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Research Article

Improved Confidence Interval Estimation for Oscillometric Blood Pressure Measurement by Combining Bootstrap-After-Jackknife Function with Non-Gaussian Models

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Confidence intervals (CIs) are generally not provided along with estimated systolic blood pressure (SBP) and diastolic blood pressure (DBP) measured using oscillometric blood pressure devices. No criteria exist to determine the CI from a small sample set of oscillometric blood pressure measurements. We provide an extended methodology to improve estimation of CIs of SBP and DBP based on a nonparametric bootstrap-after-jackknife function and a Bayesian approach. We use the nonparametric bootstrap-after-jackknife function to reduce maximum amplitude outliers. Improved pseudomaximum amplitudes (PMAs) and pseudoenvelopes (PEs) are derived from the pseudomeasurements. Moreover, the proposed algorithm uses an unfixed ratio obtained by employing non-Gaussian models based on the Bayesian technique to estimate the SBP and DBP ratios for individual subjects. The CIs obtained through our proposed approach are narrower than those obtained using the traditional Student t-distribution method. The mean difference (MD) and standard deviation (SD) of the SBP and DBP estimates using our proposed approach are better than the estimates obtained by conventional fixed ratios based on the PMA and PE (PMAE).

1. Introduction

Oscillometric blood pressure methods are widely used and monitors are commercially available. Numerous studies have been performed to improve the accuracy of oscillometric blood pressure device [1–5]. However, there are still no standard protocols for these devices and these devices provide only single estimate with no confidence interval (CI). The user may therefore not be able to distinguish statistical variability in the estimates from intrinsic variability due to physiological status. It would therefore be useful to specify the CI for blood pressure measurements. For example, a reading of 110/70 ± (SBP: 7)/(DBP: 5) would assure the user that 95% of measurements carried out under identical conditions would fall in the CI, corresponding to a systolic blood pressure (SBP) between 103 and 117 and diastolic blood pressure (DBP) between 65 and 75. If the CI is too wide-ranging, an alarm can recommend discarding the measurement and initiating another measurement. Without the CI, it is difficult to make any meaningful decision for the blood pressure estimates. Based on some aggregated statistics, in a home-based monitoring setting, the repeated wide-ranging CI can trigger an alarm and alert either the nurse station or the family doctor. Even though this is important to take the measurement uncertainties in blood pressure into account [6], few attempts have been made to study the CIs for the systolic and diastolic blood pressures with the exception of one recent study [3]. However, it is not feasible to acquire large number of measurements for each subject using a noninvasive oscillometric blood pressure measurement device, as repeatable conditions can never be guaranteed [7]. Because blood pressure varies continuously over time according to various physiological factors [8], it would be ideal to calculate CIs based on only a few measurements. This calls for an innovative methodology that can estimate CIs from a smaller sample size. Use of a bootstrap approach to

estimate CIs for oscillometric blood pressure measurements was proposed in [7]. Although CIs obtained using the bootstrap approach are narrow and have a lower standard deviation, values are often too wide or too narrow because of outliers in the maximum amplitude (MA) positions. In this paper, we describe an extended methodology to improve the CI estimates of the SBP and DBP. Specifically, improved CIs are estimated using pseudomeasurements obtained based on nonparametric bootstrap (NPB) and a jackknife influence function is used to remove MA outliers [9]. Improved pseudomaximum amplitudes (PMAs) and pseudoenvelopes (PEs) are then derived from the pseudomeasurements.

Currently, the maximum amplitude algorithm (MAA) is widely used to estimate average arterial blood pressure (ABP) under oscillometric method [2, 4, 10]. The MAA estimates the mean arterial pressure (MAP) as the cuff pressure (CP) at which maximum oscillation occurs and then linearly relates the systolic and diastolic blood pressures using experimentally obtained ratios [1, 7, 10]. As we mentioned before, Lee et al. proposed CIs estimation based on the MAA technique, which shows good performance. However, our previous method [7] is not significantly different from the conventional MAA in terms of the mean difference (MD) because of employing the fixed ratio obtained experimentally [4, 7]. Although SBP and DBP ratios (SBPR and DBPR) in the conventional method (MAA) are assumed to be fixed, these ratios are not fixed [8, 11, 12]. Recently, Raamat et al. also showed that physiological factors can have significant effects on characteristic ratios [13]. Also, Lee et al. studied the determination of blood pressure using the Bayesian approach [11]. Their method assumes a Gaussian distribution for the *a posteriori* distribution of the ratios of systolic and diastolic blood pressures. More recently, Liu et al. introduced an error mechanism of the fixed ratio based on oscillometric blood pressure measurements [14]. As recent as the last year, an alternative approach using a Gaussian mixture model (GMM) was proposed to address the fixed ratio problem of the conventional MAA algorithm [15].

In practice, for any subject, blood pressure varies continuously over time according to various physiological factors [8]. However, the fixed ratio can be viewed as a value reliant on measurements of an experimental group with a minimum mean error with respect to the auscultatory nurse measurements as a reference. If the fixed ratio obtained from the subjects in one group is used for another group, one would not be able to acquire reliable blood pressure estimates. Thus, the MAA derived by a fixed ratio is not adequate to accurately determine for the SBP and DBP, because these blood pressures (BPs) show significant continuous variability over time [8].

For these reasons, it is necessary to develop a technique that can efficiently estimate improved CIs from a small sample of blood pressure measurements and accurately determine oscillometric blood pressure measurements. In this paper, we propose a novel technique to determine these ratios and to estimate improved CIs that combines an NPB-after-jackknife influence function with non-Gaussian models based on a Bayesian approach. We confirm that the proposed methodology decreases the mean difference (MD) and the standard deviation (SD), and the CIs become tighter compared to conventional methods since we compare the results of our proposed method with those measured by the professional nurses (auscutatory results). Summarizing our technique, the goal of this paper is to reduce uncertainty for blood pressure measurements based on the following contributions:

(i) we develop a method that reduces the SD of PMA [7] using a jackknife influence function [9];

(ii) non-Gaussian models are employed based on a Bayesian technique to decrease the MD between the results of our proposed method and those of the auscutatory nurse measurements.

The proposed method consists of two main parts: obtaining the improved pseudomaximum amplitude (PMA) and the pseudoenvelope (PE) using NPB with a jackknife influence function and estimating the SBPR and DBPR by employing the non-Gaussian models based on the Bayesian approach [7, 9, 11, 16, 17].

