Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 340845, 11 pages
http://dx.doi.org/10.1155/2014/340845

Research Article

Effects of Exponential Trends on Correlations of Stock Markets

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Received 27 December 2013; Revised 11 March 2014; Accepted 11 March 2014; Published 6 April 2014

Academic Editor: Matjaz Perc

Detrended fluctuation analysis (DFA) is a scaling analysis method used to estimate long-range power-law correlation exponents in time series. In this paper, DFA is employed to discuss the long-range correlations of stock market. The effects of exponential trends on correlations of Hang Seng Index (HSI) are investigated with emphasis. We find that the long-range correlations and the positions of the crossovers of lower order DFA appear to have no immunity to the additive exponential trends. Further, our analysis suggests that an increase in the DFA order increases the efficiency of eliminating on exponential trends. In addition, the empirical study shows that the correlations and crossovers are associated with DFA order and magnitude of exponential trends.

1. Introduction

Financial markets have been known as representative complex dynamic systems, which are organized by various unexpected phenomena and affected by external factors. The study of financial markets is not easy because we do not know the control parameters that govern the systems. However, the bulk of the literature focused on the long-range correlations of financial time series in the past few years. New ideas and techniques have been introduced to measure the long-range correlation behaviors. The understanding of such long-range correlations would be helpful to design good portfolios. Long-range correlations were first observed by Barkoulas et al. in the discussion of Greek stock market [1]. In recent years, many researchers have found evidence of long-range correlations for stock markets [2–5]. To analyze the long-range correlations, previous studies presented various methods, such as rescaled range analysis (R/S analysis), wavelet transform modulus maxima (WTMM), and detrended fluctuation analysis (DFA) [6, 7]. The DFA [8] method invented by Peng et al. has become a widely used method for the determination and detection of long-range correlations in time series. It also has successfully been applied to diverse fields [9–25] such as DNA sequence, heart rate dynamics, solid state physics, long-time weather records, earthquakes, economics, and culturomics. One reason to employ the DFA method is to avoid spurious detection of correlations that are artifacts of nonstationarities in the time series. Financial time series are influenced by external trends. Strong trends in series may lead to a false detection of long-range correlations. For the reliable detecting of long-range correlations, it is essential to distinguish trends from the intrinsic long-range fluctuations.

An important concept used in the investigation by DFA method is fluctuation scaling. In case of a monofractal time series, the fluctuation scaling may be described by a single exponent H (the Hurst exponent). But there is a class of series that is much more complex and whose scaling exponents are different in different magnitudes [26–31]. The turning points of scaling changing are the so-called crossovers. In this paper, detrended fluctuation analysis (DFA) method is employed to analyze the long-rang correlations of stock markets. The main purpose of this study is to reveal how sensitive indeed are exponential trends to long-range correlation behaviors of stock markets. The effects of exponential trends on correlations of Hang Seng Index (HSI) were investigated with emphasis.

The organization of this paper is as follows. In Section 2, the methodology is introduced. In Section 3, the empirical data under consideration in this paper is described. In

Section 4, the correlation behaviors of the original time series are discussed. The results and analysis of the effects of exponential trends on correlations are presented in Section 5. Finally, it ends with a conclusion.

2. Methodology

2.1. Absolute Returns. In recent years, the studies about stock returns and volatility have attracted much attention. There are extensive literature testing for long-range correlations in stock returns and volatility. One of the important reasons is that a few papers have suggested that economic systems gave long memory properties. There are many implications for both portfolio and risk management that stem from long-range correlation characteristics in stock returns and volatility. The empirical evidence has suggested strong evidence of long-range correlations in volatility and however weaker evidence for stock returns. In our paper, we use absolute log returns as proxies for volatility. We will give the definition of absolute log returns in the following.

Let $X(t)$ $(i = 1, 2, \ldots, N)$ denote the price of a financial asset at time t. We define returns over a time horizon Δt as $r(t) = X(t + \Delta t) - X(t)$ $(t = 1, 2, \ldots, N)$. The absolute log returns over a time horizon Δt are then defined as $R(t) = |\log(X(t + \Delta t)) - \log(X(t))|$ $(t = 1, 2, \ldots, N)$.

2.2. DFA Method. The DFA procedure consists of five steps. Let us suppose that x_k is a series of length N; the DFA procedure involves the following steps.

Step 1. Determine the profile

$$Y(i) = \sum_{k=1}^{i} \left(x_k - \langle x \rangle\right). \tag{1}$$

Step 2. Cut the profile $Y(i)$ into $N_s \equiv [N/s]$ nonoverlapping segments of equal length s. Since the record length N need not be a multiple of the considered time scale s, a short part at the end of the profile will remain in most cases. In order not to disregard this part of the record, the same procedure is repeated starting from the other end of the record. Thus, $2N_s$ segments are obtained altogether.

The subtraction of the mean $\langle x \rangle$ is not compulsory, since it would be eliminated by the later detrending in the third step anyway.

Step 3. Calculate the local trend for each segment ν by a least-square fit of the data. Then we define the detrended time series for segment duration s, denoted by $Y_s(i)$, as the difference between the original time series and the fits:

$$Y_s(i) = Y(i) - p_\nu(i), \tag{2}$$

where $p_\nu(i)$ is the fitting polynomial in νth the segment. Linear, cubic, or higher order polynomials can be used in the fitting procedure (DFA1, DFA3, and higher order DFA). Since the detrending of the time series is done by the subtraction of the polynomial fits from the profile, these methods differ in their capability of eliminating trends in the series. In nth order DFA, trends of order n in the profile and of order $n - 1$ in the original series are eliminated.

We calculate—for each of the $2N_s$ segments—the variance as follows:

$$\begin{aligned} F_s^2(\nu) &= \left\langle Y_s^2(i) \right\rangle = \frac{1}{s}\sum_{i=1}^{s} Y_s^2\left[(\nu - 1)s + i\right] \\ &\qquad (\nu = 1, 2, \ldots, N_s), \\ F_s^2(\nu) &= \frac{1}{s}\sum_{i=1}^{s} Y_s^2\left[N - (\nu - N_s)s + i\right] \\ &\qquad (\nu = N_s + 1, N_s + 2, \ldots, 2N_s) \end{aligned} \tag{3}$$

of the detrended time series $Y_s(i)$ by averaging over all data points i in the νth segment.

