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Research Article

Control Synthesis of Uncertain Roesser-Type Discrete-Time Two-Dimensional Systems

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This paper is concerned with control synthesis of uncertain Roesser-type discrete-time two-dimensional (2D) systems. The mathematical model of the 2D system's parameter uncertainty, which may appear typically in many actual environment, is modeled as a convex bounded uncertain domain. By using the Lyapunov stability theory, stabilization conditions is proposed in with the purpose of ensuring the robust asymptotical stability of the underlying closed-loop uncertain Roesser-type discrete-time 2D systems. Furthermore, the obtained result of this paper is formulated in the form of linear matrix inequalities (LMIs), which can be easily solved via standard numerical software. Finally, a numerical example is also provided to demonstrate the effectiveness of the proposed result.

1. Introduction

Over the past several decades, the 2D systems have attracted considerable research interests due to their wide applications in many areas such as water stream heating, thermal processes, biomedical imaging, data processing and transmission, multidimensional digital filters, image processing, grid based wireless sensor networks [1, 2]. Recently, the problems of stability analysis and control synthesis of 2D systems have been investigated in [3–5]. Furthermore, the so-called 2D modeling theory could be applied as an efficient analysis tool to deal with other problems; for example, elimination of overflow oscillations in 2D digital filters employing saturation arithmetic has been implemented be means of LMIs in [6], LMI-based stability analysis of 2D discrete systems described by the Fornasini-Marchesini (FM) second model with state saturation has been addressed in [7], H_∞ filter design for 2D Markovian jump systems has been given in [8], and optimal guaranteed cost control of 2D discrete uncertain systems has been studied in [9], respectively. More recently, considering the fact that state saturation often appears in various 2D digital systems when its transfer function is implemented by a state-space model with the finite wordlength format, the problem of stability analysis of 2D state-space digital filters with saturation arithmetic has been deeply investigated in the literature [9–28] and less conservative LMI-based stability criteria have been persistently obtained. However, it is worth noting that most of the above results are only for certain 2D systems. As is well known, most of the practical plants can be modelled as some linear 2D systems with system's parameter uncertainties and then these existing results fail to work when system's uncertain parameters affect the 2D system.

As is well known, the Lyapunov stability theory has become an efficient tool for addressing the problem of stability analysis and control synthesis of linear or nonlinear uncertain systems. Indeed, the earlier results on stability analysis and control synthesis of linear uncertain systems were obtained by applying the common quadratic Lyapunov function (CQLF). However, the CQLF applies a single Lyapunov matrix for all uncertain submodels and the obtained results are often very conservative. With the purpose of further reducing the conservatism, an efficient affine parameter-dependent Lyapunov function (APDLF) is proposed in [21], and the derived results can be less conservative than other existing ones. More importantly, it is worth noting that the Roesser-type discrete-time 2D system's information is propagated along two independent directions and this fact makes the problem of control synthesis more complicated than the usual 1D linear uncertain systems. Therefore, due to the complexity of mathematical analysis, there has been few work on control synthesis of linear uncertain Roesser-type discrete-time 2D systems in the existing literature so far. Indeed, this problem needs to be further investigated and this fact motivates us to carry out the investigation proposed in this paper.

Motivated by the above analysis, the problem of control synthesis of uncertain Roesser-type discrete-time two-dimensional systems will be investigated based on the Lyapunov stability theory in this paper. In particular, the used mathematical model of the system's parameter uncertainty, which often appears typically in most practical environments, is modeled as a convex bounded uncertain domain. Because the Roesser-type discrete-time 2D system's information is propagated along two independent directions, and thus the derivation of control synthesis would be more complicated than the usual 1D models. Then, LMI-based stabilization conditions are proposed by applying both a Lyapunov function for the uncertain Roesser-type discrete-time 2D system and a state-feedback control law for the uncertain Roesser-type discrete-time 2D system. Under the control of obtained state-feedback control law, the uncertain Roesser-type discrete-time 2D system could be ensured to be robust asymptotically stable. More importantly, the obtained result of this paper is formulated by means of linear matrix inequalities (LMIs), which can be easily solved via standard MATLAB software. Finally, the effectiveness of the proposed approach in this paper is demonstrated by some numerical examples.

