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Research Article

Dynamical Adaptive Integral Sliding Backstepping Control of Nonlinear Nontriangular Uncertain Systems

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We present a control strategy for nonlinear nontriangular uncertain systems. The proposed control method is a synergy between the dynamic adaptive backstepping (DAB) and integral sliding mode (ISM) and is referred to as DAB-ISMC. Our main objective is to find a recursive procedure to transform a nontriangular system into an implementable form that enables designing a control law which almost eliminates the reaching-phase. The proposed method further facilitates minimization of chattering which is believed to be a shortcoming of the sliding mode control. In this methodology, the ISM, as an integrated subsystem of DAB, is introduced at the final stage of backstepping. This strategy works very well to obtain a system that is robust against model imperfections, matching and unmatching uncertainties. The DAB-ISMC method is applied on a continuous stirred tank reactor (CSTR) and simulation results obtained on Matlab are found to be very promising.

1. Introduction

Designing of robust controller for uncertain nonlinear systems is complex and challenging, particularly, in the presence of parametric uncertainties, external disturbances, and unmodeled dynamics. To achieve the desired behaviour in a plant/process, no specific method or set of analysis and design tools are available. However, several methods have been developed to control nonlinear feedback system to regulate or track the desired output. These methods have their own merits and limitations.

The sliding mode control (SMC) [1–6] technique is widely used to make the system robust against external disturbance, unmodeled dynamics, and parametric uncertainties. The SMC addresses the uncertainties which enter into the system via input channel, called the matching uncertainties. A major limitation of the classical SMC is chattering that occurs when the system is in the sliding phase. Another limitation is that of its applicability to the systems of relative degree one. However, many physical systems such as mechanical and satellite control system do not satisfy the condition of relative degree one. Therefore, various methods have been suggested [3, 7, 8] to overcome these limitations of the classical SMC method.

The backstepping algorithm [9] resolves the issue of relative degree in SMC. In this framework, an nth order dynamical system is split into n scalar subsystems and controller is synthesised in n recursive steps. Each subsystem is stabilised by designing virtual control law and parameter tuning function and the realistic control law is designed in the last step of this procedure. The whole model is then transformed into new error coordinates. The first error coordinate is generated by the difference of system output and desired output. The rest of error coordinates is generated using actual state variable as input to the corresponding subsystem and virtual control designed for respective step and the derivative of desired output function. The derivative of desired output function is applicable in the case of tracking problem. The DAB procedure [9] is widely accepted to be suitable for nontriangular uncertain nonlinear systems.

This method when applied on linearisable systems [10–12] avoids cancellation of useful nonlinearities. This is a definite advantage over the feedback linearisation method [3, 13] where some useful nonlinearities responsible for the stability may disappear. Thus, the SMC procedure addresses only matching disturbance, whereas DAB also takes care of unmatching uncertainties.

Further progress in this direction was made by Sira-Ramirez [14] who combined the fundamental adaptive backstepping algorithm [13] with dynamically input output linearisation technique [10]. This method was further extended in combination with the SMC [15–17]. This results in the global stability and improves the performance of nonlinear uncertain systems. However, this method is only applicable on the systems which can be transformed into triangular form. Later on, some interesting SMC based techniques were introduced by [18, 19] for nonlinear systems which are transferable into semistrict feedback form having unmodeled and unmatched uncertainties. In this way, the individual benefits of both SMC and adaptive backstepping were combined in one system. Recently, Khan et al. [20] reported another method (based on integral sliding mode [1]) of dynamic control for MIMO uncertain, square, and minimum phase nonlinear system. This method has synthesized dynamic sliding mode control [21, 22] and integral sliding mode strategies [1] into dynamically integral sliding mode control (DISMC) which provided the establishment of sliding mode without reaching-phase. Consequently, the robustness against uncertainties is enhanced with considerable attenuation in chattering across the integral manifold. This strategy is applicable to input-state linearisable systems.

In this paper, we propose a scheme which is more generalised in the sense that there is no restriction on the system to be in triangular or regular form. In the present scheme, we synthesize a controller for a nontriangular nonlinear uncertain system by combining DAB and integral sliding mode control (ISMC) methods. This strategy results in an increased robustness from the start against parametric uncertainty along with considerable reduction in chattering. The claim is substantiated by a practical example by applying the proposed algorithm on a chemical plant. For that, a highly nonlinear model of continuous stirred tank reactor (CSTR) [23, 24] is selected. The computer simulation results are presented to support the claims of proposed method. The design process involves the subdivision of control law into two parts: a continuous control law which can be realized by applying adaptive backstepping method and a discontinuous control law which is developed via an integral manifold. In an earlier work [25], we have shown how to obtain the parameter update law. The parameter update law is composed of two terms. The first term emerges from the adaptive backstepping method and the second term is the realization of ISMC. The significance of the paper can be highlighted as follows.

(1) The proposed scheme works well for both nontriangular and regular forms.

(2) The robustness of the system starts right from the startup time.

(3) The chattering is controlled significantly.

The paper is organized as follows: the problem description/formulation is presented in Section 2, the control law is developed in Section 3, an illustrative example is presented in Section 4, and the conclusions are drawn in Section 5.

2. Preliminaries

2.1. System Description. The DAB framework is based upon backstepping with tuning function and can be implemented without transforming a system into canonical form. The nontriangular systems are easily controllable by using DAB procedure [17] with the condition that system must be observable minimum phase. The observability is required for the existence of local nonlinear mapping which is needed for controller design and the minimum phase ensures the stability of system in close-loop. The triangular systems such as parametric strict form and parametric pure feedback form can also be controlled with DAB. The problem can be defined by considering uncertain nonlinear systems described as [24]

$$\dot{\zeta} = f_0(\zeta) + \psi(\zeta)\theta + (g_0(\zeta) + \varphi(\zeta)\theta)u, \tag{1}$$

$$y = h(\zeta), \tag{2}$$

where $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_n]^T$ is the state vector, u is the scalar control input, y is the output of interest, $\psi(\zeta_1, \zeta_2, \ldots, \zeta_i)$ and $\varphi(\zeta_1, \zeta_2, \ldots, \zeta_i) \in \mathbb{R}^{n\times p}, 1 \le i \le n$, are known and sufficiently smooth functions, $f_0(\zeta)$ and $g_0(\zeta)$ are known multivariable functions in the neighbourhood R_0 of the origin $\zeta = 0$, $f_0(0) = 0$, $g_0(0) \neq 0$, and h is a smooth scalar function also defined in R_0, and $\theta \in \mathbb{R}^p$ is the vector of unknown parameters. The control objective is to derive the system output of (1) to track or regulate desired value y_d. The procedure for nonlinear mapping and parameter tuning function design is given in the next subsection.

2.2. Adaptive Backstepping Design Development. In this study, a generalised recursive approach analogous to that of [16] is presented for the nonlinear mapping and tuning function design. The procedure starts from output $y(t)$ by taking time derivative and proceeds up to $(n-1)$ times derivative of $y(t) = h(\zeta)$,

$$\dot{y} = \frac{\partial h}{\partial \zeta}\dot{\zeta} = \frac{\partial h}{\partial \zeta}\left[f_0(\zeta) + \psi(\zeta)\theta + (g_0(\zeta) + \varphi(\zeta)\theta)u\right]. \tag{3}$$

The unknown parameter θ can be replaced by its estimated value $\widehat{\theta}$. We denote $\widetilde{\theta} = \theta - \widehat{\theta}$, where $\widetilde{\theta}$ is the error between θ and $\widehat{\theta}$. Thus (3) can be written as

$$\begin{aligned}\dot{y} &= \mathscr{L}_h^1(\zeta, \widehat{\theta}, u) \\ &= \frac{\partial h}{\partial \zeta}\left[f_0(\zeta) + \psi(\zeta)\widehat{\theta} + (g_0(\zeta) + \varphi(\zeta)\widehat{\theta})u\right] + \omega_1\widetilde{\theta},\end{aligned} \tag{4}$$

where $\omega_1 = (\partial h/\partial \zeta)(\psi(\zeta) + u\varphi(\zeta))$ and $\mathscr{L}_h^1(\zeta, \widehat{\theta}, u)$ is a Lie derivative operator. We can write (4) in a convenient Lie derivative form as

$$\dot{y} = \mathscr{L}_h^1(\zeta, \widehat{\theta}, u) = \widehat{\mathscr{L}}_h^1(\zeta, \widehat{\theta}, u) + \omega_1\widetilde{\theta} \tag{5}$$

with

$$\widehat{\mathscr{L}}_h^1\left(\zeta,\widehat{\theta},u\right)=\frac{\partial h}{\partial \zeta}\left[f_0(\zeta)+\psi(\zeta)\widehat{\theta}+\left(g_0(\zeta)+\varphi(\zeta)\widehat{\theta}\right)u\right]. \quad (6)$$

The second time derivative of the output $y(t)=h(\zeta)$,

$$\begin{aligned}\ddot{y}&=\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\zeta}\dot{\zeta}+\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial u}\dot{u}\\&=\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\zeta}\left[f_0(\zeta)+\psi(\zeta)\theta+\left(g_0(\zeta)+\varphi(\zeta)\theta\right)u\right]\\&\quad+\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial u}\dot{u}.\end{aligned} \quad (7)$$