Step 4. Average over all segments to obtain the fluctuation function:

$$F(s) = \left[\frac{1}{2N_s}\sum_{\nu=1}^{2N_s} F_s^2(\nu)\right]^{1/2}. \tag{4}$$

It is apparent that $F(s)$ will increase with increasing. Of course, $F(s)$ depends on the DFA order n. By construction, $F(s)$ is only defined for $s \geq n + 2$.

Step 5. Determine the scaling behavior of the fluctuation functions by analyzing log-log plots $F(s)$ versus s. If the series x_k are long-range power-law correlated, $F(s)$ increases, for large values of s, as a power-law

$$F(s) \sim s^h. \tag{5}$$

The exponent h is called the scaling exponents or correlation exponents, which represents the correlation properties of the signal.

2.3. Correlations of Time Series. For $\{x_i, i = 1, 2, \ldots, N\}$, we are interested in the correlation of the values x_i and x_{i+s} for different time lags, that is, correlations over different time scales s. In order to get rid of a constant offset in the data, the mean $\langle x \rangle = \sum_{i=1}^{N} /N$ is usually subtracted. Quantitatively, correlations between x-values separated by s steps are defined by the (auto-)correlation function as follows:

$$C(s) = \left\langle \overline{x}_i \overline{x}_{i+s} \right\rangle. \tag{6}$$

If $\{x_i\}$ are uncorrelated, $C(s)$ is zero for $s > 0$. Short-range correlations of $\{x_i\}$ are described by $C(s)$ declining exponentially and $C(s) \propto \exp(-s/s_x)$ with a decay time s_x. For so-called long-range correlations, $C(s)$ declines as a power-law

$$C(s) \propto s^{-\gamma} \tag{7}$$

with an exponent $0 < \gamma < 1$. A direct calculation of $C(s)$ is usually not appropriate due to noise superimposed on the collected data x_i and due to underlying trends of unknown

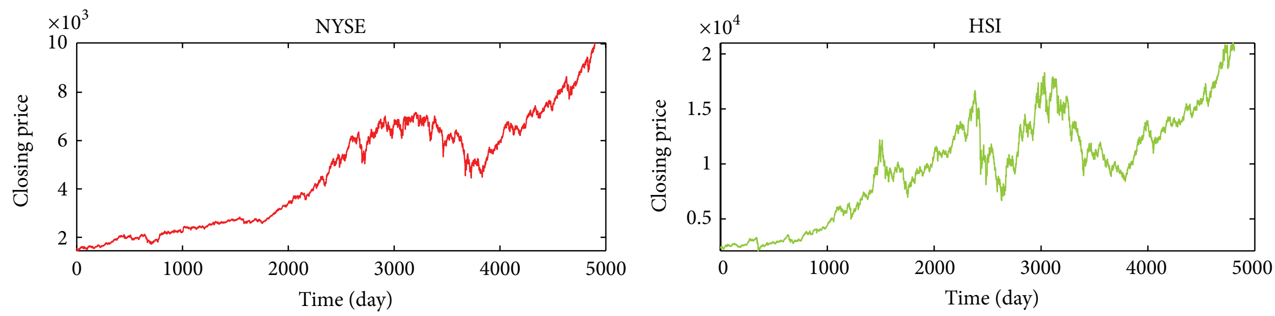

Figure 1: Daily closing prices of two stock markets NYSE and HSI during the period from 1988 to 2007.

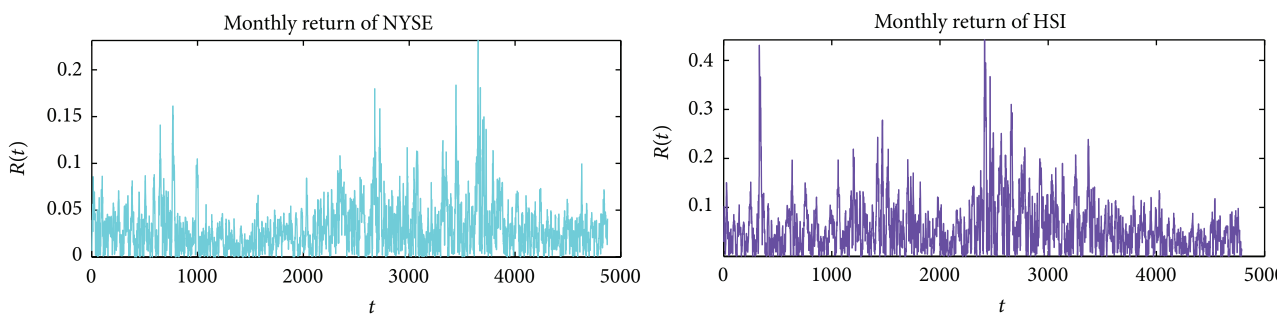

Figure 2: Monthly absolute log returns of two stock markets NYSE and HIS ($\Delta t = 22$).

origin. For example, the average $\langle x \rangle$ might be different for the first and the second half of the record, if the data are strongly long-range correlated. This makes the definition of $C(s)$ problematic. Thus, we have to determine the correlation exponent γ indirectly. A similar approximation for $F_{(n)}(s)$ is

$$F_{(n)}(s) \propto s^{1-\gamma/2}. \tag{8}$$

We find

$$\gamma = 2 - 2\alpha \quad 0 < \gamma < 1. \tag{9}$$

The correlation exponent γ can be determined by measuring the fluctuation exponent α. The correlations of stock market are calculated from realized returns on each market. Correlations tend to increase in times of large shocks to returns such as a stock market crash. Larger markets are more liquid and display more price moment than smaller counterparts. Smaller markets may not react as quickly to relevant information because some stocks may be traded infrequently. In the aftermath of October 1987 stock market crash, more attention was afforded to stock market correlation and the related concept of long-range correlation.

3. Data Description

The data used in this paper are daily closing prices of two stock indices. Form the period January 1988 to the period June 2007. The data sets consist of New York Stock Exchange Composite Index (NYSE) and Hong Kong Heng Seng Index (HSI) covering 4896 and 4810 days, respectively. The original data set used in our studies is provided by Yahoo Finance website. Two original time series are shown in Figure 1. For each stock closing price, the monthly absolute log return is calculated by $R(t) = |\log(X(t+22)) - \log(X(t))|$ $(t = 1, 2, \ldots, N-22)$. The graphs of monthly absolute log returns versus the time are shown in Figure 2.

