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Research Article

Foundation Settlement Prediction Based on a Novel NGM Model

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Prediction of foundation or subgrade settlement is very important during engineering construction. According to the fact that there are lots of settlement-time sequences with a nonhomogeneous index trend, a novel grey forecasting model called NGM $(1, 1, k, c)$ model is proposed in this paper. With an optimized whitenization differential equation, the proposed NGM $(1, 1, k, c)$ model has the property of white exponential law coincidence and can predict a pure nonhomogeneous index sequence precisely. We used two case studies to verify the predictive effect of NGM $(1, 1, k, c)$ model for settlement prediction. The results show that this model can achieve excellent prediction accuracy; thus, the model is quite suitable for simulation and prediction of approximate nonhomogeneous index sequence and has excellent application value in settlement prediction.

1. Introduction

Excessive settlement of foundation, subgrade, and so forth, especially differential settlement, can severely harm buildings and structures. Therefore, it is very important to observe settlement during engineering construction. Through analysis of settlement data, the development trend of current settlement is judged and the ultimate settlement is predicted so that corresponding measures can be timely taken to avoid damage to buildings and structures due to excessive settlement. At present, settlement forecasting methods based on measured data mainly include statistical prediction method, neural network, and grey theory [1–3]. In this paper, the work is focused on the application of grey forecasting model in settlement prediction.

The grey forecasting model has been widely applied in the field of geotechnical engineering, since it was proposed [4–6]. GM $(1, 1)$ model, as the uppermost grey forecasting model, is the most frequently used one [7, 8]. However, simulative sequences of GM $(1, 1)$ model and various improved GM $(1, 1)$ models are all homogeneous index sequences, while lots of data sequences in the field of geotechnical engineering have a nonhomogeneous index trend, such as foundation settlement-time sequences [9] and pile foundation load-displacement sequences [10]. In these cases, GM $(1, 1)$ model is just applicable for short-term prediction of settlement rather than medium- and long-term prediction. For medium- and long-term prediction, a grey forecasting model based on nonhomogeneous index sequence should be adopted.

Many scholars have studied grey forecasting models based on nonhomogeneous index sequence [11–15]. NGM $(1, 1, k)$ model is one of those models that substantially differ from other similar models. From the modeling method of NGM $(1, 1, k)$ model, it can be seen that there are only two parameters in its definition, while three parameters are set in other similar models. Then, does NGM $(1, 1, k)$ model whose parameters are one fewer than other similar models have the same prediction performance as others? Through analysis, this paper points out that parameter setting is the fatal flaw of NGM $(1, 1, k)$ model, which severely affects its application value. According to parameter settings of other similar models, this paper proposes a novel NGM model—NGM $(1, 1, k, c)$ model.

Because of the inherent deviation existed in NGM $(1, 1, k, c)$ model, this paper optimizes the parameters of the whitenization differential equation of the model to make it a better match with the grey differential equation, thus realizing prediction unbiasedness of NGM $(1, 1, k, c)$ model. Change in the whitenization differential equation is based on parameter reconstruction; the general solution of the whitenization

Table 1: The simulative values and errors of GM $(1,1)$ model and NGM $(1,1,k)$ model.

	Original value	NGM $(1,1,k)$ model		GM $(1,1)$ model	
		Simulative value $\hat{x}^{(0)}(k)$	Error $\varepsilon(k)$	Simulative value $\hat{x}^{(0)}(k)$	Error $\varepsilon(k)$
$x^{(0)}(1)$	8.600	8.600	0.00%	8.600	0.00%
$x^{(0)}(2)$	9.320	5.501	40.97%	9.211	1.17%
$x^{(0)}(3)$	10.184	9.077	10.87%	10.201	0.17%
$x^{(0)}(4)$	11.221	11.065	1.39%	11.297	0.68%
$x^{(0)}(5)$	12.465	12.171	2.36%	12.511	0.37%
$x^{(0)}(6)$	13.958	12.786	8.39%	13.855	0.74%
ε (avg)			10.66%		0.52%

