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Research Article

Simulating the Stress-Strain Relationship of Geomaterials by Support Vector Machine

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Stress-strain relationship of geomaterials is important to numerical analysis in geotechnical engineering. It is difficult to be represented by conventional constitutive model accurately. Artificial neural network (ANN) has been proposed as a more effective approach to represent this complex and nonlinear relationship, but ANN itself still has some limitations that restrict the applicability of the method. In this paper, an alternative method, support vector machine (SVM), is proposed to simulate this type of complex constitutive relationship. The SVM model can overcome the limitations of ANN model while still processing the advantages over the traditional model. The application examples show that it is an effective and accurate modeling approach for stress-strain relationship representation for geomaterials.

1. Introduction

Laboratory testing is the primary tool used by engineers and researchers for understanding geomaterials behavior. In the past few years, the use of artificial neural networks (ANN) has been introduced as an alternative approach to stress-strain relationship of geomaterials, and some of these ANN based constitutive relationships have successfully been applied to numerical analysis with improved accuracy [1–12]. However, ANN based model still has the following limitations [13].

(i) ANN does not provide information about the relative importance of the various parameters.

(ii) The knowledge acquired during the training of the ANN model is stored in an implicit manner and hence it is very difficult to have a reasonable interpretation of the overall structure of the network.

(iii) In addition, ANN model has some drawbacks such as slow convergence speed, less generalizing performance, arriving at local minimum, and overfitting problems.

Support vector machine seems to be a promising technique to circumvent these limitations. In recent years, support vector machine (SVM) methods have been rapidly developed for universal function approximations [14]. In geotechnical engineering, SVM has been applied to modeling nonlinear displacement time series of the high slope of the permanent Shiplock of the Three Gorges Project and a large landslide in China [15]. More applications of SVM can be seen in [16–18].

Therefore, in this paper, SVM will be studied as an alternative tool for ANN to simulate the stress-strain relationship for geomaterials. In Section 2, SVM approach is introduced; in Section 3, application of SVM approach in simulating the stress-strain relationship of geomaterials is given; conclusions are made in Section 4.

2. Support Vector Machine (SVM)

The SVM was firstly proposed by Vapnik and is illustrated in Figure 1 [19]. It is used to train nonlinear relationships based on the structural risk minimization principle that seeks to minimize an upper bound of the generalization error rather than to minimize the empirical error implemented in neural networks. Merit of the SVM is that training it is a uniquely solvable quadratic optimization problem. The SVM uses nonlinear mapping based on an internal integral function to

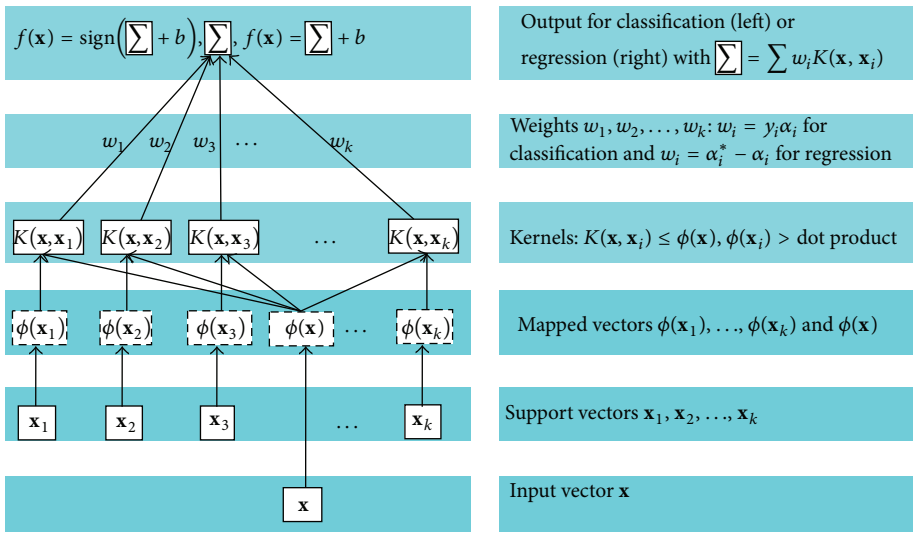

Figure 1: Support vector machine for classification and regression [19].

transform an input space to a high dimension space and then looks for a nonlinear relationship between inputs and outputs in that space. The SVM not only has theoretical support but also can find global optimum solutions for problems with small training samples, high dimensions, and nonlinear and local optima. A wide variety of applications such as pattern recognition and nonlinear regression have empirically shown the SVM's ability of generalization.

