

Research Article

Modeling Complex System Correlation Using Detrended Cross-Correlation Coefficient

Keqiang Dong,¹ Hong Zhang,² and You Gao¹

¹ College of Science, Civil Aviation University of China, Tianjin 300300, China

² Renai College, Tianjin University, Tianjin 300300, China

Correspondence should be addressed to Keqiang Dong; hongzhangdong@163.com

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The understanding of complex systems has become an area of active research for physicists because such systems exhibit interesting dynamical properties such as scale invariance, volatility correlation, heavy tails, and fractality. We here focus on traffic dynamic as an example of a complex system. By applying the detrended cross-correlation coefficient method to traffic time series, we find that the traffic fluctuation time series may exhibit cross-correlation characteristic. Further, we show that two traffic speed time series derived from adjacent sections exhibit much stronger cross-correlations than the two speed series derived from adjacent lanes. Similarly, we also demonstrate that the cross-correlation property between the traffic volume variables from two adjacent sections is stronger than the cross-correlation property between the volume variables of adjacent lanes.

1. Introduction

Many diversified complex systems are composed of constituents that mutually interact in a complex fashion. The complexity of the mutual interaction, such as the output of each constituent which depends not only on its own past but also on the past values of other constituent outputs, can be additionally studied if memory is included. Such complex systems are characterized by both long-range correlations and long-range cross-correlations. A number of studies suggest the existence of these properties in diverse systems. Applying the random matrix theory, Stanley et al. demonstrated the cross-correlation properties between individual stocks traded in the Korean stock market [1]. By analyzing 48 world financial indices, Wang et al. found the long-range power-law cross-correlations in the absolute values of returns [2]. Podobnik et al. studied the cross-correlation in successive differences of air humidity and air temperature [3]. Du et al. provided cross-correlation time delay model to improve earthquake relocation forecasts [4].

These studies provide strong empirical evidences for the existence of cross-correlations between the dynamics of natural systems. Pearson's correlation coefficient (PCC),

which is used to represent the linear correlation between two time series which are both assumed to be stationary [5, 6], is commonly used to gain insight into the dynamics of cross-correlations in time series. Nevertheless, in natural systems, the nonlinear and nonstationary characteristics are usually present [7, 8]. Therefore, PCC may not be suitable to describe the cross-correlations between time series that are nonlinear or nonstationary. To address the drawbacks of PCC, the detrended cross-correlation analysis (DCCA) method is employed in this paper.

The DCCA method, which is a modification of standard covariance analysis in which the global average is replaced by local trends [9, 10], was proposed by Podobnik and Stanleys. The performance of detrended cross-correlation analysis method was systematically tested for the effect of nonstationarities [9–11]. After that, numerous issues referring to a broad range of applications [12–15] were established to investigate cross-correlational signal in the presence of nonstationarities.

In analogy with the cross-correlation coefficient, Zebende recently introduced the detrended cross-correlation (DCCA) coefficient [6]. One of the outstanding advantages of the nonlinear cross-correlation coefficient is that it can investigate the cross-correlations at different time scales [16, 17]. After

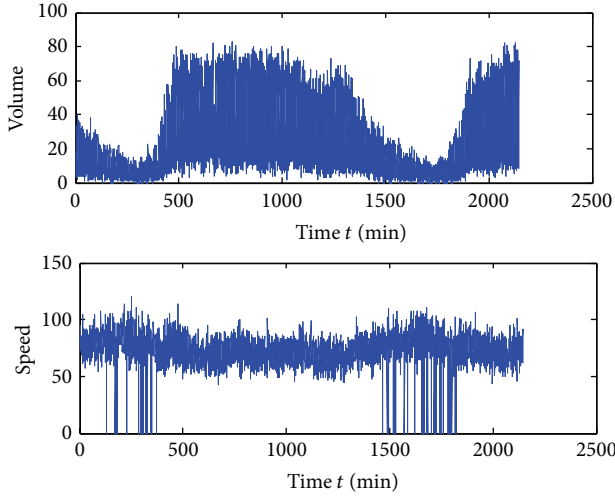


FIGURE 1: The time series plot of the speed data and volume data observed at the Beijing Third Ring Road.

that, Cao et al. adopted the DCCA coefficient to analyze and quantify cross-correlations between the Chinese exchange market and stock market [16]. Vassoler et al. quantified the cross-correlations between time series of air temperature and relative humidity by DCCA coefficient [18]. Podobnik et al. showed that the tendency of the Chinese stock market to follow the US stock market is extremely weak by using the DCCA coefficient [19]. Wang et al. studied the statistical properties of the foreign exchange network at different time scales applying the DCCA coefficient [20].

