0; \tag{58}$$

$$\begin{bmatrix} \Upsilon_{1i} & 0 & \Upsilon_{2i} & \Upsilon_{3i} & \Upsilon_{4i} & \Upsilon_{5i} & \Upsilon_{6i} \\ * & -\tilde{\gamma}^2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P_{2i} & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{P}_{2i} & 0 & 0 & 0 \\ * & * & * & * & -\tilde{P}_{2i} & 0 & 0 \\ * & * & * & * & * & -W_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{59}$$

$$\begin{bmatrix} \Upsilon_{1i} & 0 & \Upsilon_{2ij} & \Upsilon_{3ij} & \Upsilon_{4ij} & \Upsilon_{5ij} & \Upsilon_{6ij} \\ * & -\tilde{\gamma}^2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P_{2i} & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{P}_{2i} & 0 & 0 & 0 \\ * & * & * & * & -\tilde{P}_{2i} & 0 & 0 \\ * & * & * & * & * & -W_i & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad i \neq j, \tag{60}$$

$$\begin{bmatrix} P_{1i} - \kappa P_{1j} & 0 \\ 0 & P_{2i} - \kappa P_{2j} \end{bmatrix} \leq 0, \quad i \neq j. \tag{61}$$

then the system is regional mean square stable with given disturbance attenuation level and ellipsoid $\Lambda(P_i, Q)$ is contained in the mean square domain of attraction $\mathbb{Z}$ for switching signal r_k with ADT:

$$\begin{gathered} \tau_a > \tau_a^* = -\frac{\ln \kappa}{\ln\left(1 - \theta\alpha + (1-\theta)\beta\right)}, \\ -1 < -\theta\alpha + (1-\theta)\beta < 0, \end{gathered} \tag{62}$$

where

$$\Upsilon_{1i} = \begin{bmatrix} \Upsilon_{1i11} & 0 & 0 & 0 & \Upsilon_{1i15} \\ * & -\tilde{\alpha} P_{2i} & 0 & 0 & 0 \\ * & * & -\tilde{\alpha} Q_1 & 0 & 0 \\ * & * & * & -\tilde{\alpha} Q_2 & 0 \\ * & * & * & * & -I \end{bmatrix}, \qquad \Upsilon_{1i11} = -\tilde{\alpha} P_{1i} + \sum_{f=1}^{m} \left(C_i^f\right)^{\mathrm{T}} a_f C_i^f,$$

$$\Upsilon_{1i15} = \frac{\overline{C}_i^{\mathrm{T}}}{2} + \frac{\sum_{f=1}^{m} \left(C_i^f\right)^{\mathrm{T}} a_f G_f}{2}, \quad G_f = \Big[\overbrace{0, \ldots 0,}^{f-1} 1, \overbrace{0, \ldots 0,}^{m-f}\Big]^{\mathrm{T}},$$

$$\tilde{\gamma}^2 = \operatorname{diag}\left\{\gamma^2 I, \gamma^2 I\right\},$$

$$\Upsilon_{2i} = \left[\kappa^{1/2} \tilde{\alpha}^{1/2} Y_i (I - \varphi_\mu) \varphi_\vartheta \overline{C}_i \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} X_i \;\; 0 \;\; 0 \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} Y_i \varphi_\mu \;\; 0 \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} Y_i \overline{D}_i\right]^{\mathrm{T}},$$

$$\Upsilon_{3i} = \begin{bmatrix} \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{11} Y_i E_1 \overline{C}_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{1m} Y_i E_m \overline{C}_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \qquad \tilde{P}_{2i} = \operatorname{diag}\Big\{\overbrace{P_{2i}, \ldots, P_{2i}}^{m}\Big\},$$

$$\Upsilon_{4i} = \begin{bmatrix} 0 & 0 & 0 & 0 & \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{21} Y_i E_1 \overline{C}_i & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{2m} Y_i E_m \overline{C}_i & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \qquad \nu_{1f} = \sqrt{\delta_{2f} + \delta_{3f}}, \qquad \nu_{2f} = \sqrt{\delta_{1f} + \delta_{3f}},$$