differential equation is substituted into the grey differential equation to find the new parameters of the whitenization differential equation, thus establishing an optimized NGM $(1,1,k,c)$ model. Finally, the optimized NGM $(1,1,k,c)$ model is applied in settlement prediction. The results of case studies show that this model can achieve excellent prediction accuracy which is better than that of GM $(1,1)$ model and NGM $(1,1,k)$ model and also superior to Asaoka model and hyperbolic model commonly used for settlement prediction, and therefore it has certain application value in settlement prediction.

2. The Modeling Method of NGM $(1,1,k)$ Model and Its Flaw

Definition 1. Consider

$$x^{(0)}(k)+az^{(1)}(k)=kb, \tag{1}$$

where $z^{(1)}(k)=(1/2)(x^{(1)}(k)+x^{(1)}(k-1))$, is called a grey differential equation of NGM $(1,1,k)$ model (Abbreviate as NGM $(1,1,k)$), which is the defining type of NGM $(1,1,k)$ model. The parameter a in NGM $(1,1,k)$ model is called the development coefficient and kb is grey action quantity just like b in GM $(1,1)$ model.

The parameters of NGM $(1,1,k)$ model determined by least-squares method can be written as

$$[a\ \ b]^T=\left(B^TB\right)^{-1}B^TY,$$

$$B=\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \vdots & \vdots\\ -z^{(1)}(n) & 1\end{bmatrix},\qquad Y=\begin{bmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{bmatrix}. \tag{2}$$

The equation

$$\frac{dx^{(1)}(t)}{dt}+ax^{(1)}(t)=tb \tag{3}$$

is called a whitenization differential equation of NGM $(1,1,k)$ model.

The equation

$$\hat{x}^{(1)}(k)=\left(x^{(0)}(1)-\frac{b}{a}+\frac{b}{a^2}\right)\cdot e^{-a(k-1)}+\frac{b}{a}k-\frac{b}{a^2},\qquad k=1,2,\ldots,n \tag{4}$$

is the time response equation of NGM $(1,1,k)$ model.

The restored values of $x^{(0)}(k)$ can be given by

$$\begin{aligned}\hat{x}^{(0)}(k)&=\hat{x}^{(1)}(k)-\hat{x}^{(1)}(k-1)\\ &=\left(1-e^{a}\right)\left(x^{(1)}(1)-\frac{b}{a}+\frac{b}{a^2}\right)e^{-a(k-1)}+\frac{b}{a},\\ &\qquad k=2,3,\ldots,n.\end{aligned} \tag{5}$$

NGM $(1,1,k)$ model is established to broaden the application scope of grey forecasting models so that this type of model can apply to simulation and prediction of approximate nonhomogeneous index sequence. To analyze its flaw, assume that $x^{(0)}(k)=Ae^{\beta(k-1)}+B$ is a nonhomogeneous index sequence of raw data; then its 1-AGO sequence is

$$x^{(1)}(k)=Ce^{\beta(k-1)}+Bk+D, \tag{6}$$

where $C=Ae^{\beta}/(e^{\beta}-1)$ and $D=-A/(e^{\beta}-1)$.

Compare (4) with (6), if

$$B=\frac{b}{a},\qquad D=-\frac{b}{a^2},\qquad \beta=-a, \tag{7}$$

then

$$B=D\beta=\frac{-A\beta}{e^{\beta}-1}. \tag{8}$$

In the above equation, certain correlation exists among β, A, and B, or there are only two independent variables; $x^{(0)}(k)=Ae^{\beta(k-1)}+B$ is an arbitrary function with three parameters and no direct correlation exists among β, A, and B. Therefore, for NGM $(1,1,k)$ model, its simulative sequence cannot express an arbitrary nonhomogeneous index sequence, and its application scope is limited. To verify the flaw of NGM $(1,1,k)$ model, a NGM $(1,1,k)$ model is established based on nonhomogeneous index sequence $3\times 1.2^k+5$. See Table 1 for the raw data and prediction results.