Rewriting (7) as

$$\ddot{y}=\mathscr{L}_h^2\left(\zeta,\widehat{\theta},u,\dot{u}\right)=\widehat{\mathscr{L}}_h^2\left(\zeta,\widehat{\theta},u,\dot{u}\right)+\omega_2\widetilde{\theta} \quad (8)$$

with

$$\begin{aligned}\widehat{\mathscr{L}}_h^2&=\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\zeta}\left[f_0(\zeta)+\psi(\zeta)\widehat{\theta}+\left(g_0(\zeta)+\varphi(\zeta)\widehat{\theta}\right)u\right]\\&\quad+\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial u}\dot{u},\\\omega_2&=\frac{\partial\left(\mathscr{L}_h^1\right)}{\partial\zeta}\left(\psi(\zeta)+u\varphi(\zeta)\right),\end{aligned} \quad (9)$$

we can proceed in a similar way to obtain jth time derivative of the output as

$$\begin{aligned}y^{(j)}&=\mathscr{L}_h^j\left(\zeta,\widehat{\theta},u,\ldots,u^{(j-1)}\right)\\&=\widehat{\mathscr{L}}_h^j\left(\zeta,\widehat{\theta},u,\ldots,u^{(j-1)}\right)+\omega_j\widetilde{\theta}\end{aligned} \quad (10)$$

with

$$\begin{aligned}\widehat{\mathscr{L}}_h^j&=\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\zeta}\left[f_0(\zeta)+\psi(\zeta)\widehat{\theta}+\left(g_0(\zeta)+\varphi(\zeta)\widehat{\theta}\right)u\right]\\&\quad+\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\sum_{k=0}^{j-2}\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial u^{(k)}}u^{(k+1)},\end{aligned} \quad (11)$$

$$\omega_j=\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\zeta}\left(\psi(\zeta)+u\varphi(\zeta)\right). \quad (12)$$

The generalised expression for a system with well defined relative degree, that is, $1\leq\rho\leq n$, can be written by using (10) and (11)

$$\begin{aligned}y^{(j)}&=\mathscr{L}_h^j\left(\zeta,\widehat{\theta},u,\ldots,u^{(j-\rho)}\right)\\&=\widehat{\mathscr{L}}_h^j\left(\zeta,\widehat{\theta},u,\ldots,u^{(j-\rho)}\right)+\omega_j\widetilde{\theta}\end{aligned} \quad (13)$$

with

$$\begin{aligned}\widehat{\mathscr{L}}_h^j&=\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\zeta}\left[f_0(\zeta)+\psi(\zeta)\widehat{\theta}+\left(g_0(\zeta)+\varphi(\zeta)\widehat{\theta}\right)u\right]\\&\quad+\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\sum_{k=0}^{j-\rho-1}\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial u^{(k)}}u^{(k+1)}.\end{aligned} \quad (14)$$

The above procedure and following recursively defined operator can be used to find the time derivatives of the output of a nonlinear system (1) as

$$\begin{aligned}\mathscr{L}_h^0&=h(\zeta),\\\mathscr{L}_h^j&=\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\zeta}\left[f_0(\zeta)+\psi(\zeta)\theta+\left(g_0(\zeta)+\varphi(\zeta)\theta\right)u\right]\\&\quad+\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\sum_{k=0}^{j-\rho-1}\frac{\partial\left(\mathscr{L}_h^{j-1}\right)}{\partial u^{(k)}}u^{(k+1)},\end{aligned} \quad (15)$$

where $1\leq j\leq n$. With this, the control dependent nonlinear mapping of (1) can be defined as

$$z=\Xi\left(\zeta,\widehat{\theta},\ldots,u^{(n-\rho-1)}\right)=\begin{bmatrix}y\\y^{(1)}\\\vdots\\y^{(n-1)}\end{bmatrix}=\begin{bmatrix}\mathscr{L}_h^0\\\mathscr{L}_h^{(1)}\\\vdots\\\mathscr{L}_h^{(n-1)}\end{bmatrix}. \quad (16)$$

Assumption 1. The nonlinear system (1) is locally observable; that is, the mapping (16) satisfies the rank condition

$$\operatorname{rank}\left(\frac{\partial\Xi(\cdot)}{\partial\zeta}\right)=n \quad (17)$$

in a subspace $R_1\subset R_0\subset\mathbb{R}^n$.

Assumption 2. The nonlinear system (1) is minimum phase in $R_1\subset R_0\subset\mathbb{R}^n$.

The DAB procedure to achieve desired output of system (1) can be developed recursively as follows and is based on the assumption that system is observable minimum phase with smooth and bounded derivatives of desired output function. Same procedure is valid for the regulation problem of (1).

2.2.1. Step 1. Consider a new variable z_1 that represents the error between the actual output of the system and the desired output is expressed as $z_1=y_1-y_d(t)=h(\zeta)-y_d(t)$, and its derivative along (1) yields

$$\begin{aligned}\dot{z}_1&=h^{(1)}(\zeta,\theta)-\dot{y}_d(t)\\&=\frac{\partial h}{\partial\zeta}\left[f_0(\zeta)+\psi(\zeta)\theta+\left(g_0(\zeta)+\varphi(\zeta)\theta\right)u\right]-\dot{y}_d(t).\end{aligned} \quad (18)$$

In case relative degree ρ with respect to input u is greater than one, the term

$$\frac{\partial h}{\partial \zeta}\left(g_0(\zeta)+\varphi(\zeta)\theta\right)u=0. \quad (19)$$

The expression in (18) may be written in an alternate form as follows:

$$\dot{z}_1=\widehat{h}^{(1)}\left(\zeta,\widehat{\theta}\right)-\dot{y}_d(t)+\omega_1(t)\widetilde{\theta} \quad (20)$$

with

$$\widehat{h}^{(1)}\left(\zeta,\widehat{\theta}\right)=\frac{\partial h}{\partial \zeta}\left(f_0(\zeta)+\psi(\zeta)\widehat{\theta}\right),$$
$$\omega_1=\left(\frac{\partial h}{\partial \zeta}\right)\psi(\zeta), \quad (21)$$

where $\widetilde{\theta}=\theta-\widehat{\theta}$, and $\widehat{\theta}$ is the estimated value of unknown parameters. Now, a virtual control input for the stabilization of subsystem in first stage is designed by considering the Lyapunov function as

$$V_1=\frac{1}{2}z_1^2+\frac{1}{2}\widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta}, \quad (22)$$

where $\Gamma=\Gamma^T$ is a positive definite matrix used as adaptation gain. The time derivative of V_1 along (20) takes the form

$$\dot{V}_1=z_1\left(\widehat{h}^{(1)}\left(\zeta,\widehat{\theta}\right)-\dot{y}_d(t)+\omega_1(t)\widetilde{\theta}\right)-\widetilde{\theta}^T\Gamma^{-1}\dot{\widehat{\theta}}, \quad (23)$$

$$\dot{V}_1=z_1\left(\widehat{h}^{(1)}\left(\zeta,\widehat{\theta}\right)-\dot{y}_d(t)\right)+\widetilde{\theta}^T\Gamma^{-1}\left(\Gamma\omega_1^Tz_1-\dot{\widehat{\theta}}\right). \quad (24)$$

$\dot{V}_1$ can be made negative if we define the parameter tuning function of first stage as $\dot{\widehat{\theta}}=\tau_1=\Gamma\omega_1^Tz_1$ and the stabilizing control law as $\alpha_1=c_1z_1$ where $-\alpha_1(\zeta_1,\widehat{\theta},t)$ is required to be equal to $\widehat{h}^{(1)}(\zeta,\widehat{\theta})-\dot{y}_d(t)$. However, it is not exactly possible in a real scenario, so we define an error variable as follows:

$$z_2=\widehat{h}^{(1)}\left(\zeta,\widehat{\theta}\right)-\dot{y}_d(t)+c_1z_1, \quad (25)$$

where c_1 is a positive scalar design constant. By using (25) in (20) and (24) one gets

$$\dot{z}_1=-c_1z_1+z_2+\omega_1(t)\widetilde{\theta}, \quad (26)$$

$$\dot{V}_1\left(z_1,\widehat{\theta}\right)=-c_1z_1^2+z_1z_2+\widetilde{\theta}^T\Gamma^{-1}\left(\tau_1-\dot{\widehat{\theta}}\right). \quad (27)$$

2.2.2. Step 2. The time derivative of the new variable defined in (25) along (1) is expressed as

$$\dot{z}_2=\frac{\partial\left(\widehat{h}^{(1)}\right)}{\partial\zeta}\dot{\zeta}+\frac{\partial\left(\widehat{h}^{(1)}\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\frac{\partial\left(\widehat{h}^{(1)}\right)}{\partial t}+\frac{\partial\alpha_1}{\partial\zeta_1}\dot{\zeta}_1$$
$$+\frac{\partial\alpha_1}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\frac{\partial\alpha_1}{\partial t}-\ddot{y}_d(t). \quad (28)$$

In case the relative degree ρ is greater than 2 and input u does not appear in (28), we can write

$$\dot{z}_2=\frac{\partial\left(\widehat{h}^{(1)}\right)}{\partial\zeta}\left(f_0(\zeta)+\varphi(\zeta)\widehat{\theta}\right)+\frac{\partial\left(\widehat{h}^{(1)}\right)}{\partial\widehat{\theta}}\dot{\widehat{\theta}}$$
$$+\frac{\partial\left(\widehat{h}^{(1)}\right)}{\partial t}+\frac{\partial\alpha_1}{\partial\zeta_1}\left(f_0(\zeta)+\varphi(\zeta)\widehat{\theta}\right) \quad (29)$$
$$+\frac{\partial\alpha_1}{\partial\widehat{\theta}}\dot{\widehat{\theta}}+\frac{\partial\alpha_1}{\partial t}-\ddot{y}_d(t).$$