The rest of this paper is organized as follows: following the introduction, some preliminaries are provided in Section 2. LMI-based stabilization conditions of the uncertain Roesser-type discrete-time 2D systems are given in Section 3 and the task of control synthesis is achieved. In Section 4, a numerical example is given to demonstrate the effectiveness of the proposed result given in this paper. Finally, some conclusions are given in Section 5.

For simplicity, the notations used are fair standard. For example, $X > 0$ (or $X \geq 0$) denotes the matrix X as symmetric and positive definite (or symmetric and positive semidefinite); a star $*$ in a symmetric matrix represents the transposed element in the symmetric position; X^T means the transpose of X; the symbol I denotes the identity matrix with appropriate dimension.

2. Preliminaries

Consider a class of uncertain Roesser-type discrete-time 2D systems which are described as follows:

$$\mathbf{x}^{+}(k,l) = A(\alpha)\,\mathbf{x}(k,l) + B(\alpha)\,\mathbf{u}(k,l) \tag{1}$$

with

$$\mathbf{x}(k,l) = \begin{bmatrix} \mathbf{x}^h(k,l) \\ \mathbf{x}^v(k,l) \end{bmatrix}, \qquad \mathbf{x}^{+}(k,l) = \begin{bmatrix} \mathbf{x}^h(k+1,l) \\ \mathbf{x}^v(k,l+1) \end{bmatrix}, \tag{2}$$

where $\mathbf{x}^h(\cdot,\cdot)$ is the horizontal state in $\mathbf{R}^{n_1}$, $\mathbf{x}^v(\cdot,\cdot)$ is the vertical state in $\mathbf{R}^{n_2}$, where n_1 and n_2 are dimensions of the horizontal state vector and the vertical state vector, respectively. $\mathbf{u}(\cdot,\cdot)$ is the control input in $\mathbf{R}^{n_3}$. k, l are two integers in $\mathbb{Z}^{+}$. In this paper, the system's coefficient matrices $A(\alpha)$ and $B(\alpha)$ are assumed to be not precisely known and belong to the fixed structure as follows:

$$A(\alpha) = \begin{bmatrix} A^{11}(\alpha) & A^{12}(\alpha) \\ A^{21}(\alpha) & A^{22}(\alpha) \end{bmatrix}, \qquad B(\alpha) = \begin{bmatrix} B^{1}(\alpha) \\ B^{2}(\alpha) \end{bmatrix}, \tag{3}$$

where we have $A^{11}(\alpha) \in \mathbf{R}^{n_1\times n_1}$, $A^{12}(\alpha) \in \mathbf{R}^{n_1\times n_2}$, $A^{21}(\alpha) \in \mathbf{R}^{n_2\times n_1}$, $A^{22}(\alpha) \in \mathbf{R}^{n_2\times n_2}$, $B^{1}(\alpha) \in \mathbf{R}^{n_1\times n_3}$, and $B^{2}(\alpha) \in \mathbf{R}^{n_2\times n_3}$, respectively. Furthermore, the matrices $A^{11}(\alpha)$, $A^{12}(\alpha)$, $A^{21}(\alpha)$, $A^{22}(\alpha)$, $B^{1}(\alpha)$, and $B^{2}(\alpha)$ belong to a convex bounded (polytope type) uncertain domain $\mathscr{P}$ given by

$$\begin{aligned} \mathscr{P} := \Big\{ &\left(A^{11}, A^{12}, A^{21}, A^{22}, B^{1}, B^{2}\right)(\alpha) : \\ &\left(A^{11}, A^{12}, A^{21}, A^{22}, B^{1}, B^{2}\right)(\alpha) \\ &= \sum_{i=1}^{r} \alpha_i \left(A_i^{11}, A_i^{12}, A_i^{21}, A_i^{22}, B_i^{1}, B_i^{2}\right); \alpha \in \Delta_r \Big\}, \end{aligned} \tag{4}$$

where Δ_r is the unit simplex given by

$$\Delta_r = \left\{ \alpha \in \mathbf{R}^{r} : \sum_{i=1}^{r} \alpha_i = 1, \alpha_i \geq 0; i = 1, \ldots, r \right\}. \tag{5}$$

Without loss of generality, the boundary conditions along two independent directions are defined as $\mathbf{x}^h(0,l) = f(l)$, $\mathbf{x}^v(k,0) = g(k)$, where $f(l)$ and $g(k)$ are corresponding boundary conditions along two independent directions.