The block diagram of our algorithm is given in Figure 1. The PMAs and PEs are represented in the part (a), which are called pseudomaximum amplitude and envelope (PMAE) method [7]. First, the oscillometric envelopes are obtained from each subject (5 measurements × 85 subjects = 425 total measurements), respectively. The envelopes are then smoothed utilizing the Gaussian fitting and these are also used to obtain the PEs. For more detail, the MA locations using the MAA are found in Step 3. In Step 4, the MA outlier is then removed by using the jackknife influence function. We obtain the PMA locations using the NPB technique in the following step. Using the CI method, the upper, middle, and lower PMA locations are selected as shown in Step 6 of Figure 1. In Step 7, the fitted envelopes are adjusted to obtain the identical lengths, and the PEs are also achieved using the NPB technique. The upper, middle, and lower PEs are then determined in Step 8. In the previous step, the PMAs and PEs using the NPB were obtained. If the PMA locations do not match with end (start) point of the systolic (diastolic) PEs, it will be necessary to use signal processing method to ensure that the PMA locations are on the PEs as shown in Step 9 of Figure 1. For more detailed description to obtain the PMAs and PEs, the interested reader is referred to [7]. Specifically, the part (b) in Figure 1 is to estimate the systolic and diastolic ratios using non-Gaussian models based on a Bayesian approach that will be more described in (12)–(21).

2. Methods

2.1. Subjects and Data Collection. We follow the standard guidance, that is, currently available for blood pressure measurements. Our experimental dataset was obtained from 85 healthy subjects aged from 12 to 80, out of which thirty-seven were females and forty-eight were males. No recruited subject had any history of cardiovascular disease. Five sets of oscillometric BP measurements were obtained from each volunteer ($5 * 85 = 425$ total measurements; duration range: 31–95 sec., duration median: 55 sec.) using a wrist worn UFIT TEN-10 blood pressure device (Biosign Technologies Inc.,

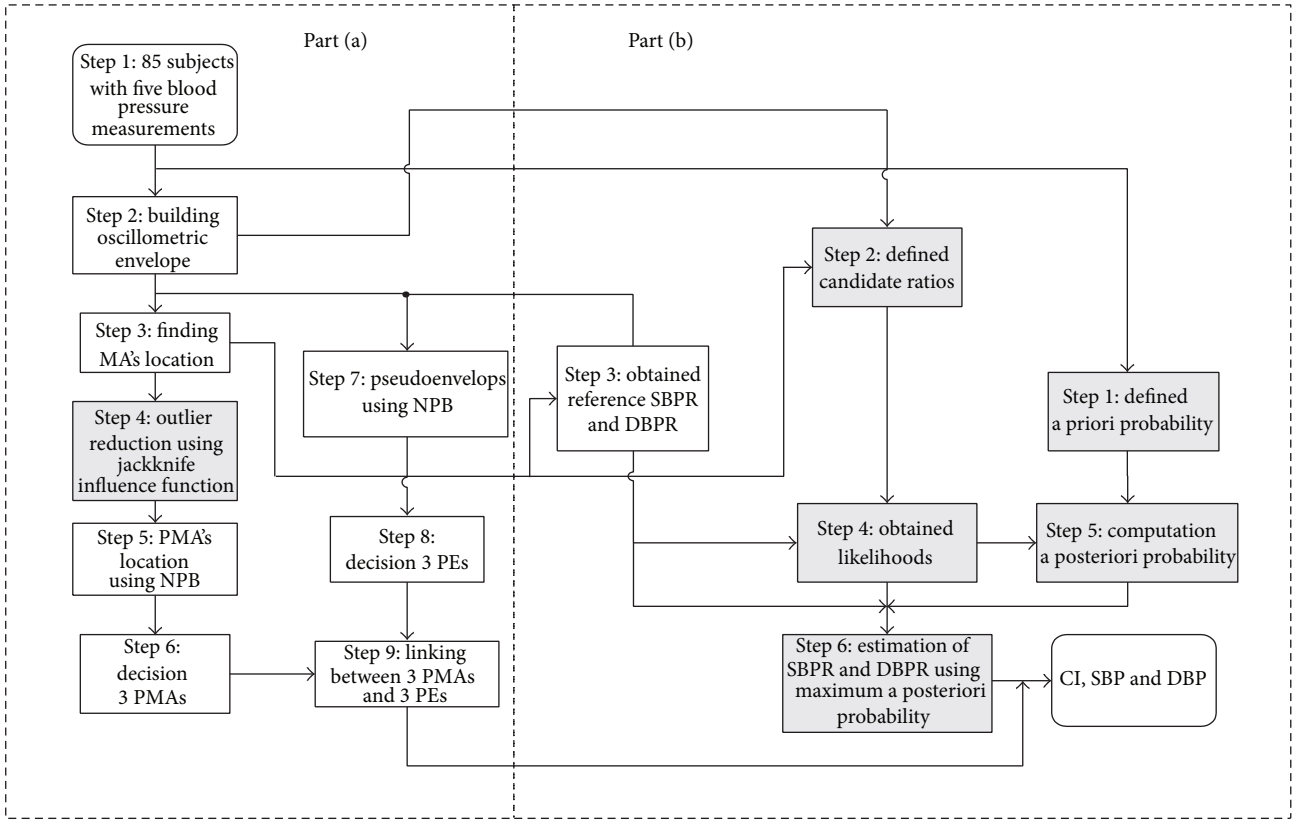

Figure 1: Block diagram for improved CI estimation combining the bootstrap-after-jackknife function influence function with a generalized Gaussian model based on a Bayesian technique. (a) CI estimation using bootstrap-after-jackknife function influence function. (b) Ratio estimation for the SBP and DBP employing non-Gaussian models based on a Bayesian technique. Note that we use different background colors, where gray box denotes the proposed approach for the PMAE (white box) [7].

Toronto, ON, Canada) at a sample rate of 100 Hz. Our number of measurements exceed the recommendations of the American National Standard Institute (ANSI) association for the advancement of medical instrumentation (AAMI) SP-10 standard which requires a minimum of 3 measurements from 85 subjects, 255 measurements in total. Corresponding to each cuff pressure waveform, two reference readings were also recorded using the auscultatory method by two independent trained observers (nurses), and these two measurements were utilized as the reference BP for each subject. Nurse measurements were used as golden standard since there are no set standard so far for oscillometric blood pressure measurements. Nurse readings were relatively stable in that the maximum difference between the two nurses was no more than 2 mmHG. This meets the recommendations of the ANSI/AAMI SP 10 standard, which requires the mean difference to be no more than 5 mmHG. Nurse reading of SBP ranged from 78 to 147 mmHG while those of DBP ranged from 42 to 99 mmHG across total 85 subjects [12]. We used oscillometric blood pressure recoding to measure BP, followed by readings of SBP and DBP with help of two trained nurse after a one-minute break. This was then followed by another one minute break. This procedure was repeated again four more times to obtain five measurements [12]. During data collection, each subject sat on an upright posture in a chair and the UFIT device's cuff was strapped to the left wrist of the subject, which was raised to heart level [12]. Another cuff for auscultatory BP measurement was placed on the upper left arm, also at heart level.