Now, define a Lyapunov function $V_2=(1/2)z_1^2+(1/2)z_2^2+(1/2)\widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta}$ and its time derivative along (26) and (29) becomes

$$\dot{V}_2\left(z_1,z_2,\widehat{\theta}\right)=-c_1z_1^2+z_2\Bigg[z_1+\left(\frac{\partial h^{(1)}}{\partial\widehat{\theta}}+\frac{\partial\alpha^{(1)}}{\partial\widehat{\theta}}\right)\left(\dot{\widehat{\theta}}-\tau_2\right)$$
$$+\frac{\partial h^{(1)}}{\partial\widehat{\theta}}\tau_2+\widehat{h}^{(2)}(\cdot)-y_d^{(2)}$$
$$+\frac{\partial\alpha_1}{\partial t}+\frac{\partial\alpha_1}{\partial\zeta}\left(f_0+\psi\widehat{\theta}\right)\Bigg]$$
$$+\widetilde{\theta}^T\Gamma^{-1}\left(\tau_2-\dot{\widehat{\theta}}\right), \quad (30)$$

where $\tau_2=\Gamma(\omega_1^Tz_1+\omega_2^Tz_2)$. Now, with procedure defined in step one, we define a third error variable of the form $z_3=\widehat{h}^{(2)}+\alpha_2(z_1,z_2,\widehat{\theta},t)-\ddot{y}_d(t)$, where

$$\alpha_2\left(z_1,z_2,\widehat{\theta},t\right)$$
$$=z_1+c_2z_2+\frac{\partial\alpha_1}{\partial\zeta}\left(f_0(\zeta)+\psi\widehat{\theta}\right) \quad (31)$$
$$+\left(\frac{\partial h^{(1)}}{\partial\widehat{\theta}}+\frac{\partial\alpha_1}{\partial\widehat{\theta}}\right)\left(\dot{\widehat{\theta}}-\tau_2\right)+\frac{\partial\alpha_1}{\partial t}+\left(\frac{\partial h^{(1)}}{\partial\widehat{\theta}}\right)\tau_2$$

reduces (30) to the following form:

$$\dot{V}_2\left(z_1,z_2,\widehat{\theta}\right)=-c_1z_1^2-c_2z_2^2+z_2z_3, \quad (32)$$

where c_1 and c_2 are positive constant, $\tau_2=\dot{\widehat{\theta}}=\Gamma(z_1\omega_1^T+z_2\omega_2^T)$, and

$$\dot{z}_2=-z_1-c_2z_2+z_3+\omega_2\widetilde{\theta}+\left(\frac{\partial h^{(1)}}{\partial\widehat{\theta}}+\frac{\partial\alpha_1}{\partial\widehat{\theta}}\right)\left(\tau_2-\dot{\widehat{\theta}}\right). \quad (33)$$

Similarly, the recursive development in the new variables can be carried up till $j-1$ derivatives in which u input does not appear explicitly. In this way a generic expression is achieved.

2.2.3. Step j ($2\le j\le\rho-1$). The procedure applied in step two is followed in this step. The variable z_j is defined as

$$z_j=\widehat{h}^{j-1}\left(\zeta,\widehat{\theta},t\right)-y_d^{(j-1)}+\alpha_{j-1} \quad (34)$$

and similarly we obtain $z_{j+1} = \hat{h}^j(\zeta, \hat{\theta}, t) - y_d^{(j)} + \alpha_j$, where

$$\begin{aligned} \alpha_j = {} & z_{j-1} + \left(\sum_{i=2}^{j-1} z_i \frac{\partial \hat{h}^{i-1}}{\partial \hat{\theta}} + \sum_{i=3}^{j-1} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right) \Gamma \omega_j^T \\ & + \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \tau_j + \frac{\partial \alpha_{j-1}}{\partial \zeta} \left(f_0(\zeta) + \psi(\zeta) \hat{\theta} \right) \\ & + c_j z_j + \frac{\partial \alpha_{j-1}}{\partial t} - y_d^{(j)}. \end{aligned} \tag{35}$$

The time derivative of z_j in closed form appears as follows:

$$\begin{aligned} \dot{z}_j = {} & -z_{j-1} - c_j z_j + z_{j+1} + \omega_j \tilde{\theta} + \left(\frac{\partial \hat{h}^{(j-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_j \right) \\ & - \left(\sum_{i=2}^{j-1} z_i \frac{\partial \hat{h}^{i-1}}{\partial \hat{\theta}} + \sum_{i=3}^{j-1} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right) \Gamma \omega_j^T. \end{aligned} \tag{36}$$

The derivative of augmentation Lyapunov function

$$V_j = V_{j-1} + \frac{1}{2} z_j^2 = \frac{1}{2} \sum_{i=1}^{j} z_i^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{37}$$

can be written as

$$\begin{aligned} \dot{V}_j = {} & -\sum_{i=1}^{j-1} c_i z_i^2 + z_j z_{j+1} + \left(\sum_{i=2}^{j} z_i \frac{\partial \hat{h}^{i-1}}{\partial \hat{\theta}} + \sum_{i=3}^{j} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right) \\ & \times \left(\dot{\hat{\theta}} - \tau_j \right) + \tilde{\theta}^T \Gamma^{-1} \left(\tau_j - \dot{\hat{\theta}} \right). \end{aligned} \tag{38}$$

2.2.4. Step k (ρ ≤ k ≤ n−1). The variable z_k is defined as

$$z_k = \hat{h}^{k-1} \left(\zeta, \hat{\theta}, t \right) - y_d^{(k-1)} + \alpha_{j-1} \tag{39}$$

and the time derivative of z_k appears as follows:

$$\begin{aligned} \dot{z}_k = {} & \hat{h}^{(k)} \left(\zeta, \hat{\theta}, u, \dots, u^{(k-\rho)}, t \right) - y_d^{(k)}(t) + \frac{\partial \alpha_{k-1}}{\partial t} \\ & + \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \tau_k + \frac{\partial \alpha_{k-1}}{\partial \zeta} \left(f_0 + \psi \hat{\theta} + \left(g_0 + \varphi \hat{\theta} \right) u \right) \\ & + \sum_{i=1}^{k-\rho} \frac{\partial \alpha_{k-1}}{\partial u^{(i-1)}} u^{(i)} + \omega_k \tilde{\theta} + \left(\frac{\partial \hat{h}^{(k-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_k \right), \end{aligned} \tag{40}$$

where

$$\begin{aligned} \hat{h}^{(k)}(\cdot) = {} & \frac{\partial \hat{h}^{(k-1)}}{\partial \hat{\theta}} \tau_k + \frac{\partial \hat{h}^{(k-1)}}{\partial \zeta} \left(f_0 + \psi \hat{\theta} + \left(g_0 + \varphi \hat{\theta} \right) u \right) \\ & + \sum_{i=1}^{k-\rho} \frac{\partial \hat{h}^{(k-1)}}{\partial u^{(i-1)}} u^{(i)} + \frac{\partial \hat{h}^{(k-1)}}{\partial t}, \\ \omega_k = {} & \left(\frac{\partial \hat{h}^{(k-1)}}{\partial \zeta} + \frac{\partial \alpha_{k-1}}{\partial \zeta} \right) (\psi + \varphi u). \end{aligned} \tag{41}$$

The Lyapunov function can be considered to design control law and tuning function τ_k as

$$V_k = V_{k-1} + \frac{1}{2} z_k^2 = \frac{1}{2} \sum_{i=1}^{k} z_i^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \tag{42}$$

Its time derivative is

$$\begin{aligned} \dot{V}_k = {} & -\sum_{i=1}^{k-1} c_i z_i^2 + \tilde{\theta}^T \Gamma^{-1} \left(-\dot{\hat{\theta}} + \tau_{k-1} + \Gamma \omega_k^T z_k \right) \\ & \times z_k \left(\frac{\partial \hat{h}^{(k-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_k \right) \\ & + \left(\sum_{i=2}^{k-1} z_i \frac{\partial \hat{h}^{(k-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{k-1} z_i \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_{k-1} \right) \\ & + z_k \left(z_{k-1} + \hat{h}^{(k)} - y_d^{(k)} + \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \tau_k \right. \\ & + \sum_{i=1}^{k-\rho} \frac{\partial \alpha_{(k-1)}}{\partial u^{(i-1)}} u^{(i)} + \frac{\partial \alpha_{k-1}}{\partial t} \\ & \left. + \frac{\partial \alpha_{k-1}}{\partial \zeta} \left(f_0 + \psi \hat{\theta} + \left(g_0 + \varphi \hat{\theta} \right) u \right) \right). \end{aligned} \tag{43}$$

The terms containing $\tilde{\theta}$ can be eliminated from (43) by choosing the tuning function

$$\dot{\hat{\theta}} = \tau_k = \tau_{k-1} + \Gamma \omega_k^T z_k. \tag{44}$$

We can manipulate $\dot{\hat{\theta}}$ as $\dot{\hat{\theta}} - \tau_{k-1} = \dot{\hat{\theta}} - \tau_k + \tau_k - \tau_{k-1}$, and inserting $\tau_k - \tau_{k-1} = \Gamma \omega_k^T z_k$ from (44), one can write

$$\dot{\hat{\theta}} - \tau_{k-1} = \dot{\hat{\theta}} - \tau_k + \Gamma \omega_k^T z_k\ . \tag{45}$$

By using (45), (43) can be rewritten as

$$\begin{aligned} \dot{V}_k = {} & -\sum_{i=1}^{k-1} c_i z_i^2 + \tilde{\theta}^T \Gamma^{-1} \left(-\dot{\hat{\theta}} + \tau_k \right) \\ & + \left(\sum_{i=2}^{k} z_i \frac{\partial \hat{h}^{(i-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{k} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_k \right) \\ & + z_k \left[\left(\sum_{i=2}^{k-1} z_i \frac{\partial \hat{h}^{(i-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{k-1} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right) \Gamma \omega_k^T \right. \\ & + \frac{\partial \alpha_{k-1}}{\partial \zeta} \left(f_0 + \psi \hat{\theta} + \left(g_0 + \varphi \hat{\theta} \right) u \right) \\ & + \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \tau_k + \sum_{i=1}^{k-\rho} \frac{\partial \alpha_{k-1}}{\partial u^{(i-1)}} u^{(i)} + \frac{\partial \alpha_{k-1}}{\partial t} \\ & \left. + z_{k-1} + \hat{h}^{(k)} - y_d^{(k)} \right]. \end{aligned} \tag{46}$$

