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Research Article

Finite-Time Adaptive Synchronization of a New Hyperchaotic System with Uncertain Parameters

Ma Yongguang and Dong Zijian

School of Control and Computer Engineering, North China Electric Power University, Baoding 071003, China

Correspondence should be addressed to Ma Yongguang; yongguang_ma@163.com

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This paper presents a finite-time adaptive synchronization strategy for a class of new hyperchaotic systems with unknown slave system's parameters. Based on the finite-time stability theory, an adaptive control law is derived to make the states of the new hyperchaotic systems synchronized in finite-time. Numerical simulations are presented to show the effectiveness of the proposed finite time synchronization scheme.

1. Introduction

Chaos synchronization has attracted increasing attention since the pioneering work of Pecora and Carroll [1] for its potential applications in secure communications, biological systems, chemical reactions, biological networks, and so on. Different notations of synchronization have been proposed, such as complete synchronization [2], generalized synchronization [3, 4], phase synchronization [5, 6], lag synchronization [7, 8], antisynchronization [9, 10], and projective synchronization [11, 12]. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system follows the output of the master system asymptotically. A wide variety of synchronization approaches have been developed such as impulsive control [13], feedback control [14], active control [15], adaptive control [16, 17], sliding mode control [18], model predictive control [19–22], and impulsive control [23, 24] and others [25, 26]. In the last thirty years, as hyperchaos has more than one positive Lyapunov exponent and has more complex dynamical behavior than chaos, many researches have focused their attention on the synchronization of hyperchaotic systems [27–31].

However, in real-world application, it is usually expected that two systems can synchronize as quickly as possible and the finite-time control is an efficient technique [32–37]. Furthermore, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties [38]. However, most of the results are derived based on the hypothesis that the system's parameters are precisely known. But in practice, most of the system's parameters cannot be exactly known in advance. The designed synchronization will be destroyed with the effects of these uncertainties. To the best of our knowledge, there is no work on the problem of finite-time adaptive synchronization of hyperchaotic systems with uncertain parameters. Motivated by the above discussions, in this paper, we are concerned with the finite-time adaptive synchronization for hyperchaotic systems. Via adaptive control method, finite-time adaptive synchronization between two identical hyperchaotic systems with unknown parameters is achieved and we prove that the suggested approach can realize finite-time synchronization. Simulation results show the effectiveness of the proposed method.

2. The Dadras System and Lemmas

Recently, a new 4D dynamical system is proposed [39], which can generate a four-ring hyperchaotic attractor and a four-wing chaotic attractor. The system is called Dadras system in this paper and it is described by