4. The DFA of Absolute Returns

We calculate the absolute returns over several intervals: 1 day, 5 days, 1 month (22 days), 6 months (132 days), corresponding to $\Delta t = 1$, $\Delta t = 5$, $\Delta t = 22$, and $\Delta t = 132$, respectively. Then we detect the correlation behaviors of absolute returns data $R(t)$ of NYSE and HSI by using DFA method. Figures 3 and 4 present the fluctuation function $F_{(n)}(s)$ as a function of time scale s. It is demonstrated that multifractal properties of NYSE and HSI are similar for different order DFA. However, the positions of crossovers depend on time intervals Δt. The time scales of crossovers increase significantly with the increase of Δt. The crossover is interpreted as a result of different correlation properties for small and large scales in the signal; however, it may also be caused by external trends.

In order to provide more details about the correlations properties of absolute log returns, we calculate the scaling exponents and correlation exponents for $\Delta t = 1$. The scaling exponents α and correlation exponents γ are shown in Tables 1 and 2. We observe that both NYSE and HSI absolute log returns show long-range correlated behaviors. It is noted that HSI is more correlated than NYSE based on DFA method.

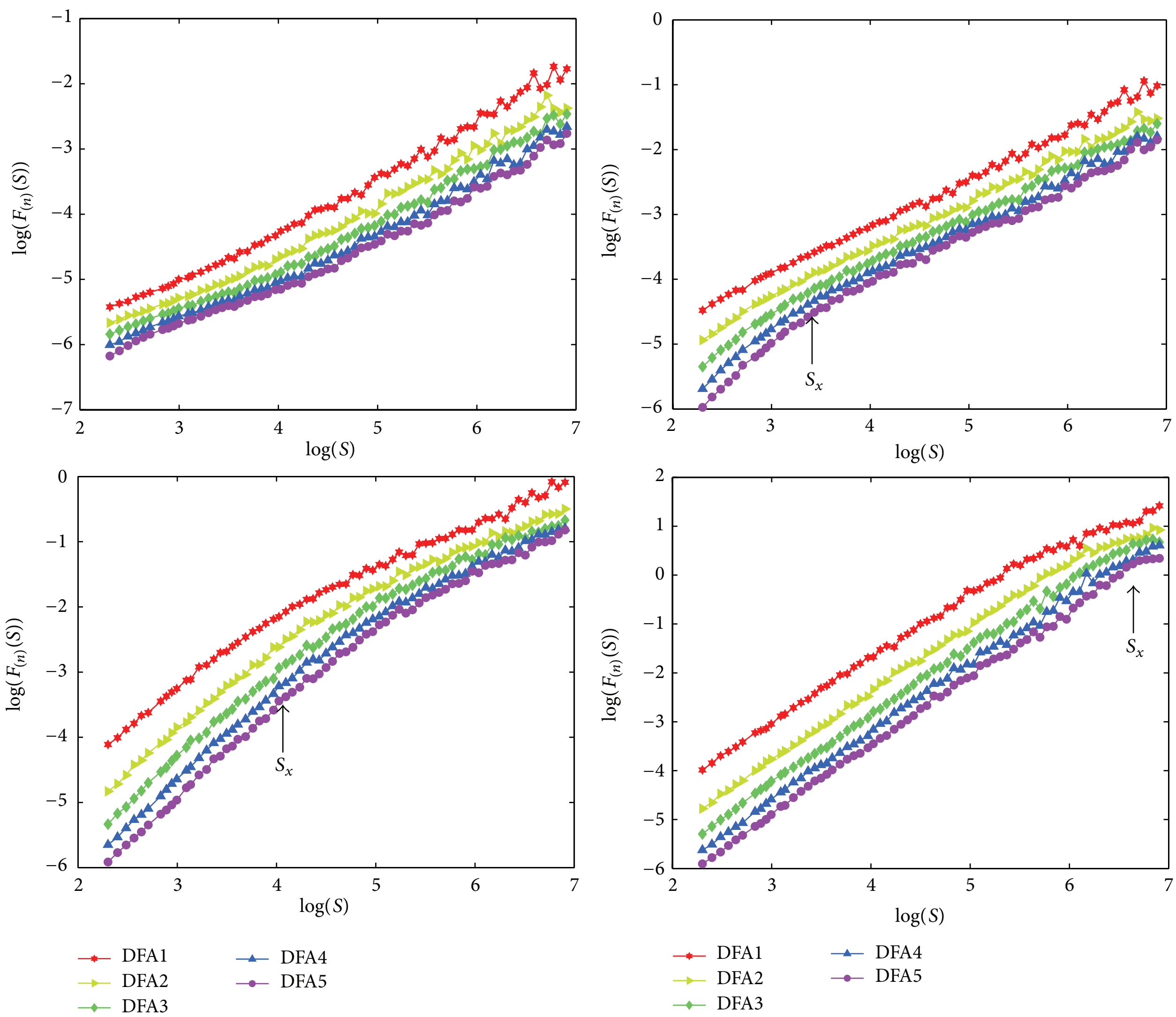

FIGURE 3: Fluctuation function $F_{(n)}(s)$ of NYSE absolute returns is plotted versus the time scales s for different time intervals Δt ($\Delta t = 1$; $\Delta t = 5$; $\Delta t = 22$; $\Delta t = 132$). The arrows point to the crossovers s_x. The positions of crossovers move from small scales to large scales with the increasing of Δt.

5. Effects of Exponential Trends on Correlations

5.1. $R(t)$ Corrupted with Exponential Trends. Time series from real world are often affected by trends, which have to be well distinguished from the original series. We have studied the correlations properties of stock market by applying the DFA method in Section 4. In this section, we survey the effect of exponential trend on DFA. The effects of exponential trends on correlations of HSI are investigated with emphasis. We attempt to generate a new artificial data by adding trends on as follows:

$$R'(t) = R(t) + 10^{p} e^{qx} \quad \text{with } x = \frac{i}{N} \quad (i = 1, 2, \ldots, N). \tag{10}$$

In this paper, $R(t)$ is the daily absolute log returns data of HSI. N is the number of the data.