$$\Upsilon_{5i} = \left[\kappa^{1/2} \tilde{\alpha}^{1/2} A_i \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} B_{1i}^u C_i^c \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} A_i^d \;\; 0 \;\; 0 \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} B_i^w \;\; 0\right]^{\mathrm{T}},$$

$$\Upsilon_{6i} = \left[C_i^z \;\; B_{2i}^u C_i^c \;\; C_i^d \;\; 0 \;\; 0 \;\; D_i^w \;\; 0\right]^{\mathrm{T}},$$

$$\Upsilon_{2ij} = \left[\kappa^{1/2} \tilde{\alpha}^{1/2} Y_j (I - \varphi_\mu) \varphi_\vartheta \overline{C}_i \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} X_j \;\; 0 \;\; 0 \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} Y_j \varphi_\mu \;\; 0 \;\; \kappa^{1/2} \tilde{\alpha}^{1/2} Y_j \overline{D}_i\right]^{\mathrm{T}},$$

$$\Upsilon_{3ij} = \begin{bmatrix} \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{11} Y_j E_1 \overline{C}_i & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{1m} Y_j E_m \overline{C}_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \qquad \Upsilon_{4ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{21} Y_j E_1 \overline{C}_i & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \kappa^{1/2} \tilde{\alpha}^{1/2} \nu_{2m} Y_j E_m \overline{C}_i & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\Upsilon_{5ij} = \left[\kappa^{1/2}\widetilde{\alpha}^{1/2}A_i \ \ \kappa^{1/2}\widetilde{\alpha}^{1/2}B_{1i}^{u}C_j^{c} \ \ \kappa^{1/2}\widetilde{\alpha}^{1/2}A_i^{d} \ \ 0 \ \ 0 \ \ \kappa^{1/2}\widetilde{\alpha}^{1/2}B_i^{w} \ \ 0\right]^{\mathrm{T}},$$

$$\Upsilon_{6ij} = \left[C_i^{z} \ \ B_{2i}^{u}C_j^{c} \ \ C_i^{d} \ \ 0 \ \ 0 \ \ D_i^{w} \ \ 0\right]^{\mathrm{T}}. \tag{63}$$

The designed controller gain can be obtained as $A_i^c = X_i P_{2i}^{-1}$ *and* $B_i^c = Y_i P_{2i}^{-1}$, C_i^c.

Proof. Constraint (55) is established by denoting $W_i = P_{1i}^{-1}$. Inequality (56) can be directly obtained from $0 < \varphi_a < I$ because φ_a is a diagonal matrix. Noting that $\Lambda(P_i, Q) \subset \mathbb{F}(\varphi_a \overline{C}_i G)$, we have

$$\begin{aligned}
&\Lambda(P_i) \subset \mathbb{F}\left(\varphi_a \overline{C}_i G\right) \\
&\quad\Longleftrightarrow \left\{\eta \mid \eta^{\mathrm{T}} P_i \eta \le 1\right\} \subset \left\{\eta \mid \eta^{\mathrm{T}} G^{\mathrm{T}} \left(C_i^f\right)^{\mathrm{T}} a_f a_f C_i^f G \eta \le 1\right\} \\
&\quad\Longleftrightarrow P_i \ge G^{\mathrm{T}} \left(C_i^f\right)^{\mathrm{T}} a_f a_f C_i^f G \\
&\quad\Longleftrightarrow -P_i + G^{\mathrm{T}} \left(C_i^f\right)^{\mathrm{T}} a_f a_f C_i^f G \le 0 \\
&\quad\Longleftrightarrow \begin{bmatrix} -P_i & G^{\mathrm{T}}\left(C_i^f\right)^{\mathrm{T}} a_f \\ * & -I \end{bmatrix} \le 0 \Longleftrightarrow \text{condition (57)}.
\end{aligned} \tag{64}$$

With the property of polytope [30], we obtain $\Lambda(Q) \subset \mathbb{F}(\varphi_a \overline{C}_i G)$ which yields condition (58).

Conditions (61) and (62) come directly from (46) and (47).