Suppose that we are given a set of observation data (samples) $(\mathbf{X_1}, y_1), (\mathbf{X_2}, y_2), \ldots, (\mathbf{X_k}, y_k)$, $X_i \in R^n$, $y_i \in R$. For the regression problem based on the SVM, we can get the following regression function:

$$f(X) = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(X \cdot X_i) + b, \tag{1}$$

where $K()$ is the kernel function; that is, $K(X_i, X_j) = \phi(X_i)\phi(X_j)$. α, α^*, and b are obtained by solving the following quadratic programming problem:

maximize

$$W(\alpha, \alpha^*) = -\frac{1}{2} \sum_{i,j=1}^{n} (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) K(X_i \cdot X_j) + \sum_{i=1}^{n} y_i (\alpha_i - \alpha_i^*) - \varepsilon \sum_{i=1}^{n} (\alpha_i + \alpha_i^*) \tag{2}$$

subject to

$$\sum_{i=1}^{n} (\alpha_i - \alpha_i^*) = 0, \quad 0 \leq \alpha_i, \ \alpha_i^* \leq C, \ i = 1, 2, \ldots n. \tag{3}$$

The constant $C > 0$ determines the tradeoff between the flatness of f and the amount up to which deviations larger than ε are tolerated.

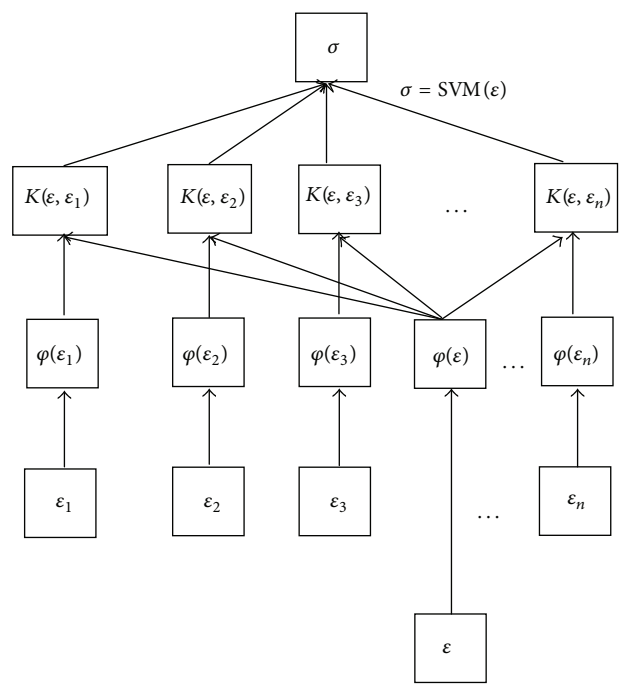

Figure 2: SVM-based stress-strain relationship.

3. The Stress-Strain Model Based on SVM

In order to have a SVM-based stress-strain relationship for a geomaterial, firstly, laboratory or numerical test of the behavior of rocks and clay under different loading patterns is conducted, and then the obtained data are used to train a SVM model. If the training data contains enough relevant information, the trained SVM is supposed to be able to generalize the stress-strain relationship and predict the behavior of materials under new loading circumstances.