If τ_k is the tuning function and the relation

$$\left(\sum_{i=2}^{k-1} z_i \frac{\partial \widehat{h}^{(i-1)}}{\partial \widehat{\theta}} + \sum_{i=3}^{k-1} z_i \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\right)\Gamma\omega_k^T + \frac{\partial \alpha_{k-1}}{\partial \zeta}\left(f_0 + \psi\widehat{\theta} + \left(g_0 + \varphi\widehat{\theta}\right)u\right) + \frac{\partial \alpha_{k-1}}{\partial \widehat{\theta}}\tau_k + \sum_{i=1}^{k-\rho}\frac{\partial \alpha_{k-1}}{\partial u^{(i-1)}}u^{(i)} + \frac{\partial \alpha_{k-1}}{\partial t} + z_{k-1} + \widehat{h}^{(k)} - y_d^{(k)} = -c_k z_k \tag{47}$$

is satisfied, then $\dot{V} = -\sum_{i=1}^{k} c_i z_i^2$ with the c_i's being positive scalar design constants. Since aforementioned relation is not valid, we define a new error variable

$$z_{k+1} = \widehat{h}^{(k)}\left(\zeta, \widehat{\theta}, u, \ldots, u^{(k-\rho)}, t\right) - y_d^{(k)} + \alpha_k\left(\zeta, \widehat{\theta}, u, \ldots, u^{(k-\rho)}, t\right) \tag{48}$$

with

$$\alpha_k = z_{k-1} + \left(\sum_{i=2}^{k-1} z_i \frac{\partial \widehat{h}^{(i-1)}}{\partial \widehat{\theta}} + \sum_{i=3}^{k-1} z_i \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\right)\Gamma\omega_k^T + \frac{\partial \alpha_{k-1}}{\partial \widehat{\theta}}\tau_k + \frac{\partial \alpha_{k-1}}{\partial \zeta}\left(f_0 + \psi\widehat{\theta} + \left(g_0 + \varphi\widehat{\theta}\right)u\right) + \sum_{i=1}^{k-\rho}\frac{\partial \alpha_{k-1}}{\partial u^{(i-1)}}u^{(i)} + \frac{\partial \alpha_{k-1}}{\partial t} + c_k z_k. \tag{49}$$

The time derivative of the z_k becomes

$$\dot{z}_k = -z_{k-1} - c_k z_k + z_{k+1} + \omega_k\widetilde{\theta} + \left(\frac{\partial h^{(k-1)}}{\partial \widehat{\theta}} + \frac{\partial \alpha_{k-1}}{\partial \widehat{\theta}}\right)\left(\dot{\widehat{\theta}} - \tau_k\right) - \left(\sum_{i=2}^{k-1} z_i \frac{\partial \widehat{h}^{i-1}}{\partial \widehat{\theta}} + \sum_{i=3}^{k-1} z_i \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\right)\Gamma\omega_k^T. \tag{50}$$

Now, the time derivative of a Lyapunov candidate function V_k takes the form

$$\dot{V}_k = -\sum_{i=1}^{k} c_i z_i^2 + z_k z_{k+1} + \widetilde{\theta}^T\Gamma^{-1}\left(\tau_k - \dot{\widehat{\theta}}\right) + \left(\sum_{i=2}^{k} z_i \frac{\partial \widehat{h}^{(i-1)}}{\partial \widehat{\theta}} + \sum_{i=3}^{k} z_i \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\right)\left(\dot{\widehat{\theta}} - \tau_k\right). \tag{51}$$

2.2.5. Step n. In the final step actual parameter update law and control law are designed by putting $k = n - 1$ in (48). The derivative z_n is

$$\dot{z}_n = \widehat{h}^{(n)}\left(\zeta, \widehat{\theta}, u, \ldots, u^{(n-\rho)}, t\right) - y_d^{(n)}(t) + \frac{\partial \alpha_{n-1}}{\partial t} + \frac{\partial \alpha_{n-1}}{\partial \widehat{\theta}}\tau_n + \frac{\partial \alpha_{n-1}}{\partial \zeta}\left(f_0 + \psi\widehat{\theta} + \left(g_0 + \varphi\widehat{\theta}\right)u\right) + \omega_n\widetilde{\theta} + \sum_{i=1}^{n-\rho}\frac{\partial \alpha_{n-1}}{\partial u^{(i-1)}}u^{(i)} + \left(\frac{\partial \widehat{h}^{(n-1)}}{\partial \widehat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \widehat{\theta}}\right)\left(\dot{\widehat{\theta}} - \tau_n\right), \tag{52}$$

where

$$\widehat{h}^{(n)}(\cdot) = \frac{\partial \widehat{h}^{(n-1)}}{\partial \widehat{\theta}}\tau_n + \frac{\partial \widehat{h}^{(n-1)}}{\partial \zeta}\left(f_0 + \psi\widehat{\theta} + \left(g_0 + \varphi\widehat{\theta}\right)u\right) + \sum_{i=1}^{n-\rho}\frac{\partial \widehat{h}^{(n-1)}}{\partial u^{(i-1)}}u^{(i)} + \frac{\partial \widehat{h}^{(n-1)}}{\partial t},$$
$$\omega_n = \left(\frac{\partial \widehat{h}^{(n-1)}}{\partial \zeta} + \frac{\partial \alpha_{n-1}}{\partial \zeta}\right)(\psi + \varphi u). \tag{53}$$

The Lyapunov function can be considered to design control law and tuning function τ_n as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2}\widetilde{\theta}^T\Gamma^{-1}\widetilde{\theta}. \tag{54}$$

Its time derivative is

$$\dot{V}_n = -\sum_{i=1}^{n-1} c_i z_i^2 + \widetilde{\theta}^T\Gamma^{-1}\left(-\dot{\widehat{\theta}} + \tau_{n-1} + \Gamma\omega_k^T z_n\right) + z_n\left(\frac{\partial \widehat{h}^{(n-1)}}{\partial \widehat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \widehat{\theta}}\right)\left(\dot{\widehat{\theta}} - \tau_n\right) + \left(\sum_{i=2}^{n-1} z_i \frac{\partial \widehat{h}^{(n-1)}}{\partial \widehat{\theta}} + \sum_{i=3}^{n-1} z_i \frac{\partial \alpha_{i-1}}{\partial \widehat{\theta}}\right)\left(\dot{\widehat{\theta}} - \tau_{n-1}\right) + z_n\left[z_{n-1} + \widehat{h}^{(n)} - y_d^{(n)} + \frac{\partial \alpha_{n-1}}{\partial \widehat{\theta}}\tau_n + \sum_{i=1}^{n-\rho}\frac{\partial \alpha_{(n-1)}}{\partial u^{(i-1)}}u^{(i)} + \frac{\partial \alpha_{n-1}}{\partial t} + \frac{\partial \alpha_{n-1}}{\partial \zeta}\left(f_0 + \psi\widehat{\theta} + \left(g_0 + \varphi\widehat{\theta}\right)u\right)\right]. \tag{55}$$

The terms containing $\widetilde{\theta}$ can be eliminated from (55) by choosing the parameter update law

$$\dot{\widehat{\theta}} = \tau_n = \tau_{n-1} + \Gamma\omega_k^T z_n = \Gamma W^T z, \tag{56}$$

where the regressor matrix W^T is composed of the regressor vectors as $W^T = [\omega_1^T, \omega_2^T, \ldots, \omega_n^T]$. We can manipulate $\dot{\hat{\theta}}$ as in (44) and (45)

$$\dot{\hat{\theta}} - \tau_{n-1} = \dot{\hat{\theta}} - \tau_n + \tau_k - \tau_{n-1} = \dot{\hat{\theta}} - \tau_n + \Gamma\omega_n^T z_n; \tag{57}$$

thus (55) can be rewritten as

$$\begin{aligned} \dot{V}_n = & -\sum_{i=1}^{n-1} c_i z_i^2 + \tilde{\theta}^T \Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_n\right) \\ & + \left(\sum_{i=2}^{n} z_i \frac{\partial \hat{h}^{(i-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{n} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\right)\left(\dot{\hat{\theta}} - \tau_n\right) \\ & + z_n \Bigg[\left(\sum_{i=2}^{n-1} z_i \frac{\partial \hat{h}^{(i-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{n-1} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\right)\Gamma\omega_n^T \\ & \quad + \frac{\partial \alpha_{n-1}}{\partial \zeta}\left(f_0 + \psi\hat{\theta} + \left(g_0 + \varphi\hat{\theta}\right)u\right) \\ & \quad + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\tau_n + \sum_{i=1}^{n-\rho} \frac{\partial \alpha_{n-1}}{\partial u^{(i-1)}} u^{(i)} + \frac{\partial \alpha_{n-1}}{\partial t} \\ & \quad + z_{n-1} + \hat{h}^{(n)} - y_d^{(n)}\Bigg]. \end{aligned} \tag{58}$$

In order to achieve $\dot{V} = \dot{V}_n = -\sum_{i=1}^{n} c_i z_i^2 \leq 0$ the bracketed term of (58) that is being multiplied with z_n must be equal to $-c_n z_n$,

$$\begin{aligned} & \left(\sum_{i=2}^{n-1} z_i \frac{\partial \hat{h}^{(i-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{n-1} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\right)\Gamma\omega_n^T \\ & \quad + \frac{\partial \alpha_{n-1}}{\partial \zeta}\left(f_0 + \psi\hat{\theta} + \left(g_0 + \varphi\hat{\theta}\right)u\right) \\ & \quad + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\tau_n + \sum_{i=1}^{n-\rho} \frac{\partial \alpha_{n-1}}{\partial u^{(i-1)}} u^{(i)} + \frac{\partial \alpha_{n-1}}{\partial t} \\ & \quad + z_{n-1} + \hat{h}^{(n)}\left(\zeta, \hat{\theta}, u, \ldots, u^{(n-\rho)}, t\right) - y_d^{(n)} = -c_n z_n. \end{aligned} \tag{59}$$