Here, using the DCCA coefficient method, we model the traffic data collected on the Beijing Third Ring Road as the input data which can be readily observed from conventional point detectors. The preliminary test results demonstrate that the cross-correlation property between the traffic series from two adjacent sections is stronger than the cross-correlation property between the series of adjacent lanes and disjoint lanes. The scaling results suggest the feasibility of estimating cross-correlations in traffic variables using point detector data via the proposed approach.

The organization of this paper is as follows. In the next section, we present the dataset and DCCA coefficient method. In Section 3, we show the main empirical results and discussion. Finally, we draw some conclusions in Section 4.

2. Data and Methodology

2.1. The Dataset. Traffic systems have a number of parameters that can be measured. The speed and volume are employed in collecting and studying traffic data here. The data was observed on the Beijing Third Ring Road (BTRR) over a period of about 7 days, from 0:00 AM on March 21, 2011, to 23:30 PM on March 27, 2011. Figure 1 shows the time series plot of the speed data and volume data observed at the Beijing Third Ring Road.

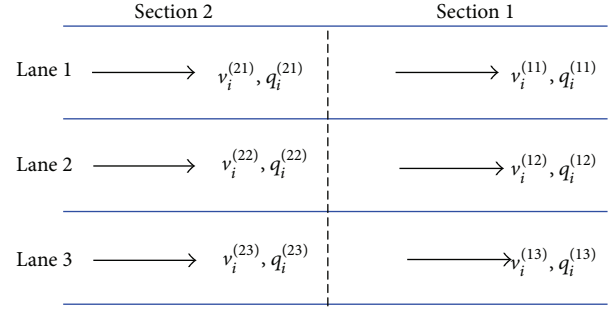


FIGURE 2: The twelve datasets of traffic time series.

The BTRR is a closed road system without any traffic-signal control. There are three main lanes as well as one or two auxiliary lanes related to on-and-off ramps for each direction. The data were downloaded from the Highway Performance Measurement Project (FPMP). The periodic time of detecting is 2 min and the distance between two adjacent detectors is about 500 m. For investigating the cross-correlations in traffic time series, we will analyze twelve datasets as follows (see Figure 2):

- (1) $\{v_i^{mn} : m = 1, 2; n = 1, 2, 3\}$: the speed series of Lane n in Section m ;
- (2) $\{q_i^{mn} : m = 1, 2; n = 1, 2, 3\}$: the volume series of Lane n in Section m .

2.2. DCCA Coefficient Method. DCCA coefficient method is an extension of detrended cross-correlation analysis (DCCA) and detrended fluctuation analysis (DFA) method, and both methods are based on random walk theory [6, 21, 22]. For two nonstationary time series $\{x_k\}$ and $\{y_k\}$, $k = 1, 2, \dots, T$, where T is the length of data, the DCCA coefficient is given as follows.

Step 1. Compute the profiles of underlying time series using

$$\begin{aligned} X(i) &= \sum_{k=1}^i (x_k - \langle x \rangle), \\ Y(i) &= \sum_{k=1}^i (y_k - \langle y \rangle), \end{aligned} \quad (1)$$

where $\langle x \rangle = (1/k) \sum_{j=1}^k x_j$ and $\langle y \rangle = (1/k) \sum_{j=1}^k y_j$ are the mean.

Step 2. Cut the profiles X and Y into $N_s = [N/s]$ nonoverlapping segments of equal length s , respectively. In each segment v , we calculate the local trend by a least-square fit of the data and obtain the difference between the original time series and the fits.