Let us denote $X_N = \sup\{\|\mathbf{x}(k,l)\| : N = k + l\}$, and then we give out the definition of robust asymptotical stability for uncertain Roesser-type discrete-time 2D systems (1) as follows.

Definition 1. The uncertain Roesser-type discrete-time 2D systems (1) are robust asymptotically stable if $\lim_{k\to\infty, l\to\infty} X_N = 0$ with the initial and boundary conditions $\mathbf{x}^h(0,l) = f(l)$ and $\mathbf{x}^v(k,0) = g(k)$.

Then, we end this section with a well-known lemma which plays an important role in the proof of our main result.

Lemma 2 (see [22]). *Given matrices $Q = Q^T$, $R = R^T$, S with appropriate dimensions, the matrix inequality $\left(\begin{smallmatrix} Q & S \\ S^T & R \end{smallmatrix}\right) > 0$ is equivalent to the following two-matrix inequality: $R > 0$, $Q - SR^{-1}S^T > 0$.*

3. Main Results

Considering the characteristics of the uncertain Roesser-type discrete-time 2D systems (1), a kind of state-feedback control law for implementing the task of control synthesis is proposed as follows:

$$\mathbf{u}(k,l) = K\mathbf{x}(k,l), \tag{6}$$

where we have $K = [K_1 \;\; K_2]$ with $K_1 \in \mathbf{R}^{n_3\times n_1}$ and $K_2 \in \mathbf{R}^{n_3\times n_2}$.

Therefore, the closed-loop uncertain Roesser-type discrete-time 2D systems (1) become

$$\mathbf{x}^{+}(k,l) = (A(\alpha) + B(\alpha)K)\,\mathbf{x}(k,l). \tag{7}$$

With the purpose of conceiving the robust asymptotic stability of the closed-loop uncertain Roesser-type discrete-time 2D systems, stabilization conditions will be developed by applying a Lyapunov function for uncertain Roesser-type discrete-time 2D systems in the following sequel.

Theorem 3. *Under the control of the state-feedback control law* (6), *the closed-loop uncertain Roesser-type discrete-time 2D systems* (7) *are robust asymptotically stable if there exist appropriately dimensional matrices X and, with*

$$\begin{aligned} X &= \begin{bmatrix} X^1 & * \\ X^3 & X^2 \end{bmatrix}, \quad X^1 \in \mathbf{R}^{n_1\times n_1}, \\ X^2 &\in \mathbf{R}^{n_2\times n_2}, \quad X^3 \in \mathbf{R}^{n_2\times n_1}, \\ F &= [F_1 \;\; F_2], \quad F_1 \in \mathbf{R}^{n_3\times n_1}, \quad F_2 \in \mathbf{R}^{n_3\times n_2}, \end{aligned} \tag{8}$$

such that the following LMIs hold:

$$\begin{bmatrix} -X & * \\ A_iX + B_iF & -X \end{bmatrix} < 0, \quad i = 1,2,\ldots,r. \tag{9}$$

Moreover, the controller gain matrices could be calculated as follows: $K = FX^{-1}$.

Proof. Consider a Lyapunov function for closed-loop uncertain Roesser-type discrete-time 2D systems (7) as follows:

$$V(\mathbf{x}(k,l)) = \mathbf{x}^T(k,l)\,P\mathbf{x}(k,l), \tag{10}$$

where the Lyapunov matrix P is a positive definite matrix and with the following structure: $P = \begin{bmatrix} P^1 & * \\ P^3 & P^2 \end{bmatrix}$, $P^1 \in \mathbf{R}^{n_1\times n_1}$, $P^2 \in \mathbf{R}^{n_2\times n_2}$, $P^3 \in \mathbf{R}^{n_2\times n_1}$.

Then, the variation of $V(\mathbf{x}(k,l))$ can be written as

$$\Delta V(\mathbf{x}(k,l)) = \mathbf{x}^T(k,l)\left(\Xi^T P \Xi - P\right)\mathbf{x}(k,l), \tag{11}$$

where $\Xi = A(\alpha) + B(\alpha)K$.

