# Dynamic Model of a Wind Turbine for the Electric Energy Generation 

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#### Abstract

A novel dynamic model is introduced for the modeling of the wind turbine behavior. The objective of the wind turbine is the electric energy generation. The analytic model has the characteristic that considers a rotatory tower. Experiments show the validity of the proposed method.


## 1. Introduction

Researchers are often trying to improve the total power of a wind turbine. The dynamic model of a wind turbine plays an important role in some applications as the control, classification, or prediction.

Some authors have proposed the equations to model the dynamic behavior of the wind turbine as shown in [1-5].

This paper presents a novel dynamic model of a wind turbine; the first dynamic model is for the wind turbine, the second is for the tower, and both are related. Because the rotatory tower can turn, it may help the wind turbine to increase the air intake. Some companies propose wind turbines that consider rotatory blades; however, in this study, a rotatory tower is proposed instead of the rotatory blades because the first dynamic model is easier to obtain than the second.

The proposed model is linear in the states as the works analyzed by [6-10] show. The technique provides an acceptable approximation of the wind turbine behavior.

The paper is structured as follows. In Section 2, the dynamic model of a windward wind turbine of three blades with a rotatory tower is introduced. In Section 3, the simulations of the dynamic model are compared with the real
data obtained by a real wind turbine prototype. Finally, in Section 4, the conclusion and future research are detailed.

## 2. Dynamic Model of the Wind Turbine with a Rotatory Tower

This section is divided into four parts: the first is the description of the mechanic model, the second is the description of the aerodynamic model, the third is the description of the electric model, and, finally, the fourth is the combination of the aforementioned models to obtain the final dynamic model.

The dynamic model of the wind turbine is, first, the equations that represent the change between the wind energy and mechanic energy and, second, the equations that represent the change between the mechanic energy and electric energy.
2.1. The Mechanic Model. A windward wind turbine of three blades with a rotatory tower is considered. First, the Euler Lagrangian method [11, 12] is used to obtain the model that represents the change from the wind energy to mechanic energy for the wind turbine and the change from the electric energy to mechanic energy for the tower. The masses are


Figure 1: Lateral view of the wind turbine.


Figure 2: Upper view of the wind turbine.
concentrated at the center of mass. Consider the lateral view of Figure 1 and upper view of Figure 2.

From Figures 1 and 2, it can be seen that

$$
\begin{gather*}
x_{2}=-l_{c 2} S_{2}\left(-C_{1}\right)=l_{c 2} S_{2} C_{1}, \\
y_{2}=-l_{c 2} S_{2}\left(-S_{1}\right)=l_{c 2} S_{2} S_{1}, \tag{1}
\end{gather*}
$$

where $S_{1}=\sin \left(\theta_{1}\right), S_{2}=\sin \left(\theta_{2}\right), C_{1}=\cos \left(\theta_{1}\right), C_{2}=\cos \left(\theta_{2}\right)$, $\theta_{1}$ is the angular position of the tower motor in rad, $\theta_{2}$ is the angular position of a wind turbine blade in rad, and $l_{c 2}$ is the length to the center of the wind turbine blade in $m$.

Consequently, the kinetic energy $K_{1}, K_{2}$ and potential energy $P_{1}, P_{2}$ are given as

$$
\begin{gather*}
K_{1}=0, \\
K_{2}=\frac{1}{2} m_{2} l_{c 2}^{2} S_{2}^{2} \dot{\theta}_{1}^{2}+\frac{1}{2} m_{2} l_{c 2}^{2} \dot{\theta}_{2}^{2},  \tag{2}\\
P_{1}=m_{1} g l_{c 1} \\
P_{2}=l_{1}+m_{2} g l_{c 2} C_{2},
\end{gather*}
$$

where $m_{1}$ is the tower mass in $\mathrm{kg}, m_{2}$ is the blade mass in $\mathrm{kg}, g$ is the acceleration gravity in $\mathrm{m} / \mathrm{s}^{2}, l_{1}$ is the constant length of the tower in m , and $l_{c 1}$ is the length to the center of the tower in m . The torques $\tau_{T 1 a}$ and $\tau_{T 2 a}$ are given as

$$
\begin{align*}
& \tau_{T 1 a}=\tau_{1 a}-k_{b 1} \theta_{1}-b_{b 1} \dot{\theta}_{1}, \\
& \tau_{T 2 a}=\tau_{2 a}-k_{b 2} \theta_{2}-b_{b 2} \dot{\theta}_{2}, \tag{3}
\end{align*}
$$

