

Research Article

Joint Application of Bilinear Operator and F-Expansion Method for (2 + 1)-Dimensional Kadomtsev-Petviashvili Equation

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The bilinear operator and F-expansion method are applied jointly to study (2 + 1)-dimensional Kadomtsev-Petviashvili (KP) equation. An exact cusped solitary wave solution is obtained by using the extended single-soliton test function and its mechanical feature which blows up periodically in finite time for cusped solitary wave is investigated. By constructing the extended double-soliton test function, a new type of exact traveling wave solution describing the assimilation of solitary wave and periodic traveling wave is also presented. Our results validate the effectiveness for joint application of the bilinear operator and F-expansion method.

1. Introduction

In the past few decades, much effort has been devoted to the investigation of dynamical behaviours of nonlinear evolution equation. Traveling wave, one of the spatial dynamics analyses, always plays a significant role and attracts more and more of the experts' and scholars' attention. There has been much literature on traveling wave of nonlinear evolution equation due to the abundant type of nonlinear traveling wave and some well-known concepts (e.g., solitary wave [1-3], periodic wave [4, 5], kink wave [6], cusped wave [7], etc.) have been used and generalized extensively. To understand the inherent essence and evolution mechanism of these nonlinear traveling waves, seeking the exact traveling wave solutions has been recognized. In recent years, much efforts have been spent on this task and many significant methods have been established such as variational iteration method [8], homotopy perturbation method [9, 10], Fan subequation method [11, 12], exp-function method [13], Hirota's bilinear method [14, 15], G'/G-expansion method [16, 17], and F-expansion method [18–21]. In most of the existing literature, authors always study the improvement of the adopted method to obtain more forms of solutions. However, to the best of our knowledge, how to realize the joint applications of different methods is still challenging and open work. In this paper, we choose the classical nonlinear evolution equation,

(2 + 1)-dimensional Kadomtsev-Petviashvili (KP) equation, as an example to validate the effectiveness of the proposed method.

The (2 + 1)-dimensional KP equation [14] is written as

$$u_{xt} - u_{xxxx} - 3\left(u^2\right)_{xx} = 3p^2 u_{yy},\tag{1}$$

where $u : R_x \times R_y \times R_t \rightarrow R$ and $p^2 = \pm 1$ measure the positive and negative transverse dispersion effects. Equation (1) with $p^2 = 1$ and $p^2 = -1$ are called KP-I equation and KP-II equation, respectively. In recent years, kinds of research fields and solution types of KP equation have been studied extensively in various aspects [22–24]; exact multiple solitary wave solution, periodic solitary wave solution, quasiperiodic solutions, and so forth have been obtained. In the past works, the resonance interaction phenomenon between periodic solitary wave and line soliton was investigated and spatial-temporal bifurcation and deflexion of solitary wave were exhibited [14, 15].

The rest of the paper is organized as follows. In Section 2, we combine the bilinear operator with F-expansion method to solve KP equation. By single-soliton test approach, a new type of solitary wave solution which possesses cusped structure is obtained. In Section 3, an exact expression describing the interaction of solitary wave and periodic traveling wave

is obtained by the extending of double-soliton test approach. Conclusions are drawn in Section 4.

2. Cusped Solitary Wave Solution

Introduce an independent transformation

$$u(x, y, t) = 2(\ln F)_{xx},$$
 (2)

where F = F(x, y, t) is an unknown real function. Substituting (2) into (1), we can obtain the bilinear form of KP equation:

$$\left(D_x D_t - D_x^4 - D_x^2 - p^2 D_y^2\right) F \cdot F = 0, \tag{3}$$

where Hirota's bilinear operator "D" is defined by

$$D_{x}^{k} D_{y}^{m} D_{t}^{n} f(x, y, t) \cdot g(x, y, t)$$

$$= \frac{\partial^{k}}{\partial s^{k}} \frac{\partial^{m}}{\partial \sigma^{m}} \frac{\partial^{n}}{\partial \tau^{n}}$$

$$\times \left(f(x + s, y + \sigma, t + \tau) g(x - s, y - \sigma, t - \tau) \right) \Big|_{s=0,\sigma=0,\tau=0}.$$
(4)