2.2. Conventional MAA Based on the Oscillometric Envelope. The MAA is generally utilized to estimate the SBP and DBP based on the oscillometric waveform (OMW). Indeed, the MAA needs to find the point of envelope that corresponds to the MAP, which is estimated to be the maximum amplitude (MA) position on the envelope of the OMW as shown in Figure 2(c). This position on the deflation curve (cuff pressure) gives us to acquire the MAP in mmHg [8]. The left part of the MAP represents the SBP side and the right part of the MAP denotes the DBP side. The MAA then uses fixed characteristic ratios to find the points that correspond to the SBP and DBP, respectively. The MA is multiplied by the fixed SBPR and DBPR [4, 7] to obtain the amplitudes of the systolic position and the diastolic position, respectively, as follows:

$$\begin{aligned} \widehat{s}_{(i,j)} &= m_{(i,j)} \cdot \widehat{r}_s, \\ \widehat{d}_{(i,j)} &= m_{(i,j)} \cdot \widehat{r}_d, \end{aligned} \tag{1}$$

where $\widehat{s}_{(i,j)}$ and $\widehat{d}_{(i,j)}$ denote the oscillometric amplitudes corresponding to the SBP and DBP, respectively, $m_{(i,j)}$ denotes

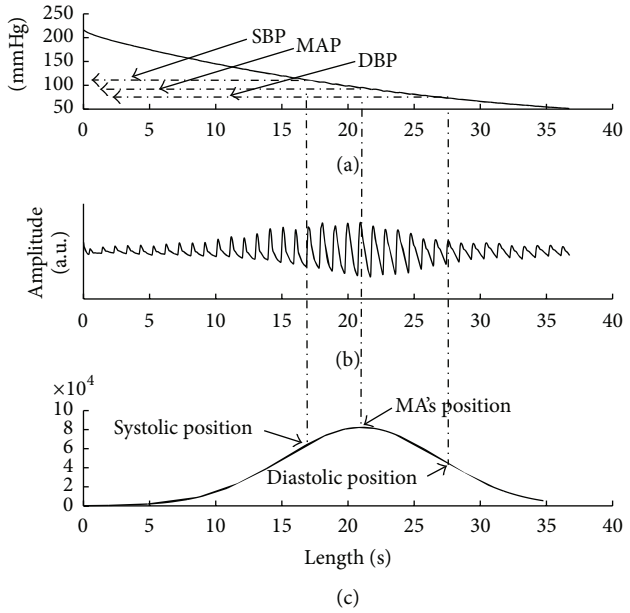

FIGURE 2: The fundamental concept of the MAA based on the oscillometric waveform (OMW). (a) Cuff pressure (CP). (b) OMW. (c) Envelope of smoothed OMW.

the MA based on the oscillometric envelope, and $\widehat{r}_s$ and $\widehat{r}_d$ denote the fixed SBPR and DBPR. Also, $i = 1, \ldots, N$, $j = 1, \ldots, M$; N and M denote the number of subjects and the number of measurements per subject, respectively. Thus, points of the oscillometric amplitudes corresponding to the SBP and DBP are mapped back to the cuff pressure to obtain the SBP and DBP values in mmHg as shown in Figure 2.

2.3. Review of CI Estimation Based on the Bootstrap Technique. In this subsection, we introduce the fundamental concept of the bootstrap technique, which is a computational tool used to improve the accuracy of estimates from a small number of measurements in situations where conventional methodologies fail [7, 16, 18]. The basic idea behind the bootstrap technique is to provide a large number of independent BP parameters by resampling the original blood pressure estimate, $\Psi = (\psi_1, \psi_2, \ldots, \psi_n)$ of say n measurements at random from an unknown probability distribution Ω_Ψ. A bootstrap resample $\Psi^* = (\psi_1^*, \psi_2^*, \ldots, \psi_n^*)$ is obtained as a random sample of size n drawn randomly with replacement from the original measurement set Ψ with elements occurring zero, once, or multiple times. Let $\widehat{\mu} = \widehat{\mu}(\Psi)$ be an estimator of the mean of Ω_Ψ. The goal is to find characteristics of $\widehat{\mu}(\Psi)$ similar to the distribution of $\mu(\Psi)$. Thus, the distribution of the estimated mean $\widehat{\mu}$ is approximated by the distribution of the pseudoestimated mean $\widehat{\mu}$ from the bootstrap resample $\widehat{\Omega}_{\Psi^*}$.

Based on the principle of the bootstrap technique, the CI can be obtained by using the NPB technique. Let $\widehat{\mu}_\alpha^*$ denote the 100 αth percentile [7] of N bootstrap replications $\widehat{\mu}^*(1), \widehat{\mu}^*(2), \ldots, \widehat{\mu}^*(N)$. The percentile band $\widehat{\mu}_l$, $\widehat{\mu}_u$ of intended coverage $1 - 2\alpha$ is defined by

$$\left[\widehat{\mu}_l, \widehat{\mu}_u\right] = \left[\widehat{\mu}_\alpha^*, \widehat{\mu}_{1-\alpha}^*\right], \tag{2}$$

where l and u denote the lower and upper limits of the CI, respectively.

2.4. Improved PMA Estimates Using a Bootstrap-After-Jackknife Function Influence Function. In this subsection, we obtain the maximum amplitudes and the length of occurrence of the maxima from all the five BP measurements for each subject. These preliminary MA values are used to determine improved PMAs based on the NPB technique after implementation of a jackknife influence function [7, 9]. The principle of our proposed algorithm is similar to that underlying the PMA technique [7]. The difference is that we use the jackknife influence function to remove the MA outlier. The main objective of this subsection is to describe how to remove MA outlier using the jackknife influence function [9]. Suppose that $\mathbf{x} = \{x_1, \ldots, x_5\}$ is based on the oscillometric BP envelope. Let $\mathbf{x}_{(k)}$ denote the remaining length positions of the MAs with the kth observation removed such that

$$\mathbf{x}_{(k)} = \left\{x_1, x_2, \ldots, x_{k-1}, x_{k+1}, \ldots, x_K\right\}. \tag{3}$$

For a given set, the jackknife influence function is defined by

$$u_k(\mu) = (K-1)\left[\mu_{(\cdot)} - \mu_{(k)}\right], \tag{4}$$

where $\mu_{(k)} \equiv \mu(\mathbf{x}_{(k)})$, $\mu_{(\cdot)}$ is the mean of the jackknifed values as $\mu_{(\cdot)} \equiv \sum_{k=1}^{K} \mu_{(k)}/K$, and K is the number of BP measurements. From $u_k(\mu)$, the relative jackknife influence function is given by

$$u_k^+(\mu) = \frac{u_k(\mu)}{\sqrt{\sum u_i(\mu)^2/(K-1)}}, \tag{5}$$

where $\sup_k(|u_k^+(\mu)|) < 2$ indicates a robust statistic value. The statistic values of the relative jackknife influence function suggested by Efron [9] can be used to evaluate the extremity of the position of the suspect MA outliers.