Next, we pay our attention to obtaining (59). By using Schur complement to (44), one has

$$\Theta_i = \begin{bmatrix} \overline{\Upsilon}_{1i} & \overline{\Upsilon}_{2i} & \overline{\Upsilon}_{3i} & \overline{\Upsilon}_{4i} & \overline{\Upsilon}_{5i} \\ * & -P_{2i} & 0 & 0 & 0 \\ * & * & -\widetilde{P}_{2i} & 0 & 0 \\ * & * & * & -\widetilde{P}_{2i} & 0 \\ * & * & * & * & -P_{1i} \end{bmatrix}, \tag{65}$$

where

$$\overline{\Upsilon}_{1i} = \begin{bmatrix} \overline{\Upsilon}_{1i11} & \overline{\Upsilon}_{1i12} & \overline{\Upsilon}_{1i13} & \overline{\Upsilon}_{1i14} & 0 \\ * & \overline{\Upsilon}_{1i22} & 0 & \overline{\Upsilon}_{1i24} & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & \overline{\Upsilon}_{1i44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\overline{\Upsilon}_{1i11} = \kappa\widetilde{\alpha}\widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i - \widetilde{\alpha} P_i + \left(\overline{C}_i^{z}\right)^{\mathrm{T}} \overline{C}_i^{z} - G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} \varphi_a \overline{C}_i G,$$

$$\overline{\Upsilon}_{1i12} = \kappa\widetilde{\alpha}\widetilde{A}_i^{\mathrm{T}} P_{1i} \widetilde{A}_i^{d} + \left(\overline{C}_i^{z}\right)^{\mathrm{T}} \overline{C}_i^{d},$$

$$\overline{\Upsilon}_{1i13} = \frac{G^{\mathrm{T}} \overline{C}_i^{\mathrm{T}} (I + \varphi_a)}{2},$$

$$\overline{\Upsilon}_{1i14} = \kappa\widetilde{\alpha}\widetilde{A}_i^{\mathrm{T}} P_{1i} B_i^{w} \left(\overline{C}_i^{z}\right)^{\mathrm{T}} D_i^{w},$$

$$\overline{\Upsilon}_{1i22} = -\widetilde{\alpha} Q + \left(\overline{C}_i^{d}\right)^{\mathrm{T}} \overline{C}_i^{d},$$

$$\overline{\Upsilon}_{1i24} = \kappa\widetilde{\alpha}\left(\widetilde{A}_i^{d}\right)^{\mathrm{T}} P_{1i} B_i^{w} + \left(\overline{C}_i^{d}\right)^{\mathrm{T}} D_i^{w},$$

$$\overline{\Upsilon}_{1i44} = -\gamma^2 I + \kappa\widetilde{\alpha}\left(B_i^{w}\right)^{\mathrm{T}} P_{1i} B_i^{w} + \left(D_i^{w}\right)^{\mathrm{T}} D_i^{w},$$

$$\overline{\Upsilon}_{2i} = \left[\kappa^{1/2}\widetilde{\alpha}^{1/2} P_{2i} \overline{R}_i \ \ 0 \ \ \kappa^{1/2}\widetilde{\alpha}^{1/2} P_{2i} B_i^{c} \varphi_{\mu} \ \ 0 \ \ \kappa^{1/2}\widetilde{\alpha}^{1/2} P_{2i} B_i^{c} \overline{D}_i\right]^{\mathrm{T}},$$

$$\overline{\Upsilon}_{3i} = \begin{bmatrix} \kappa^{1/2}\widetilde{\alpha}^{1/2}\nu_{11} P_{2i} B_i^{c} E_1 \overline{C}_i G & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \kappa^{1/2}\widetilde{\alpha}^{1/2}\nu_{1m} P_{2i} B_i^{c} E_m \overline{C}_i G & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\overline{\Upsilon}_{4i} = \begin{bmatrix} 0 & 0 & \kappa^{1/2}\widetilde{\alpha}^{1/2}\nu_{21} P_{2i} B_i^{c} E_1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \kappa^{1/2}\widetilde{\alpha}^{1/2}\nu_{2m} P_{2i} B_i^{c} E_m & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\overline{\Upsilon}_{5i} = \left[0 \ \ \kappa^{1/2}\widetilde{\alpha}^{1/2} P_{1i} \widetilde{A}_i^{d} \ \ 0 \ \ 0 \ \ 0\right]^{\mathrm{T}}. \tag{66}$$