TABLE 1: The values of scaling exponents α when $\Delta t = 1$.

	α − DFA1	α − DFA2	α − DFA3	α − DFA4	α − DFA5
NYSE	0.8152	0.7629	0.7416	0.7201	0.7069
HSI	0.8328	0.7916	0.7696	0.7521	0.7466

TABLE 2: The values of correlation exponents γ when $\Delta t = 1$.

	γ − DFA1	γ − DFA2	γ − DFA3	γ − DFA4	γ − DFA5
NYSE	0.3696	0.4743	0.5169	0.5598	0.5862
HSI	0.3343	0.4167	0.4608	0.4959	0.5068

5.2. DFA on Exponential Trends. We consider the exponential trends in the form $y = 10^{p} e^{qx}$ with $x = i/N$ $(i = 1, 2, \ldots, N)$. In the case of $p = 0$ and $q = 1, 2, 3, 4$, we demonstrate the effects of exponential trends on DFA in Figure 5 and Table 3.

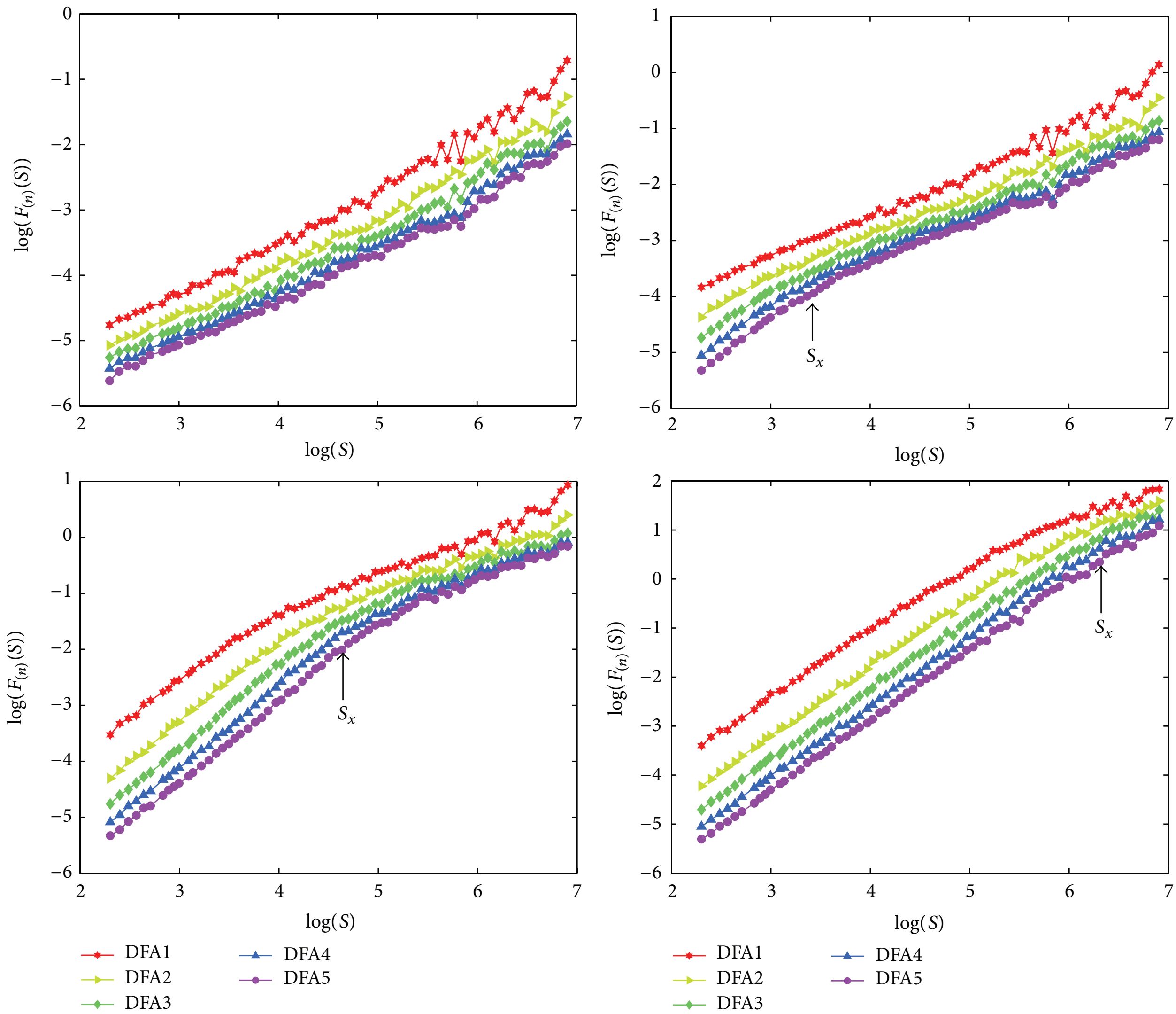

Figure 4: Fluctuation function $F_{(n)}(s)$ of HSI absolute returns is plotted versus the time scales s for different time intervals Δt ($\Delta t = 1$; $\Delta t = 5$; $\Delta t = 22$; $\Delta t = 132$). The arrows point to the crossovers s_x. The positions of crossovers move from small scales to large scales with the increasing of Δt.

