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Research Article

Asymptotic Stabilization by State Feedback for a Class of Stochastic Nonlinear Systems with Time-Varying Coefficients

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This paper investigates the problem of state-feedback stabilization for a class of upper-triangular stochastic nonlinear systems with time-varying control coefficients. By introducing effective coordinates, the original system is transformed into an equivalent one with tunable gain. After that, by using the low gain homogeneous domination technique and choosing the low gain parameter skillfully, the closed-loop system can be proved to be globally asymptotically stable in probability. The efficiency of the state-feedback controller is demonstrated by a simulation example.

1. Introduction

Consider a class of upper-triangular stochastic nonlinear systems with time-varying control coefficients described by

$$\begin{aligned} dx_1 &= \left(d_1(t)\,x_2 + f_1(\tilde{x}_3)\right)dt + g_1^T(\tilde{x}_3)\,d\omega, \\ dx_2 &= \left(d_2(t)\,x_3 + f_2(\tilde{x}_4)\right)dt + g_2^T(\tilde{x}_4)\,d\omega, \\ &\vdots \\ dx_{n-2} &= \left(d_{n-2}(t)\,x_{n-1} + f_{n-2}(\tilde{x}_n)\right)dt + g_{n-2}^T(\tilde{x}_n)\,d\omega, \\ dx_{n-1} &= d_{n-1}(t)\,x_n dt, \\ dx_n &= d_n(t)\,u dt, \end{aligned} \tag{1}$$

where $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$ are the measurable state and the input of system, respectively. $\tilde{x}_i = (x_i, \ldots, x_n)^T$. ω is an r-dimensional standard Wiener process defined on a probability space $(\Omega, \mathscr{F}, P)$, with Ω being a sample space, $\mathscr{F}$ being a filtration, and P being a probability measure. The functions $f_i : \mathbb{R}^{n-i-1} \to \mathbb{R}$ and $g_i : \mathbb{R}^{n-i-1} \to \mathbb{R}^r$, $i = 1, \ldots, n-2$, are assumed to be $\mathscr{C}^1$ with their arguments and $f_i(0) = 0$, $g_i(0) = 0$. $d_i : R_+ \to R$, $i = 1, \ldots, n$, are unknown time-varying control coefficients with known sign.

In recent years, the global controller design for stochastic nonlinear systems has been attracting more and more attention. According to the difference of selected Lyapunov functions, the existing literature on controller design can be mainly divided into two types. One type is to derive the backstepping controller design by using quadratic Lyapunov function and a risk-sensitive cost criterion [1–3]. Another essential improvement belongs to Krstić and Deng. By introducing the quartic Lyapunov function, [4–12] present asymptotical stabilization control under the assumption that the nonlinearities equal zero at the equilibrium point of the open-loop system. Subsequently, for several classes of stochastic high-order nonlinear systems, by combining Krstić and Deng's method with stochastic analysis, [13, 14] study the problem of state-feedback stabilization and the output-feedback stabilization problem is considered in [15, 16].

The study of stabilization control for upper-triangular nonlinear systems has long been recognized as difficult due to the inherent nonlinearity. In the existing literature, most results are established using the nested-saturation method [17, 18] and forwarding technique [19]. When no a priori

information of the system nonlinearities is known, the work [20] proposes a universal stabilizer for feedforward nonlinear systems by employing a switching controller. Note that the listed results above do not consider the stochastic noise. However, from both practical and theoretical points of view, it is more important to study the control of upper-triangular stochastic nonlinear systems with time-varying control coefficients. Therefore, in this paper, under some appropriate assumptions, we consider the stabilization for system (1). To the best of the authors' knowledge, there are not any results about this topic.

In this paper, based on the low gain homogeneous domination technique, for system (1), we design a stabilization state-feedback controller, under which the closed-loop systems can be proved to be globally asymptotically stable in probability.

The contributions of this paper are highlighted as follows.

(i) This paper is the first result about state-feedback stabilization of upper-triangular stochastic nonlinear systems with time-varying control coefficients.

(ii) Due to the complex of upper-triangular system structure, how to deal with stochastic noise and time-varying control coefficients in the controller design is a nontrivial work.