$$\dot{x} = ax - yz + w$$

$$\dot{y} = xz - by$$

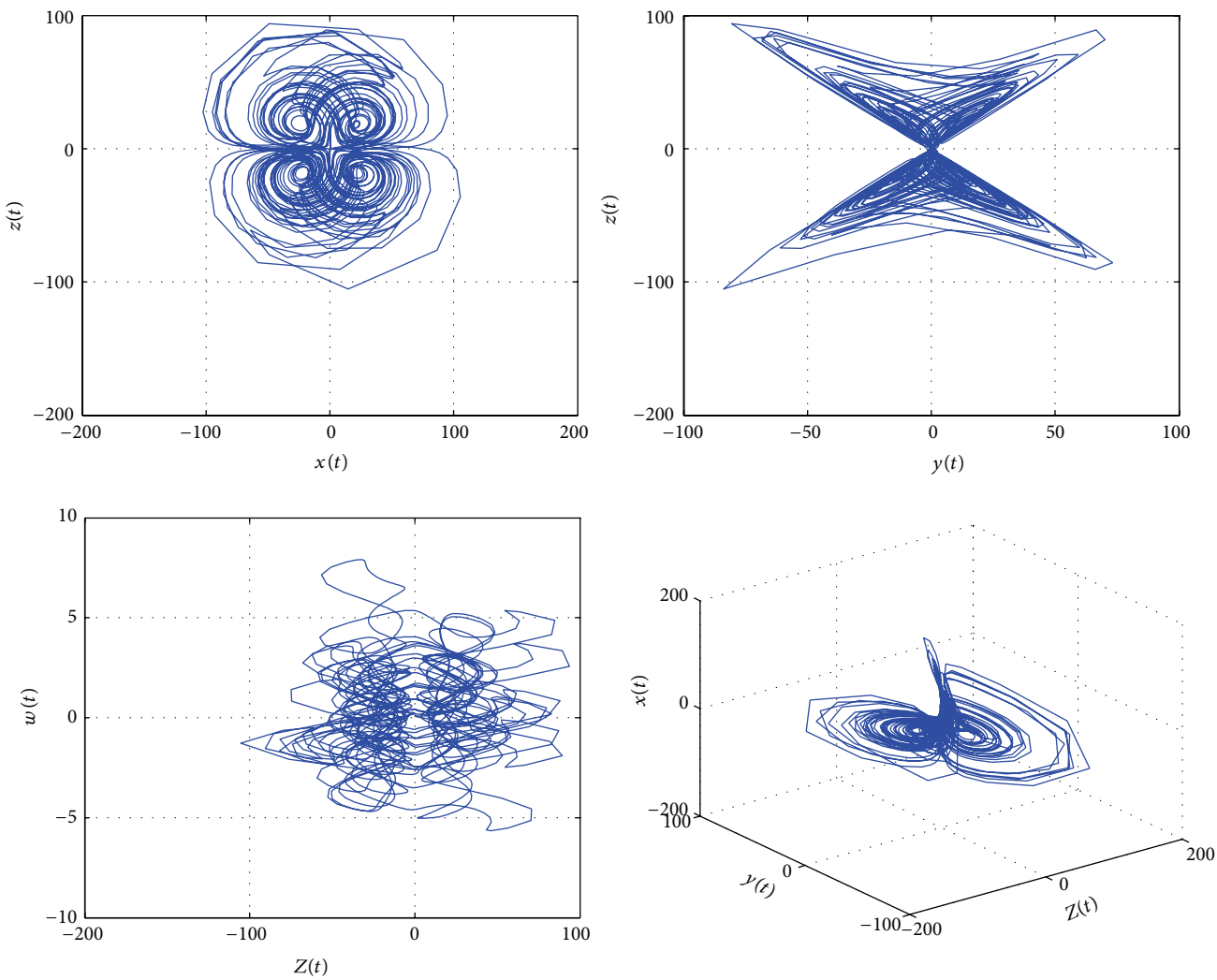

Figure 1: Phase portraits of the four-wing hyperchaotic attractor for $a = 8$, $b = 40$, and $c = 14.9$.

$$\dot{z} = xy - cz + xw$$
$$\dot{w} = -y, \tag{1}$$

where $[x, y, z, w]^T \in R^4$ is the state vector and a, b, and c are positive constant parameters of the system. When $a = 8$, $b = 40$, $c = 14.9$, and the initial condition is set to $[10, 1, 10, 1]^T$, the system has generated a four-wing hyperchaotic attractor which is shown in Figure 1.

Lemma 1 (see [40]). *Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:*

$$\dot{V}(t) \le -\gamma V^{\eta}(t) \quad \forall t \ge t_0,\ V(t_0) \ge 0, \tag{2}$$

where $\gamma > 0$, $0 < \eta < 1$, are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - \gamma(1-\eta)(t - t_0), \quad t_0 \le t \le t_1, \tag{3}$$

$$V(t) \equiv 0 \quad \forall t \ge t_1 \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\gamma(1-\eta)}. \tag{5}$$

3. The Proposed Synchronization Method

In order to achieve master-slave synchronization of Dadras system, the master and slave systems are defined with the subscripts m and s below, respectively:

$$\begin{aligned}
\dot{x}_m &= ax_m - y_m z_m + w_m \\
\dot{y}_m &= x_m z_m - by_m \\
\dot{z}_m &= x_m y_m - cz_m + x_m w_m \\
\dot{w}_m &= -y_m,
\end{aligned} \tag{6}$$