From the results in Tables 1 and 2, it can be seen that the prediction accuracy of NGM $(1,1,k)$ model which is intended to be applicable for modeling of approximate nonhomogeneous index sequence is not as good as that of GM $(1,1)$ model, let alone nonhomogeneous index sequence which is selected in the case study of this paper, while GM $(1,1)$ model is only applicable for modeling of approximate homogeneous index sequence. Either simulation accuracy or

Table 2: The predictive values and errors of GM $(1,1)$ model and NGM $(1,1,k)$ model.

	Original value	NGM $(1,1,k)$ model		GM $(1,1)$ model	
		Predictive value $\widehat{x}^{(0)}(k)$	Error $\varepsilon(k)$	Predictive value $\widehat{x}^{(0)}(k)$	Error $\varepsilon(k)$
$x^{(0)}(7)$	15.750	13.129	16.642	15.344	2.576
$x^{(0)}(8)$	17.899	13.319	25.591	16.992	5.067
$x^{(0)}(9)$	20.479	13.425	34.448	18.818	8.111

prediction accuracy of NGM $(1,1,k)$ model is unsatisfactory or the relative error is more than 10%. As to GM $(1,1)$ model, its simulation accuracy and one-step prediction accuracy are both higher, indicating that the applicability of GM $(1,1)$ model is stronger than that of NGM $(1,1,k)$ model.

3. A Novel NGM Model—NGM $(1,1,k,c)$ Model and Its Optimization

3.1. The Modeling Method of NGM $(1,1,k,c)$ Model. The above theoretical analyses and case study suggest that NGM model has its own flaw; that is, its simulative sequence cannot express an arbitrary nonhomogeneous index sequence. Compare it with other similar grey forecasting models.

The whitenization differential equation of the grey model proposed by Yu and Wei [13] is $(dx^{(1)}/dp)+ax^{(1)}=b+c(p-1)$.

The whitenization differential equation of the grey model proposed by Zhang and Gu [14] is $(dx^{(1)}/dt)+ax^{(1)}=bt+c$.

The above models both have three parameters and the parameters of solutions of their whitenization differential equations are all independent from each other. Thus, these models are both applicable for simulation and prediction of approximate nonhomogeneous index sequence, which are relatively effective models. According to similar models, a parameter must be added to NGM $(1,1,k)$ model.

Definition 2. The equation

$$x^{(0)}(k)+az^{(1)}(k)=kb+c \tag{9}$$

is called a grey differential equation of NGM $(1,1,k,c)$ model (abbreviated as NGM $(1,1,k,c)$), which is the defining type of NGM $(1,1,k,c)$ model. The parameter a in the NGM $(1,1,k,c)$ model is called the development coefficient and $kb+c$ is grey action quantity just like b in GM $(1,1)$ model.

The parameters of NGM $(1,1,k,c)$ model calculated by least-square method can be written as

$$\begin{bmatrix} a & b & c\end{bmatrix}^T=\left(B^TB\right)^{-1}B^TY, \tag{10}$$

where

$$B=\begin{bmatrix} -z^{(1)}(2) & 2 & 1\\ -z^{(1)}(3) & 3 & 1\\ \vdots & \vdots & \vdots\\ -z^{(1)}(n) & n & 1\end{bmatrix},\qquad Y=\begin{bmatrix} x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{bmatrix}. \tag{11}$$

The first-order differential equation

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=bt+c \tag{12}$$

is called a whitenization differential equation of NGM $(1,1,k,c)$ model.

The equation

$$\widehat{x}^{(1)}(k)=\left(x^{(1)}(1)-\frac{b}{a}+\frac{b}{a^2}-\frac{c}{a}\right)e^{-ak}+\frac{b}{a}k-\frac{b}{a^2}+\frac{c}{a} \tag{13}$$

is called a time response equation of NGM $(1,1,k,c)$ model.