The remainder of this paper is organized as follows. Section 2 offers some preliminary results. The state-feedback controller is designed and analyzed in Section 3. After that, in Section 4, a simulation example is presented to show the effectiveness of the state-feedback controller. Finally, the paper is concluded in Section 5.

2. Preliminary Results

The following notation will be used throughout the paper. $\mathbb{R}_+$ denotes the set of all nonnegative real numbers. For a given vector or matrix X, X^T denotes its transpose, $\mathrm{Tr}\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X. $\mathscr{C}^i$ denotes the set of all functions with continuous ith partial derivatives. $\mathscr{K}$ denotes the set of all functions: $\mathbb{R}_+ \to \mathbb{R}_+$, which are continuous, strictly increasing, and vanishing at zero; $\mathscr{K}_\infty$ denotes the set of all functions which are of class $\mathscr{K}$ and unbounded; $\mathscr{KL}$ denotes the set of all functions $\beta(s,t)$: $\mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, which are of $\mathscr{K}$ for each fixed t and decrease to zero as $t \to \infty$ for each fixed s.

Consider the following stochastic nonlinear system:

$$dx = f(x)\,dt + g^T(x)\,d\omega, \tag{2}$$

where $x \in \mathbb{R}^n$ is the state of the system and ω is an r-dimensional standard Wiener process defined on the probability space $(\Omega, \mathscr{F}, P)$. The Borel measurable functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g^T : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are local Lipschitz in $x \in \mathbb{R}^n$.

The following definitions and lemma will be used throughout the paper.

Definition 1 (see [5]). For any given $V(x) \in \mathscr{C}^2$ associated with stochastic system (2), the differential operator $\mathscr{L}$ is defined as

$$\mathscr{L}V(x) \triangleq \frac{\partial V(x)}{\partial x} f(x) + \frac{1}{2}\mathrm{Tr}\left\{g(x)\frac{\partial^2 V(x)}{\partial x^2} g^T(x)\right\}. \tag{3}$$

Definition 2 (see [5]). For the stochastic system (2) with $f(0) = 0$, $g(0) = 0$, the equilibrium $x(t) = 0$ of (2) is globally asymptotically stable (GAS) in probability if, for any $\varepsilon > 0$, there exists a class $\mathscr{KL}$ function $\beta(\cdot,\cdot)$ such that $P\{|x(t)| < \beta(|x_0|, t)\} \geq 1 - \varepsilon$ for any $t \geq 0$ and $x_0 \in \mathbb{R}^n \setminus \{0\}$.

Lemma 3 (see [5]). *Consider the stochastic system* (2)*; if there exist a $\mathscr{C}^2$ function $V(x)$, class $\mathscr{K}_\infty$ functions α_1 and α_2, constants $c_1 > 0$ and $c_2 \geq 0$, and a nonnegative function $W(x)$ such that*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \qquad \mathscr{L}V \leq -c_1 W(x) + c_2, \tag{4}$$

then

(a) *for* (2)*, there exists an almost surely unique solution on* $[0,\infty)$*;*

(b) *when $c_2 = 0$, $f(0) = 0$, $g(0) = 0$, and $W(x) = \alpha_3(|x|)$, where $\alpha_3(\cdot)$ is a class $\mathscr{K}$ function, then the equilibrium $x = 0$ is GAS in probability and $P\{\lim_{t\to\infty}|x(t)| = 0\} = 1$.*

3. Controller Design and Stability Analysis

The following assumptions are made on system (1).

Assumption 1. For $i = 1,\ldots,n$, there exists a constant $b > 0$ such that

$$\begin{aligned} |f_i(\tilde{x}_{i+2})| &\leq b(|x_{i+2}| + \cdots + |x_n|), \\ |g_i(\tilde{x}_{i+2})| &\leq b(|x_{i+2}| + \cdots + |x_n|). \end{aligned} \tag{5}$$

Assumption 2. Without loss of generality, the sign of $d_i(t)$ is assumed to be positive, and there exist known positive constants λ_i and μ_i such that, for any $t \in \mathbb{R}^+$ and $i = 1,\ldots,n$,

$$0 < \lambda_i \leq d_i(t) \leq \mu_i. \tag{6}$$

Remark 4. From Assumption 1, the system investigated has an upper-triangular form. Due to the complex of upper-triangular system structure and the effect of stochastic noise, the stabilization of such systems is usually very difficult. In this paper, by using the low gain homogeneous domination approach, the state-feedback stabilization problem is investigated for the first time.