According to the previous observation, we obtain a new set $\mathbf{x}^+ = \{x_1, \ldots, x_{K-1}\}$ of length positions of the MAs after removing the largest value $\max|u_k^+(\mu)|$ for each individual subject. To obtain the new set $\mathbf{y}^+ = \{y_1, \ldots, y_{K-1}\}$ of corresponding MA values, we perform the same procedure described above. In practice, we obtain three improved positions of PMAs using the NPB, as described below, to estimate improved CIs of the SBP and DBP. By subjecting the new sets to the NPB, we generate a number of $N(= 1000)$ resamples, $\mathbf{x}_n^*$, $\mathbf{y}_n^*$, $n = 1, \ldots, N$, where $\mathbf{x}_n^* = \{x_{1n}^*, \ldots, x_{4n}^*\}$ and $\mathbf{y}_n^* = \{y_{1n}^*, \ldots, y_{4n}^*\}$, respectively. Next, we calculate the mean of all measurements in $\mathbf{x}_n^*$ and $\mathbf{y}_n^*$ to obtain $\widehat{\mu}_{\mathbf{x}(n)}^*$ and $\widehat{\mu}_{\mathbf{y}(n)}^*$, given by

$$\begin{aligned} \widehat{\mu}_{\mathbf{x}(n)}^* &= \frac{1}{K-1}\sum_{k=1}^{K-1} x_{k,n}^*, \\ \widehat{\mu}_{\mathbf{y}(n)}^* &= \frac{1}{K-1}\sum_{k=1}^{K-1} y_{k,n}^*, \end{aligned} \tag{6}$$

where $K = 5$ and $n = 1, \ldots, N$. The distributions of the bootstrap estimates $\widehat{\mu}^*_{\mathbf{x}(n)}$ and $\widehat{\mu}^*_{\mathbf{y}(n)}$ are shown in [7], which indicate the length of occurrence of the pseudomaxima and PMAs from all five measurements per subject, respectively. We then sort the bootstrap estimates, $\widehat{\mu}^*_{\mathbf{x}(n)}$ and $\widehat{\mu}^*_{\mathbf{y}(n)}$ in increasing order. Thus, the sorted PMAs are given by $\widehat{\mu}^*_{\mathbf{y}(1)} \leq \widehat{\mu}^*_{\mathbf{y}(2)} \leq \widehat{\mu}^*_{\mathbf{y}(3)} \cdots \leq \widehat{\mu}^*_{\mathbf{y}(N-1)} \leq \widehat{\mu}^*_{\mathbf{y}(N)}$ and the length locations of the PMAs are given by $\widehat{\mu}^*_{\mathbf{x}(1)} \leq \widehat{\mu}^*_{\mathbf{x}(2)} \leq \widehat{\mu}^*_{\mathbf{x}(3)} \cdots \leq \widehat{\mu}^*_{\mathbf{x}(N-1)} \leq \widehat{\mu}^*_{\mathbf{x}(N)}$. The desired $100 \cdot (1-\alpha)\%$ nonparametric CIs for the position of the PMAs are given by $(\widehat{\mu}^*_{\mathbf{x}(Q_1)}, \widehat{\mu}^*_{\mathbf{x}(Q_2)})$ and $(\widehat{\mu}^*_{\mathbf{y}(Q_1)}, \widehat{\mu}^*_{\mathbf{y}(Q_2)})$, respectively, where Q_1 is the quotient of $N \cdot \alpha/2$, $Q_2 = N - Q_1 + 1$, and $Q_3 = N/2$. For $\alpha = 0.05$ and $N = 1000$, we obtain $Q_1 = 25$, $Q_2 = 976$, and $Q_3 = 500$. Thus, we take the three positions of the PMAs that will be used by the algorithm to estimate CIs of the SBP and DBP.

2.5. Review of Pseudoenvelopes (PEs) Using NPB. To obtain the PEs to estimate the CIs of the SBP and DBP using NPB, we construct a BP measurement matrix $\mathbf{B}$ as in (7). This matrix consists of envelopes for five measurements for the systolic and diastolic part of each subject as described in [7]:

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdot & \cdot & b_{1K} \\ b_{21} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{L1} & \cdot & \cdot & \cdot & b_{LK} \end{bmatrix}, \tag{7}$$

where L denotes the length of the PE, $K(= 5)$ denotes the number of envelopes, and each column denotes an envelope of oscillometric measurements. Particularly, all measurements are forced to be of length L by either extrapolating to length L if the measurement is shorter or truncating the length to L if the measurement is longer. From the BP envelope matrix $\mathbf{B}$, we achieve N resample envelope matrices $\mathbf{B}^*_1, \ldots, \mathbf{B}^*_N$ by employing NPB method. The envelopes in the nth resample envelope matrix $\mathbf{B}^*_n$ are given by

$$\mathbf{B}^*_n = \begin{bmatrix} b^*_{11} & b^*_{12} & \cdot & \cdot & b^*_{1K} \\ b^*_{21} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ b^*_{L1} & \cdot & \cdot & \cdot & b^*_{LK} \end{bmatrix}, \tag{8}$$

where $n = 1, \ldots, N(= 1000)$. The SBP and DBP parts of the envelope are identified by utilizing the peak of the envelope. The section from the beginning to the peak of the BP envelope (corresponding to decreasing cuff pressure) represents SBP, while the section from the peak to the end of the BP envelope represents DBP. We then reorder resampled BP envelope matrices (for systolic and diastolic parts of the envelopes) using ascending and descending sort techniques (for the SBP and DBP parts of the envelopes, resp.). Each of the sorted matrices has five columns and each column corresponds to a BP measurement of length L. We then obtain a single BP envelope per subject as the last step of the PE process, where the desired $100 \cdot (1-\alpha)\%$ NPB CIs are given by $(\mathbf{PE}_{Q_1}, \mathbf{PE}_{Q_2})$ [7]. Here, $Q_1 = \lfloor N \cdot \alpha/2 \rfloor$ is the quotient of $N \cdot \alpha/2$, $Q_2 = N - Q_1 + 1$, and $Q_3 = \lfloor N/2 \rfloor$ is the quotient of $N/2$. As a result, we have

$$\begin{aligned} \mathbf{PE}_{Q_1} &= K^{-1} \sum_{k=1}^{K} \mathbf{B}^*_{Q_1}(:, k), \\ \mathbf{PE}_{Q_2} &= K^{-1} \sum_{k=1}^{K} \mathbf{B}^*_{Q_2}(:, k), \\ \mathbf{PE}_{Q_3} &= K^{-1} \sum_{k=1}^{K} \mathbf{B}^*_{Q_3}(:, k), \end{aligned} \tag{9}$$

where $\mathbf{B}^*_{Q_1}(:, i)$, $\mathbf{B}^*_{Q_2}(:, i)$, and $\mathbf{B}^*_{Q_3}(:, i)$ denote the ith column of matrices $\mathbf{B}^*_{Q_1}$, $\mathbf{B}^*_{Q_2}$, and $\mathbf{B}^*_{Q_3}$ and $i = 1, \ldots, N$. Thus, we obtain upper, lower, and middle PEs that can be used to estimate the CIs for SBP and DBP utilizing the systolic and diastolic BP envelope matrices, respectively [7].