Consider the traveling wave transformation

$$F(x, y, t) = F(\xi), \qquad \xi = kx + ly + \omega t + \gamma_0, \qquad (5)$$

where k, l, and ω are nonzero constants and γ_0 is a phase constant. Equation (3) is converted to an ODE:

$$(k^{2} + l^{2}p^{2} - k\omega)F'^{2} - 3k^{4}F''^{2} + 4k^{4}F'F''' - F((k^{2} + l^{2}p^{2} - k\omega)F'' + k^{4}F^{(4)}) = 0.$$
(6)

Generally, letting $F(\xi) = 1 + e^{\xi}$, we can obtain an exact single solitary wave solution of bilinear equation (3). In this case, we consider the extended single-soliton test function

$$F(\xi) = \sum_{j=1}^{n} a_j G^j(\xi) ,$$
 (7)

where $G(\xi)$ satisfies the following auxiliary equation:

$$(G'(\xi))^{2} = \sum_{k=0}^{r} b_{j} G^{j}(\xi) .$$
(8)

The coefficients a_j , b_k (j = 0, 1, ..., n; k = 0, 1, ..., r) are undetermined constants and *n* and *r* are undetermined positive integers. To determine the values of *n* and *r*, balancing the lowest order nonlinear term with the highest nonlinear terms in (6), we have a relation of *n* and *r*:

$$2n + r - 2 = 2n + 2r - 4. \tag{9}$$

From (9), we conclude that r = 2 and n is an arbitrary positive integer. As a test, n = 2 is taken into account; (7) and (8) are reduced into

$$F(\xi) = a_0 + a_1 G(\xi) + a_2 G^2(\xi),$$

$$\left(G'(\xi)\right)^2 = h_0 + h_1 G(\xi) + h_2 G^2(\xi),$$
(10)

where $a_2 \neq 0$ and $h_2 \neq 0$.

Substituting (10) into (6), setting the coefficients of all powers of $G(\xi)$ to zero, we get a nonlinear algebraic system of coefficients a_0, a_1, a_2, h_0, h_1 , and h_2 and k, l, and ω . Solving it, we obtain

$$a_{0} = a_{0}, \qquad a_{1} = a_{1}, \qquad a_{2} = \frac{a_{1}h_{2}}{h_{1}},$$

$$h_{0} = \frac{h_{1}^{2}}{4h_{2}}, \qquad h_{1} = h_{1}, \qquad h_{2} = h_{2}, \qquad (11)$$

$$\omega = \frac{-k^{2} - L^{2}p^{2} - 4k^{4}h_{2}}{k},$$

where a_0, a_1, h_1, k , and *L* are arbitrary nonzero reals and $h_2 > 0$. Under the condition of $h_0 = h_1^2/4h_2$ to solve (10), we get

$$G(\xi) = -\frac{h_1}{2h_2} + e^{\sigma\sqrt{h_2}\xi},$$
 (12)

where $\sigma = \pm 1$.

Substituting (11) and (12) into (10), by (2), we obtain an exact traveling wave solution of KP equation as follows:

$$u(x, y, t) = \frac{32k^2 a_1 h_1 h_2^3 (4a_0 h_2 - a_1 h_1) e^{2\sigma \sqrt{h_2(kx + Ly - \omega t + \gamma_0)}}}{(4a_0 h_1 h_2 - a_1 h_1^2 + 4a_1 h_2^2 e^{2\sigma \sqrt{h_2(kx + Ly - \omega t + \gamma_0)}})^2}.$$
(13)