Then, it is easy to see that the closed-loop uncertain Roesser-type discrete-time 2D systems (7) are asymptotically stable if the following inequality holds:

$$(A(\alpha) + B(\alpha)K)^T P (A(\alpha) + B(\alpha)K) - P < 0. \tag{12}$$

Using Lemma 2 (i.e., the well-known Schur complement lemma given in [22]) in (12), it could be concluded that (12) is equivalent to the inequality as follows:

$$\begin{bmatrix} -P & * \\ P(A(\alpha) + B(\alpha)K) & -P \end{bmatrix} < 0. \tag{13}$$

Pre- and postmultiplying both sides of (13) by $\mathrm{diag}\{P^{-1}, P^{-1}\}$ and applying the change of related matrix variables as $X = P^{-1}$, $F = KX$, the inequality (13) becomes

$$\begin{bmatrix} -X & * \\ A(\alpha)X + B(\alpha)F & -X \end{bmatrix} < 0. \tag{14}$$

On the other hand, reordering the expression of $\Theta = \begin{bmatrix} -X & * \\ A(\alpha)X+B(\alpha)F & -X \end{bmatrix}$, one gets

$$\Theta = \sum_{i=1}^{r} \alpha_i \left(\begin{bmatrix} -X & * \\ A_iX + B_iF & -X \end{bmatrix} \right). \tag{15}$$

From (13)–(15), if the LMI-based stabilization condition (9) holds, the inequality (12) evidently holds, which guarantees the robust asymptotical stability for the closed-loop uncertain Roesser-type discrete-time 2D systems (7).

This completes the proof. □

Remark 4. By using the Lyapunov stability theory, LMI-based stabilization conditions are given in Theorem 3 and the robust asymptotical stability for the closed-loop uncertain Roesser-type discrete-time 2D systems (1) is ensured. Here, it is worth noting that the Roesser-type discrete-time 2D system's information is propagated along two independent directions and this fact makes the problem of control synthesis more complicated than before. However, the obtained result of this paper is formulated in the form of linear matrix inequalities, which can be easily solved via standard MATLAB software.

Remark 5. It is worth noting that the used Lyapunov matrix in (10) is a single Lyapunov matrix for all uncertain submodels and the corresponding results are often very conservative. To further reduce the conservatism, several new Lyapunov functions may be developed to overcome these drawbacks [21]. In the future, the search for relaxed stabilization conditions would be investigated by extending the methods given in [23] and [24].

4. Numerical Examples

Consider a class of uncertain Roesser-type discrete-time two-dimensional systems which could be described as follows:

$$\begin{aligned} &\begin{bmatrix} x^h(k+1,l) \\ x^v(k,l+1) \end{bmatrix} \\ &\quad = \sum_{i=1}^{2} \alpha_i \left(A_i \begin{bmatrix} x^h(k,l) \\ x^v(k,l) \end{bmatrix} + B_i u_c(k,l) \right), \end{aligned} \tag{16}$$

where $A_1 = \begin{bmatrix} 1+a_1T_1 & (a_1a_2+a_0)T_1 \\ T_2 & 1+a_2T_2 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1+a_1T_1 & a_1a_2T_1 \\ T_2 & 1+a_2T_2 \end{bmatrix}$, and $B_1 = B_2 = \begin{bmatrix} bT_1 \\ 0 \end{bmatrix}$. And the following parameter values about a_0, a_1, a_2, b, T_1, and T_2 are given: $a_0 = -2$, $a_1 = -3$, $a_2 = -2.8$, $b = -1$, $T_1 = 0.3$, and $T_2 = 0.6$. Furthermore, the initial and boundary conditions of the above uncertain Roesser-type discrete-time two-dimensional systems are set as $x^h(0,l) = 3\cos(l)$ for $l < 40$, $x^v(k,0) = 2\sin(k)$ for $k < 40$, $x^h(0,l) = 0$ for $l \geq 40$, and $x^v(k,0) = 0$ for $k \geq 40$.

Let $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$; Figures 1 and 2 show the state trajectories of the system state variables $x^h(k,l)$ and $x^v(k,l)$, respectively. It is easy to see that the state trajectories of $x^h(k,l)$ and $x^v(k,l)$ are not asymptotically stable from Figures 1 and 2.