where $\tau_{2 a}$ is the torque of the generator moved by the blade in $\mathrm{kg} \mathrm{m}{ }^{2} \mathrm{rad} / \mathrm{s}^{2}, \tau_{1 a}$ is the torque of the motor used to move the tower in $\mathrm{kg} \mathrm{m}{ }^{2} \mathrm{rad} / \mathrm{s}^{2}, k_{b 1}$ and $k_{b 2}$ are the spring effects presented when the blade is near to a stop in $\mathrm{kg} \mathrm{m} \mathrm{m}^{2} / \mathrm{s}^{2}$, and $b_{b 1}$ and $b_{b 2}$ are the shock absorbers in $\mathrm{kg} \mathrm{m}^{2} \mathrm{rad} / \mathrm{s}$. Using the Euler Lagrange method in [11, 12] gives the following equations:

$$
\begin{gather*}
m_{2} l_{c 2}^{2} S_{2}^{2} \ddot{\theta}_{1}+2 m_{2} l_{c 2}^{2} S_{2} C_{2} \dot{\theta}_{1} \dot{\theta}_{2}+b_{b 1} \dot{\theta}_{1}+k_{b 1} \theta_{1}=\tau_{1 a}  \tag{4}\\
m_{2} l_{c 2}^{2} \ddot{\theta}_{2}+m_{2} g l_{c 2} S_{2}+b_{b 2} \dot{\theta}_{2}+k_{b 2} \theta_{2}=\tau_{2 a}
\end{gather*}
$$

The angular position of the blade 1 is related to the angular position of the blades 2 and 3 as follows:

$$
\begin{align*}
& \theta_{3}=\theta_{2}+\frac{2}{3} \pi \mathrm{rad}  \tag{5}\\
& \theta_{4}=\theta_{2}+\frac{4}{3} \pi \mathrm{rad}
\end{align*}
$$

where $\theta_{3}$ and $\theta_{4}$ are the angular positions for the blades 2 and 3 , respectively. Then, using (4), it yields the equations for blades 2 and 3 as a function of $\theta_{2}$ as follows:

$$
\begin{gather*}
m_{2} l_{c 2}^{2} S_{2+(2 / 3) \pi}^{2} \ddot{\theta}_{1}+2 m_{2} l_{c 2}^{2} S_{2+(2 / 3) \pi} C_{2+(2 / 3) \pi} \dot{\theta}_{1} \dot{\theta}_{2} \\
+b_{b 1} \dot{\theta}_{1}+k_{b 1} \theta_{1}=\tau_{1 b} \\
m_{2} l_{c 2}^{2} \ddot{\theta}_{2}+m_{2} g l_{c 2} S_{2+(2 / 3) \pi}+b_{b 2} \dot{\theta}_{2}+k_{b 2}\left(\theta_{2}+\frac{2}{3} \pi\right)=\tau_{2 b}  \tag{6}\\
m_{2} l_{c 2}^{2} S_{2+(4 / 3) \pi}^{2} \ddot{\theta}_{1}+2 m_{2} l_{c 2}^{2} S_{2+(4 / 3) \pi} C_{2+(4 / 3) \pi} \dot{\theta}_{1} \dot{\theta}_{2} \\
+b_{b 1} \dot{\theta}_{1}+k_{b 1} \theta_{1}=\tau_{1 c} \\
m_{2} l_{c 2}^{2} \ddot{\theta}_{2}+m_{2} g l_{c 2} S_{2+(4 / 3) \pi}+b_{b 2} \dot{\theta}_{2}+k_{b 2}\left(\theta_{2}+\frac{4}{3} \pi\right)=\tau_{2 c} \tag{7}
\end{gather*}
$$

where $\tau_{1 b}, \tau_{2 b}, \tau_{1 c}$, and $\tau_{2 c}$ are the torques applied to move the blades 2 and 3, respectively. Adding $S_{2}^{2}+S_{2+(2 / 3) \pi}^{2}+S_{2+(4 / 3) \pi}^{2}$,
$S_{2} C_{2}+S_{2+(2 / 3) \pi} C_{2+(2 / 3) \pi}+S_{2+(4 / 3) \pi} C_{2+(4 / 3) \pi}$, and $S_{2}+S_{2+(2 / 3) \pi}+$ $S_{2+(4 / 3) \pi}$, respectively, it gives

$$
\begin{gather*}
S_{2}^{2}+S_{2+(2 / 3) \pi}^{2}+S_{2+(4 / 3) \pi}^{2}=1.5  \tag{8}\\
S_{2} C_{2}+S_{2+(2 / 3) \pi} C_{2+(2 / 3) \pi}+S_{2+(4 / 3) \pi} C_{2+(4 / 3) \pi}=0  \tag{9}\\
S_{2}+S_{2+(2 / 3) \pi}+S_{2+(4 / 3) \pi}=0 \tag{10}
\end{gather*}
$$