Noting that

$$\widetilde{A}_i = \left[A_i \ \ 0\right], \qquad \widetilde{A}_i^{d} = \left[A_i^{d} \ \ 0\right],$$

$$\overline{B}_i = \begin{bmatrix} \overline{B}_{1i} \\ \overline{B}_{2i} \end{bmatrix} = \begin{bmatrix} B_i^{w} & 0 \\ 0 & B_i^{c} \overline{D}_i \end{bmatrix},$$

$$\overline{C}_i^{\mathrm{T}} \varphi_a \overline{C}_i = \sum_{f=1}^{m} \left(C_i^f\right)^{\mathrm{T}} a_f C_i^f \qquad \overline{C}_i^{\mathrm{T}} \varphi_a = \sum_{f=1}^{m} \left(C_i^f\right)^{\mathrm{T}} a_f G_f, \tag{67}$$

and by repeatedly using Schur complement, we can get inequality (59). The detailed proof is omitted owing to limited space. Following the same lines of the proof of (59), we can get inequality (60). Thus, the proof is completed. □

Remark 13. It should be noted that the conditions stated in Theorem 12 are a set of LMIs with matrix inverse constraints. Although they are nonconvex, which are difficult to solve, we develop the following cone complementary linearization (CCL) algorithm to solve such matrix inequalities.

Algorithm 14. Consider the following steps.

Step 1. Initialize index $g = 0$ and specify the number of iterations g_N. Find matrices $P_{1i}(g)$ and $W_i(g)$, such that LMIs (56)~(61) and

$$\begin{bmatrix} P_{1i} & I \\ I & W_i \end{bmatrix} \ge 0, \tag{68}$$

hold.

Step 2. For P_{1i} and W_i obtained in the previous step, find $P_{1i}(g+1)$ and $W_i(g+1)$ such that the following minimization problem has solutions:

$$\begin{aligned} &\text{Minimize} \quad \{\operatorname{trace}(P_{1i}(g)W_i + W_i(g)P_{1i})\} \\ &\text{subject to} \quad \text{LMIs } (56) \sim (61) \text{ and } (68). \end{aligned} \tag{69}$$

Step 3. Check whether the solutions satisfy (59) and (60) via replacing W_i by P_{1i}^{-1} and check whether condition (55) is satisfied as precise as possible. If (59) and (60) are satisfied, Step 2 ceases and returns the value of

$$P_{1i}, P_{2i}, Q_1, Q_2, X_i, Y_i, C_i^c, a_f. \tag{70}$$

Else, increment $g = g + 1$ and get back to Step 2.

When $g = g_N$, iterative process stops and feasible solutions could not be found.

4. Numerical Example

Consider a discrete-time switched time-delay system (1) with matrix parameters:

$$\begin{gathered} A_1 = \begin{bmatrix} 0.88 & -0.05 \\ 0.40 & -0.72 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1.00 & 0.24 \\ 0.80 & 0.32 \end{bmatrix}, \\ A_1^d = \begin{bmatrix} -0.20 & 0.10 \\ 0.20 & 0.15 \end{bmatrix}, \qquad A_2^d = \begin{bmatrix} -0.60 & 0.40 \\ 0.20 & 0.60 \end{bmatrix}, \\ B_1^w = \begin{bmatrix} 0.01 \\ 0.09 \end{bmatrix}, \qquad B_2^w = \begin{bmatrix} 0.02 \\ 0.14 \end{bmatrix}, \\ D_1^w = 0.2, \qquad D_2^w = 0.3, \\ B_{11}^u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad B_{12}^u = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \\ B_{21}^u = 0.4, \qquad B_{22}^u = -0.5. \end{gathered} \tag{71}$$

The considered sensor model is given with parameters:

$$\begin{gathered} C_1^1 = [1 \;\; 0.5], \qquad C_1^2 = [1 \;\; 1], \\ C_2^1 = [0.5 \;\; 0.4], \qquad C_2^2 = [0.7 \;\; 0.6], \\ \overline{D}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \qquad \overline{D}_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \end{gathered} \tag{72}$$

The exogenous disturbance is taken as $w_k = 0.5e^{-0.5k}$. The measurement noises are taken as $v_k^1 = v_k^2 = 2\cos(0.3k)/5(k+1)$. Time delay is taken as $d = 3$.