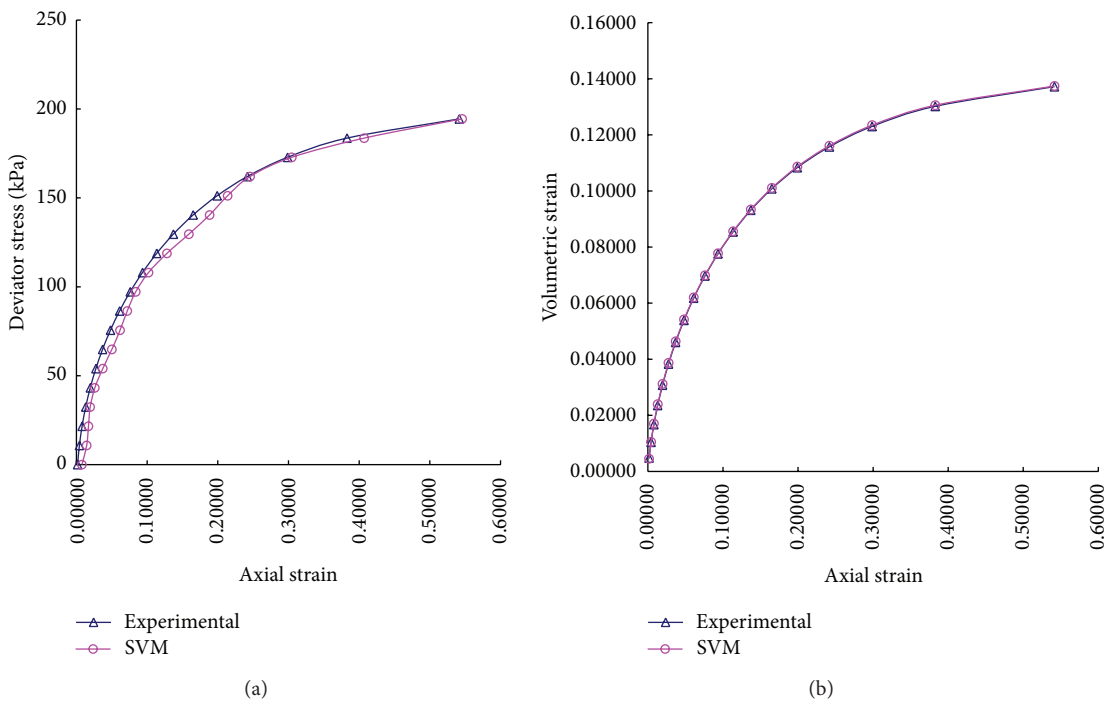

Figure 3: Comparison of the clay behavior predicted by SVM and numerical results based modified Cam model.

SVM modeling involves determination of input variables, output variables, and parameters of SVM (such as kernel function and constant C mentioned above). An appropriate selection of the input and output is also important for a successful simulation using SVM. In strain-controlled triaxial compression tests, axial strains are given as input, and the corresponding stress responses are considered output. The simplest SVM-based stress-strain relationship of geomaterial is shown in Figure 2 as an example. Based on support vector machine, the stress-strain relationship can be expressed as

$$\sigma = \mathrm{SVM}(\varepsilon). \tag{4}$$

The stress-strain relationship under stress-controlled circumstance can be written in the following form alternatively:

$$\begin{aligned} {\varepsilon_1}^n &= \mathrm{SVM}\left(p^n, q^n, {\varepsilon_1}^{n-1}, {\varepsilon_v}^{n-1}, e^n\right) \\ {\varepsilon_v}^n &= \mathrm{SVM}\left(p^n, q^n, {\varepsilon_1}^{n-1}, {\varepsilon_v}^{n-1}, e^n\right), \end{aligned} \tag{5}$$

where n and $n-1$ denote values at different load time steps.

3.1. Stress-Strain Relationship of Clay Based on SVM. In order to examine the ability of SVM model, a numerical test of consolidated-drained (CD) triaxial compress of normally consolidated clays was calculated using modified Cam clay model. The test was used to calculate the consolidated-drained triaxial behavior of a normally consolidated clay specimen subjected to a confining pressure 137.8 kPa [20]. The soil parameters are bulk modulus $\kappa = 0.026$, Lame constant $\lambda = 0.174$, and initial void ratio $e_0 = 0.889$. It is well known that stress-strain relationship of geomaterials is greatly influenced by such important factors as stress path. Therefore past history of stress and strain is part of the input data.

Finally, 19 samples were generated based on modified Cam model (Table 1). Figure 3 shows the excellent generalization performance of SVM.

3.2. Stress-Strain Relationship of Rock Based on SVM. SVM representation of the stress-strain relationship for rock is investigated in this section.

To provide the input data for training and validating a SVM model, experimental tests were conducted in laboratory. In laboratory, rock specimens (taken from a Liangbei coal mine site in Henan, China) were subjected to triaxial compression tests.

Triaxial compression tests under various confined pressures were carried out (see Table 2). For the purpose of demonstration, stress-strain relationships are shown in Figure 4. This indicates high nonlinearity between axial stress and strain.

The test data sets B-2-5 were used as the training data to build the support vector machine model. There are 558 test records in specimen B-2-5 and compose 556 sample for support vector machine. The sample was randomly separated into two groups, that is, training samples and testing samples. There are 277 training samples and 279 testing samples. When the stress-strain relationship was built based on test data form specimen B-2-5, the model can predict the stress-strain relationship of other specimens.