Table 3: Scaling exponents α of DFA1–5 with fixing $p = 0$ and varying q.

q	DFA1	DFA2	DFA3	DFA4	DFA5
1	2.0008	3.0064	4.0168	$-0.2017 \xrightarrow{s_x=20} 4.9378$	$0.1868 \xrightarrow{s_x=90} 5.8625$
2	1.9951	3.0007	4.0109	$-0.1237 \xrightarrow{s_x=14} 4.9845$	$0.1603 \xrightarrow{s_x=60} 5.9457$
3	1.988	2.9935	4.0037	$-0.4453 \xrightarrow{s_x=11} 4.9707$	$0.1425 \xrightarrow{s_x=43} 5.9479$
4	1.9807	2.9859	3.996	4.997	$0.0081 \xrightarrow{s_x=33} 5.9163$

Table 4: Scaling exponents α of DFA1–5 with fixing $q = 1.2$ and varying p.

q	DFA1	DFA2	DFA3	DFA4	DFA5
0	1.9998	3.0055	4.0158	$-0.0546 \xrightarrow{s_x=20} 4.9713$	$0.216 \xrightarrow{s_x=90} 5.95$
1	1.9998	3.0055	4.0158	$-0.0218 \xrightarrow{s_x=20} 4.9702$	$0.2199 \xrightarrow{s_x=90} 5.9484$
2	1.9998	3.0055	4.0158	$-0.0362 \xrightarrow{s_x=20} 4.9707$	$0.2289 \xrightarrow{s_x=90} 5.9479$
3	1.9998	3.0055	4.0158	$-0.033 \xrightarrow{s_x=20} 4.971$	$0.2213 \xrightarrow{s_x=90} 5.949$

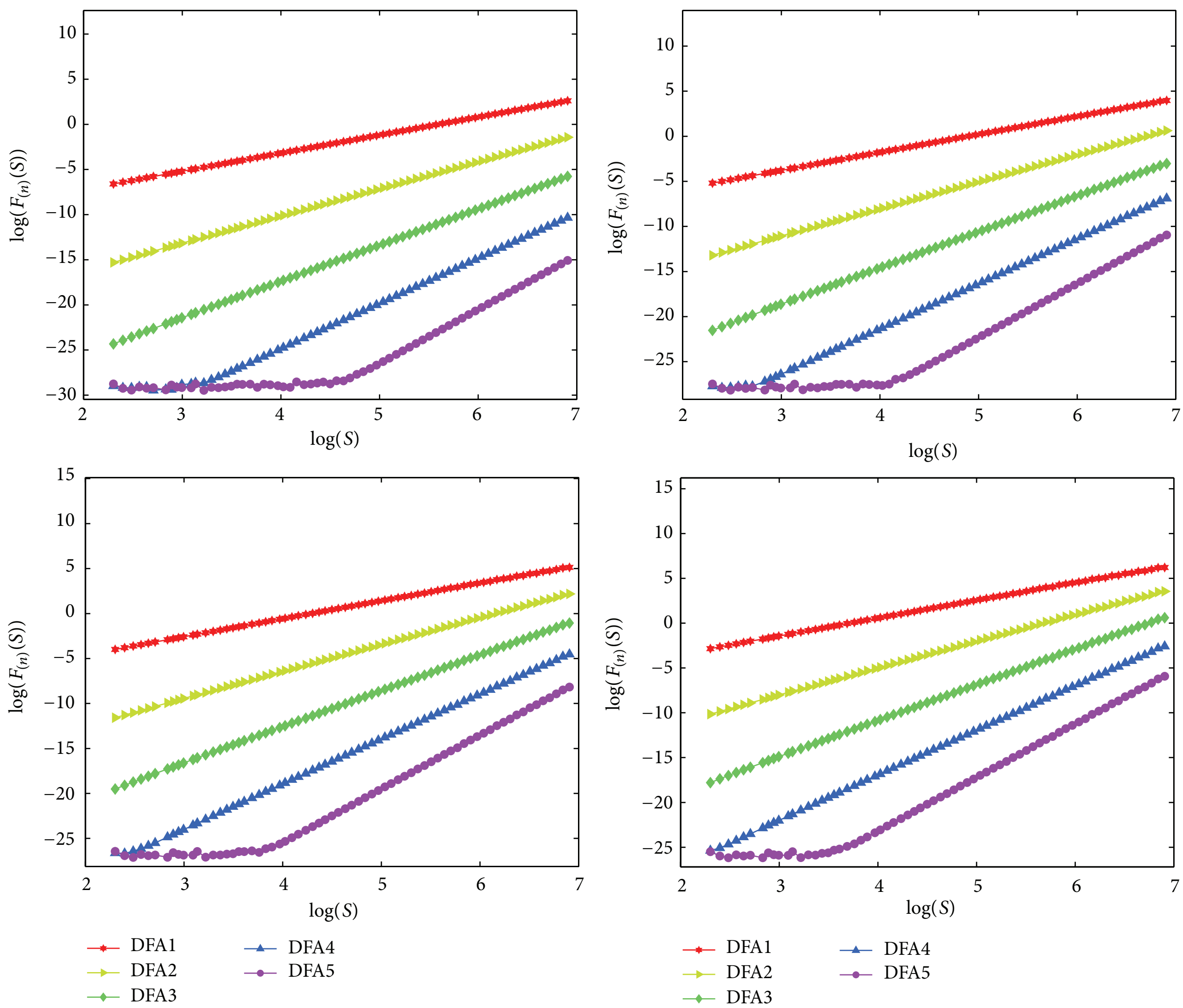

Figure 5: Fluctuation function $F_{(n)}(s)$ of DFA1 to DFA5 on exponential functions with different q. The results show that the correlations are significantly changed for different order DFA. Positions of the crossovers move to small scales with increasing the parameter q.

In the case of $p = 0, 1, 2, 3$ and $q = 1.2$, we show the effects of exponential trends on DFA in Figure 6 and Table 4. In this paper, $a \xrightarrow{s_x=c} b$ presents the scaling exponents before the crossover $s_x = c$ is a and after the crossover is b. For example, in Table 3, $-0.2017 \xrightarrow{s_x=20} 4.9378$ means that the scaling exponents before crossover $s_x = 20$ is -0.2017 and after crossover is 4.9378.

In the first case, we find that the slope of the fluctuation function $F_{(n)}(s)$ versus the scales s obtained from the DFA method slightly depends on the value of parameter q. When we increase the parameter q, the positions of the crossovers move to small scales, that is to say, the bigger the values of q, the smaller the scales of the crossovers. In the second case, we obtain that scaling characteristics of fluctuation function $F_{(n)}(s)$ versus scales s are immune to parameters p. There is no visible difference among the results obtained for all values of p.

5.3. Effects of DFA on Exponential Trends

5.3.1. Effects of DFA on Strong Exponential Trends. In this section, to reliably describe the effects of DFA method on strong exponential trends, we construct artificial time series by adjusting the values of $p = 0$ and q. For artificial data $R'(t)$ with trends $p = 0$ and $q = 1, 2, 3$, the DFA fluctuation $F_{(n)}(s)$ versus the time scales is plotted in Figure 7. The curve exhibits a crossover at $s_x = 40$ in Figure 7(a), with a slope $\alpha = 0.9181$ for $s < s_x$ and $\alpha = 0.9155$ for $s > s_x$. For $s < s_x$, the curve presents a long-range correlated behavior, but, for $s > s_x$, there is no evidence of long-range correlation.