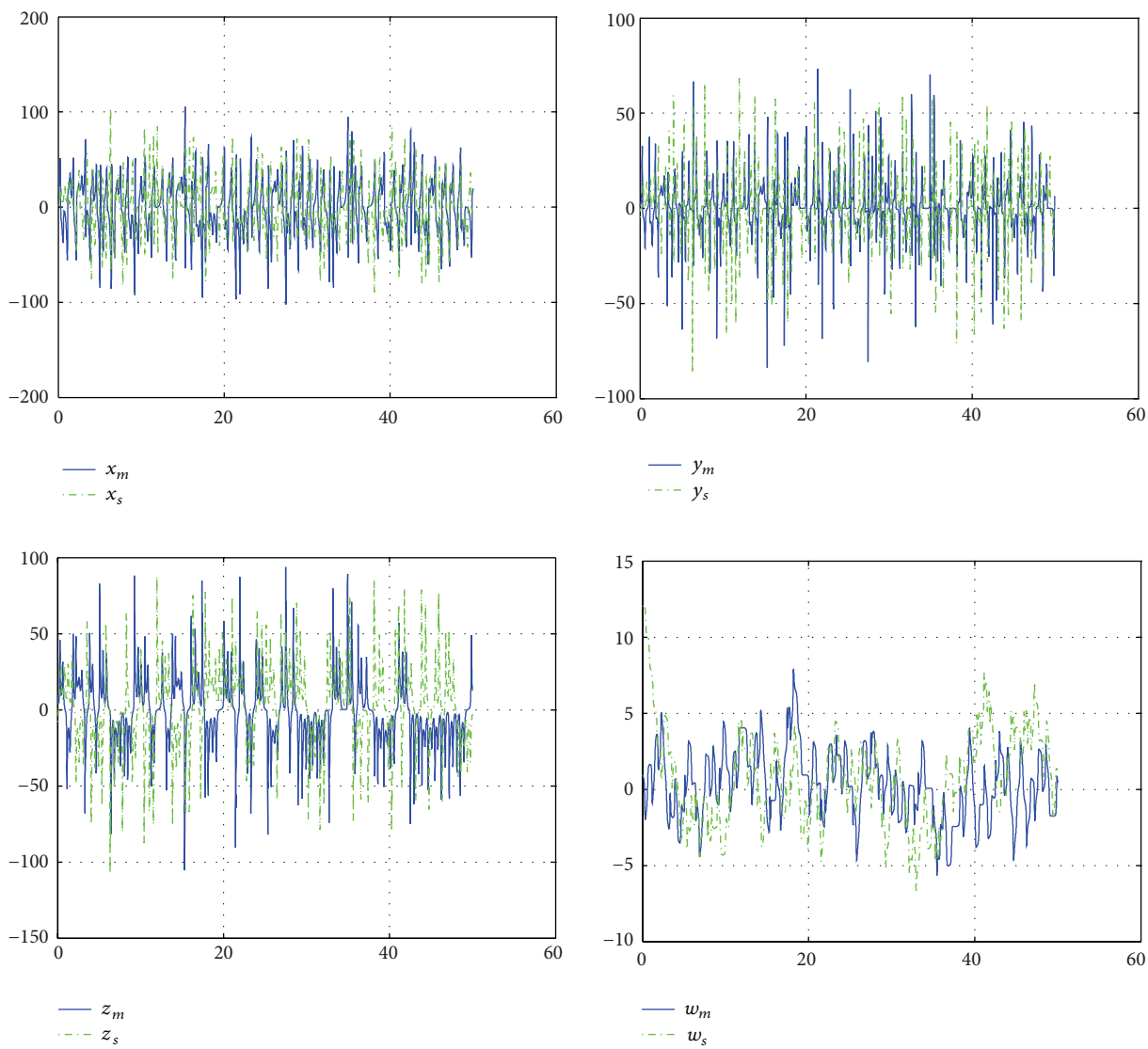

FIGURE 2: States of the master and slave systems without control.

$$\begin{aligned}\dot{x}_s &= a_s x_s - y_s z_s + w_s + u_1\\ \dot{y}_s &= x_s z_s - b_s y_s + u_2\\ \dot{z}_s &= x_s y_s - c_s z_s + x_s w_s + u_3\\ \dot{w}_s &= -y_s + u_4,\end{aligned} \tag{7}$$

where a_s, b_s, and c_s are uncertain parameters, which need to be estimated in the slave system, and u_1, u_2, u_3, and u_4 are the designed controllers to realize the two hyperchaotic systems' finite-time synchronization.