SVM model is obtained by being trained with the stress and strain data generated from triaxial compression test, and

Table 1: The data for training SVM.

	p (kPa)	q (kPa)	e	${\varepsilon_1}^{n-1}$	${\varepsilon_v}^{n-1}$	${\varepsilon_1}^{n}$	${\varepsilon_v}^{n}$
1	137.8	0	0.88900	0.00000	0.00000	0.00160	0.00481
2	141.4	10.8	0.87991	0.00481	0.00160	0.00424	0.01044
3	145	21.6	0.86933	0.01044	0.00424	0.00804	0.01674
4	148.6	32.4	0.85756	0.01674	0.00804	0.01314	0.02357
5	152.2	43.2	0.84487	0.02357	0.01314	0.01960	0.03080
6	155.8	54	0.83153	0.03080	0.01960	0.02753	0.03833
7	159.4	64.8	0.81774	0.03833	0.02753	0.03700	0.04606
8	163	75.6	0.80369	0.04606	0.03700	0.04814	0.05391
9	166.6	86.4	0.78953	0.05391	0.04814	0.06110	0.06181
10	170.2	97.2	0.77538	0.06181	0.06110	0.07611	0.06973
11	173.8	108	0.76133	0.06973	0.07611	0.09346	0.07761
12	177.4	118.8	0.74745	0.07761	0.09346	0.11359	0.08543
13	181	129.6	0.73378	0.08543	0.11359	0.13716	0.09317
14	184.6	140.4	0.72036	0.09317	0.13716	0.16513	0.10081
15	188.2	151.2	0.70723	0.10081	0.16513	0.19911	0.10833
16	191.8	162	0.69438	0.10833	0.19911	0.24183	0.11574
17	195.4	172.8	0.68183	0.11574	0.24183	0.29872	0.12302
18	199	183.6	0.66959	0.12302	0.29872	0.38282	0.13017
19	202.6	194.4	0.65764	0.13017	0.38282	0.54161	0.13719

Table 2: Database used for training and testing in the SVM modelling.

Test ID	Diameter (mm)	Height (mm)	Confined pressure (Mpa)
B-2-5	54.1	102.2	50
B-4-15	53.4	100	150
C-1-5	54	96.7	50
C-3-15	54.5	96.4	150
D-4-5	53.9	99.4	50
D-2-15	53.8	101.6	150
E-5-10	52	91.2	100
E-3-15	54	99.4	150
E-2-20	54	100.9	200
E-2-15	53.5	99.4	150
F-5-5	47.7	98.5	50
F-4-10	47.7	99.4	100
F-2-15	47.6	99.9	150
F-1-25	47.7	93.8	250

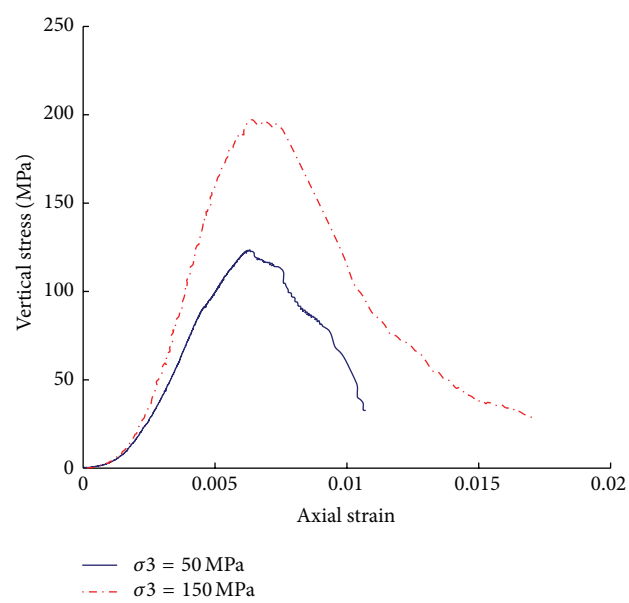

Figure 4: Stress-strain relationship with different confining pressures in tests.

this SVM model is supposed to represent the relationship between stresses and strains. Here, the SVM model was tested with both trained and untrained data to examine its performance in generalization and prediction. Figures 5 and 6 show the stress-strain relationship measured in the experiments and generated by the SVM model for trained and untrained samples. It can be observed from the figure that SVM model is capable of describing the complex, nonlinear stress-strain relationship of geomaterials such as rock. Figures 7, 8, 9, and 10 demonstrate further the prediction capability of the SVM model under various confining pressures by comparisons with experimental results. It can be seen that the obtained SVM model can represent the stress-strain relationship of geomaterials. To the special phenomenon such as soften feature, it needs more data (information) to build the relationship.