Step 3. Calculate the covariance of the residuals in each segment:

$$\begin{aligned}
 f_{\text{DCCA}}^2(s, \nu) &= \frac{1}{s} \sum_{i=1}^s (X_{(\nu-1)s+i} - \tilde{X}_{i,\nu}) (Y_{(\nu-1)s+i} - \tilde{Y}_{i,\nu}), \\
 f_{\text{DFA},\{x_i\}}^2(s, \nu) &= \frac{1}{s} \sum_{i=1}^s (X_{(\nu-1)s+i} - \tilde{X}_{i,\nu})^2, \\
 f_{\text{DFA},\{y_i\}}^2(s, \nu) &= \frac{1}{s} \sum_{i=1}^s (Y_{(\nu-1)s+i} - \tilde{Y}_{i,\nu})^2,
 \end{aligned} \tag{2}$$

for each segment $\nu = 1, 2, \dots, N_s$, and

$$\begin{aligned}
 f_{\text{DCCA}}^2(s, \nu) &= \frac{1}{s} \sum_{i=1}^s (X_{N-(\nu-N_s)s+i} - \tilde{X}_{i,\nu}) \\
 &\quad \times (Y_{N-(\nu-N_s)s+i} - \tilde{Y}_{i,\nu}), \\
 f_{\text{DFA},\{x_i\}}^2(s, \nu) &= \frac{1}{s} \sum_{i=1}^s (X_{N-(\nu-N_s)s+i} - \tilde{X}_{i,\nu})^2, \\
 f_{\text{DFA},\{y_i\}}^2(s, \nu) &= \frac{1}{s} \sum_{i=1}^s (Y_{N-(\nu-N_s)s+i} - \tilde{Y}_{i,\nu})^2,
 \end{aligned} \tag{3}$$

for each segment $\nu = N_s + 1, N_s + 2, \dots, 2N_s$. Here $\tilde{X}_{i,\nu}$ and $\tilde{Y}_{i,\nu}$ are the fitting polynomials in segment ν , respectively. Then the averages over all segments to obtain the fluctuation function are as follows:

$$f_{\text{DCCA}}^2(s) = \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} f_{\text{DCCA}}^2(s, \nu), \tag{4}$$

$$f_{\text{DFA},\{x_i\}}(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} f_{\text{DFA},\{x_i\}}^2(s, \nu) \right\}^{1/2}, \tag{5}$$

$$f_{\text{DFA},\{y_i\}}(s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} f_{\text{DFA},\{y_i\}}^2(s, \nu) \right\}^{1/2}. \tag{6}$$

Step 4. For the two nonstationary time series $\{x_i\}$ and $\{y_i\}$, the DCCA coefficient is defined as the ratio between the detrended covariance function $f_{\text{DCCA}}^2(s)$ of (4) and two detrended variance functions $f_{\text{DFA}}(s)$ of (5) and (6):

$$\rho_{\text{DCCA}}(s) = \frac{f_{\text{DCCA}}^2(s)}{f_{\text{DFA},\{x_i\}}(s) f_{\text{DFA},\{y_i\}}(s)}, \tag{7}$$

where $\rho_{\text{DCCA}}(s)$ ranges from -1 to 1 [6, 20]. A value of $\rho_{\text{DCCA}}(s) = 1$ or $\rho_{\text{DCCA}}(s) = -1$ implies that the two nonstationary time series $\{x_i\}$ and $\{y_i\}$ are completely cross-correlated or anti-cross-correlated, at the time scale s ,

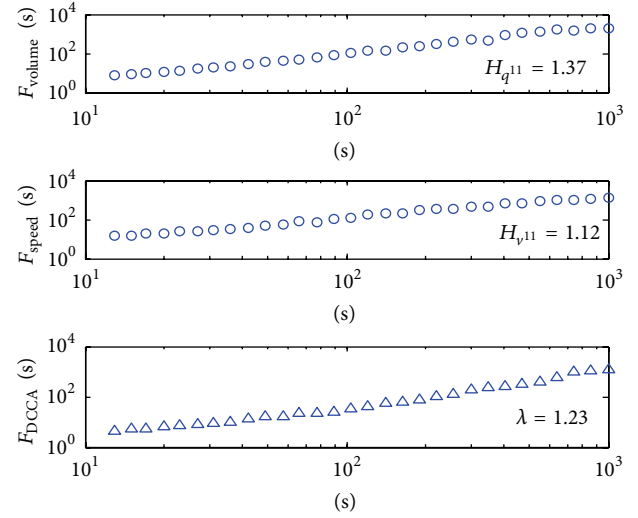


FIGURE 3: The DFA and DCCA fluctuate function for traffic speed fluctuation series $\{v_i^{(11)}\}$ and the traffic volume fluctuation series $\{q_i^{(11)}\}$.

whereas a value of $\rho_{\text{DCCA}}(s) = 0$ indicates that there is no cross-correlation between the two time series $\{x_i\}$ and $\{y_i\}$ [6, 19]. Obviously, the DCCA coefficient $\rho_{\text{DCCA}}(s)$ is a function of the different window size s of data, which means that it can investigate the cross-correlations between two time series $\{x_i\}$ and $\{y_i\}$ at different window scales.