In the previous subsection, we obtained the value of the improved PMA by using NPB with a jackknife influence function. As the PMA estimates may not connect with the end (start) point of the systolic (diastolic) PEs or amplitudes, we take advantage of signal processing (padding and clipping) to ensure that the location values (both amplitude and length) of the PMAs are based on the PEs. In the final step, we need to obtain the mean cuff pressure to find the CI estimates of SBP and DBP [7]. To find the SBP and DBP, the SBPR and DBPR are estimated using the generalized Gaussian model-based Bayesian technique as described in detail in the following subsection.

2.6. Ratio Estimation Employing Non-Gaussian Models Based on a Bayesian Technique. We previously described the Bayesian technique to determine systolic and diastolic ratios [11, 12]. Using this approach, we determine the systolic and diastolic ratios for any ith subject and any jth measurement [11]. However, this approach assumes a Gaussian distribution for *a posteriori* distribution of the ratios of the systolic and diastolic blood pressures [11]. The proposed methodology herein adopts a more tractable model, namely, non-Gaussian models [19] for *a posteriori* distribution because the random process may not adhere to the Gaussian model and there is a potential for *a posteriori* distribution to be successfully characterized by the non-Gaussian models which includes the Laplacian and Cauchy-Lorentz (CL) models as special cases [19]. Among the various extensions of the Gaussian models, the generalized Gaussian model is one of the most popular models such that

$$f_x = \frac{j\eta}{2\Gamma(1/j)} \exp^{(-\eta|x-\beta|)^2}, \tag{10}$$

where $\Gamma(\cdot)$ denotes the Gamma function $\Gamma(x) = \int_0^\infty t^{x-1} \exp^{-t} dt$ and η denotes a constant given by

$$\eta = \sigma^{-1} \sqrt{\Gamma\left(\frac{3}{j}\right)\left(\Gamma\left(\frac{1}{j}\right)\right)^{-1}}, \tag{11}$$

where σ represents the standard deviation as the scale parameter of the distribution $\sigma > 0$ while the impulsiveness is determined by the parameter $j > 0$. A special case of the generalized Gaussian model is the well-known Laplacian model in the case $j = 1$.

According to the non-Gaussian models, we then choose the value of the likelihood function that maximizes *a posteriori* probability using the Bayesian approach [20]. An equally likely *a priori* assumption is utilized to derive a likelihood function based on blood pressure values acquired by the MAA algorithm for each *priori* probability.

In this paper, the systolic and diastolic ratios acquired using the non-Gaussian model-based Bayesian technique are used to estimate the SBP and DBP for any ith subject as follows:

$$\widehat{r}_s(i) = \frac{\widehat{s}(i)}{m(i)}, \qquad \widehat{r}_d(i) = \frac{\widehat{d}(i)}{m(i)}, \tag{12}$$

where $\widehat{r}_s$ and $\widehat{r}_d$ denote the estimated systolic and diastolic ratios, respectively, for each subject, $\widehat{s}(i)$ and $\widehat{d}(i)$ denote the oscillometric amplitude corresponding to the SBP and DBP, respectively, while $m(i)$ is the MA based on the oscillometric envelope, and $i = 1, \ldots, I$ and $I(= 85)$ denote the number of subjects. We assume that the systolic and diastolic ratios of the current measurement have no dependence on any of the previous measurements and are only reliant on the physiological status, h, of the person, as in [11]. Furthermore, it is assumed that the physiological processes of a person are random processes [7, 8] and that *a priori* probability of the ratios $P(\widehat{r}(i) \mid h)$ is uniformly distributed between the known *a priori* minimum and maximum values.

Let $\mathbf{r}_{cs}$ and $\mathbf{r}_{cd}$ be vectors of possible candidates for the SBP and DBP ratios, respectively. Consider

$$\mathbf{r}_{cs} = [\alpha_1, \alpha_2, \ldots, \alpha_K], \qquad \mathbf{r}_{cd} = [\beta_1, \beta_2, \ldots, \beta_K], \tag{13}$$

where $\mathbf{r}_{cs}$ and $\mathbf{r}_{cd}$ represent the vectors of possible candidates for the SBP and DBP ratios, respectively, and K is determined in *a priori* manner where K is the number of candidate ratios. In this work, $K = 31$, ($\alpha_1 = 0.65$ to $\alpha_K = 0.95$ and $\beta_1 = 0.30$ to $\beta_K = 0.60$) for the SBP and DBP, respectively, in increments of 0.01. Consider

$$\mathbf{p}_s = [\gamma_1, \gamma_2, \ldots, \gamma_K], \qquad \mathbf{p}_d = [\delta_1, \delta_2, \ldots, \delta_K], \tag{14}$$

where $\mathbf{p}_s$ and $\mathbf{p}_d$ represent *a priori* probability vectors and the elements of the vector are $1/K = 0.032$. Because we have no *a priori* information, equal *a priori* probabilities are assigned to all candidate ratios. Note that $\mathbf{r}_{cs}(i) = \mathbf{r}_{cs}$, $\mathbf{r}_{cd}(i) = \mathbf{r}_{cd}$, $\mathbf{p}_s(i) = \mathbf{p}_s$, and $\mathbf{p}_d(i) = \mathbf{p}_d$ for all i.