Let

$$\begin{aligned}e_1 &= x_s(t) - x_m(t)\\ e_2 &= y_s(t) - y_m(t)\\ e_3 &= z_s(t) - z_m(t)\\ e_4 &= w_s(t) - w_m(t).\end{aligned} \tag{8}$$

Then the error dynamical system between (2) and (3) is

$$\begin{aligned}\dot{e}_1 &= a_s x_s - y_s z_s + w_s - a x_m + y_m z_m - w_m + u_1\\ \dot{e}_2 &= x_s z_s - b_s y_s - x_m z_m + b y_m + u_2\\ \dot{e}_3 &= x_s y_s - c_s z_s + x_s w_s - x_m y_m + c z_m - x_m w_m + u_3\\ \dot{e}_4 &= -y_s + y_m + u_4.\end{aligned} \tag{9}$$

Our goal is to design controllers u_i $(i = 1, 2, 3, 4)$ to realize finite-time synchronization between the master system (6) and the slave system (7) with the uncertain slave system parameters; that is, $\|e(t)\| = 0$ when $t > T_0$, where $e = [e_1, e_2, e_3, e_4]^T$.

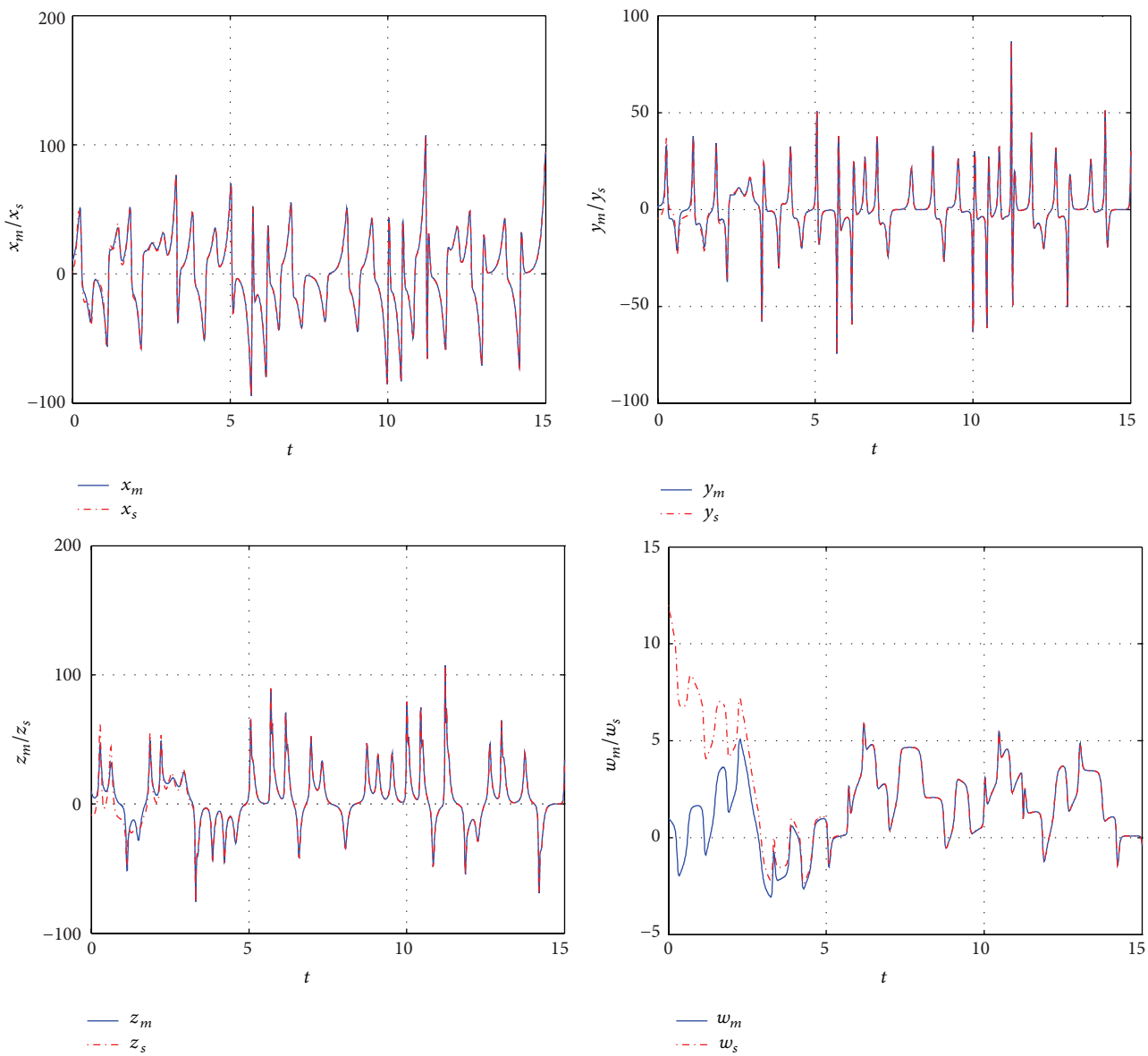

FIGURE 3: States of the master and slave systems with the designed controller.

Define

$$\begin{aligned} e_a &= a_s - a \\ e_b &= b_s - b \\ e_c &= c_s - c. \end{aligned} \tag{10}$$