The restored values are

$$\widehat{x}^{(0)}(k)=\left(x^{(1)}(1)-\frac{b}{a}+\frac{b}{a^2}-\frac{c}{a}\right)\left(1-e^{a}\right)e^{-ak}+\frac{b}{a}, \tag{14}$$

where $C(1-e^a)$, b/a, and a are independent from each other, satisfying the requirement. Therefore, NGM $(1,1,k,c)$ model can be taken for simulation and prediction of approximate nonhomogeneous index sequence.

3.2. The Flaw of NGM $(1,1,k,c)$ Model. According to the above modeling method of NGM $(1,1,k,c)$ model, a problem similar to that in GM $(1,1)$ model can be found; that is, its grey differential equation and white differential equation do not match strictly, which results in inherent deviation of the model. This problem will be analyzed below.

On the interval $[k-1,k]$, the integral form of whitenization differential equation (12) is

$$\int_{k-1}^{k}\frac{dx^{(1)}}{dt}dt+a\int_{k-1}^{k}x^{(1)}dt=\int_{k-1}^{k}(bt+c)\,dt; \tag{15}$$

then

$$x^{(1)}(k)-x^{(1)}(k-1)+a\int_{k-1}^{k}x^{(1)}dt=\frac{(2k-1)\,b}{2}+c. \tag{16}$$

Comparison between (16) and grey differential equation (15) shows that they are differences in k and $(2k-1)/2$ as well as background value $z^{(1)}(k)=(1/2)(x^{(1)}(k)+x^{(1)}(k-1))$ and $\int_{k-1}^{k}x^{(1)}dt$. Therefore, the grey differential equation and the white differential equation in NGM $(1,1,k,c)$ model do not match strictly.

3.3. Optimization of NGM $(1,1,k,c)$ Model. An optimization method similar to that for GM $(1,1)$ model, such as improved grey derivative or background value equation [16–23], would disobey the definition of NGM $(1,1,k,c)$ model. Reference [7] mentioned that "GM $(1,1)$ white model itself and all results derived out from the white model just establish only when they are not contradictory with the defining type, otherwise, invalid." Based on this idea, this paper improves the whitenization differential equation to make it a better match with the grey differential equation, so as to realize unbiasedness of the prediction model under the condition that the definition of NGM $(1,1,k,c)$ model is obeyed and to establish an optimized NGM $(1,1,k,c)$ model (abbreviated as ONGM $(1,1,k,c)$) as well.

Parameters of the whitenization differential equation are changed to match it with the grey differential equation; the whitenization differential equation with redefined parameters is written as

$$\frac{dx^{(1)}}{dt} + mx^{(1)} = nt + p. \tag{17}$$

The general solution of the above equation is

$$x^{(1)}(k) = Ae^{-m(k-1)} + \frac{n}{m}k - \frac{n}{m^2} + \frac{p}{m}. \tag{18}$$

1-IAGO sequence of $x^{(1)}(k)$ is

$$x^{(0)}(k) = A\left(1-e^{m}\right)e^{-m(k-1)} + \frac{n}{m}. \tag{19}$$

Apparently, if (18) and (19) satisfy grey differential equation (9), the whitenization differential equation can match with the grey differential equation at this point. Substituting (18) and (19) into grey differential equation (9) gives

$$\begin{aligned} &Ae^{-m(k-1)}\left[\left(1-e^{m}\right) + \frac{a\left(1+e^{m}\right)}{2}\right] + \frac{an}{m}k \\ &\quad + \frac{n}{m} - \frac{an}{2m} + a\left(\frac{p}{m} - \frac{n}{m^2}\right) = bk + c. \end{aligned} \tag{20}$$