The control law can be retrieved from (59) by solving for $\sum_{i=1}^{n-\rho}(\partial\alpha_{n-1}/\partial u^{(i-1)})u^{(i)}$ as under

$$\begin{aligned} \dot{v}_1 &= v_2 \\ \dot{v}_2 &= v_3 \\ &\vdots \\ \dot{v}_{n-\rho} &= \frac{1}{\left(\left(\partial\hat{h}^{n-1}/\partial v_{n-\rho}\right) + \left(\partial\alpha_{n-1}/\partial v_{n-\rho}\right)\right)} \\ & \times \Bigg[-z_{n-1} + y_d^{(n)} - \left(\frac{\partial \hat{h}^{(n-1)}}{\partial \zeta} + \frac{\partial \alpha_{n-1}}{\partial \zeta}\right) \\ & \times \left\{f_0 + \psi\hat{\theta} + \left(g_0 + \varphi\hat{\theta}\right)v_1\right\} - \frac{\partial \hat{h}^{n-1}}{\partial t} \\ & - \frac{\partial \alpha_{n-1}}{\partial t} - \left(\frac{\partial \hat{h}^{(n-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\right)\tau_n \\ & - \sum_{i=2}^{n-1}\left(\frac{\partial \hat{h}^{(n-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\right) z_i \Gamma\omega_n^T \\ & - \sum_{i=1}^{n-\rho-1}\left(\frac{\partial \hat{h}^{(n-1)}}{\partial v_i} + \frac{\partial \alpha_{n-1}}{\partial v_i}\right) v_{i+1} - c_n z_n\Bigg], \end{aligned} \tag{60}$$

where $v_1 = u$, and its derivatives $\dot{u}, \ddot{u}, \ldots$ are replaced by the extended state variables $v_2, v_3, \ldots$, respectively.

3. Controller Design via DAB-ISMC

In this section, we discuss the design of a control law for a nonlinear nontriangular system by combining DAB and ISMC, as our main contribution. The design is carried out by making use of adaptive backstepping integral sliding mode control and is discussed in sequel.

3.1. Controller Design via Integral Sliding Mode. The controller for a nonlinear system is designed by augmenting ISM method with DAB procedure. The control law appears as sum of a continuous and discontinuous component which may take the following mathematical form:

$$u = u_0 + u_1, \tag{61}$$

where the continuous part, u_0, acts as an ideal control of unperturbed nominal system and is usually designed by linear method. The discontinuous part, u_1, acts as control to cancel the bounded uncertainties which keeps the state trajectories on sliding surface. It is also known as enforcing sliding mode [1]. The main advantage of this technique is that sliding mode is enforced from the very beginning which enhances robustness against uncertainties. In addition, the system operates in sliding mode under the action of the continuous control component $u_0 \in \mathbb{R}$ [20] which is quite robust in this development because it is designed via the adaptive backstepping technique. The discontinuous component $u_1 \in \mathbb{R}$ comes into action, when the system is in the vicinity of the sliding manifold. In addition, the parameter updates law is formulated as

$$\dot{\hat{\theta}} = \dot{\hat{\theta}}_0 + \dot{\hat{\theta}}_1, \tag{62}$$

where the first term comes from adaptive backstepping while the second term from the sliding mode approaches. Note that the update law (56) refers to $\dot{\hat{\theta}}_0$.

3.1.1. Designing Adaptive Backstepping Controller u_0. The development of the continuous control component is presented in the form of the following proposition.

Proposition 3. *For a nonlinear system with auxiliary variables $[z_1, z_2, \ldots, z_n]$, as specified in (48), and given that the set of differential equations (60) yield the desired control function u, the parameter update law $\dot{\hat{\theta}}_0 = \tau_n = \tau_{n-1} + \Gamma\omega_n^T z_n$ ensures that the energy of the transformed system decays to zero asymptotically.*

Proof. Consider a Lyapunov candidate function of the following form:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2. \tag{63}$$

Calculating the time derivative of V_n along (52) and then inserting the value of the control input u from (60) and parameter update law from (56), one has

$$\dot{V}_n \leq -\sum_{i=1}^{n} c_i z_i^2 \tag{64}$$

which indicates that $V \to 0$ asymptotically (analogous to that of [19]), which ensures that the continuous control component is responsible for steering the actual system output to the desired output asymptotically. □

3.1.2. Designing Discontinuous Component u_1. In [19], a classical sliding surface is used for traditional adaptive sliding mode which is by definition a Hurwitz polynomial of the states. However, in the present development, an integral manifold is designed which results in reaching-phase-free sliding mode. The integral manifold under study is defined as follows:

$$\sigma(z) = \sigma_0(z) + \varrho, \tag{65}$$

where $\sigma_0(z)$ is the sliding manifold which usually appears as a linear combination of the states; that is, $\sigma_0(z) = \sum_{i=1}^{n} k_i z_i$, where $k_i > 0$, $i = 1, \ldots, n-1$ with $k_n = 1$ are the designer parameters which are chosen according to the performance of the system. The second term on the right hand side, that is, ϱ, is the integral term which always contains the nominal dynamics of the system. The design of ϱ is presented in the following theorem.

Theorem 4. *Consider the transformed system with the state vector $z = [z_1, z_2, \ldots, z_n]^T$. If the integral manifold is defined according to (65), the integral dynamics is chosen according to the following equation:*

$$\begin{aligned}
\dot{\varrho} = {} & (-k_1 z_2 + k_1 c_1 z_1 - k_2 z_3 + k_2 c_2 z_2 + \cdots - k_{n-1} z_n \\
& + k_{n-1} c_{n-1} z_{n-1}) + (k_2 z_1 + \cdots + k_{n-1} z_{n-2}) \\
& - k_{n-1} \left\{ \left(\frac{\partial \hat{h}^{(n-2)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{n-2}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} + \tau_{n-1} \right) \right. \\
& \left. + \left(\sum_{i=2}^{n-2} z_i \frac{\partial \hat{h}^{(i-1)}}{\partial \hat{\theta}} + \sum_{i=3}^{n-2} z_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \right) \Gamma \omega_k^T \right\} \\
& - \frac{\hat{h}^{(n-1)}}{\partial t} - \frac{\partial \alpha_{n-1}}{\partial t} - \left(\frac{\partial \hat{h}^{(n-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \right) \tau_n \\
& - \left(\frac{\partial \hat{h}^{(n-1)}}{\partial \hat{\theta}} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \right) \left(\dot{\hat{\theta}} - \tau_n \right) \\
& - \left(\frac{\partial \hat{h}^{(n-1)}}{\partial \zeta} + \frac{\partial \alpha_{n-1}}{\partial \zeta} \right) \left(f_0 + \psi \hat{\theta} + \left(g_0 + \varphi \hat{\theta} \right) u \right) \\
& - \sum_{i=1}^{n-\rho} \left(\frac{\partial \hat{h}^{(n-1)}}{\partial u_0^{(i-1)}} + \frac{\partial \alpha_{n-1}}{\partial u_0^{(i-1)}} \right) u_0^{(i)} + y^{(n)}
\end{aligned} \tag{66}$$

and the discontinuous control component is selected as

$$u_1^{(i)} = \frac{-K \cdot \operatorname{sign}(\sigma)}{\left(\partial \hat{h}^{(n-1)} / \partial u^{(n-\rho-1)} \right) + \left(\partial \alpha_{n-1} / \partial u^{(n-\rho-1)} \right)}, \tag{67}$$

where K is a positive constant. Expression (67) ensures that the sliding mode is forced along the integral manifold asymptotically.

Proof. We consider the Lyapunov function of the following form:

$$V_2 = \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \tag{68}$$

The time derivative of the above Lyapunov function along (65) becomes

$$\dot{V}_2 = \sigma \left(k_1 \dot{z}_1 + \cdots + k_{k-1} \dot{z}_{k-1} + \dot{z}_n + \dot{\varrho} \right) - \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}. \tag{69}$$

Substituting the values of $\dot{z}_i$ from (50) where $i = 1, 2, \ldots, n$, one has

$$\begin{aligned}
\dot{V}_2 & \\
= \sigma \Big[& k_1 \left(-c_1 z_1 + z_2 + \omega_1 \tilde{\theta} \right) \\
& + k_2 \left(-z_1 - c_2 z_2 + z_3 + \omega_2 \tilde{\theta} \right) + \\
& \vdots \\
& + k_{n-1} \Big(-z_{n-2} - c_{n-1} z_{n-1} + z_n + \omega_{n-1} \tilde{\theta}
\end{aligned}$$

$$+\left(\frac{\partial\widehat{h}^{(n-2)}}{\partial\widehat{\theta}}+\frac{\partial\alpha_{n-2}}{\partial\widehat{\theta}}\right)\left(\dot{\widehat{\theta}}-\tau_{n-1}\right)$$
$$-\left(\sum_{i=2}^{n-2}z_i\frac{\partial\widehat{h}^{(i-1)}}{\partial\widehat{\theta}}+\sum_{i=3}^{n-2}z_i\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}}\right)\Gamma\omega_k^T\Bigg)$$
$$+\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial\zeta}+\frac{\partial\alpha_{n-1}}{\partial\zeta}\right)$$
$$\times\left(f_0+\psi\widehat{\theta}+\left(g_0+\varphi\widehat{\theta}\right)u\right)$$
$$+\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial\widehat{\theta}}+\frac{\partial\alpha_{n-1}}{\partial\widehat{\theta}}\right)\tau_n+\frac{\partial\widehat{h}^{n-1}}{\partial t}+\frac{\partial\alpha_{n-1}}{\partial t}$$
$$+\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial\widehat{\theta}}-\frac{\partial\alpha_{n-1}}{\partial\widehat{\theta}}\right)\left(\dot{\widehat{\theta}}-\tau_n\right)-y_d^{(n)}+\omega_n\widetilde{\theta}$$
$$+\sum_{i=1}^{n-\rho}\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial u^{(i-1)}}+\frac{\partial\alpha_{n-1}}{\partial u^{(i-1)}}\right)u_0^{(i)}+u_1^{(i)}+\dot{\varrho}\Bigg]-\widetilde{\theta}^T\Gamma^{-1}\dot{\widehat{\theta}}. \tag{70}$$