Remark 5. By Assumption 2, we know that $d_i(t)$s are time-varying control coefficients; how to effectively deal with them in the design process is nontrivial work.

Firstly, introduce the following coordinate transformation:

$$z_i = \frac{x_i}{\varepsilon^{i-1}}, \quad v = \frac{u}{\varepsilon^n}, \quad i = 1,\ldots,n, \tag{7}$$

where $0 < \varepsilon < 1$ is a parameter to be designed. System (1) can be rewritten as

$$\begin{aligned} dz_1 &= \left(\varepsilon d_1(t) z_2 + \overline{f}_1(\widetilde{z}_3)\right) dt + \overline{g}_1^T(\widetilde{z}_3) d\omega, \\ dz_2 &= \left(\varepsilon d_2(t) z_3 + \overline{f}_2(\widetilde{z}_4)\right) dt + \overline{g}_2^T(\widetilde{z}_4) d\omega, \\ &\vdots \\ dz_{n-2} &= \left(\varepsilon d_{n-2}(t) z_{n-1} + \overline{f}_{n-2}(\widetilde{z}_n)\right) dt + \overline{g}_{n-2}^T(\widetilde{z}_n) d\omega, \\ dz_{n-1} &= \varepsilon d_{n-1}(t) z_n dt, \\ dz_n &= \varepsilon d_n(t) v dt, \end{aligned} \tag{8}$$

where $\overline{f}_i(\widetilde{z}_{i+2}) = f_i(\widetilde{x}_{i+2})/\varepsilon^{i-1}$, $\overline{g}_i(\widetilde{z}_{i+2}) = g_i(\widetilde{x}_{i+2})/\varepsilon^{i-1}$.

The nominal system for (8) is

$$\begin{aligned} dz_1 &= d_1(t) z_2, \\ dz_2 &= d_2(t) z_3, \\ &\vdots \\ dz_{n-2} &= d_{n-2}(t) z_{n-1}, \\ dz_{n-1} &= d_{n-1}(t) z_n dt, \\ dz_n &= d_n(t) v dt. \end{aligned} \tag{9}$$

Theorem 6. *For nominal system* (9)*, with Assumption 2, one can design a stabilizing state-feedback controller to guarantee that*

(1) *the closed-loop system has an almost surely unique solution on* $[0, \infty)$*;*

(2) *the equilibrium of the closed-loop system is GAS in probability.*

Proof. The controller design process proceeds step by step.

Step 1. Defining $\xi_1 = z_1$ and choosing $V_1 = (1/4)z_1^4$, from (9), it follows that

$$\mathscr{L}V_1 \le d_1(t) z_1^3 z_2. \tag{10}$$

Suppose that $z_2^* = -z_1\alpha_1 = -\xi_1\alpha_1$, where $\alpha_1 \ge 0$ is a constant to be chosen. Thus, by Assumption 2, we have

$$d_1(t) z_1^3 z_2^* \le \lambda_1 z_1^3 z_2^* \le 0. \tag{11}$$

By (10) and (11), one gets

$$\mathscr{L}V_1 \le \lambda_1 z_1^3 z_2^* + d_1(t) z_1^3 (z_2 - z_2^*). \tag{12}$$