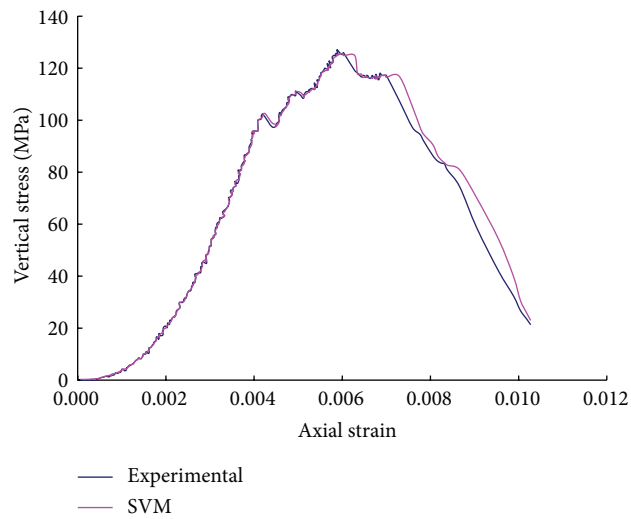

Figure 5: Comparison of stress-strain relationship predicted by SVM with experimental measurements (trained data).

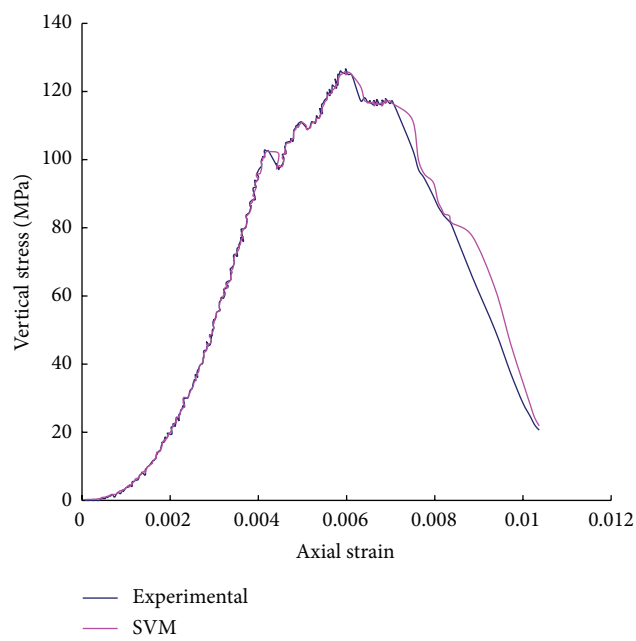

Figure 6: Comparison of stress-strain relationship predicted by SVM with experimental measurements (untrained specimen: B).

From the preceding procedure, it is evident that with the SVM modeling one does not need to find a series of material parameters describing mathematical equations associated with a constitutive model. This is one of the important advantages of the SVM model over a traditional constitutive model, since the computation of material parameters is usually a very tedious and difficult process and therefore prone to error.

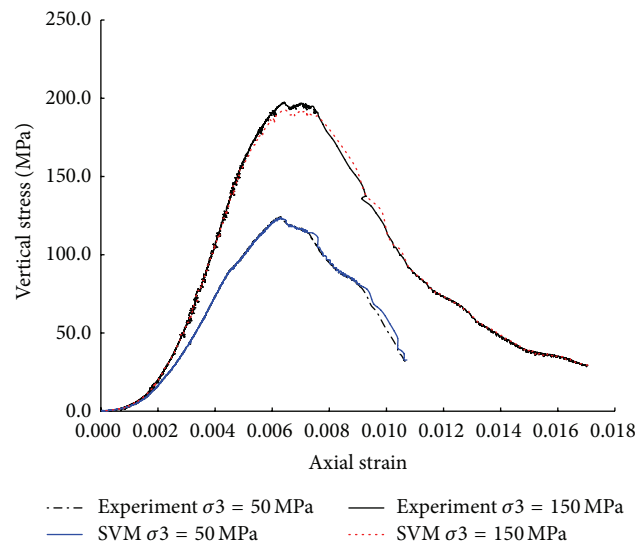

Figure 7: Comparison of stress-strain relationship predicted by SVM with experimental measurements (untrained specimen: C).