Now, adding the three equations of (4), (6), and (7) and using (8), (9), and (10), it gives

$$
\begin{gather*}
4.5 m_{2} l_{c 2}^{2} \ddot{\theta}_{1}+3 b_{b 1} \dot{\theta}_{1}+3 k_{b 1} \theta_{1}=\tau_{1},  \tag{11}\\
3 m_{2} l_{c 2}^{2} \ddot{\theta}_{2}+3 b_{b 2} \dot{\theta}_{2}+3 k_{b 2} \theta_{2}+2 \pi k_{b 2}=\tau_{2}
\end{gather*}
$$

where $\tau_{1}=\tau_{1 a}+\tau_{1 b}+\tau_{1 c}$ and $\tau_{2}=\tau_{2 a}+\tau_{2 b}+\tau_{2 c}$. From Figures 1 and $2, \tau_{2}$ of (11) is defined as follows:

$$
\begin{gather*}
\tau_{2}=C_{1} F_{2}, \\
F_{2}=F_{2 a}+F_{2 b}+F_{2 c}, \tag{12}
\end{gather*}
$$

where $C_{1}=\cos \left(\theta_{1}\right) F_{2 a}$ and $F_{2 b}$ and $F_{2 c}$ are the force of the air received by the three blades. Equation (12) describes the assumption that the air goes in one direction; if $\theta_{1}=0$, then the maximum air intake moves the blades of the wind turbine, but if the tower turns to the left or to the right and $\theta_{1}$ changes, then the wind turbine turns and the air intake decreases. From Figures 1 and 2 and [12], $\tau_{1}$ of (11) is defined as follows:

$$
\begin{equation*}
\tau_{1}=k_{m} i_{1} \tag{13}
\end{equation*}
$$

where $k_{m}$ is a motor magnetic flux constant of the tower in Wb and $i_{1}$ is the motor armature current of the tower in A . Equations (11), (12), and (13) are the main equations of the mechanic model that represents the wind turbine and tower.
2.2. Aerodynamic Model. The aerodynamic model is the dynamic model of the torque applied to the blades. The mechanic power captured by the wind turbine $P_{a}$ is given by [2-5]

$$
\begin{equation*}
P_{a}=F_{2 a} \dot{\theta}_{2}=\frac{1}{2} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3} \tag{14}
\end{equation*}
$$

where $\rho$ is the air density in $\mathrm{Kg} / \mathrm{m}^{3}, A=\pi R^{2}$ is the area swept by the rotor blades in $\mathrm{m}^{2}$ with radius $R$ in $\mathrm{m}, V_{\omega}$ is the wind speed in $\mathrm{m} / \mathrm{s}$, and $C_{p}(\lambda, \beta)$ is the performance coefficient of the wind turbine, whose value is a function of the tip speed ratio $\lambda$, defined as [2-5]

$$
\begin{equation*}
\lambda=\frac{\dot{\theta}_{2} R}{V_{\omega}} \tag{15}
\end{equation*}
$$

For the purpose of simulation, the following model of $C_{p}(\lambda, \beta)$ is presented [2-5]:

$$
\begin{equation*}
C_{p}(\lambda, \beta)=c_{1}\left(\frac{c_{2}}{\lambda_{i}}-c_{3} \beta-c_{4}\right) e^{-c_{5} / \lambda_{i}}+c_{6} \lambda \tag{16}
\end{equation*}
$$



Figure 3: The wind turbine generator.