Choosing the virtual smooth control z_2^* as

$$z_2^* = -\frac{n}{\lambda_1}\xi_1 \triangleq -\xi_1\alpha_1, \tag{13}$$

which substitutes into (12), yields

$$\mathscr{L}V_1 \le -n\xi_1^4 + d_1(t) z_1^3 (z_2 - z_2^*). \tag{14}$$

Deductive Step. Assume that, at step $k-1$, there are a $\mathscr{C}^2$, proper and positive definite Lyapunov function V_{k-1}, and the virtual controllers z_j^* defined by

$$\begin{aligned} z_1^* &= 0, \qquad \xi_1 = z_1 - z_1^*, \\ z_2^* &= -\xi_1\alpha_1, \qquad \xi_2 = z_2 - z_2^*, \\ &\vdots \\ z_k^* &= -\xi_{k-1}\alpha_{k-1}, \qquad \xi_k = z_k - z_k^*, \end{aligned} \tag{15}$$

where $\alpha_i \ge 0$, $1 \le i \le k-1$, are positive constants, such that

$$\mathscr{L}V_{k-1}(\overline{z}_{k-1}) \le -(n-k+2)\sum_{i=1}^{k-1}\xi_i^4 + d_{k-1}(t)\xi_{k-1}^3(z_k - z_k^*), \tag{16}$$

where $\overline{z}_{k-1} = (z_1, \ldots, z_{k-1})^T$. To complete the induction, at the kth step, one can choose the following Lyapunov function:

$$V_k(\overline{z}_k) = V_{k-1}(\overline{z}_{k-1}) + \frac{1}{4}\xi_k^4, \tag{17}$$

where $\overline{z}_k = (z_1, \ldots, z_k)^T$.

By (15)–(17), one has

$$\begin{aligned} \mathscr{L}V_k(\overline{z}_k) \le{}& -(n-k+2)\sum_{i=1}^{k-1}\xi_i^4 + d_{k-1}(t)\xi_{k-1}^3\xi_k \\ &+ \xi_k^3\left(d_k(t) z_{k+1} - \sum_{i=1}^{k-1}\frac{\partial z_k^*}{\partial z_i} d_i(t) z_{i+1}\right). \end{aligned} \tag{18}$$

By using Young's inequality and Assumption 2, one has

$$\begin{aligned} d_{k-1}(t)\xi_{k-1}^3\xi_k &\le \frac{1}{2}\xi_{k-1}^4 + c_k\xi_k^4, \\ -\xi_k^3\sum_{i=1}^{k-1}\frac{\partial z_k^*}{\partial z_i} d_i(t) z_{i+1} &\le c_{k1}|\xi_k|^3\sum_{i=1}^{k}|z_i| \le \frac{1}{2}\sum_{i=1}^{k-1}\xi_i^4 + \widehat{c}_k\xi_k^4, \end{aligned} \tag{19}$$

where $c_k > 0$, $c_{k1} > 0$, and $\widehat{c}_k > 0$ are constants. Suppose that

$$z_{k+1}^* = -\xi_k\alpha_k, \tag{20}$$

where $\alpha_k \ge 0$ is a constant to be chosen. Then, by Assumption 2, one has

$$d_k(t)\xi_k^3 z_{k+1}^* \le \lambda_k\xi_k^3 z_{k+1}^*. \tag{21}$$

Substituting (19) and (21) into (18) yields

$$\begin{aligned} \mathscr{L}V_k(\overline{z}_k) \le{}& -(n-k+1)\sum_{i=1}^{k-1}\xi_i^4 + d_k(t)\xi_k^3(z_{k+1} - z_{k+1}^*) \\ &+ \lambda_k\xi_k^3 z_{k+1}^* + (c_k + \widehat{c}_k)\xi_k^4. \end{aligned} \tag{22}$$

Choosing the virtual smooth control

$$z_{k+1}^{*} = -\frac{1}{\lambda_k}\left(n-k+1+c_k+\hat{c}_k\right)\xi_k \triangleq -\xi_k\alpha_k, \tag{23}$$

which substitutes into (22), yields

$$\mathscr{L}V_k\left(\bar{z}_k\right) \le -\left(n-k+1\right)\sum_{i=1}^{k}\xi_i^4 + d_k\left(t\right)\xi_k^3\left(z_{k+1}-z_{k+1}^{*}\right). \tag{24}$$

Step n. By choosing the actual control law

$$v = -\xi_n\alpha_n, \tag{25}$$

where $\alpha_n \ge 0$ is a constant and $\xi_n = x_n - x_n^{*}$, one gets

$$\mathscr{L}V_n\left(\bar{z}_n\right) \le -\sum_{i=1}^{n}\xi_i^4, \tag{26}$$

where

$$V_n\left(\bar{z}_n\right) = V_{n-1}\left(\bar{z}_{n-1}\right) + \frac{1}{4}\xi_n^4. \tag{27}$$

Finally, based on (26) and (27), by Lemma 3, one immediately gets the conclusion. □

Now, we are in a position to get the main results of this paper.