A solitary wave which possesses a cusped structure is shown by (13), whose amplitude oscillates with the evolution of time. To guarantee the regularity of solitary wave, we should avoid the denominator of (13) equalling zero. So, an inequality is taken into account:

$$\left(4a_0h_1h_2 - a_1h_1^2\right)a_1h_2^2 > 0.$$
⁽¹⁴⁾

For simplicity, if $a_1 > 0$, we can conclude that $a_0 < a_1h_1/4h_2$ when $h_1 < 0$; $a_0 > a_1h_1/4h_2$ when $h_1 > 0$. Choosing a set of parameters

k = 0.2, L = 1.2, p = 1, $\gamma_0 = 0.5,$ $a_1 = 0.2,$ $h_1 = 0.8,$ $h_2 = 0.4,$ (15) $a_0 = 0.2,$ $\sigma = 1,$

we exhibit a waveform of regular cusped solitary wave expressed by (13). From Figure 1, it is observed that the amplitude of cusped solitary wave periodically oscillates along the x-axis.

However, if $(4a_0h_1h_2 - a_1h_1^2)a_1h_2^2 < 0$, it is inevitable that the denominator of (13) equals zero for some values of *x*, *y*, and *t*. In other words, the equation

$$kx + Ly - \omega t + \gamma_0 = \frac{1}{2\sigma\sqrt{h_2}} \ln\left(\frac{h_1(a_1h_1 - 4a_0h_2)}{4a_1h_2^2}\right)$$
(16)

is satisfied; the irregularity of cusped solitary wave appears. From Figures 2, 3, and 4, these singular phenomena exhibit that irregular solitary wave blows up in finite time, where we only change $a_0 = 0.2$ to $a_0 = -0.2$ in (15).



FIGURE 1: A snapshot of regular cusped solitary wave expressed by (13) at t = 1.2, where $x \in [-60, 70]$ and $y \in [-40, 20]$.



FIGURE 2: A snapshot of irregular cusped solitary wave expressed by (13) at t = 0.84, where $x \in [-50, 50]$ and $y \in [-30, 10]$.

3. Interaction of Solitary Wave and Periodic Wave

Let

$$F(\xi,\eta) = 1 + e^{\xi} + e^{\eta} + Ae^{\xi+\eta},$$
(17)

where $\eta = k_1 x + L_1 y - \omega_1 t + \gamma_1$ and $\xi = k_2 x + L_2 y - \omega_2 t + \gamma_2$. Theoretically, we can obtain the double solitary wave solution of KP equation (1).

In this case, we introduce an extended double-soliton test function:

$$F(x, y, t) = 1 + e^{\eta} + G(\xi) + AG(\xi) e^{\eta},$$
(18)

where $\eta = k_1 x + L_1 y - \omega_1 t + \gamma_1$ and $\xi = k_2 x + L_2 y - \omega_2 t + \gamma_2$. The unknown real function $G(\eta)$ satisfies the following auxiliary equation:

$$\left(G'(\xi)\right)^2 = h_0 + h_1 G(\xi) + h_2 G^2(\xi).$$
⁽¹⁹⁾

The parameters k_1 , k_2 , L_1 , L_2 , ω_1 , and ω_2 are nonzero constants to be determined, and γ_1 , and γ_2 are phase constants. A is a real number that stands for the resonant factor of traveling wave.

Substituting (18) and (19) into (3) and collecting the coefficients of e^{η} , $G(\xi)$, and $G'(\xi)$, one yields a nonlinear algebraic system of parameters $h_0, h_1, h_2, k_1, k_2, L_1, L_2, \omega_1$, and ω_2 .



FIGURE 3: A snapshot of irregular cusped solitary wave expressed by (13) at t = 0.85.



FIGURE 4: A snapshot of irregular cusped solitary wave expressed by (13) at t = 1.1.