To make (20) hold true, we need

$$\begin{gathered} \left(1-e^{m}\right) + \frac{a\left(1+e^{m}\right)}{2} = 0, \\ \frac{an}{m} = b, \qquad \frac{n}{m} - \frac{an}{2m} + a\left(\frac{p}{m} - \frac{n}{m^2}\right) = c; \end{gathered} \tag{21}$$

then

$$m = \ln\frac{2+a}{2-a}, \qquad n = \frac{mb}{a}, \qquad p = \frac{mc}{a} - \frac{n}{a} + \frac{n}{2} + \frac{n}{m}. \tag{22}$$

The above are the parameters after the whitenization differential equation of NGM $(1,1,k,c)$ model is optimized. The time response equation of the model is

$$\begin{aligned} \hat{x}^{(1)}(k) &= \left(x^{(1)}(1) - \frac{n}{m} + \frac{n}{m^2} - \frac{p}{m}\right)e^{-m(k-1)} \\ &\quad + \frac{n}{m}k - \frac{n}{m^2} + \frac{p}{m}. \end{aligned} \tag{23}$$

The restored values are

$$\hat{x}^{(0)}(k) = \left(1-e^{a}\right)\left(x^{(1)}(1) - \frac{n}{m} + \frac{n}{m^2} - \frac{p}{m}\right)e^{-m(k-1)} + \frac{n}{m}. \tag{24}$$

4. Validation of White Exponential Law Coincidence

Similar to the flaw of GM $(1,1)$ model, the whitenization differential equation and the grey differential equation does not strictly match, which results in that NGM $(1,1,k,c)$ model cannot completely fit a pure nonhomogeneous index sequence.

A pure nonhomogeneous index sequence $X^{(0)} = \{dq + o, dq^2 + o, dq^3 + o, dq^4 + o, dq^5 + o\}$ is used to verify the unbiasedness of prediction of ONGM $(1,1,k,c)$ model.

Accumulating the generation of $X^{(0)}$, we get

$$\begin{aligned} X^{(1)} = &\left\{dq + o, d\left(q+q^2\right) + 2o, d\left(q+q^2+q^3\right) + 3o,\right. \\ &\left. d\sum_{i=1}^{4} q^i + 4o, d\sum_{i=1}^{5} q^i + 5o\right\}. \end{aligned} \tag{25}$$

Substituting all data values into the calculation formula (10) for parameters gives

$$B = \begin{bmatrix} -d\left(\sum_{i=1}^{1} q^i + \frac{q^2}{2}\right) - \frac{3o}{2} & 2 & 1 \\ -d\left(\sum_{i=1}^{2} q^i + \frac{q^3}{2}\right) - \frac{5o}{2} & 3 & 1 \\ -d\left(\sum_{i=1}^{3} q^i + \frac{q^4}{2}\right) - \frac{7o}{2} & 4 & 1 \\ -d\left(\sum_{i=1}^{4} q^i + \frac{q^5}{2}\right) - \frac{9o}{2} & 5 & 1 \end{bmatrix}, \tag{26}$$

$$Y = \begin{bmatrix} dq^2 + o \\ dq^3 + o \\ dq^4 + o \\ dq^5 + o \end{bmatrix}^{T},$$

$$\begin{aligned} [a \ \ b \ \ c]^T &= \left(B^T B\right)^{-1} B^T Y \\ &= \left[\frac{2-2q}{1+q} \quad \frac{(2-2q)\,o}{1+q} \quad \frac{2q(d+o)}{1+q}\right]. \end{aligned} \tag{27}$$

Now we can obtain the parameters of whitenization differential equation (23) as follows:

$$\begin{aligned} m &= \ln\frac{2+a}{2-a} = \ln\frac{2+(2-2q)/(1+q)}{2-(2-2q)/(1+q)} = \ln\frac{1}{q}, \\ n &= \frac{mb}{a} = \frac{(2-2q)\,o/(1+q)}{(2-2q)/(1+q)}\ln\frac{1}{q} = o\ln\frac{1}{q}, \\ p &= \frac{q(d+o)}{1-q}\ln\frac{1}{q} - \frac{o\ln(1/q)}{(2-2q)/(1+q)} + \frac{o\ln(1/q)}{2} + o. \end{aligned} \tag{28}$$

Table 3: The observed values and simulative values of the above forecasting models.