Then (5) can be converted to the following form:

$$\begin{aligned} \dot{e}_1 &= e_a x_s + a e_1 - y_m e_3 - z_s e_2 + e_4 + u_1 \\ \dot{e}_2 &= x_s e_3 + z_m e_1 - b_s e_2 - y_m e_b + u_2 \\ \dot{e}_3 &= x_s e_2 + y_m e_1 - c_s e_3 - z_m e_c + x_s e_4 + w_m e_1 + u_3 \\ \dot{e}_4 &= -e_2 + u_4. \end{aligned} \tag{11}$$

In order to achieve the synchronization, we select the following control laws and the update rules for three uncertain parameters a_s, b_s, and c_s:

$$\begin{aligned} u_1 &= -a e_1 + y_m e_3 + z_s e_2 - e_4 - e_1^{\beta} \\ u_2 &= -x_s e_3 - z_m e_1 + b_s e_2 - e_2^{\beta} \\ u_3 &= -x_s e_2 - y_m e_1 + c_s e_3 - x_s e_4 - w_m e_1 - e_3^{\beta} \\ u_4 &= e_2 - e_4^{\beta}, \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{e}_a &= -e_1 x_s - e_a^{\beta} \\ \dot{e}_b &= y_m e_2 - e_b^{\beta} \\ \dot{e}_c &= z_m e_3 - e_c^{\beta}. \end{aligned} \tag{13}$$

Then, the following result is obtained.