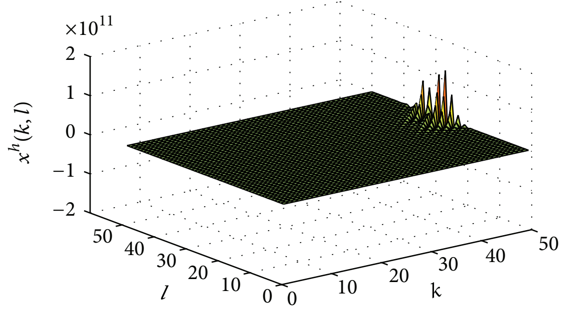

FIGURE 1: The state trajectory of $x^h(k,l)$ without control input.

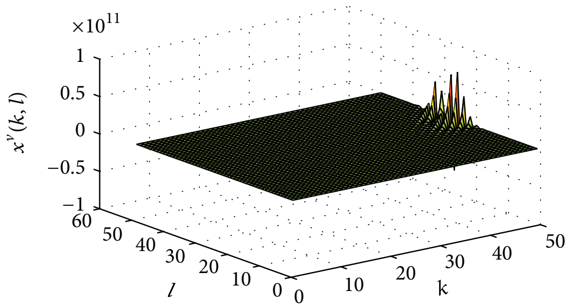

FIGURE 2: The state trajectory of $x^v(k,l)$ without control input.

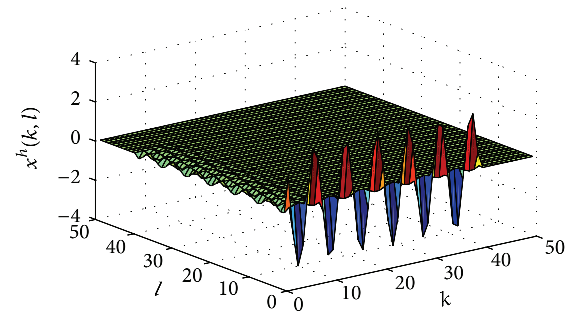

FIGURE 3: The state trajectory of $x^h(k,l)$ with control input.

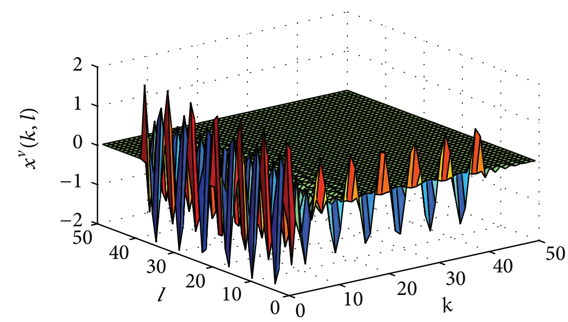

FIGURE 4: The state trajectory of $x^v(k,l)$ with control input.

Using Theorem 3, the following control matrices could be calculated by solving the LMI-based stabilization conditions (9):

$$F = \begin{bmatrix} 1.9268 & 5.5571 \end{bmatrix},$$
$$X = \begin{bmatrix} 0.8458 & 0.2223 \\ 0.2223 & 0.7410 \end{bmatrix}, \quad (17)$$
$$K = \begin{bmatrix} 0.3333 & 7.4000 \end{bmatrix}.$$

Under the control of (6), Figures 3 and 4 show the state trajectories of the system state variables $x^h(k,l)$ and $x^v(k,l)$, respectively. Now, it is easy to see that the state trajectories of the system state variables are robust asymptotically stable from Figures 3 and 4; that is, the effectiveness of the proposed result given in Theorem 3 has been illustrated via this numerical example.

5. Conclusions

In this paper, the problem of control synthesis of uncertain Roesser-type discrete-time 2D systems has been investigated via the well-known Lyapunov stability theory. The mathematical model of the system's parameter uncertainty, which often appears typically in practical environment, is modeled by a convex bounded uncertain domain. With the purpose of conceiving the robust asymptotic stability of the closed-loop uncertain Roesser-type discrete-time 2D systems, stabilization conditions have been developed by proposing a Lyapunov function for uncertain Roesser-type discrete-time 2D systems. The obtained stabilization conditions are in terms of LMIs and can be easily solved via standard MATLAB software. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed result.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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