Number	Time (day)	Observed value (cm)	Predictive value (cm) ONGM $(1, 1, k, c)$ model	NGM $(1, 1, k, c)$ model	NGM $(1, 1, k)$ model	GM $(1, 1)$ model	Asaoka model	Hyperbolic model
1	10	23.36	23.36	23.36	23.36	23.36	23.36	25.55
2	20	43.19	42.18	29.70	31.97	60.71	43.44	44.85
3	30	58.73	59.25	49.25	49.97	65.80	59.58	59.94
4	40	70.87	72.84	64.82	64.63	71.31	72.55	72.06
5	50	83.71	83.64	77.22	76.58	77.28	82.99	82.01
6	60	92.91	92.23	87.08	86.30	83.75	91.38	90.33
7	70	99.73	99.07	94.94	94.23	90.76	98.12	97.39
8	80	105.08	104.50	101.20	100.68	98.36	103.54	103.45
9	90	109.73	108.83	106.18	105.94	106.60	107.90	108.71
10	100	112.19	112.27	110.14	110.22	115.52	111.41	113.32
11	110	113.45	115.00	113.30	113.71	125.20	114.23	117.39

Table 4: The comparison of prediction performance and predictive values of ultimate settlement among the above forecasting models.

	ONGM $(1, 1, k, c)$ model	NGM $(1, 1, k, c)$ model	NGM $(1, 1, k)$ model	GM $(1, 1)$ model	Asaoka model	Hyperbolic model
Mean absolute error	0.73	5.07	5.08	6.77	1.05	1.87
Mean absolute relative error	0.94%	7.60%	7.31%	9.30%	1.18%	2.82%
Mean squared error	0.88	39.95	36.71	69.73	1.45	4.18
Predictive value of ultimate settlement (cm)	125.63	125.63	129.03	$+\infty$	125.79	183.28

Substitute (28) into the time response equation (23), then we can get

$$\begin{aligned}\hat{x}^{(1)}(k) &= \frac{q^2 d}{q-1} e^{-(\ln(1/q))(k-1)} + ok + \frac{qd}{q-1} \\ &= \frac{dq^2}{q-1} q^{k-1} + ok + \frac{qd}{q-1}.\end{aligned} \quad (29)$$

The restored values are

$$\hat{x}^{(0)}(k) = \frac{dq^2}{q-1} q^{k-2}(q-1) + o = dq^k + o. \quad (30)$$

$\hat{x}^{(0)}(k)$ is exactly equal to $x^{(0)}(k)$, while d, q, and o can take any value. Therefore, as long as the sequence of raw data has an approximate nonhomogeneous index trend, the optimized NGM $(1, 1, k, c)$ model can be used for simulation and prediction.

5. Application of ONGM $(1, 1, k, c)$ Model in Prediction of Foundation Settlement

5.1. Case Study 1. Close neighboring the Yangtze River, the northwest stockyard of a dock is soft subsoil; the bearing capacity of which cannot satisfy the stacking requirement of the stockyard. Gravel compaction piles with the diameter of 0.5 m which are arranged in a regular triangle and the drainage consolidation method of staged loading are adopted for reinforcement. The reinforced area covers the area within 10 m away from the stockyard [24]. 11 sets of observed values of postconstruction settlement (observation interval: 10 d) at Point H16 in Area B of the stockyard are chosen as the samples, and $\Delta t = 10$ d is denoted by $\Delta k = 1$; ONGM $(1, 1, k, c)$ model, NGM $(1, 1, k, c)$ model, NGM $(1, 1, k)$ model, and GM $(1, 1)$ model are established for prediction of the ultimate settlement, so as to provide basis for engineering construction. Meanwhile, Asaoka model (graphic method) [9] and hyperbolic model [25] commonly used for settlement prediction are established for comparison with ONGM $(1, 1, k, c)$ model presented in this paper. See Table 3 for the raw data and simulation results, and see Table 4 for the prediction results of ultimate settlement.