3. Empirical Results and Analysis

3.1. The Cross-Correlation of the Speed and Volume Series. For two nonstationary cross-correlated time series $\{x_i\}$ and $\{y_i\}$, the power-law relationship $f_{\text{DCCA}}^2(s) \sim s^{2\lambda}$ exists. The scaling exponent λ represents the degrees of the cross-correlation between the two time series $\{x_i\}$ and $\{y_i\}$. For time series $x_i = y_i$, the DCCA fluctuate function reduces to the DFA fluctuate function $f_{\text{DFA}}(s)$.

In order to study the dynamics of the traffic time series over time, we first consider two time series, both of which can be considered as two outputs of traffic system: the traffic speed fluctuation series $\{v_i^{(11)}\}$ and the traffic volume fluctuation series $\{q_i^{(11)}\}$. Here $\{v_i^{(11)}\}$ are the speeds of Lane 1 in Section 1 and $\{q_i^{(11)}\}$ are the volumes of Lane 1 in Section 1.

Figure 3 displays the DFA and DCCA curve obtained between traffic speed fluctuation series $\{v_i^{(11)}\}$ and the traffic volume fluctuation series $\{q_i^{(11)}\}$. The curves exhibit obvious power-law behavior with DFA exponent $H_{q^{(11)}} = 1.37$, $H_{v^{(11)}} = 1.12$ and the DCCA exponent $\lambda = 1.23$, implying long-range autocorrelation and cross-correlations in traffic dynamics.

It is apparent that the traffic flow series can be characterized by a local variability of the DCCA coefficient as shown in Figure 4. The small fluctuations exhibited by the $\rho_{\text{DCCA}}(s)$ provide evidence that a more complex evolution dynamics characterizes the traffic flow.

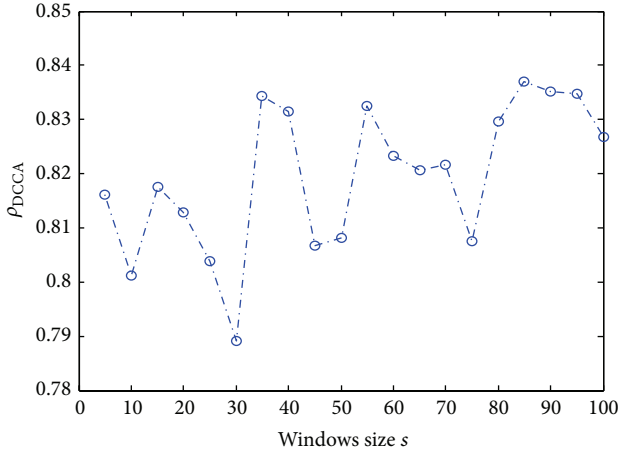
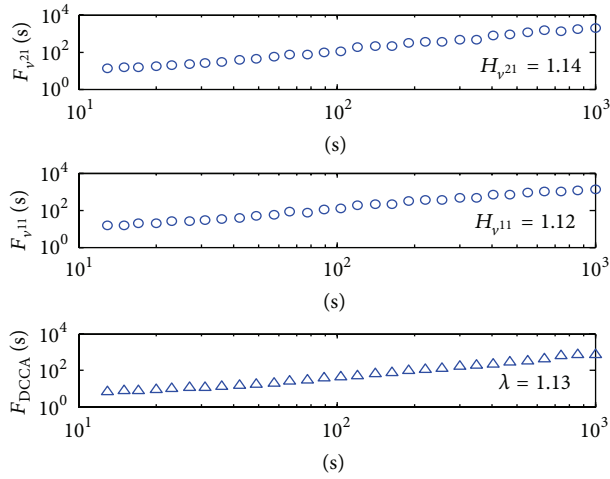


FIGURE 4: The DCCA coefficient between traffic time series.

FIGURE 5: The DFA and DCCA curve of traffic speed variables $\{v_i^{(11)}\}$ and $\{v_i^{(21)}\}$.

3.2. The Cross-Correlation of the Speed Series. It is worth noticing the fact that, according to the definition of cross-correlation [9], each of the two variables at any time depends not only on its own past values but also on past values of the other variable.