where

$$
\begin{equation*}
\frac{1}{\lambda_{i}}=\frac{1}{\lambda+0.08 \beta}-\frac{0.035}{\beta^{3}+1} \tag{17}
\end{equation*}
$$

and the coefficients are $c_{1}=0.5176, c_{2}=116, c_{3}=0.4, c_{4}=5$, $c_{5}=21, c_{6}=0.0068$, and $\beta$ is the blade pitch angle in rad. Using (12), (14), (15), (16), and (17) gives the dynamic model of the torque applied to the wind turbine blades as follows:

$$
\begin{align*}
& \tau_{2}=C_{1}\left(F_{2 a}+F_{2 b}+F_{2 c}\right), \\
& F_{2 a}=\frac{1}{2 \dot{\theta}_{2}} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3}, \\
& F_{2 b}=\frac{1}{2 \dot{\theta}_{2}} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3},  \tag{18}\\
& F_{2 c}=\frac{1}{2 \dot{\theta}_{2}} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3},
\end{align*}
$$

where $C_{1}$ is defined in (12), $\lambda$ is defined in (15), $C_{p}(\lambda, \beta)$ is defined in (16), and $\lambda_{i}$ is defined in (17).
2.3. The Electric Model. Now, analyze the change from the mechanic to electric energy for the wind turbine generator and the electric energy to mechanic energy for the tower motor. Figure 3 shows the wind turbine generator and Figure 4 shows the tower motor.

In $[12,13]$, they presented the model of a motor, and similarly, from Figure 3, a model for the generator is obtained
by using the Kirchhoff voltage law. Therefore, the dynamic models of the motor and generator are as follows:

$$
\begin{align*}
& V_{1}=R_{1} i_{1}+L_{1} \dot{i}_{1}+k_{1} \dot{\theta}_{1} \\
& k_{2} \dot{\theta}_{2}=R_{2} i_{2}+L_{2} \dot{i}_{2}+V_{2} \tag{19}
\end{align*}
$$

where $k_{1}$ is the motor back emf constant in $\mathrm{Vs} / \mathrm{rad}, k_{2}$ is the generator back emf constant in $\mathrm{V} \mathrm{s} / \mathrm{rad}, R_{1}$ is the motor armature resistance in $\Omega, R_{2}$ is the generator armature resistance in $\Omega, L_{1}$ is the motor armature inductance in H , $L_{2}$ is the generator armature inductance in $\mathrm{H}, V_{1}$ is the motor armature voltage in $\mathrm{V}, V_{2}$ is the generator armature voltage in $\mathrm{V}, i_{1}$ is the motor armature current in A , and $i_{2}$ is the generator armature current in A . For the generator of this paper, $V_{2}=R_{e} i_{2}$. Thus, (19) becomes

$$
\begin{gather*}
V_{1}=R_{1} i_{1}+L_{1} \dot{i}_{1}+k_{1} \dot{\theta}_{1} \\
k_{2} \dot{\theta}_{2}=\left(R_{2}+R_{e}\right) i_{2}+L_{2} \dot{i}_{2}  \tag{20}\\
V_{2}=R_{e} i_{2}
\end{gather*}
$$

2.4. The Final Dynamic Model. Thus, (11), (13), and (18) that represent the change between the wind energy and mechanic energy and (20) that represents the change between the mechanic energy and electric energy are considered as the wind turbine dynamic model with a rotatory tower.

Define the state variables as $x_{1}=i_{2}, x_{2}=\theta_{2}, x_{3}=\dot{\theta}_{2}$, $x_{4}=i_{1}, x_{5}=\theta_{1}, x_{6}=\dot{\theta}_{1}$, the inputs as $u_{1}=F_{2}$ and $u_{2}=V_{1}$, and the output as $y=V_{2}$. Consequently, the dynamic model of (11), (13), (15), (18), and (20) becomes

$$
\begin{gather*}
\dot{x}_{1}=-\frac{\left(R_{2}+R_{e}\right)}{L_{2}} x_{1}+\frac{k_{2}}{L_{2}} x_{3}, \\
\dot{x}_{2}=x_{3}, \\
\dot{x}_{3}=-\frac{k_{b 2}}{m_{2} l_{c 2}^{2}} x_{2}-\frac{b_{b 2}}{m_{2} l_{c 2}^{2}} x_{3}-\frac{2 \pi k_{b 2}}{3 m_{2} l_{c 2}^{2}}+\frac{\cos \left(x_{5}\right)}{3 m_{2} l_{c 2}^{2}} u_{1}, \\
\dot{x}_{4}=-\frac{R_{1}}{L_{1}} x_{4}-\frac{k_{1}}{L_{1}} x_{6}+\frac{1}{L_{1}} u_{2} \\
\dot{x}_{5}=x_{6} \\
\dot{x}_{6}=-\frac{3 k_{b 1}}{4.5 m_{2} l_{c 2}^{2}} x_{5}-\frac{3 b_{b 1}}{4.5 m_{1} l_{c 2}^{2}} x_{6}+\frac{k_{m}}{4.5 m_{2} l_{c 2}^{2}} x_{4}  \tag{21}\\
y=R_{e} x_{1}, \\
u_{1}=F_{2 a}+F_{2 b}+F_{2 c} \\
F_{2 a}=\frac{1}{2 x_{3}} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3} \\
F_{2 b}=\frac{1}{2 x_{3}} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3} \\
F_{2 c}=\frac{1}{2 x_{3}} \rho A C_{p}(\lambda, \beta) V_{\omega}^{3} \\
\lambda=\frac{x_{3} R}{V_{\omega}}
\end{gather*}
$$