The probabilities are taken as $\mu_1 = 0.3$, $\mu_2 = 0.4$, $\vartheta_1 = 0.7$, $\vartheta_2 = 0.75$, and $\theta = 0.7$. The given disturbance attenuation level is $\gamma = 1.2$. Some other parameters are chosen as $\kappa = 1.01$, $\alpha = 0.1$, and $\beta = 0.21$. The initial system state and controller state are taken as $x_0 = [0.3 \quad 0.1]^{\mathrm{T}}$ and $x_{c0} = [0.3 \quad 0.1]^{\mathrm{T}}$.

By employing Algorithm 14, we obtain the controller gain:

$$\begin{gathered} A_1^c = \begin{bmatrix} 0.4299 & 0.0000 \\ 0.0000 & 0.4299 \end{bmatrix}, \qquad A_2^c = \begin{bmatrix} 0.4263 & -0.0039 \\ -0.0039 & 0.4259 \end{bmatrix}, \\ B_1^c = \begin{bmatrix} 0.0011 & -0.0003 \\ -0.0003 & 0.0018 \end{bmatrix}, \\ B_2^c = 10^{-3} \times \begin{bmatrix} 0.6379 & -0.1280 \\ -0.1280 & 0.4329 \end{bmatrix}, \\ C_1^c = 10^{-3} \times [-0.6263 \;\; -0.5686], \\ C_2^c = [0.0047 \;\; 0.0053]. \end{gathered} \tag{73}$$

The ellipsoid parameters are obtained as

$$\begin{gathered} P_1 = \begin{bmatrix} 1.3903 & -0.0375 & 0 & 0 \\ -0.0375 & 0.9906 & 0 & 0 \\ 0 & 0 & 1.3942 & -0.0264 \\ 0 & 0 & -0.0264 & 0.9873 \end{bmatrix}, \\ P_2 = \begin{bmatrix} 9.4604 & -2.1459 \times 10^{-5} & 0 & 0 \\ -2.1459 \times 10^{-5} & 9.4604 & 0 & 0 \\ 0 & 0 & 9.4605 & -2.1028 \times 10^{-5} \\ 0 & 0 & -2.1028 \times 10^{-5} & 9.4605 \end{bmatrix}, \\ Q_1 = \begin{bmatrix} 9.2132 & -0.2616 \\ -0.2616 & 9.9121 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 9.0000 & 0 \\ 0 & 9.0000 \end{bmatrix}. \end{gathered} \tag{74}$$

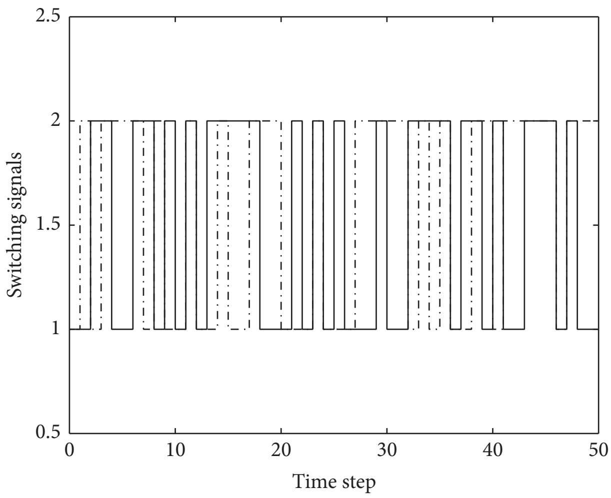

Figure 1: Asynchronous switching signals (the solid line represents the switching of the system modes and the dash-dotted line represents the switching of the controllers).