Here, we firstly investigate the cross-correlations between two traffic speed fluctuation variables $\{v_i^{(11)}\}$ and $\{v_i^{(21)}\}$, which are derived from two adjacent sections of a highway and simultaneously recorded every two minutes (see Figure 2). Figure 5 displays the DFA and DCCA curve for traffic speed fluctuation variables $\{v_i^{(11)}\}$ and $\{v_i^{(21)}\}$. The curves also exhibit obvious power-law behavior with DFA exponent $H_{v^{(21)}} = 1.14$, $H_{v^{(11)}} = 1.12$ and the DCCA exponent $\lambda = 1.13$, implying long-range autocorrelation and cross-correlations in traffic speed time series.

The DCCA coefficient curve is given in Figure 6. We find that $\rho_{DCCA}(s)$ fluctuate around the value $\rho_{DCCA} = 0.97$ and show that the cross-correlated behavior between the time series $\{v_i^{(11)}\}$ and $\{v_i^{(21)}\}$ is very strong.

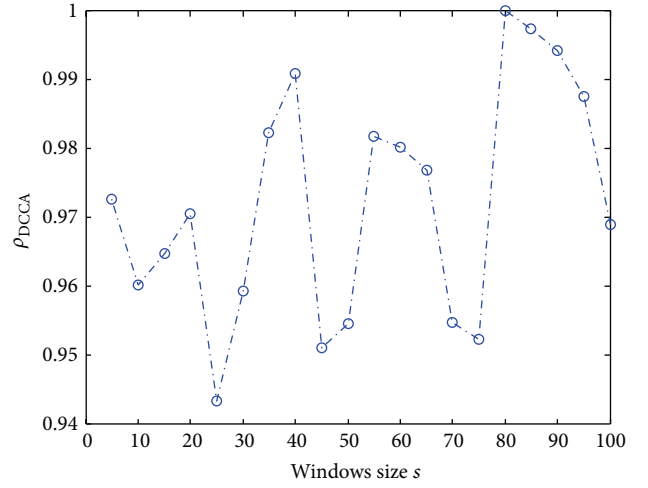
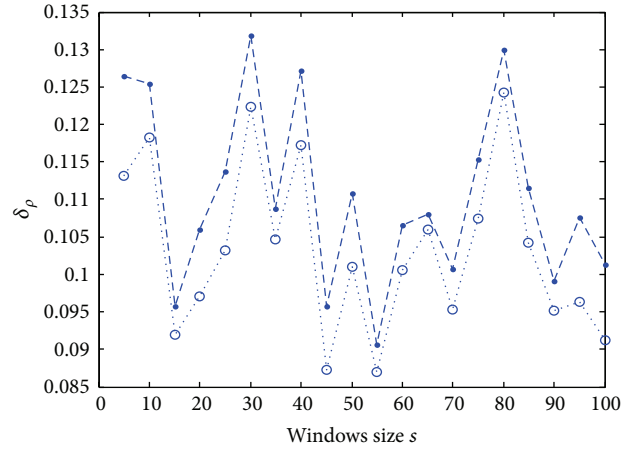


FIGURE 6: The cross-correlations between two traffic speed fluctuation variables from two adjacent sections.

FIGURE 7: The error function $\delta(s)$ of traffic speed series for the data from two adjacent sections and the data from two adjacent lanes (circles), and $\delta'(s)$ for the data from two adjacent sections and the data from two disjoint lanes (filled dots).

And then, we consider the case when two time series of variables $\{v_i^{(11)}\}$ and $\{v_i^{(12)}\}$ are derived from two adjacent lanes (see Figure 2). For convenience, we study the difference between the DCCA coefficient of the data from two adjacent sections of one lane and the data from two adjacent lanes by using the error function.

The error function is defined as $\delta(s) = \rho_{DCCA}(s) - \rho_{DCCA}^{(1)}(s)$, where $\rho_{DCCA}(s)$ is the DCCA coefficient of traffic speed fluctuation variables $\{v_i^{(11)}\}$ and $\{v_i^{(21)}\}$ and $\rho_{DCCA}^{(1)}(s)$ is the DCCA coefficient of traffic speed fluctuation variables $\{v_i^{(11)}\}$ and $\{v_i^{(12)}\}$.

From Figure 7, we can see that the error function $\delta(s) > 0$ (circles) indicates that the cross-correlation of speed series between two adjacent lanes is weaker than the time series of two adjacent sections.