Theorem 7. *If Assumptions 1 and 2 hold for the upper-triangular stochastic nonlinear systems* (1)*, with the coordinate transformation* (7)*, by appropriately choosing the parameter* $0 < \varepsilon < 1$*, then, under the state-feedback controller* (25)*, one has the following:*

(1) *the closed-loop system has an almost surely unique solution on* $[0,\infty)$*;*

(2) *the equilibrium of the closed-loop system is GAS in probability.*

Proof. For system (8), with the state-feedback controller (25) and Lyapunov function (27), one has

$$\mathscr{L}V_n\left(\bar{z}_n\right) \le -\varepsilon\sum_{i=1}^{n}\xi_i^4 + \sum_{i=1}^{n}\frac{\partial V_n}{\partial z_i}\bar{f}_i\left(\tilde{z}_{i+2}\right) + \frac{1}{2}\operatorname{Tr}\left\{G\frac{\partial^2 V_n}{\partial z^2}G^T\right\}, \tag{28}$$

where $z = (z_1,\ldots,z_n)^T$, $G = (\bar{g}_1,\ldots,\bar{g}_{n-2},0,0)$. From (15) and (27), one has

$$V_n\left(\bar{z}_n\right) = \frac{1}{4}\sum_{i=1}^{n}\xi_i^4 = \frac{1}{4}\sum_{i=1}^{n}\left(z_i + c_{i,i-1}z_{i-1} + \cdots + c_{i,1}z_1\right)^4, \tag{29}$$

where $c_{i,j}$, $j = 1,\ldots,i-1$, are constants. By (7), (15) and Assumption 1, one can get

$$\begin{aligned}\left|\bar{f}_i\left(\tilde{z}_{i+2}\right)\right| &= \left|\frac{f_i\left(\tilde{x}_{i+2}\right)}{\varepsilon^{i-1}}\right| \le b\varepsilon^2\sum_{j=i+2}^{n}\left|z_j\right| \\ &\le b\varepsilon^2\sum_{j=i+2}^{n}\left(\left|\xi_j\right| + \alpha_{j-1}\left|\xi_{j-1}\right|\right).\end{aligned} \tag{30}$$

By Young's inequality, using (29) and (30), one has

$$\sum_{i=1}^{n}\frac{\partial V_n}{\partial z_i}\bar{f}_i\left(\tilde{z}_{i+2}\right) \le b_f\varepsilon^2\sum_{i=1}^{n}\xi_i^4. \tag{31}$$

Similarly, one can prove that

$$\frac{1}{2}\operatorname{Tr}\left\{G\frac{\partial^2 V_n}{\partial z^2}G^T\right\} \le b_g\varepsilon^2\sum_{i=1}^{n}\xi_i^4. \tag{32}$$

Substituting (31) and (32) into (28), one has

$$\begin{aligned}\mathscr{L}V_n\left(\bar{z}_n\right)|_{(8)} &\le -\varepsilon\sum_{i=1}^{n}\xi_i^4 + \left(b_f+b_g\right)\varepsilon^2\sum_{i=1}^{n}\xi_i^4 \\ &= -\varepsilon\left(1-\left(b_f+b_g\right)\varepsilon\right)\sum_{i=1}^{n}\xi_i^4.\end{aligned} \tag{33}$$

By choosing $0 < \varepsilon < 1$ appropriately, (33) can be written as

$$\mathscr{L}V_n\left(\bar{z}_n\right)|_{(8)} \le -c_0\sum_{i=1}^{n}\xi_i^4, \tag{34}$$

where $c_0 > 0$ is a constant.