The prediction performance of these models are compared by utilizing three indexes, namely, mean absolute error, mean absolute relative error, and mean squared error. See Table 4 for the results.

First of all, four grey forecasting models are compared. From the results in Table 4, it can be seen that the values of the indexes of GM $(1, 1)$ model are the biggest, and the prediction performance of this model is the poorest; the values of indexes of ONGM $(1, 1, k, c)$ model are much smaller than those of other models, and the prediction performance of this model is the best; NGM $(1, 1, k)$ model and NGM $(1, 1, k, c)$ model both have inherent deviation, and their prediction performance is quite close to each other in this case. This indicates that for approximate nonhomogeneous index sequence, ONGM $(1, 1, k, c)$ model has a good prediction performance.

The results in Table 4 show that the predictive value of ultimate settlement of GM $(1, 1)$ model goes to infinity, which goes against the actual situation; the predictive value

Table 5: The observed values and simulative values of the above forecasting models.

Number	Time (day)	Observed value (cm)	Predictive value (cm) ONGM $(1, 1, k, c)$ model	NGM $(1, 1, k, c)$ model	NGM $(1, 1, k)$ model	GM $(1, 1)$ model	Asaoka model	Hyperbolic model
1	85	64	64	64	64	64	64	57.47
2	95	73	73.12	64.58	43.53	76.19	72.93	76.24
3	105	81	80.69	73.27	70.51	80.78	80.63	85.56
4	115	87	87.26	80.81	84.94	85.64	87.27	91.13
5	125	93	92.96	87.36	92.66	90.80	92.99	94.83
6	135	98	97.90	93.03	96.79	96.27	97.92	97.47
7	145	102	102.18	97.96	99.00	102.07	102.17	99.45
8	155	106	105.90	102.23	100.18	108.22	105.83	100.99

Table 6: The comparison of prediction performance and predictive values of ultimate settlement among the above forecasting models.

	ONGM $(1, 1, k, c)$ model	NGM $(1, 1, k, c)$ model	NGM $(1, 1, k)$ model	GM $(1, 1)$ model	Asaoka model	Hyperbolic model
Mean absolute error	0.139	5.094	6.548	1.373	0.142	3.548
Mean absolute relative error	0.158%	5.855%	8.214%	1.561%	0.160%	5.544%
Mean squared error	0.029	31.987	128.429	3.100	0.035	15.786
Predictive value of ultimate settlement (cm)	130.21	130.21	101.54	$+\infty$	128.72	113.24

of ultimate settlement of ONGM $(1, 1, k, c)$ model is equal to that of NGM $(1, 1, k, c)$ model, with the only difference in settlement convergence trend; the predictive value of ultimate settlement of NGM $(1, 1, k)$ is greater than that of the former two models.

From the comparison among ONGM $(1, 1, k, c)$ model, Asaoka model and hyperbolic model, it can be seen that all indexes of ONGM $(1, 1, k, c)$ model are smaller than those of the other two models, suggesting that ONGM $(1, 1, k, c)$ model has the best prediction performance among these three models. The prediction result of Asaoka model is closer to that of ONGM $(1, 1, k, c)$ model, because these two models are essentially exponential models which are different only in modeling mechanism. Compared with the other two models, the prediction result of ultimate settlement from hyperbolic model is significantly greater.