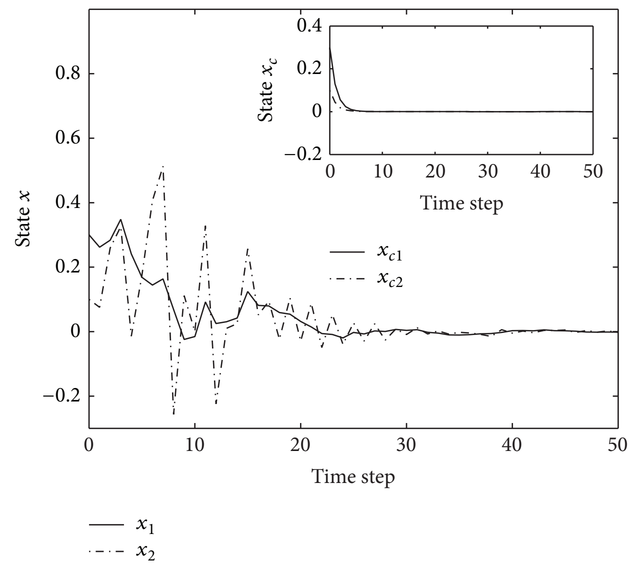

Figure 2: State responses under the asynchronous control (system state and controller state).

Since we have

$$P_{11}W_1 = \begin{bmatrix} 1.0220 & 0.0134 \\ 0.0091 & 1.0056 \end{bmatrix},$$
$$P_{12}W_2 = \begin{bmatrix} 1.8536 & 0.6470 \\ 0.4484 & 1.3399 \end{bmatrix}, \quad (75)$$

we could find that the obtained controller is feasible.

The mean square domain of attraction could be described as

system state: $\Lambda(P_{1i}, Q_1) = \{x \in \mathbb{R}^n : x^{\mathrm{T}}(\lambda_1 P_{11} + \lambda_2 P_{12} + \lambda_3 Q_1)x \le 1\}$,

controller state: $\Lambda(P_{2i}, Q_2) = \{x_c \in \mathbb{R}^n : x_c^{\mathrm{T}}(\lambda_1 P_{21} + \lambda_2 P_{22} + \lambda_3 Q_2)x_c \le 1\}$.

The switching signals generated in Figure 1 depict the operating of the system modes and controllers, where the solid line represents the switching of the system modes and the dash-dotted line represents the switching of the controllers. The eigenvalues of modes A_1 and A_2 could be obtained as

$$\operatorname{eig}(A_1) = \begin{bmatrix} 0.8674 \\ -0.7074 \end{bmatrix}, \qquad \operatorname{eig}(A_2) = \begin{bmatrix} 1.2146 \\ 0.1054 \end{bmatrix}; \quad (76)$$

therefore the input-free switched system (with $u_k = 0$) could be unstable. By solving ADT constraint (62), we get $\tau_a^* = 1.4165$. We also could obtain that the real ADT of the switching mode is $\tau_a = 1.7857$, which satisfies ADT constraint $\tau_a > \tau_a^*$. Figure 2 shows the trajectories of system state and controller state under the asynchronous control move. The controlled system is mean square stable, which satisfactorily justifies the effectiveness of the proposed control method. The mean square domains of attraction including system states and controller states are shown in Figure 3.

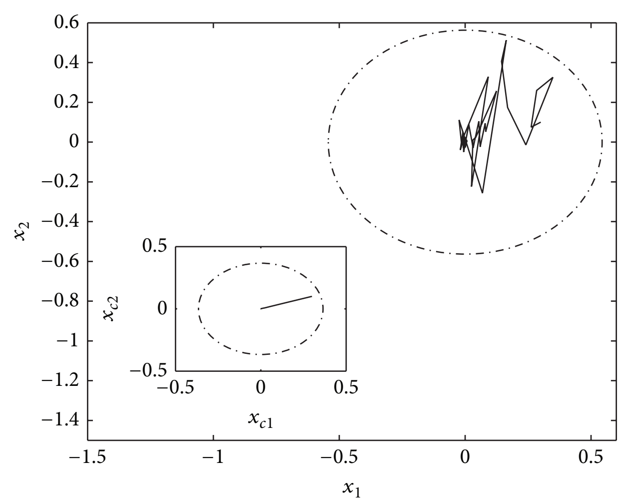

Figure 3: Mean square domains of attraction.

5. Conclusions

The problems of asynchronous H_∞ dynamic output feedback control for a class of time-delay switched systems subject to sensor nonlinearity and missing measurements are investigated in this study. New results on the regional stability, l_2 gain analysis, and regional controller design for the underlying system are given by allowing the Lyapunov-like function to increase with a random probability during the unmatched period of the switching mode and controller. A convex combination of a set of ellipsoids is employed to estimate the domain of attraction of the system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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