By (34) and the coordinate transformation (7), using Lemma 3, the conclusions hold. □

Remark 8. Theorems 6 and 7 provide us a new perspective to deal with the state-feedback control problem for upper-triangular stochastic nonlinear systems with time-varying coefficients. The main technical obstacle in the Lyapunov design for stochastic upper-triangular systems is that Itô stochastic differentiation involves not only the gradient but also the higher order Hessian term. The traditional design methods are invalid to deal with these terms. However, with the design methodology provided in Theorems 6 and 7, there is no need to estimate the bounds of drift and diffusion terms step by step. Based on this technique, a homogeneous nonlinear controller for the nominal nonlinear system is firstly constructed. Then we will design a scaled controller which can effectively dominate the drift and diffusion terms by taking advantage of the homogenous structure of the controller.

4. A Simulation Example

Consider the following system:

$$\begin{aligned}dx_1 &= \left(\left(2-\sin^2 t\right)x_2 + x_3\sin x_3\right)dt + x_3\cos x_3 d\omega, \\ dx_2 &= \left(2-\cos t\right)x_3 dt, \\ dx_3 &= \left(1+\sin^2 t\right)u dt.\end{aligned} \tag{35}$$

Obviously, Assumptions 1 and 2 hold.

Introduce the following coordinate transformation:

$$z_1 = x_1, \qquad z_2 = \frac{x_2}{\varepsilon}, \qquad z_3 = \frac{x_3}{\varepsilon^2}, \qquad v = \frac{u}{\varepsilon^3}, \tag{36}$$

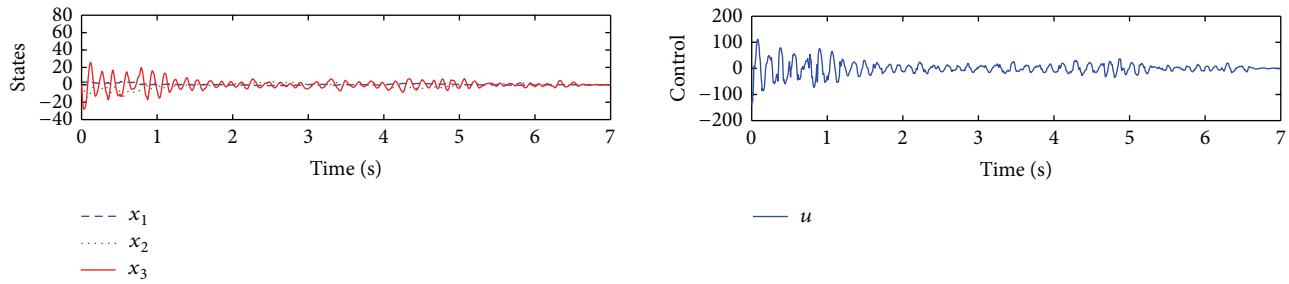

Figure 1: The response of closed-loop system (35)–(38).

where $0 < \varepsilon < 1$ is a design parameter. Then (35) can be written as

$$\begin{aligned} dz_1 &= \left(\varepsilon\left(2-\sin^2 t\right)z_2 + \varepsilon^2 z_3 \sin^2\left(\varepsilon^2 z_3\right)\right)dt \\ &\quad + \varepsilon^2 z_3 \cos\left(\varepsilon^2 z_3\right) d\omega, \\ dz_2 &= \varepsilon\left(2-\cos t\right) z_3 dt, \\ dz_3 &= \varepsilon\left(1+\sin^2 t\right) v dt. \end{aligned} \tag{37}$$

By following the design procedure in Section 3, one gets

$$v\left(z_1, z_2, z_3\right) = -1310\left(372 z_1 + 124 z_2 + z_3\right). \tag{38}$$

By choosing $\varepsilon = 0.001$, with the initial values $z_1(0) = 3$, $z_2(0) = 2$, and $z_3(0) = 5$, Figure 1 gives the system response of the closed-loop system consisting of (35)–(38), from which the efficiency of the tracking controller is demonstrated.

5. Concluding Remarks

For a class of upper-triangular stochastic nonlinear systems with time-varying control coefficients, this paper investigates the state-feedback stabilization problem. The designed controller can guarantee that the closed-loop system has a unique solution and the closed-loop system can be proved to be GAS in probability.

There are many related problems to be investigated, for example, how to generalize the result in this paper to more general stochastic upper-triangular nonlinear systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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