In particular, $h_0 = -qr$, $h_1 = q + r$, and $h_2 = -1$ are taken to solve the nonlinear algebraic system whose q and r are undetermined constants; we obtain

$$q = -2 - r,$$

$$L_{1} = \frac{-\sqrt{3p^{2}k_{2}^{2}(k_{1}^{2} + k_{2}^{2}) + p^{2}k_{1}k_{2}L_{2}}}{p^{2}k_{2}^{2}},$$

$$A = 0,$$

$$\omega_{1} = \frac{-k_{1}k_{2}^{3} - 4k_{1}^{3}k_{2}^{3} + 2\sqrt{3p^{2}k_{2}^{2}(k_{1}^{2} + k_{2}^{2})L_{2} - p^{2}k_{1}k_{2}L_{2}^{2}}}{k_{2}^{3}},$$

$$\omega_{2} = \frac{-k_{2}^{2} + 4k_{2}^{4} - p^{2}L_{2}^{2}}{k_{2}}.$$
(20)

For (19), when $h_0 = -qr$, $h_1 = q + r$, and $h_2 = -1$, it allows the fundamental solution as follows:

$$G(\xi) = \frac{1}{2} (q + r - (q - r) \cos(\xi)).$$
 (21)

According to the expressions of ω_1 , and L_1 in (20) and the significance of coefficient *p* for KP equation, it is necessary



FIGURE 5: A snapshot for assimilation of solitary wave and periodic traveling wave expressed by (22) at t = 3, where $x \in [-8, 8]$ and $y \in [-8, 8]$.

that $p^2 = 1$. Substituting (20) and (21) into (18), by (2), we obtain an exact traveling wave solution for KP-I equation:

$$u(x, y, t) = 2 (1 + r) \left(k_1^2 e^{\eta} \cos(\xi) + 2k_1 k_2 e^{\eta} \sin(\xi) - k_2^2 \left(1 + r + e^{\eta} \cos(\xi) \right) \right)$$
(22)

$$\times \left(\left(e^{\eta} + (1 + r) \cos(\xi) \right)^2 \right)^{-1},$$

where $\eta = k_1 x + L_1 y - \omega_1 t + \gamma_1$ and $\xi = k_2 x + L_2 y - \omega_2 t + \gamma_2, k_1, k_2, L_2$, and *r* are arbitrary nonzero real constants, and ω_1, ω_2 , and L_1 satisfy

$$\omega_{1} = \frac{2\sqrt{3}k_{2}\left(k_{1}^{2} + k_{2}^{2}\right)L_{2} - k_{1}k_{2}^{2} - 4k_{1}^{3}k_{2}^{2} - k_{1}L_{2}^{2}}{k_{2}^{2}},$$

$$\omega_{2} = \frac{4k_{2}^{4} - k_{2}^{2} - L_{2}^{2}}{k_{2}},$$
(23)

$$L_1 = -\frac{\sqrt{3k_2(k_1^2 + k_2^2) - k_1 L_2}}{k_2}.$$

The solution expressed by (22) is a new type of traveling wave solution for KP-I equation. Choosing a set of parameters

$$k_1 = 0.2,$$
 $k_2 = 0.8,$ $L_2 = 0.4,$
 $\gamma_1 = 0,$ $\gamma_2 = 0,$ $r = 1,$ (24)

the assimilation of solitary wave and periodic traveling wave is exhibited by Figures 5 and 6.

4. Conclusions

In this paper, we consider the joint application of bilinear operator and F-expansion method. Choosing the KP equation as an example, we obtain a new type of solitary wave solution which possesses cusped structure by the singlesoliton test approach. The regular and irregular parametric relationships of cusped solitary wave solution are discussed; an interesting phenomenon is found where irregular cusped



FIGURE 6: A snapshot for assimilation of solitary wave and periodic traveling wave expressed by (22) at t = -3.

solitary wave periodically blows up in finite time. Furthermore, an extended double-soliton test method is applied to obtain a new type of exact traveling wave solution of KP-I equation. By numerical simulation of waveform, a nonlinear phenomenon describing the dynamical behavior of assimilation for solitary wave and periodic traveling wave is found. To our knowledge, it has not yet been found until now. The above results obtained in this paper validate the effectiveness of joint application of bilinear operator and Fexpansion method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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