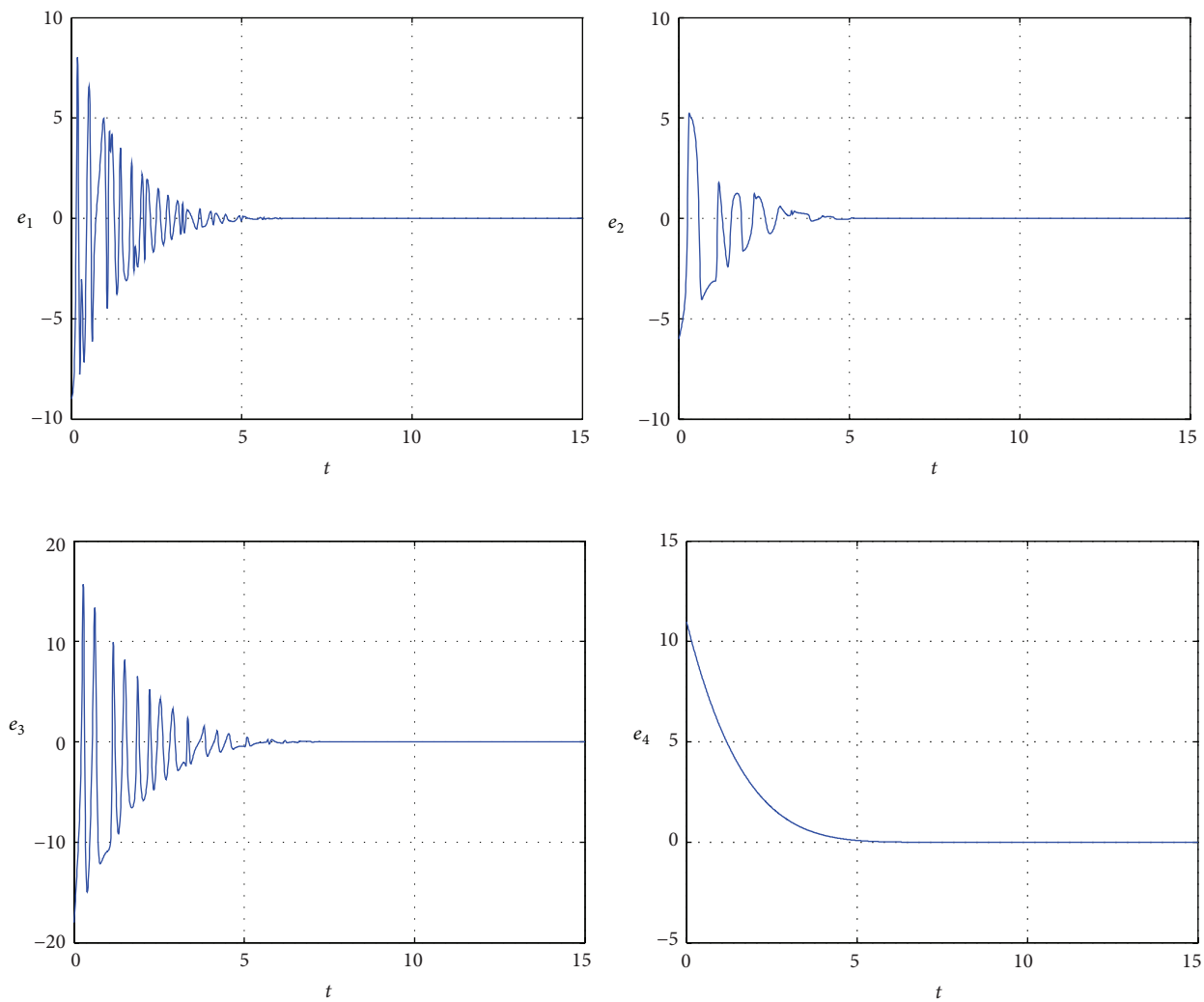

Figure 4: Synchronization errors between the master and slave systems with the designed controller.

Theorem 2. *For any initials, the two systems* (6) *and* (7) *realize finite-time adaptive synchronization under the control laws* (12) *and the parameters' update laws* (13).

Proof. Choose the following Lyapunov function candidate:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2\right). \tag{14}$$

The differential of the Lyapunov function along the trajectory of the error system (13) is