where $C_{p}(\lambda, \beta)$ is defined in (16) and $\lambda_{i}$ is defined in (17).


Figure 5: Prototype of a wind turbine with rotatory tower.

Table 1: Parameters of the prototype.

| Parameter | Value |
| :--- | :---: |
| $l_{c 2}$ | 0.5 m |
| $m_{2}$ | 0.5 kg |
| $k_{b 2}$ | $1 \times 10^{-6} \mathrm{kgm}^{2} / \mathrm{s}^{2}$ |
| $b_{b 2}$ | $1 \times 10^{-1} \mathrm{kgm}^{2} \mathrm{rad} / \mathrm{s}$ |
| $k_{2}$ | $0.45 \mathrm{Vs} / \mathrm{rad}$ |
| $R_{2}$ | $6.96 \Omega$ |
| $L_{2}$ | $6.031 \times 10^{-1} \mathrm{H}$ |
| $R$ | $l_{c 2} \mathrm{~m}$ |
| $\rho$ | $1.225 \mathrm{~kg} / \mathrm{m}^{3}$ |
| $g$ | $9.81 \mathrm{~m} / \mathrm{s}^{2}$ |
| $R_{e}$ | $30 \Omega$ |
| $k_{m}$ | 0.09 Wb |
| $k_{b 1}$ | $1 \times 10^{-6} \mathrm{kgm}^{2} / \mathrm{s}^{2}$ |
| $b_{b 1}$ | $1 \times 10^{-1} \mathrm{kgm}^{2} \mathrm{rad} / \mathrm{s}$ |
| $k_{1}$ | $0.0045 \mathrm{Vs} / \mathrm{rad}^{2}$ |
| $R_{1}$ | $18 \Omega$ |
| $L_{1}$ | $6.031 \times 10^{-1} \mathrm{H}$ |
| $V_{\omega}$ | $5 \mathrm{~m} / \mathrm{s}$ |
| $\beta$ | 0.5 rad |

Remark 1. The model of this research considers a direct current motor and a direct current generator. This wind turbine is proposed to be used on the house roof for the daily usage to feed a light as a renewable source. If more than one wind turbine is used, more electricity is generated. The proposed model could be extended to other kinds of machines by changing (20).

Remark 2. Note that the dynamic model states of the wind turbine with a rotatory tower (21) is a linear system [6-10].

## 3. Experiments to Validate the Dynamic Model

Figure 5 shows the prototype of a wind turbine with a rotatory tower which is considered for the simulations of the dynamic model. This prototype has three blades with a rotatory tower which does not use a gear box. Table 1 shows the parameters of the prototype. The parameters $m_{2}$ and $l_{c 2}$ are obtained from


Figure 6: Inputs and output of the wind turbine of Example 1.
the wind turbine blades. The parameters $R_{1}, L_{1}$, and $k_{1}$ are obtained from the tower motor. The parameters $k_{2}, R_{2}, R_{e}$, and $L_{2}$ are obtained from the wind turbine generator. The parameters $R, \rho, V_{\omega}$, and $\beta$ are obtained from [2-5].

The dynamic model of the wind turbine with a rotatory tower is given by (21) with the parameters of Table $1.1 \times 10^{-5}$ are considered as the initial conditions for the plant states $x_{1}=i_{2}, x_{2}=\theta_{2}, x_{3}=\dot{\theta}_{2}, x_{4}=i_{1}, x_{5}=\theta_{1}$, and $x_{6}=\dot{\theta}_{1}$. The root mean square error (RMSE) is used, and it is given as [13-15]

$$
\begin{equation*}
\operatorname{RMSE}=\left(\frac{1}{T} \int_{0}^{T} e^{2} d \tau\right)^{1 / 2} \tag{22}
\end{equation*}
$$

where $e^{2}=e_{u j}^{2}=\left(u_{j r}-u_{j}\right)^{2}, e^{2}=e_{x i}^{2}=\left(x_{i r}-x_{i}\right)^{2}$, or $e^{2}=$ $e_{y}^{2}=\left(y_{r}-y\right)^{2}$ and $u_{j r}, x_{i r}$, and $y_{r}$ are the real data of $u_{j}, x_{i}$, and $y$, respectively, $i=1,2, \ldots, 6, j=1,2$.
3.1. Example 1: The First Behavior. Figures 6 and 7 show the wind turbine inputs, outputs, and states. Table 2 shows the RMSE for the errors.