Now, inserting (66) in the above expression, one has

$$\dot{V}_2=\sigma\left[\sum_{i=1}^{n-\rho}\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial u^{(i-1)}}+\frac{\partial\alpha_{n-1}}{\partial u_1^{(i-1)}}\right)u_1^{(i)}+\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial\widehat{\theta}}-\frac{\partial\alpha_{n-1}}{\partial\widehat{\theta}}\right)\left(\dot{\widehat{\theta}}-\tau_n\right)\right]+\widetilde{\theta}^T\Gamma^{-1}\left(\tau_n-\dot{\widehat{\theta}}\right). \tag{71}$$

The terms $\widetilde{\theta}$ in (71) vanishes by using parameter update law $\dot{\widehat{\theta}}=\tau_n$ where $\tau_n=\Gamma\sigma(\sum_{i=1}^{n-1}\omega_i^Tk_i+\omega_n)$. Therefore, the above expression reduces to

$$\dot{V}_2=\sigma\sum_{i=1}^{n-\rho}\left(\frac{\partial\widehat{h}^{(n-1)}}{\partial u^{(i-1)}}+\frac{\partial\alpha_{n-1}}{\partial u^{(i-1)}}\right)u_1^{(i)} \tag{72}$$

and using (67), the expression in (72) takes the form

$$\dot{V}_2=-k\left|\sigma\right| \tag{73}$$

which is a decreasing function. Thus, according to LaSalle-Yoshizawa theorem [13] output tracking error is zero which proves that the sliding mode is enforced in finite time. However, the parameter behaves in an asymptotic convergence. Thus, the overall system is asymptotically convergent. □

Note that the parameter update law retrieved from (71) is the second component of the parameter update law in (62). Thus the algebraic sum of the two parameter update laws is the final expression of the parameter update law.

4. Illustrative Example

This section is dedicated to verifying the aforementioned claims by applying proposed algorithm on CSTR. Therefore, consider a third order nonlinear system reported in [23] which is in the nontriangular form.

$$\begin{aligned}\dot{\zeta}_1&=1-\left(1+D_{a1}\right)\zeta_1+D_{a2}\zeta_2^2,\\ \dot{\zeta}_2&=D_{a1}\zeta_1-\zeta_2-\left(D_{a2}+D_{a3}\right)\zeta_2^2+u,\\ \dot{\zeta}_3&=D_{a3}\zeta_2^2-\zeta_3,\\ y&=\zeta_3,\end{aligned} \tag{74}$$

where

ζ_1: normalized concentration C_A/C_{AF} of a species A,
ζ_2: normalized concentration C_B/C_{AF} of a species B,
ζ_3: normalized concentration C_C/C_{AF} of a species C,
C_{AF}: the feed concentration of the species A (mol $\cdot$ m^{-1}),
u: the ratio of the per unit volumetric molar feed, rate of species B, denoted by N_{BF}, and the feed concentration C_{AF}, that is, $u=N_{BF}/FC_{AF}$,
F: volumetric feed rate (m$^3\cdot$ s^{-1}),
$D_{a1}=k_1V/F$ constant parameter,
$D_{a2}=k_2VC_{AF}/F$ constant parameter,
$D_{a3}=k_3VC_{AF}/F$ constant parameter,
V: volume of the reactor (m^3)
k_1,k_2,k_3: first order rate constants (s^{-1}).

The system has a constant stable equilibrium point, for every constant volumetric feed rate value $u=U$, which is located in minimum phase region of the system. Consider

$$\begin{aligned}\chi_1&=\frac{1+D_{a2}\chi_2^2}{1+D_{a1}},\\ \chi_2&=\left(1+D_{a1}\right)\left(\frac{-1+\sqrt{1+4\left(U+\Phi\right)\Psi}}{2\left(D_{a2}+D_{a3}+D_{a1}D_{a3}\right)}\right),\\ \chi_3&=D_{a3}\chi_2^2,\end{aligned} \tag{75}$$

where auxiliary variables are $\Phi=D_{a1}/(1+D_{a1})$ and $\Psi=(D_{a2}+D_{a3}+D_{a1}D_{a3})/(1+D_{a1})$. The operating region of the system is, evidently, the strict orthant in $\mathbb{R}^3$, where all concentrations are positive. In other words $\chi=\{\zeta_i\in\mathbb{R}^2$ s.t. $\zeta_i>0$ for $i=1,2,3\}$; we assume that the constant parameters D_{a1}, D_{a2}, and D_{a3} are all constant but unknown. Thus the system (74) can be rewritten as

$$\begin{aligned}\dot{\zeta}_1&=1-\zeta_1+\varphi_1^T\left(\zeta_1,\zeta_2\right)\theta,\\ \dot{\zeta}_2&=-\zeta_2+u+\varphi_2^T\left(\zeta_1,\zeta_2\right)\theta,\\ \dot{\zeta}_3&=-\zeta_3+\varphi_3^T\left(\zeta_2\right)\theta,\\ y&=\zeta_3,\end{aligned} \tag{76}$$

where $\theta=[\theta_1\ \theta_2\ \theta_3]^T=[D_{a1}\ D_{a2}\ D_{a3}]^T$ is the unknown parameter vector, $\varphi_1^T=[-\zeta_1\ \zeta_2^2\ 0]$, $\varphi_2^T=[\zeta_1\ -\zeta_2^2\ -\zeta_2^2]$, and

$\varphi_3^T = [0\ 0\ \zeta_2^2]$. DAB-SMC controller [24] is applied on system (76) to reproduce results for comparison, which resulted in coordinate transformation as

$$\begin{aligned} z_1 &= y - \chi_3 = \zeta_3 - \chi_3, \\ z_2 &= -\zeta_3 + \varphi_3^T(\zeta_2)\hat{\theta} + c_1 z_1, \\ z_3 &= \alpha(\zeta,\hat{\theta}) + \frac{\partial \varphi_3^T}{\partial \zeta_2}\hat{\theta}u, \end{aligned} \tag{77}$$

and derivatives of z's are written as

$$\begin{aligned} \dot{z}_1 &= -\zeta_3 + \varphi_3^T\hat{\theta} + \varphi_3^T\tilde{\theta}, \\ \dot{z}_2 &= \zeta_3(1-c_1) - \frac{\partial \varphi_3^T}{\partial \zeta_2}\zeta_2\hat{\theta} + \frac{\partial \varphi_3^T}{\partial \zeta_2}u\hat{\theta} \\ &\quad + \varphi_3^T\dot{\hat{\theta}} + \omega_2^T\hat{\theta} + \omega_2^T\tilde{\theta}, \\ \dot{z}_3 &= \frac{\partial \alpha_1}{\partial \zeta_1}(1-\zeta_1) - \frac{\partial \alpha_1}{\partial \zeta_3}\zeta_3 + \frac{\partial \varphi_3^T}{\partial \zeta_2}\dot{u}\hat{\theta} \\ &\quad + \left(\frac{\partial \alpha_1}{\partial \zeta_2} + \frac{\partial^2 \varphi_2^T}{\partial \zeta_2^2}u\hat{\theta}\right)(u-\zeta_2) \\ &\quad + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}} + \frac{\partial \varphi_3^T}{\partial \zeta_2}u\right)\tau_3 + \omega_3^T\hat{\theta} + \omega_3^T\tilde{\theta}. \end{aligned} \tag{78}$$

Virtual control is designed as

$$\begin{aligned} \alpha_1(\zeta,\hat{\theta}) &= z_1 - (c_1-1)\zeta_3 - \frac{\partial \varphi_3^T}{\partial \zeta_2}\hat{\theta}\zeta_2 + \omega_2^T\hat{\theta} \\ &\quad + \varphi_3^T\Gamma(z_1\varphi_3 + z_2\omega_2) + c_2 z_2, \end{aligned} \tag{79}$$

where c_1 and c_2 are constant positive design parameters. The parameter update law is designed as

$$\begin{aligned} \dot{\hat{\theta}} &= \tau_3 = \tau_2 + \Gamma\sigma(k_1\varphi_3 + k_2\omega_2 + \omega_3) \\ &= \Gamma[z_1\varphi_3 + z_2\omega_2 + \sigma(k_1\varphi_3 + k_2\omega_2 + \omega_3)], \end{aligned} \tag{80}$$

where σ is the sliding manifold and is designed as

$$\sigma = k_1 z_1 + k_2 z_2 + z_3 \tag{81}$$

with ω_2 and ω_3 in (80) being defined as

$$\begin{aligned} \omega_2^T &= (c_1-1)\varphi_3^T(\zeta_3) + \frac{\partial \varphi_3^T}{\partial \zeta_2}\hat{\theta}\varphi_2^T(\zeta_1,\zeta_2), \\ \omega_3^T &= \frac{\partial \alpha_1}{\partial \zeta_1}\varphi_1^T(\zeta_1,\zeta_2) + \frac{\partial \alpha_1}{\partial \zeta_3}\varphi_3^T(\zeta_2) \\ &\quad + \left(\frac{\partial \alpha_1}{\partial \zeta_2} + \frac{\partial^2 \varphi_3^T}{\partial \zeta_2^2}u\hat{\theta}\right)\varphi_2^T(\zeta_1,\zeta_2). \end{aligned} \tag{82}$$

The following is the control law reported in [24], formulated in the last step of DAB-SMC procedure:

$$\begin{aligned} \dot{u} = \frac{1}{(\partial \varphi_3^T/\partial \zeta_2)\hat{\theta}} \Bigg[&-z_1 - (k_2+z_2)\varphi_3^T(\tau_3-\tau_2) \\ &- k_1(-c_1 z_1 + z_2) - k_2(-z_1 - c_2 z_2 + z_3) \\ &- \left(\frac{\partial \alpha_1}{\partial \zeta_2} + \frac{\partial^2 \varphi_2^T}{\partial \zeta_2^2}u\hat{\theta}\right)(u-\zeta_2) \\ &- \frac{\partial \alpha_1}{\partial \zeta_1}(1-\zeta_1) - \left(\frac{\partial \alpha_1}{\partial \hat{\theta}} + u\frac{\partial \varphi_3^T}{\partial \zeta_2}\right)\tau_3 \\ &+ \frac{\partial \alpha_1}{\partial \zeta_3}\zeta_3 - \omega_3^T\hat{\theta} - \kappa(\sigma + \beta\,\mathrm{sgn}(\sigma)) \Bigg], \end{aligned} \tag{83}$$

where $\Gamma = \Gamma^T$ is a positive definite diagonal matrix containing adoption parameters gains. The output $y = \zeta_3$ converges asymptotically to the desired value χ_3.