Subsequently, *a posteriori* probability (POP) for every k, $k = 1, \ldots, K$, is found as

$$p\left(\mathbf{r}(k)_{cs(i)} \mid \widetilde{r}_{s(i)}\right) = \frac{\mathbf{p}(k)_{s(i)} f\left(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cs(i)}\right)}{\sum_{k=1}^{K} \mathbf{p}(k)_{s(i)} f\left(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cs(i)}\right)}, \qquad p\left(\mathbf{r}(k)_{cd(i)} \mid \widetilde{r}_{s(i)}\right) = \frac{\mathbf{p}(k)_{d(i)} f\left(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cd(i)}\right)}{\sum_{k=1}^{K} \mathbf{p}(k)_{d(i)} f\left(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cd(i)}\right)}, \tag{15}$$

where $\mathbf{p}(k)_{s(i)}$ and $\mathbf{p}(k)_{d(i)}$ represent *a priori* probabilities for the kth candidate ratio and $f(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{rs(i)})$ and $f(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cd(i)})$ represent the likelihoods for the SBP and DBP at the chosen ratio, respectively. The distributions of conditional BP measurements, $\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cs(i)}$ and $\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cd(i)}$, are Gaussian with a known mean and variance. Their densities are defined such that

$$f\left(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cs(i)}\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp^{-(1/2\sigma^2)(\widetilde{r}_{s(i)} - \mathbf{r}(k)_{cs(i)})^2}, \qquad f\left(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cd(i)}\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp^{-(1/2\sigma^2)(\widetilde{r}_{s(i)} - \mathbf{r}(k)_{cd(i)})^2}, \tag{16}$$

where σ represents the standard deviation (SD). We performed an experiment by varying σ from 0.02 to 0.2 for the chosen range of systolic and diastolic ratios, namely, 0.65 to 0.95 and 0.30 to 0.60, respectively. The results confirmed that the likelihood function was largely unaffected by changes in σ as mentioned in [20].

The likelihoods $f(\widetilde{r}_{s(i)} \mid \mathbf{r}(k)_{cs(i)})$ of each ratio are the values of the measurement distribution at a measurement value, where $\widetilde{r}_{s(i)}$ are the ratios of the pressure values obtained from the auscultatory nurse measurements and the maximum amplitude as given by (17). The rationale behind our approach is to find the SBP ratio $\mathbf{r}(k)_{cs(i)}$ that maximizes the likelihood ratio for the available SBP auscultatory nurse measurements for each subject. The same idea holds for the DBP ratio. Because two auscultatory nurse measurements are available, we use average SBP and DBP measurements as references to obtain the SBP and DBP ratios. Reference SBP and DBP ratios are obtained for the ith subject as follows:

$$\widetilde{r}_{s(i)} = \frac{\widehat{s}_{(i)}}{m_{(i)}}, \tag{17}$$

$$\widetilde{r}_{d(i)} = \frac{\widehat{d}_{(i)}}{m_{(i)}}, \tag{18}$$

where $\widetilde{r}_{s(i)}$ and $\widetilde{r}_{d(i)}$ represent the reference SBP and DBP ratios obtained from the averaged nurse measurements and $\widehat{s}_{(i)}$ and $\widehat{d}_{(i)}$ denote the amplitudes of the SBP and DBP identified on the OMW's envelope through the deflation curve giving the auscultatory average nurse measurements for the SBP and DBP, respectively. $m_{(i)}$ denotes the MA in the OMW [11, 12].

The proposed methodology is an extended version of that in our previous paper [7, 11, 12] and applies non-Gaussian models such as Laplacian and Cauchy-Lorentz (CL) functions to obtain *a posteriori* probability (POP) for every k,

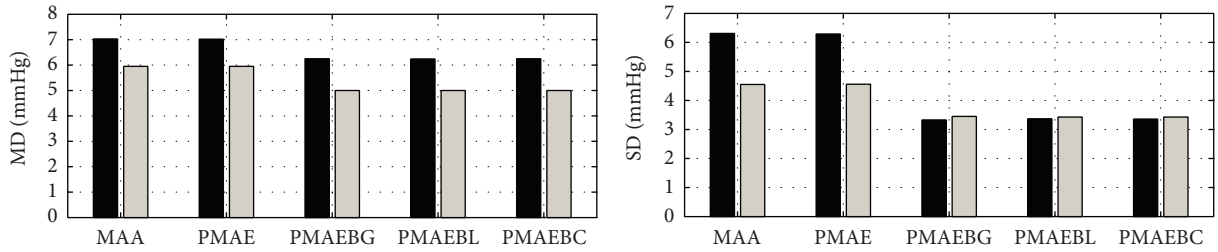

Figure 3: Summary of the MD and SD obtained using the MAA, PMAE, PMAE with BG (PMAEBG), PMAE with BL (PMAEBL), and PMAE with BC (PMAEBC) relative to the auscultatory nurse measurements, where BG, BL, and BC represent the Gaussian, Laplacian, Cauchy-Lorentz models based on the Bayesian technique, respectively, where black color denotes the results of SBP and where gray color denotes the results of DBP. (a) Summary of the MD. (b) Summary of the SD.

$k = 1, \ldots, K$. We first apply the Laplacian model to obtain the likelihoods of each ratio [19] as follows:

$$f_L\left(\widetilde{r}_{s(i)} \mid \mathbf{r}\left(k\right)_{cs(i)}\right) = \frac{1}{\sqrt{2}\sigma}\exp^{-(\sqrt{2}/\sigma)\left|\widetilde{r}_{s(i)} - \mathbf{r}(k)_{cs(i)}\right|},$$
$$f_L\left(\widetilde{r}_{d(i)} \mid \mathbf{r}\left(k\right)_{cd(i)}\right) = \frac{1}{\sqrt{2}\sigma}\exp^{-(\sqrt{2}/\sigma)\left|\widetilde{r}_{d(i)} - \mathbf{r}(k)_{cd(i)}\right|}. \tag{19}$$

Then, we use the Cauchy-Lorentz (CL) function to acquire the likelihood for the ratios of SBP and DBP:

$$f_{\mathrm{CL}}\left(\widetilde{r}_{s(i)} \mid \mathbf{r}\left(k\right)_{cs(i)}\right) = \frac{1}{\Pi}\frac{\gamma}{\gamma^2 + \left(\widetilde{r}_{s(i)} - \mathbf{r}\left(k\right)_{cs(i)}\right)^2},$$
$$f_{\mathrm{CL}}\left(\widetilde{r}_{d(i)} \mid \mathbf{r}\left(k\right)_{cd(i)}\right) = \frac{1}{\Pi}\frac{\gamma}{\gamma^2 + \left(\widetilde{r}_{d(i)} - \mathbf{r}\left(k\right)_{cd(i)}\right)^2}, \tag{20}$$

where $\gamma = 1$ represents the dispersion and is similar to the variance in a Gaussian model [19].

We also obtain SBP and DBP estimates using non-Gaussian models based on a Bayesian technique [19, 20] and then compare the results of the conventional MAA and the proposed method. Specifically, we estimate SBP and DBP ratios using the non-Gaussian models based on a Bayesian technique.

(1) Step 1: we use the ranges of the SBP and DBP ratios, which are initially found experimentally [2, 4].

(2) Step 2: *a priori* probability (P) is computed as shown in (14).