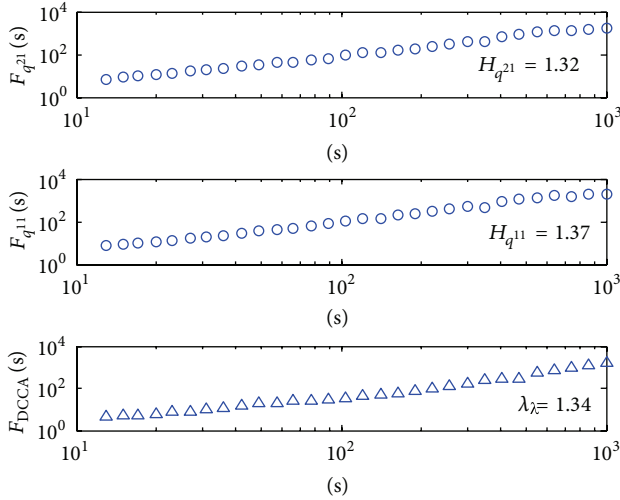


FIGURE 8: The DFA and DCCA curve of traffic volume variables $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$.

In addition, we also find that cross-correlation exists between the two time series of variables $\{v_i^{(11)}\}$ and $\{v_i^{(13)}\}$, which are derived from Lane 1 and Lane 3 (see Figure 2). We employ the error function $\delta'(s) = \rho_{DCCA}(s) - \rho_{DCCA}^{(2)}(s)$ once again, where $\rho_{DCCA}^{(2)}(s)$ is the DCCA coefficient of traffic speed fluctuation variables $\{v_i^{(11)}\}$ and $\{v_i^{(13)}\}$. For comparison, the error function $\delta'(s) = \rho_{DCCA}(s) - \rho_{DCCA}^{(2)}(s)$ is also plot in Figure 7 (filled dots). Obviously, the error function $\delta'(s) > 0$ (filled dots) indicates that the cross-correlation between speed series from two disjoint lanes is weaker than the cross-correlation between the time series of two adjacent sections.

To analyze the statistical properties of the speed time series, we compute the P value for $\rho_{DCCA}(s)$ and $\rho_{DCCA}^{(1)}(s)$. The result $P = 6.38 \times 10^{-16}$ indicates that the difference between two quantities is statistically significant. Similarly, the P value of $\rho_{DCCA}(s)$ and $\rho_{DCCA}^{(2)}(s)$ also shows significant difference ($P = 2.51 \times 10^{-22}$).

3.3. The Cross-Correlation of the Traffic Volume Series. Next, we investigate the cross-correlations between two traffic volume time series $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$ (see Figure 2). The DFA curves in Figure 8 show that each of two volume time series $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$ exhibits autocorrelated behavior by DFA exponent $H_{q_i^{(21)}} = 1.32, H_{q_i^{(11)}} = 1.37$. Figure 8 also illuminates that the cross-correlated behavior between $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$ exists by DCCA exponent.

Figure 9 shows the DCCA coefficient of traffic volume fluctuation variables $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$. The DCCA coefficient $\rho_{DCCA}(s)$ fluctuates around the value $\rho_{DCCA} = 0.78$ and shows that the cross-correlations between $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$ exists.

Further, we investigate the case when two time series of variables $\{q_i^{(11)}\}$ and $\{q_i^{(12)}\}$ are derived from two adjacent lanes (see Figure 2). The error function is employed once again. In Figure 10, we give the error function

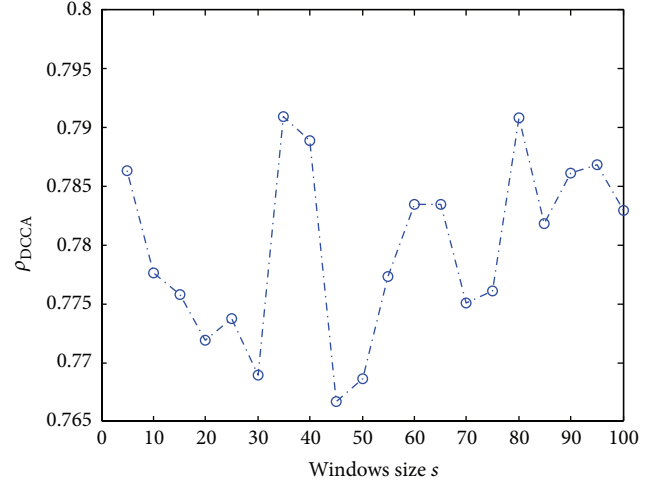


FIGURE 9: The cross-correlations between two traffic volume fluctuation variables from two adjacent sections.