From Figures 6 and 7, it can be seen that the dynamic model has good behavior described as follows: (1) from 0 s to 2 s , both inputs are fed; consequently, the tower moves far of the maximum air intake, the generator current is decreased, and the wind turbine blades stop moving; (2) from 2 s to 4 s , both inputs are not fed; consequently, current is not generated and both the tower and wind turbine blades do not move; (3) from 4 s to 6 s , both inputs are fed, but the air intake is positive and tower voltage is negative; consequently, the tower returns to the maximum air intake, the generator current is increased, and the wind turbine blades move; (4) from 6 s to 8 s , both

Table 2: The RMSE for Example 1.

|  | RMSE |
| :--- | :---: |
| $e_{u 1}^{2}$ | 0.0204 |
| $e_{u 2}^{2}$ | 1.1835 |
| $e_{y}^{2}$ | 0.0171 |
| $e_{x 1}^{2}$ | $5.7158 \times 10^{-4}$ |
| $e_{x 2}^{2}$ | 0.9843 |
| $e_{x 3}^{2}$ | 0.0470 |
| $e_{x 4}^{2}$ | 0.0658 |
| $e_{x 5}^{2}$ | 0.0872 |
| $e_{x 6}^{2}$ | 0.0163 |

inputs are not fed; consequently, current is not generated and the tower and wind turbine blades do not move. The dynamic model is a good approximation of the wind turbine behavior because the signals of the first are near to the signals of the second. From Table 2, it is shown that the dynamic model is a good approximation of the real process because the RMSE is near to zero.
3.2. Example 2: The Second Behavior. Figures 8 and 9 show the wind turbine inputs, outputs, and states. Table 3 shows the RMSE for the errors.

From Figures 8 and 9, it can be seen that the dynamic model has good behavior described as follows: (1) from 0 s to 2 s , the input air is fed and the tower input is not fed; consequently, the tower remains in the maximum air intake, the generator current is maximum, and the wind turbine blades have motion; (2) from 2 s to 4 s , the air is not fed and


Figure 7: States of the wind turbine of Example 1.


Figure 8: Inputs and output of the wind turbine of Example 2.


FIgure 9: States of the wind turbine of Example 2.

Table 3: The RMSE for Example 2.

|  | RMSE |
| :--- | :---: |
| $e_{u 1}^{2}$ | 0.0288 |
| $e_{u 2}^{2}$ | 0.9423 |
| $e_{y}^{2}$ | 0.0280 |
| $e_{x 1}^{2}$ | $9.3212 \times 10^{-4}$ |
| $e_{x 2}^{2}$ | 1.7209 |
| $e_{x 3}^{2}$ | 0.0766 |
| $e_{x 4}^{2}$ | 0.0523 |
| $e_{x 5}^{2}$ | 0.1324 |
| $e_{x 6}^{2}$ | 0.0162 |

the tower input is fed; consequently, current is not generated, the tower moves far of the maximum air intake, and the wind turbine blades do not have motion; (3) from 4 s to 6 s , the air is fed and the tower input is not fed; consequently, the tower does not move, the generator current is minimum, and the wind turbine blades almost do not move; (4) from 6 s to 8 s , the air is not fed and the tower input is fed with a negative voltage; consequently, current is not generated, the tower returns to the maximum air intake, and the wind turbine blades do not have motion. The dynamic model is a good approximation of the wind turbine behavior because
the signals of the first are near to the signals of the second. From Table 2, it is shown that the dynamic model is a good approximation of the real process because the RMSE is near to zero.

## 4. Conclusion

In this paper, a dynamic model of a wind turbine with a rotatory tower was introduced; the simulations using the parameters of a prototype showed that the proposed dynamic model has an acceptable approximation of the wind turbine behavior. The proposed dynamic model could be used on control, prediction, or classification. As a future research, the modeling will be improved using other interesting methods as the least squares in [16-26], neural networks in [14, 27-32], or fuzzy systems in [15, 33-35].

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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