(3) Step 3: the reference SBP (DBP) ratio is obtained from the auscultatory nurse measurements, which are themselves obtained using cuff pressure, reference auscultatory measurements, and the maximum amplitude for each subject.

(4) Step 4: *a priori* likelihoods are also obtained using non-Gaussian models such as the Laplacian (L) and Cauchy-Lorentz (CL) models.

(5) Step 5: *a posteriori* probability (POP) is calculated to find the final ratios of SBP and DBP using (15).

(6) Step 6: the ratios of SBP and DBP obtained using the maximum *a posteriori* probability in (21) are considered as the best ratios for the measurement. Consider

$$\widehat{r}_{s(i)} = \arg\max_{\mathbf{r}(k)_{cs(i)}} p\left(\mathbf{r}\left(k\right)_{cs(i)} \mid \widetilde{r}_{s(i)}\right),$$
$$\widehat{r}_{d(i)} = \arg\max_{\mathbf{r}(k)_{cd(i)}} p\left(\mathbf{r}\left(k\right)_{cd(i)} \mid \widetilde{r}_{d(i)}\right). \tag{21}$$

Using these ratios, SBP and DBP points are finally obtained on the oscillometric envelope and they are mapped back to deflation curve resulting in the SBP and DBP values in millimeter of mercury [15].

3. Results and Discussion

To verify the performance of BP estimation, we calculated and compared the MDs and SDs of estimated BP and auscultatory nurse measurements based on the recommended AAMI standard protocol [21]. Indeed, we used five measurements to evaluate the MDs and SDs of our algorithm, whereas average ratios of five measurements were used to obtain the CIs for each of 85 subjects. A device is considered acceptable according to the AAMI criteria if the measurement error has a mean value of less than 5 mmHg with a SD of no more than 8 mmHg [21]. Thus, lower values of MD correspond to better overall performance. We compared the MD of the SBP and DBP obtained using our proposed algorithm to those obtained using the conventional methods. The results, as shown in Figure 3(a), confirm that our proposed method has an effect on the error of the estimate. We also used SD as a measure of error variability between the auscultatory nurse measurements and the estimates obtained using our proposed method. The SD between the proposed method (PMAEBG) and the auscultatory nurse measurements was found to be 3.33 mmHg for the SBP and 3.45 mmHg for the DBP, respectively, which were superior to those obtained from the nurse measurements and the conventional methods as shown in Figure 3(b). Additionally, the CI results for the PMAEBG were smaller than those obtained using the conventional methods of MAAST and MAAGUM (Table 1).

This is the first study to describe analysis of automated oscillometric blood pressure measurement by combining the bootstrap-after-jackknife function influence function with

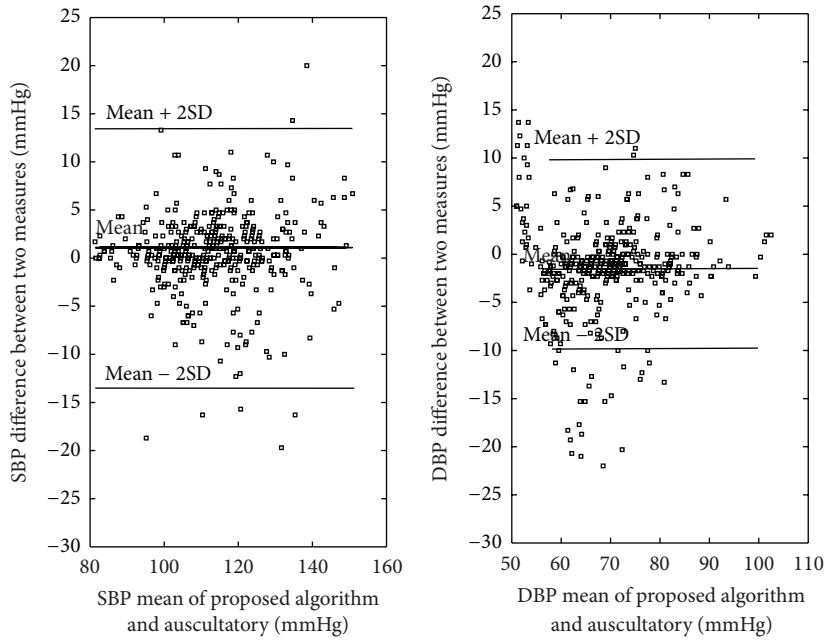

Figure 4: The Bland-Altman plots comparison of the performance between the (PMAEBG) and auscultatory nurse measurements. (a) Bland-Altman plot for the SBP. (b) Bland-Altman plot for the DBP.

Table 1: Comparison of average results (85 subjects with five measurements) with respect to the upper limit and lower limit of CIs (95%) for SBP and DBP using the MAA with Student's t-distribution (MAAST) [7], MAA with GUM (MAAGUM), PMAE, PMAEBG, PMAEBL, and PMAEBC, where σ is a standard deviation and GUM is the guide to the expression of uncertainty in measurement (GUM) [22].

BP (mmHg)	SBP (σ) Lower limit	SBP (σ) Upper limit	DBP (σ) Lower limit	DBP (σ) Upper limit
MAAST	106.7 (14.3)	120.2 (16.5)	62.4 (10.4)	71.7 (11.0)
MAAGUM	106.4 (14.3)	120.5 (16.4)	62.0 (10.4)	72.1 (10.9)
PMAE	112.4 (13.9)	115.0 (14.9)	66.7 (10.5)	68.2 (9.9)
PMAEBG	110.7 (13.0)	113.6 (13.9)	67.1 (10.3)	68.8 (10.2)
PMAEBL	110.7 (13.0)	113.6 (13.9)	67.1 (10.3)	68.8 (10.2)
PMAEBC	110.7 (13.0)	113.6 (13.9)	67.1 (10.3)	68.8 (10.2)

non-Gaussian models using a Bayesian technique to determine ratios and estimate the CIs of MAA for the SBP and DBP. We evaluated the accuracy of the readings obtained using our proposed method and those obtained using the ausculatory nurse method by comparing the MD and SD values obtained using these two approaches [21]. As shown in Figure 3(a), our proposed PMAEBL had an effect on the mean difference (MD), compared with the conventional methods. The MDs obtained using our proposed method are smaller than those obtained using conventional methods for both SBP and DBP. Note that the PMAEBL resulted in lower MDs for both SBP and DBP (6.25 and 5.00 mmHg, resp.).