5.2. Case Study 2. To construct a high-grade highway on soft silt, postconstruction settlement and embankment stability are two critical problems that must be solved. Prediction of ultimate settlement through analysis of postconstruction settlement is of great significance for the evaluation of embankment stability. The observed data of settlement of a cross-section after loading on the test section of a highway in the Pearl River Delta Region are taken as samples [26], and $\Delta t = 10$ d is denoted by $\Delta k = 1$; ONGM $(1, 1, k, c)$ model, NGM $(1, 1, k, c)$ model, NGM $(1, 1, k)$ model, GM $(1, 1)$ model, Asaoka model (graphic method), and hyperbolic model are established for prediction of the ultimate settlement. See Table 5 for the raw data and simulation results, and see Table 6 for the prediction results of ultimate settlement.

First of all, four grey forecasting models are compared. From the results in Table 6, it can be seen that the values of the indexes of NGM $(1, 1, k)$ model are the biggest, and the prediction performance of this model is the poorest; the values of indexes of ONGM $(1, 1, k, c)$ model are much smaller than those of other models, and the prediction performance of this model is the best; the prediction performance of NGM $(1, 1, k, c)$ model is slightly better than that of NGM $(1, 1, k)$ model; the prediction performance of GM $(1, 1)$ model is only second to ONGM $(1, 1, k, c)$ model but cannot forecast the ultimate settlement. The results in Table 6 also indicate that for approximate nonhomogeneous index sequence, ONGM $(1, 1, k, c)$ model has a good prediction performance.

The results in Table 6 show that the predictive value of ultimate settlement of NGM $(1, 1, k, c)$ model is identical to that of NGM $(1, 1, k, c)$ model, which is 130.21 cm bigger than the last observed value of settlement which is 106 cm; this implies that settlement is still unstable when observation is finished. The predictive value of ultimate settlement of NGM $(1, 1, k)$ model is smaller than the last observed value of settlement, which goes against the actual situation.

From the comparison among ONGM $(1, 1, k, c)$ model, Asaoka model, and hyperbolic model, it can be seen that all indexes of ONGM $(1, 1, k, c)$ model are almost equal to but still smaller than those of Asaoka model and also smaller than hyperbolic model, suggesting that ONGM $(1, 1, k, c)$ model has the best prediction performance among these three models. Compared with the other two models, the prediction result of ultimate settlement from the hyperbolic model is significantly smaller.

6. Conclusions

GM $(1, 1)$ model has been successfully applied in many fields, but its prediction performance is sometimes unsatisfactory since its simulative sequence is homogeneous index sequence. For a lot of settlement-time sequences with a nonhomogeneous index trend, GM $(1, 1)$ model is not appropriate for medium- and long-term prediction of foundation settlement.

The simulative sequence of NGM $(1, 1, k)$ model cannot express an arbitrary nonhomogeneous index sequence and sometimes it cannot achieve a satisfactory prediction performance since this model only has two undetermined parameters. Using other similar models for reference, this paper adds a parameter to NGM model and establishes NGM $(1, 1, k, c)$ model which is a novel NGM model.

Like GM $(1, 1)$ model, NGM $(1, 1, k, c)$ model also has the flaw of mismatching between its grey differential equation and whitenization differential equation. Based on the idea that "GM (1, 1) white model itself and all results derived out from the white model just establish only when they are not contradictory with the defining type, otherwise invalid"; this paper realizes prediction unbiasedness of NGM $(1, 1, k, c)$ model through optimizing the parameters of the whitenization differential equation of this model.

Finally, the optimized NGM $(1, 1, k, c)$ model is applied in settlement prediction. The results of case studies show that the optimized NGM $(1, 1, k, c)$ model has good prediction performance which is better than that of GM $(1, 1)$ model, NGM $(1, 1, k)$ model, and NGM $(1, 1, k, c)$ model and also superior to Asaoka model and hyperbolic model commonly used for settlement prediction; it has an excellent application value for approximate nonhomogeneous index sequence; thus, it is suitable for settlement prediction in geotechnical engineering.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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