DAB-ISMC. In this part, the proposed augmented DAB-ISMC algorithm is applied to formulate controller for regulation of CSTR system (76). The DAB part of DAB-ISMC is the same as formulated in the previous section (Section 3). The coordinate transformation achieved previously is applied in the formulation of ISMC part of DAB-ISMC algorithm. The integral sliding mode manifold can be designed as

$$\begin{aligned} \sigma &= k_1 z_1 + k_2 z_2 + z_3 + \varrho, \\ \dot{\sigma} &= k_1 \dot{z}_1 + k_2 \dot{z}_2 + \dot{z}_3 + \dot{\varrho}. \end{aligned} \tag{84}$$

Consider a Lyapunov function as follows to design $\dot{\varrho}$,

$$V_2 = \frac{1}{2}\sigma^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}, \tag{85}$$

with the discontinuous control component as follows:

$$\dot{u}_1 = -\frac{k\cdot \mathrm{sign}(\sigma)}{\left((\partial \varphi_3^T/\partial \zeta_2)\hat{\theta}\right)}. \tag{86}$$

The expression of the integral compensator dynamics is given by

$$\begin{aligned} \dot{\varrho} = &-k_1 z_2 + k_1 c_1 z_1 \\ &+ k_2\Bigg(z_2 - \varphi_3^T\hat{\theta} - c_1 z_1(1-c_1) \\ &\qquad - (z_3-\alpha_1) - \varphi_3^T\tau_2 - \omega_2^T\hat{\theta} + \frac{\partial \varphi_3^T}{\partial \zeta_2}\zeta_2\hat{\theta}\Bigg) \\ &- \left(\frac{\partial \alpha_1}{\partial \zeta_2} + \frac{\partial^2 \varphi_2^T}{\partial \zeta_2^2}u\hat{\theta}\right)(-\zeta_2 + u) + \frac{\partial \alpha_1}{\partial \zeta_3}\zeta_3 \\ &- \left(\frac{\partial \alpha_1}{\partial \hat{\theta}} + \frac{\partial \varphi_3^T}{\partial \zeta_2}u\right)\tau_3 - \omega_3^T\hat{\theta} \\ &- \frac{\partial \varphi_3^T}{\partial \zeta_2}\dot{u}_0\hat{\theta} - \frac{\partial \alpha_1}{\partial \zeta_1}(1-\zeta_1), \end{aligned} \tag{87}$$

where the parameters update law in case of adaptive integral sliding mode is given by

$$\dot{\hat{\theta}}_1 = \Gamma\sigma\left(\varphi_3^T k_1 + \omega_2 k_2 + \omega_3\right). \tag{88}$$

The final parameter update law can then become

$$\tau_{is} = \dot{\hat{\theta}}_0 + \dot{\hat{\theta}}_1 \tag{89}$$

and the final expression of the control law becomes

$$\dot{u} = \dot{u}_0 + \dot{u}_1, \tag{90}$$

where

$$\begin{aligned}\dot{\hat{\theta}}_0 &= \Gamma\left(\varphi_3^T z_1 + \omega_2 z_2 + \omega_3 z_3\right),\\ \dot{u}_0 &= \frac{-1}{\left(\partial\varphi_3^T/\partial\zeta_2\right)\hat{\theta}}\left[z_2 + \frac{\partial\alpha_1}{\partial\zeta_1}\left(1-\zeta_1\right) - \frac{\partial\alpha_1}{\partial\zeta_3}\zeta_3\right.\\ &\quad + \left(\frac{\partial\alpha_1}{\partial\zeta_2} + \frac{\partial^2\varphi_2^T}{\partial\zeta_2^2}u\hat{\theta}\right)\left(u-\zeta_2\right) + \omega_2\hat{\theta}\\ &\quad \left. + \left(\frac{\partial\alpha_1}{\partial\hat{\theta}} + \frac{\partial\varphi_3^T}{\partial\zeta_2}u\right)\tau_3 - c_3 z_3\right].\end{aligned} \tag{91}$$

Using the approach presented in Theorem 4, one gets

$$\dot{V}_2 \le -\lambda\left|\sigma\right|. \tag{92}$$

This expression shows that the sliding mode is enforced in finite time. However parameter convergence is asymptotic. Thus the overall system is asymptotically convergent.

Now we present computer simulations for the regulation of CSTR using DAB-SMC and DAB-ISMC design methods. In this simulation, the control law and parameter update law have the following unknown parameters:

$$D_{a1} = 3.0, \qquad D_{a2} = 0.5, \qquad D_{a3} = 1.0 \tag{93}$$

and the desired equilibrium values are given as

$$X_1 = 0.3467, \qquad X_2 = 0.8796, \qquad X_3 = 0.7737 \tag{94}$$

which corresponds to a constant value of input u as given by $U = 1$. The design parameters for DAB-SMC law have been selected as

$$\begin{gathered} c_1 = 2, \qquad c_2 = 1, \qquad k_1 = 1, \qquad k_2 = 2,\\ \kappa = 2, \qquad \Gamma = 2I, \qquad \beta = 1, \end{gathered} \tag{95}$$

and the state initial values have been selected as

$$x(0) = \left[0.2 \;\; 0.8 \;\; 0.5 \;\; 2.7 \;\; 0.6 \;\; 1.2 \;\; 2\right] \tag{96}$$

and for DAB-ISMC, we have

$$\begin{gathered} c_1 = 2\times 10^{-5}, \qquad c_2 = 2.5, \qquad c_3 = 1,\\ k_1 = 30, \qquad k_2 = 47, \qquad \kappa = 0.7,\\ \Gamma = 0.4I, \qquad \omega = 800, \end{gathered} \tag{97}$$

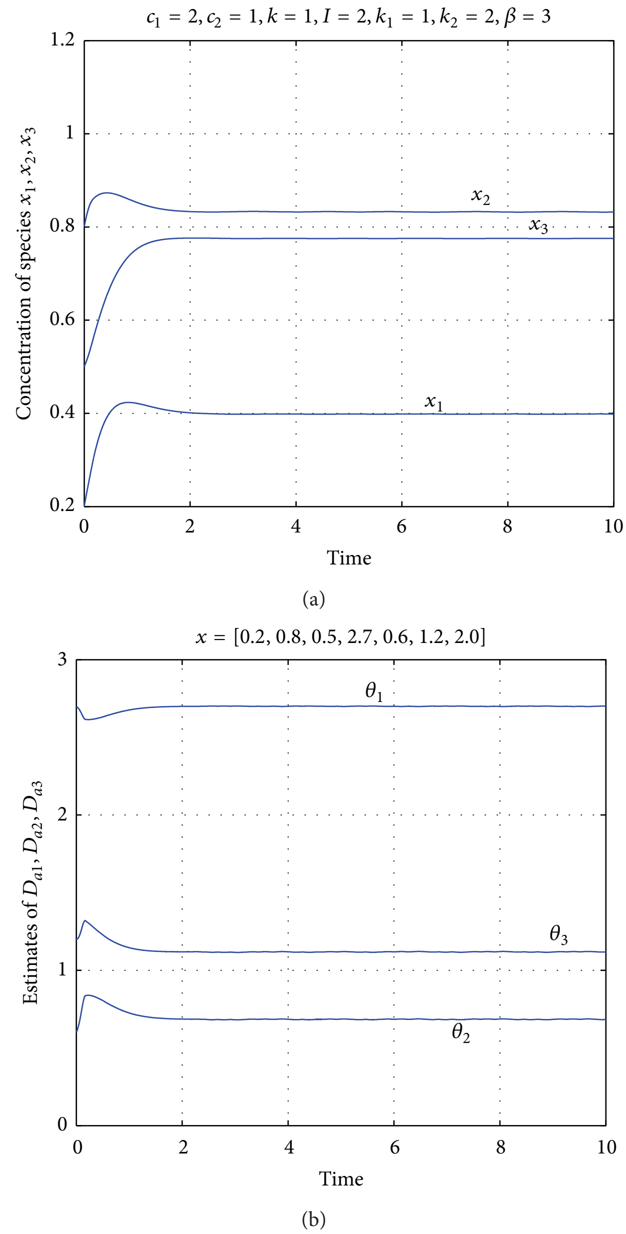

Figure 1: DAB-SMC: concentration of species and estimated parameters.

and the state initial values have been selected as

$$x(0) = \left[0.2 \;\; 0.8 \;\; 0.5 \;\; 2.7 \;\; 0.6 \;\; 1.2 \;\; 2 \;\; 0 \;\; -2.5\right]. \tag{98}$$

Figures 1 and 2 show the DAB-SMC controlled CSTR output, estimated parameters, and sliding surface responses.