In addition, we used SD to assess error variability between the auscultatory nurse measurements and the estimates obtained using our proposed method. The SDs of our proposed methods were 3.33 mmHg for SBP and 3.45 mmHg for DBP, which are superior to those obtained from the auscultatory nurse measurements and conventional methods [2] as shown in Figure 3(b). In contrast, our proposed method (PMAEBL) also showed improved performance relative to that of the PMAE [7]; the proposed method decreased the SBP and DBP estimation error by 2.98 and 1.10 mmHg, respectively. These results that our proposed method has an large effect on the variability of the estimate compared to the conventional methods [7]. Bland-Altman plots comparing the performance of PMAEBL and the auscultatory nurse measurements (425 measurements) are presented in Figure 4 [23]. The performance of conventional MAA and auscultatory nurse measurements (425 measurements) was also compared by Bland-Altman plots (Figure 5). The limits of agreement (see bold horizontal lines in Figures 4 and 5) that we used are (MD $\pm$ 2 $\times$ SD) for all plots. Bias (see horizontal center lines), for all plots, was negligible ($\leq$ $\pm$1.5 mmHg). This indicates that the BP estimates obtained by MAA and PMAEBL were in close agreement with those obtained by auscultatory nurse measurements without being overly biased in any particular direction. Note that the vertical spreads of the proposed algorithm (PMAEBL) for the SBP and DBP were smaller than those of the conventional MAA method, as shown in Figures 4 and 5. Clearly, the proposed algorithm (PMAEBL) improves oscillometric BP estimation. The consequences of such improvements could be very significant given that the AAMI standard protocol

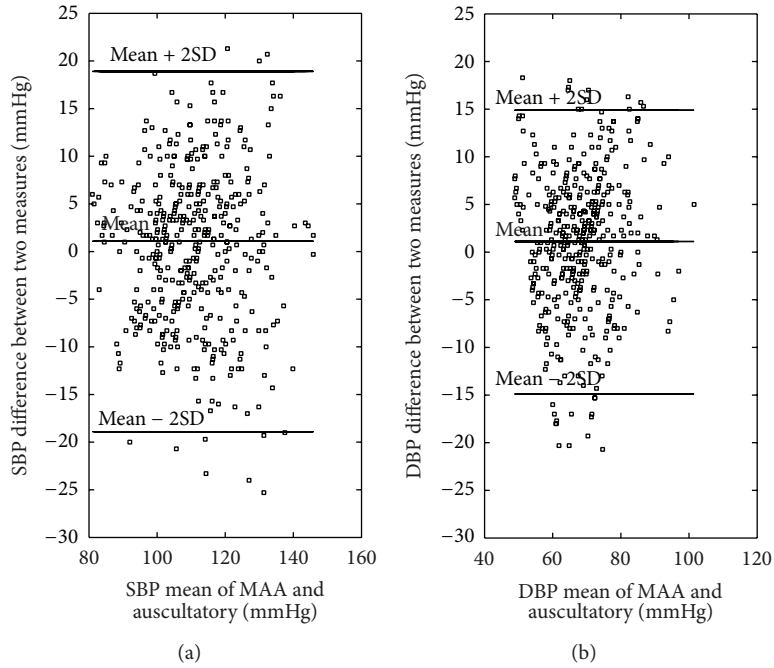

Figure 5: The Bland-Altman plots comparison of the performance between the MAA and auscultatory nurse measurements. (a) Bland-Altman plot for the SBP. (b) Bland-Altman plot for the DBP.

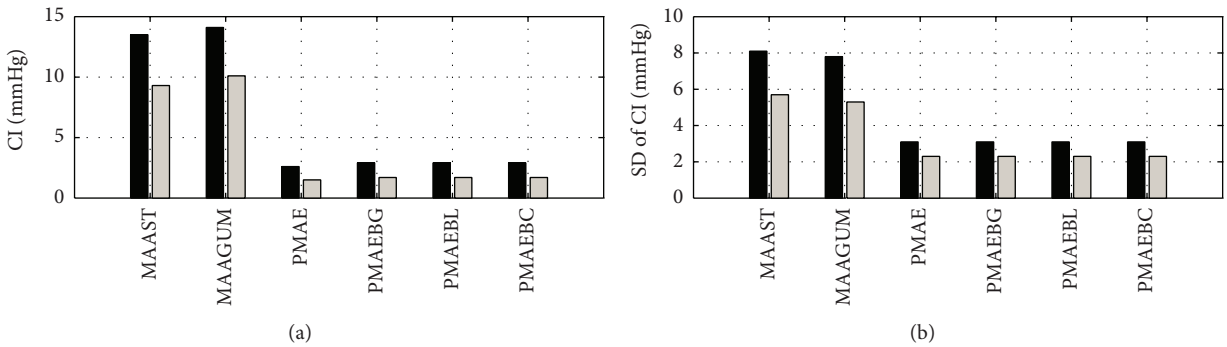

Figure 6: Comparison of average results (85 subjects with five measurements) with respect to the CIs (95%) for SBP and DBP using the MAAST, MAAGUM, PMAE, PMAEBG, PMAEBL, and PMAEBC, where σ is a standard deviation.

recommends an accuracy of standard deviation (SD) of no more than 8 mmHg for an automated BP device [21].

The experimental results for the CIs of the SBP and DBP using PMAEBL, PMAE [7], and the conventional MAA are compared in Figure 6. The CIs obtained using the PMAEBL are similar to those obtained using the PMAE, because these algorithms estimate CIs based on increasing the pseudomeasurements using the NPB technique with the average results for 85 subjects. However, the MD and SD of PMAEBL were smaller than the MD and SD of PMAE, respectively, as shown in Figure 3. This indicates that our proposed methodology reduces both the MD and SD, thereby improving accuracy. Consequently, the proposed methodology may reduce the uncertainty of blood pressure measurements. Note that the PMAEBG, PMAEBL, and PMAEBC results were very similar. This implies that the proposed methodology is robust to the use of different likelihood functions. In addition, the SDs of the SBP and DBP of the PMAEBL are similar to those of the SBP and DBP of the MAA and PMAE as shown in Table 1. These results clearly show that the distribution of the SBP and DBP using PMAEBL accurately represents the sampling distribution of the original measurement [24]. Practically, these results lead us to the conclusion that the CIs can provide a basis of decision with respect to health risks in the future for patient.

4. Conclusion

We demonstrated that the CI obtained using the proposed method is narrower and has a narrower standard deviation than CIs obtained using other methods. This is the first known work that combines the bootstrap-after-jackknif with

non-Gaussian models to estimate individualized ratios and these are then utilized to estimate SBP and DBP and the CIs for SBP and DBP. By combining bootstrap-after-jackknife function influence function with non-Gaussian function based on the Bayesian technique, we decreased the MD and SD of the CIs of the SBP and DBP compared with those obtained using conventional methods. Our approach is the first way to explicitly address small sample measurement sizes of SBP and DBP relative to currently used methods, while concurrently estimating the CIs of the SBP and DBP.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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