The positions of s_x move to small scales for the same order DFA when we increase q. As we expected, the strong exponential trends are markedly filtered by higher order DFA, although apparent crossovers are still observed for large time scales s. The positions of the crossovers and the changes of the scaling exponents before and after the crossovers are

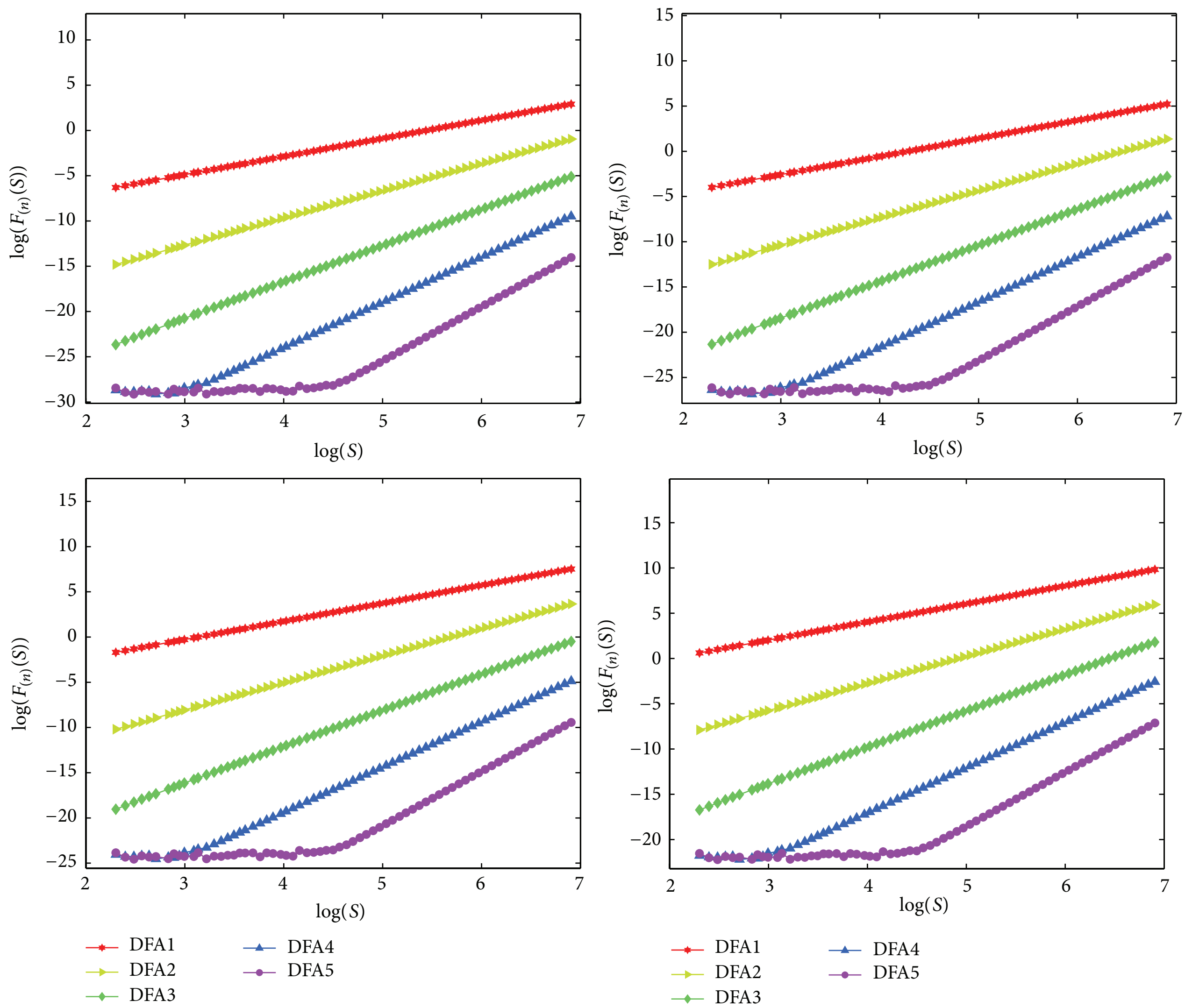

FIGURE 6: Fluctuation function $F_{(n)}(s)$ of DFA1 to DFA5 on exponential functions with different values p. There is no visible distinction among the results for all values of p.

shown in Table 5. The positions of the crossovers have been determined from the intersection of linear fits done on both sides of the crossovers. We can get the same results from Table 5, compared to Figure 7. Furthermore, the results are more credible.

Secondly, we study how different trends $p = 0, 1, 3, 5$ and $q = 1.2$ affect the scaling behavior of $R'(t)$ by applying DFA1 to DFA5. For artificial data $R'(t)$ corrupted with trends $p = 0, 1, 3, 5$ and $q = 1.2$, the fluctuation functions versus the time scales s are shown in Figure 8. In Table 5, the detail values of the crossovers s_x and the changes of scaling exponents α are presented.

In Figure 8 ($p = 0, q = 1.2$), the curve exhibits a crossover at $s_x = 27$, with a slope $\alpha = 8922$ for $s < s_x$ and $\alpha = 1.9109$ for $s > s_x$. For $s < s_x$, the curve presents a long-range correlated behavior, but, for $s > s_x$, there is no evidence of long-rang correlation. We can find from Figure 8 and Table 6 that in case $p = 0, 1$, for $n < \lceil p + q \rceil$, the exponential trends cannot be eliminated; for $n = \lceil p + q \rceil$, the exponential trends are mostly eliminated; for $n > \lceil p + q \rceil$ the exponential trends are significantly eliminated. In case $p = 2, 3, 4, 5$, for $n < \lceil p + q \rceil - 1$, the exponential trends cannot be eliminated; for $n = \lceil p + q \rceil - 1$, the exponential trends are mostly eliminated; for $n > \lceil p + q \rceil - 1$, the exponential trends are significantly eliminated.