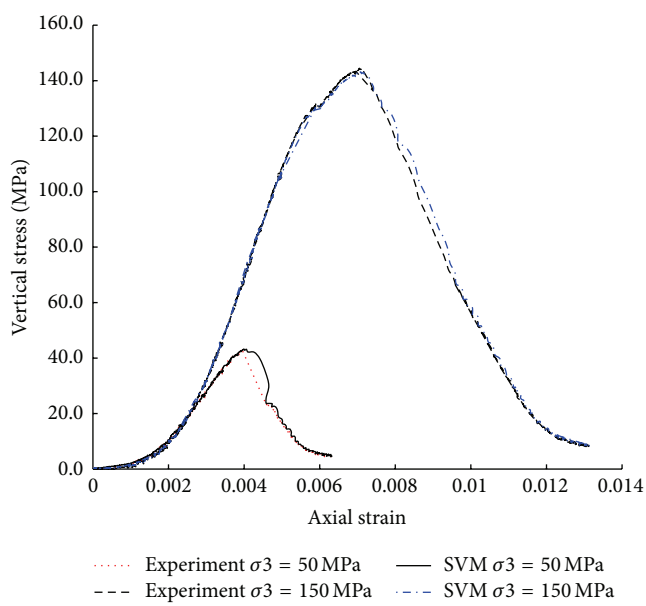

Figure 8: Comparison of stress-strain relationship predicted by SVM with experimental measurements (untrained specimen D).

4. Conclusions and Discussions

Support vector machine provides an effective alternative for modeling mechanical behavior of geomaterials by overcoming some drawbacks of ANN model. The excellent performance of the SVM model is demonstrated by successful simulation and prediction of stress-strain relationship of geomaterials under various confining pressures. Meanwhile, similar to ANN model, the SVM model has some advantages over traditional approaches. Firstly, the SVM model is essentially based on experimental data only. No assumptions are made, which allows the model to become more objective. In

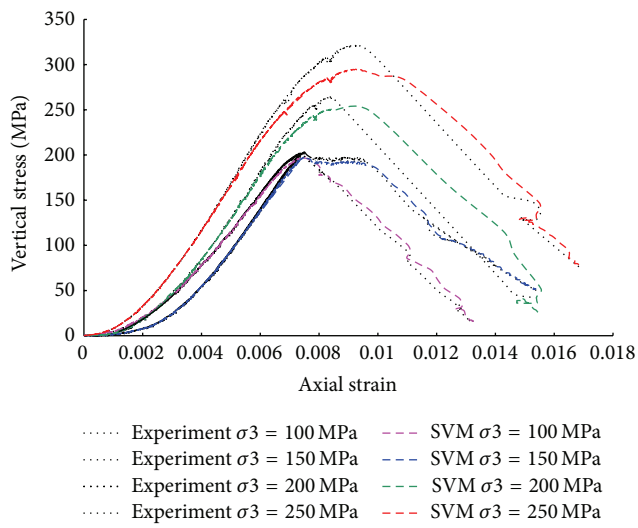

Figure 9: Comparison of stress-strain relationship predicted by SVM with experimental measurements (untrained specimen: E).

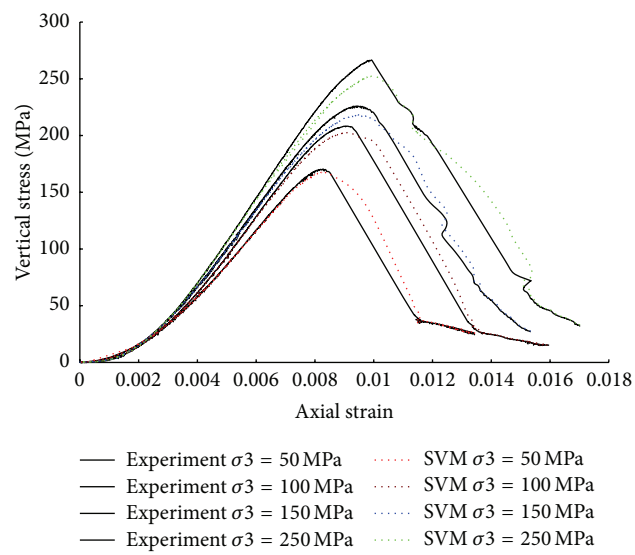

Figure 10: Comparison of stress-strain relationship predicted by SVM with experimental measurements (untrained specimen: F).

other words, the SVM model is not to be influenced by the shape of stress-strain curves. This feature is of particular significance in dealing with geomaterials constitutive behavior. Secondly, the SVM model is set up without any calculation of parameters required by a mathematical constitutive model. Therefore, the SVM model is simple and effective for stress-strain relationship modeling, if appropriate experimental data are available for the geomaterial. In this paper, it only proposes a general frame of stress-strain using SVM which can be used like the traditional stress-strain relationship. To the application of model in numerical analysis, it will be considered in later research.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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