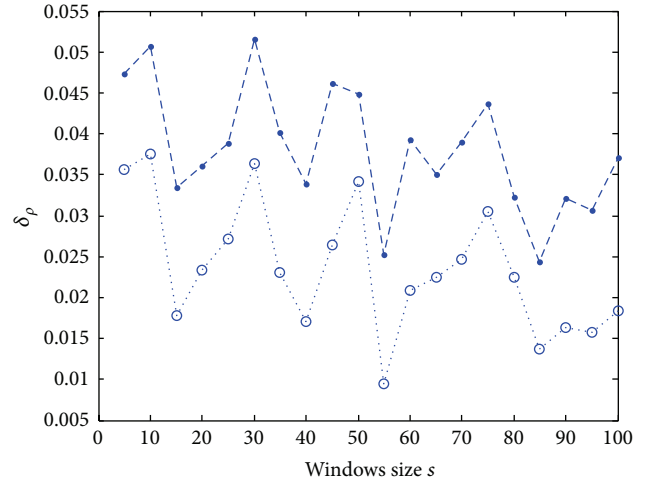


FIGURE 10: The error function $\delta(s)$ of traffic volume series for the data from two adjacent sections and the data from two adjacent lanes (circles) and $\delta'(s)$ for the data from two adjacent sections and the data from two disjoint lanes (filled dots).

$\delta(s) = \rho_{DCCA}(s) - \rho_{DCCA}^{(1)}(s)$, where $\rho_{DCCA}(s)$ is the DCCA coefficient of traffic volume variables $\{q_i^{(11)}\}$ and $\{q_i^{(21)}\}$ and $\rho_{DCCA}^{(1)}(s)$ is the DCCA coefficient of traffic volume fluctuation variables $\{q_i^{(11)}\}$ and $\{q_i^{(12)}\}$. The error function $\delta(s) > 0$ (circles) demonstrates that the cross-correlation of volume fluctuation series between two adjacent lanes is weaker than the time series of two adjacent sections.

For convenience, Figure 10 also shows the error function $\delta'(s) = \rho_{DCCA}(s) - \rho_{DCCA}^{(2)}(s)$, where $\rho_{DCCA}^{(2)}(s)$ is the DCCA coefficient of traffic volume series $\{q_i^{(11)}\}$ and $\{q_i^{(13)}\}$. Similarly, it is apparent that the cross-correlation of volume series between two disjoint lanes is weaker than the time series of two adjacent sections by direct observation of the error function $\delta'(s) > 0$ (filled dots).

In the statistical analysis, the P value for $\rho_{\text{DCCA}}(s)$ and $\rho_{\text{DCCA}}^{(1)}(s)$ is 2.22×10^{-8} which indicates that the difference between two quantities is statistically significant. $\rho_{\text{DCCA}}(s)$ and $\rho_{\text{DCCA}}^{(2)}(s)$ of volume time series are also statistically significant based on permutation testing ($P = 6.61 \times 10^{-14}$).

4. Conclusion

In the paper, we consider DCCA coefficients method to understand the complexity of traffic dynamic. The technique has been implemented on the time series of the original traffic variables from adjacent lanes and adjacent sections. For the traffic speed time series and volume time series, the DCCA coefficients fluctuate around the value $\rho_{\text{DCCA}} = 0.82$ and provide evidence that cross-correlation characteristic exists in traffic dynamic. Then, we apply DCCA coefficients method to study the cross-correlation between traffic speed series. We find that two traffic speed fluctuation parameters derived from adjacent sections exhibit much stronger correlation than the traffic parameters derived from adjacent lanes and disjoint lanes. Similarly, by applying DCCA coefficients method to traffic volume series, the cross-correlation property between the volume variables from two adjacent sections is stronger than the cross-correlation property between the volume variables of adjacent lanes and disjoint lanes.

The relationship of traffic series between two adjacent sections or lanes in China is investigated with the data from BTRR. The results that the traffic series between two adjacent sections or lanes exhibit cross-correlation are attributable to each of the two variables at any time depending not only on its own past values but also on past values of the other variable. Therefore, the findings presented here encourage us to think that this method reveals the relation in anomalous traffic conditions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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