5.3.2. Effects of DFA on Weak Exponential Trends. For weak trends, we perform the similar procedure as we do in studying strong trends. For $n \geq 2$, the results of DFAn on $R'(t)$ are the same as DFAn on $R(t)$; hence, we only give the corresponding scaling exponents α of DFA1. The corresponding scaling exponents α of DFA1 by means of fixing $p = -1$ and varying q are revealed in Table 7. The corresponding scaling exponents α by means of fixing $q = 1.2$ and varying p are shown in Table 8.

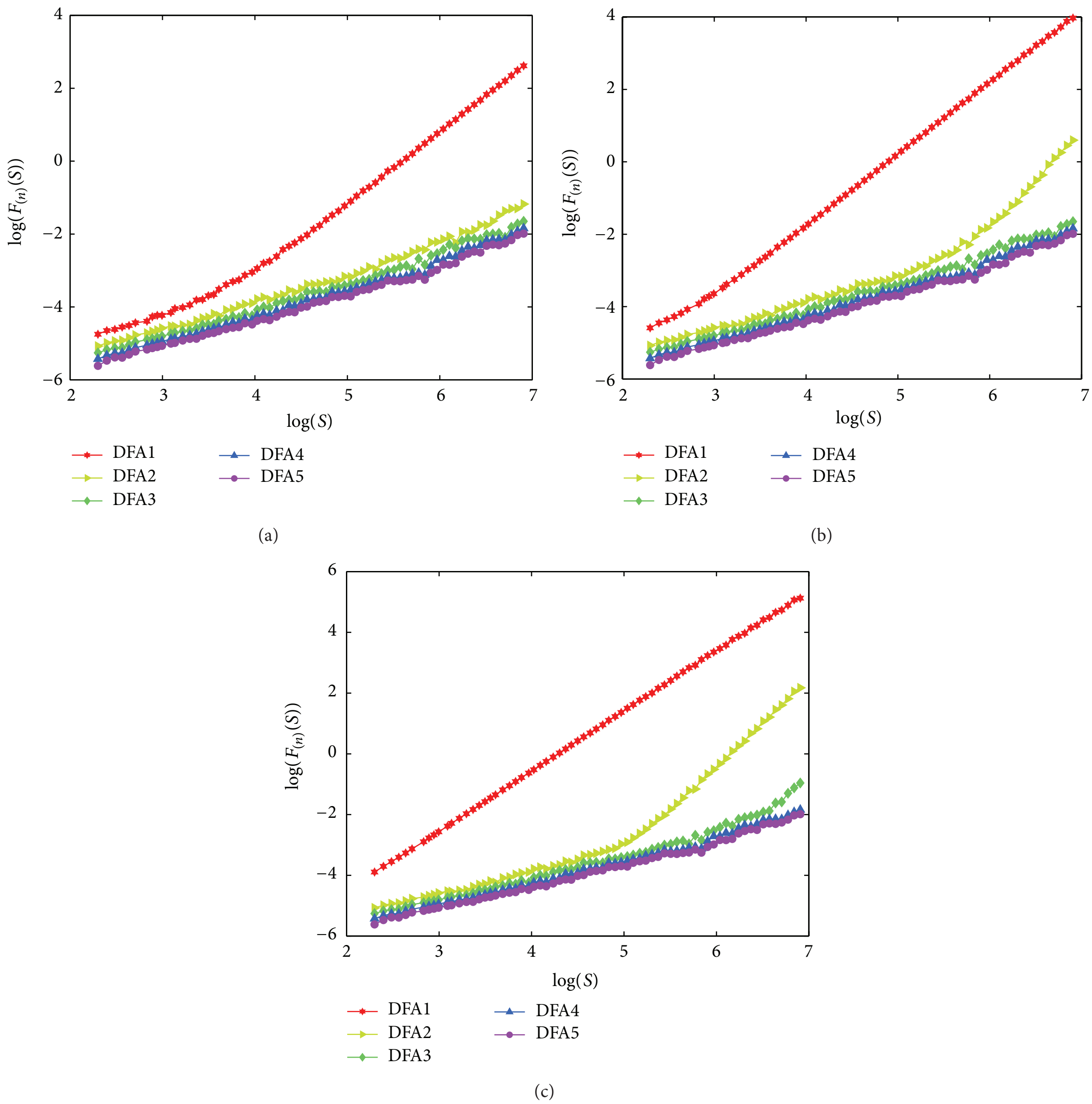

FIGURE 7: Fluctuation function $F_{(n)}(s)$ of $R'(t)$ with fixing $p = 0$ and varying q ($p = 0, q = 1$; $p = 0, q = 2$; $p = 0, q = 3$). It seems that the exponential trends are completely eliminated if $n \geq q + 1$.

TABLE 5: Scaling exponents α of DFA1 to DFA5 with fixing $p = 0$ and varying q.

q	DFA1	DFA2	DFA3	DFA4	DFA5
1	1.9181 $\rightarrow$ 1.9155	0.8145	0.7696	0.7521	0.7466
2	1.5619 $\rightarrow$ 1.9799	0.7516 $\rightarrow$ 2.3956	0.7077	0.7521	0.7466
3	1.9777	0.749 $\rightarrow$ 2.8355	0.7544 $\rightarrow$ 3.0535	0.7521	0.7466

When $p = -1$ and q is small, there are no obvious crossovers and the values of scaling exponents are decreasing with the reducing of the values of q. When $q = 0.01$, the value of fluctuation exponents is 0.8328, which is the same as the value of the original time series. When $q = 1.2$ and p is small, the similar consequences can be concluded. When $p = -2.55$ the value of fluctuation exponents is 0.8328. The results indicate that if $p + q < -0.3$, there are no obvious crossovers; if $p + q < -1.5$, the exponential trends have no effects on scaling exponents of each order DFA.