It can be noticed in Figure 1 that the DAB-SMC controlled response gives good transient and convergence output while in Figure 2 chattering and reaching-phase phenomenon are evident. The response of DAB-ISMC controller is depicted in Figures 3 and 4 which verify the removal of reaching-phase. Consequently, the robustness has been improved. In comparison to DAB-SMC, it can be noticed that the DAB-ISMC removes chattering completely and enhances

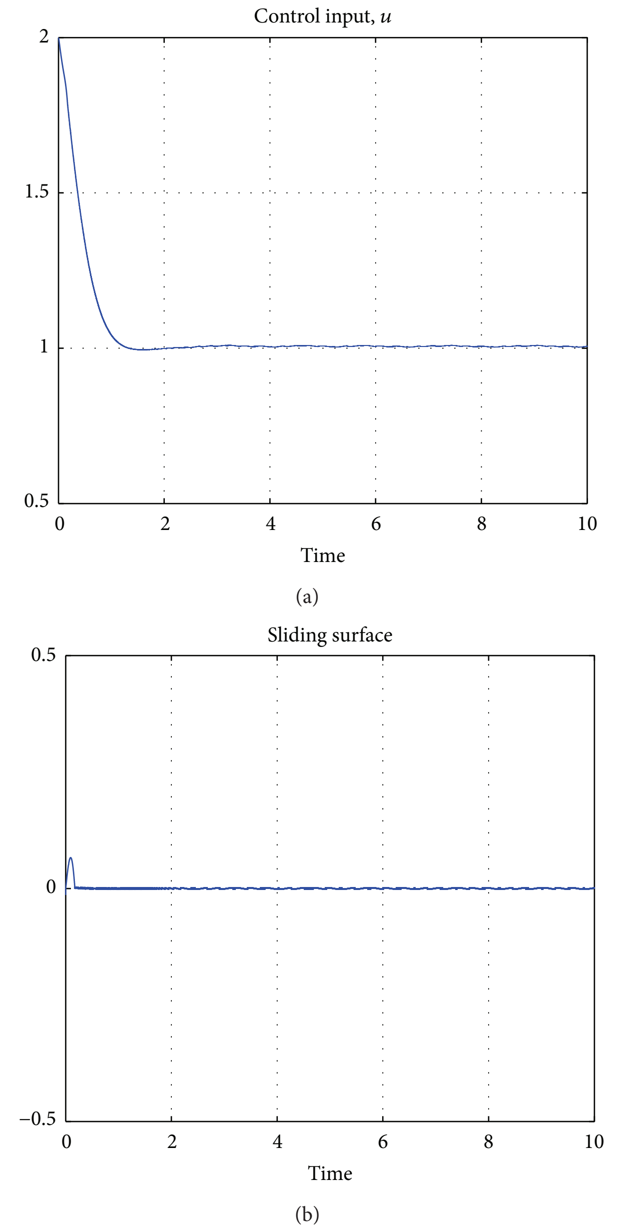

FIGURE 2: DAB-SMC: applied control input and sliding surface.

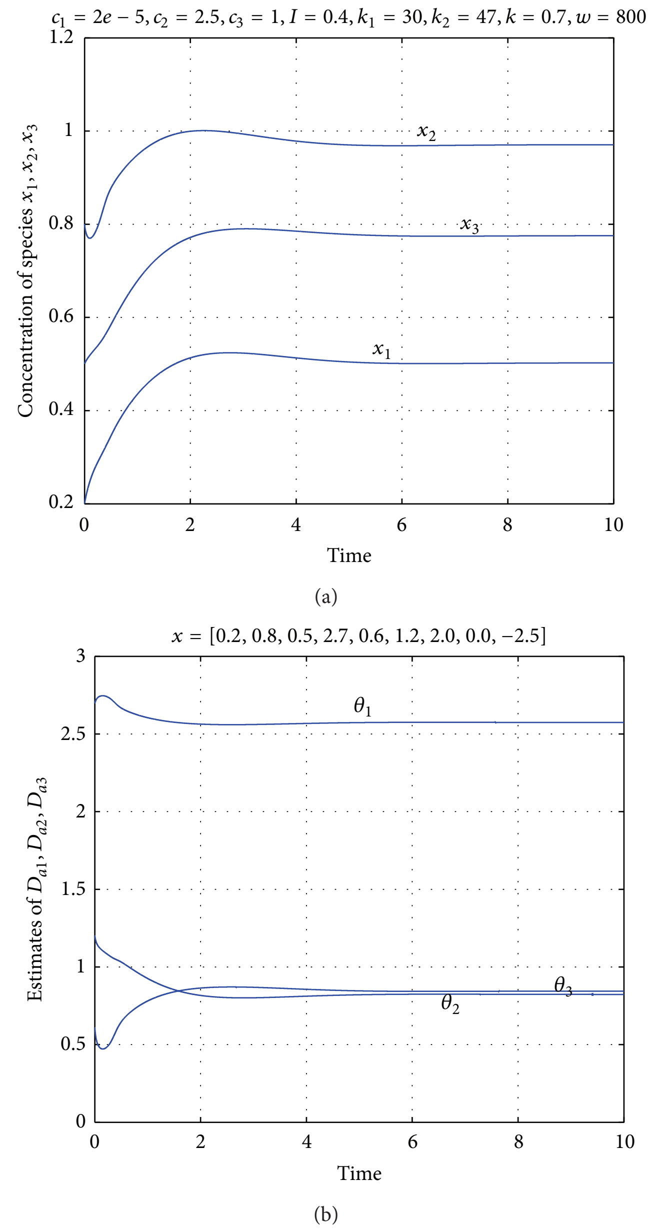

FIGURE 3: DAB-ISMC: concentration of species and estimated parameters.

robustness implicitly. However, the transient performance is affected by increase in settling time which can be trade-off with the robustness requirements.

From the aforementioned discussion of simulated results, it can be seen that the proposed technique has provided increased robustness at the beginning stage and insensitivity to parametric variations owing to integral manifold approach and adaptive backstepping, respectively. It is, therefore, claimed that the above development significantly outperforms the existing adaptive sliding mode techniques in robustness. Proposed design technique is recursive in nature; therefore, it is costly in computation as compared to the traditional adaptive sliding mode. However, the competitive advantage of this method is that its control action is global and robust.

5. Conclusion

In this work, two different combined algorithms for the control of uncertain, nonlinear, nontriangular system with unmatched uncertainties have been presented: the dynamic adaptive backstepping-SMC (DAB-SMC) and the dynamic adaptive backstepping-integral SMC (DAB-ISMC). In DAB-SMC, a dynamic adaptive backstepping method has been combined with sliding mode control to synthesize a robust system against matching and unmatching uncertainties. In

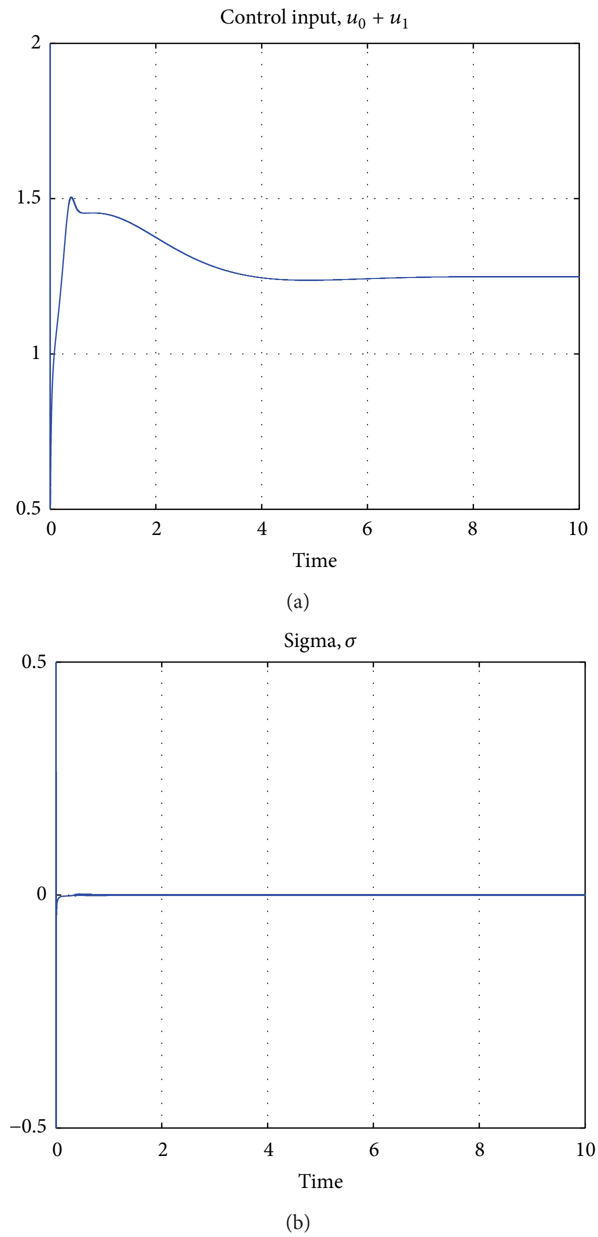

FIGURE 4: DAB-ISMC: applied control input and sliding surface.

the proposed DAB-ISMC, we rather exploit an integral sliding mode control which resulted in a more robust controller than DAB-SMC. Simulation experiments have been carried out to compare the robustness of the addressed two methods for continuous stirred tank reactor (which is highly nonlinear in nature). It has been observed that the proposed method (DAB-ISMC) offers significant improvement in terms of robustness and near elimination of chattering. The enhanced robustness is achieved by the elimination of reaching-phase in sliding mode where the sliding manifold has been designed by using adaptively developed state variables. Moreover, both the DAB-SMC and DAB-ISMC laws are synthesized in a systemic manner and the stability is proved by using a quadratic Lyapunov function.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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