$$\begin{aligned}
\frac{dV}{dt} &= e_1\left(e_a x_s - e_1^{\beta}\right) + e_2\left(-y_m e_b - e_2^{\beta}\right) + e_3\left(-z_m e_c - e_3^{\beta}\right) \\
&\quad + e_4\left(-e_4^{\beta}\right) + e_a\left(-e_1 x_s - e_a^{\beta}\right) + e_b\left(y_m e_2 - e_b^{\beta}\right) \\
&\quad + e_c\left(z_m e_3 - e_c^{\beta}\right) \\
&= -e_1^{1+\beta} - e_2^{1+\beta} - e_3^{1+\beta} - e_4^{1+\beta} - e_a^{1+\beta} - e_b^{1+\beta} - e_c^{1+\beta} \\
&= -2^{(\beta+1)/2}\left(\left(\frac{1}{2}e_1^2\right)^{(\beta+1)/2} + \left(\frac{1}{2}e_2^2\right)^{(\beta+1)/2}\right. \\
&\qquad + \left(\frac{1}{2}e_3^2\right)^{(\beta+1)/2} + \left(\frac{1}{2}e_4^2\right)^{(\beta+1)/2} \\
&\qquad + \left(\frac{1}{2}e_a^2\right)^{(\beta+1)/2} + \left(\frac{1}{2}e_b^2\right)^{(\beta+1)/2} \\
&\qquad \left. + \left(\frac{1}{2}e_c^2\right)^{(\beta+1)/2}\right) \\
&\le -2^{(\beta+1)/2}\left(\frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_4^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2\right. \\
&\qquad \left. + \frac{1}{2}e_c^2\right)^{(\beta+1)/2} \\
&= -2^{(\beta+1)/2} V^{(\beta+1)/2}.
\end{aligned} \tag{15}$$

□

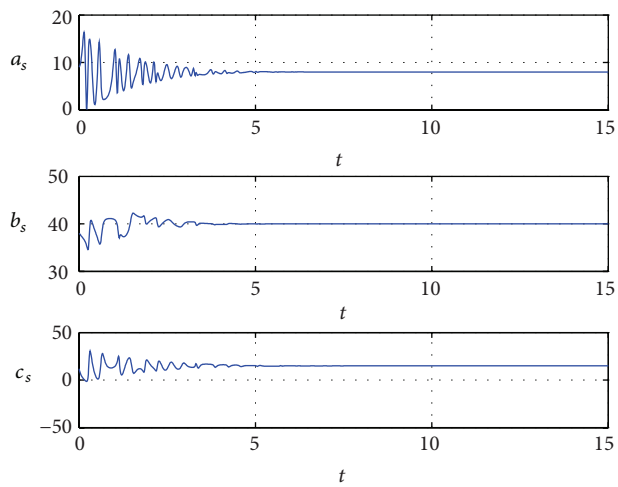

FIGURE 5: Adaptive parameters a_s, b_s, and c_s.

From Lemma 1, it follows that the error system (11) is finite-time stabilized. Then the uncertain slave system (7) can synchronize the master system (6) in finite time.

Remark 3. From the proof of Theorem 2, it is found that $dV/dt \leq -2^{(\beta+1)/2}V^{(\beta+1)/2}$. Using Lemma 1, we can get $V(t) \equiv 0, \forall t \geq t_1$, where $t_1 = 2^{(1-\beta)/2} \cdot (1/(1-\beta))[V(0)]^{(1-\beta)/2}$. So the synchronization time is influenced by not only the parameters and initial values' mismatch but also the control parameter β.

Remark 4. Although the synchronization scheme is designed for Dadras systems, it can be used in two other identical hyperchaotic systems and even two different hyperchaotic systems.

4. Simulation Results

Numerical simulation results are presented to show the effectiveness of the proposed finite-time synchronization method. Fourth-order Runge-Kutta method is used and the time step size is 0.001 s. The master system's parameters and initial conditions are the same as in Figure 1. The initial states of the response system are $x_s(0) = 1$, $y_s(0) = -5$, $z_s(0) = -8$, and $w_s(0) = 12$. Furthermore, the initial values of estimated parameters are $a = 9$, $b = 38$, and $c = 12$ and the parameter $\beta = 0.8$. The simulations of the two Dadras systems without control are shown in Figure 2, followed by the simulation with the designed finite-time adaptive control shown in Figure 3. Figure 4 shows that the trajectories of $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ tended to zero in finite time. The changes of parameters of a_s, b_s, and c_s are shown in Figure 5. Obviously, the synchronization errors converge to zero and the estimations of parameters converge to some constants in finite time.

5. Conclusion

This paper has addressed the finite-time adaptive synchronization of Dadras hyperchaotic systems. Based on finite-time stability theory, the proposed scheme can assure the states of slave system to track the states of the master system in finite time. From the process of proof we can see that this method can be extended to other hyperchaotic systems such as Rössler hyperchaotic system and Lü hyperchaotic system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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