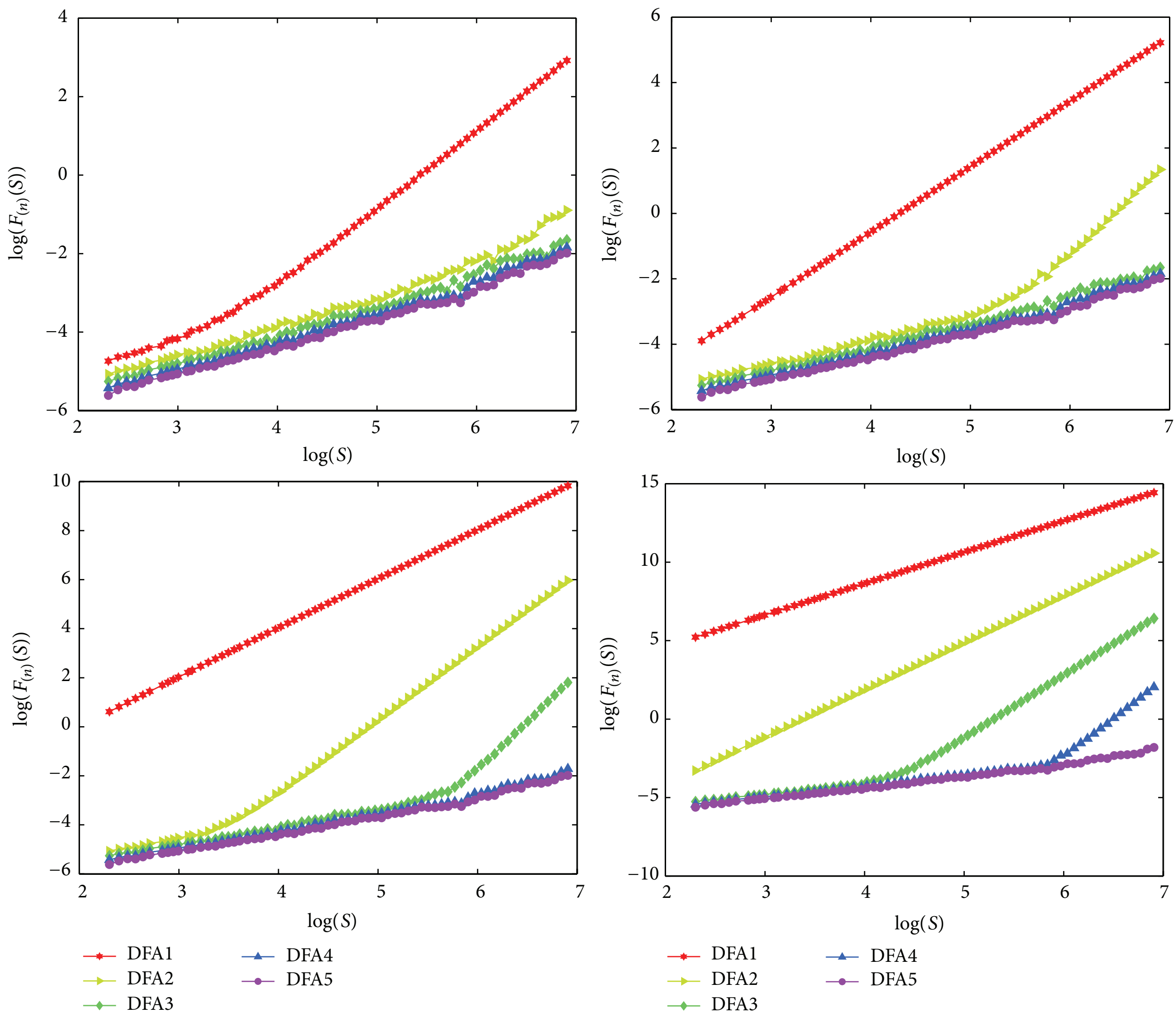

FIGURE 8: Fluctuation function $F_{(n)}(s)$ of DFA1 to DFA5 of $R'(t)$ with fixing $q = 1.2$ and varying p ($p = 0, q = 1.2$; $p = 1, q = 1.2$; $p = 3$, $q = 1.2$; $p = 5, q = 1.2$). For different values of p and q, the fluctuation function $F_{(n)}(s)$ presents striking diversity.

TABLE 6: Scaling exponents α of DFA1 to DFA5 with fixing $q = 1.2$ and varying p.

p	DFA1	DFA2	DFA3	DFA4	DFA5
0	0.8922 → 1.9109	0.8445	0.7696	0.7521	0.7466
1	1.9895	0.7296 → 2.8496	0.7714	0.7521	0.7466
3	1.9998	0.7671 → 2.9853	0.7272 → 3.8545	0.7572	0.7466
5	1.9998	3.0046	0.706 → 3.9892	0.7032 → 4.7769	0.7529

TABLE 7: Scaling exponents α of DFA1 with $p = -1$ and varying q.

q	0.7	0.5	0.2	0.1	0.05	0.02	0.01
α	0.95	0.8941	0.8417	0.8349	0.8333	0.8329	0.8328

TABLE 8: Scaling exponents α of DFA1 with $q = 1.2$ and varying p.

p	−1.5	−1.7	−1.9	−2	−2.2	−2.5	−2.55
α	0.9081	0.867	0.8463	0.8408	0.8353	0.8329	0.8328

6. Conclusion

In this paper, the long-range correlation behaviors in absolute log returns are investigated by applying DFA. We find that the absolute log returns of NYSE and HSI are long-range correlated; however, the long-range correlation of NYSE is stronger than HSI. The effect of different exponential trends on the DFA scaling behavior and the crossovers of long-range correlated signals are also studied. The results suggest that long-range correlation behaviors of higher order DFA are not affected by exponential trends, while the effects of exponential trends depend on parameters p and q. The real financial data are often influenced by different trends which are caused by external effects. So it is of great importance to investigate the properties of the trends and the effects of the trends on the dynamical changes of original signal. There are several issues for further research. The first issue concerns application of our method to other types of trends. The second issue considers predicting the stock markets based on our current results due to the crossover being the representation of economic cycle in some sense. Our studies are relevant for a better understanding of the stock market mechanism and may be useful for identifying economic turning points and forecasting stock volatility. Further studies are necessary from theoretical and empirical perspective.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant nos. 61304145, 61071142, and 61371130), the China Postdoctoral Science Foundation (Grant no. 2012M520156), the Fundamental Research Funds for Central Universities of China (Grant no. 2013JBM089), the Research Fund for the Doctoral Program of Higher Education (Grant no. 20130009120016), and the National High Technology Research Development Program of China (Grant no. 2011AA110306).

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