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Research Article

Output-Feedback Control for a Class of Stochastic High-Order Feedforward Nonlinear Systems with Delay

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The problem of global output-feedback stabilization for a class of stochastic high-order time-delay feedforward nonlinear systems with different power orders is investigated. By combining the adding one power integrator technique with the homogeneous domination approach, an output-feedback controller design is proposed, which ensures the global asymptotical stability in probability of the closed-loop system.

1. Introduction

In the last decades, stochastic systems have received much more attention since stochastic modeling has come to play an important role in many branches of science and engineering applications. For this type of systems, the authors in [1–10] presented the basic stability theory of the stochastic control systems.

Global stabilization of triangular structural stochastic nonlinear systems has been a major issue in control theory over the last decades. For lower-triangular systems (namely, feedback systems), the stochastic asymptotic stabilization has been studied by using backstepping approach; see [11–13] and the references therein. Upper-triangular systems, which are also called feedforward systems, have been fully used to model many physical devices, such as the ball-beam with a friction term [14] and the cart-pendulum system [15]. By using a homogeneous domination approach, the problem of global output-feedback stabilization was addressed in [16] for a class of upper-triangular systems with higher-order nonlinearities. Then, the case of lower-order nonlinearities was considered in [17]. With the help of a generalized definition of homogeneity, the problem of using small controls to globally stabilize a class of upper-triangular systems was investigated in [18]. Inspired by the works [19, 20], the problem of using a sampled data controller to globally stabilize a class of uncertain upper-triangular systems was considered in [21]. When parameter uncertainties appeared in a system model, an adaptive stabilizer for feedforward nonlinear systems with general dynamic order in [22] was extended to [23].

On the other hand, time delays may arise naturally, which are usually the key factors that influence the stability of nonlinear systems. Many researchers have paid more attention to studying the stochastic nonlinear time-delay systems over the last decades and various results concerning stochastic lower-triangular nonlinear systems with time delays have been reported in [24–27]. For the case where the delays are of unknown length, an adaptive control design was presented in [28] for a system in feedforward form. An approach for compensating input delay of arbitrary length was presented [29] for forward complete and strict-feedforward nonlinear systems. When the nonlinear functions were higher order in states, the problem of global stabilization by state feedback was addressed in [30, 31]. It should be pointed out that these results were obtained for the case of state measurement. Naturally, one may ask a challenging and interesting question: if not all the states are measurable, how do we design an output-feedback controller for stochastic high-order feedforward systems with time delay? To the best of our knowledge, no output-feedback stabilization control scheme has so far been proposed for stochastic high-order feedforward systems with time delay.

Inspired by the aforementioned discussion, we deal with the problem of global output-feedback stabilization for a class of stochastic high-order feedforward nonlinear systems with time-varying delay and different power orders. Firstly, we design a state-feedback controller for the nominal system using the adding one power integrator technique. Then, by designing a homogeneous observer for the nominal system, we explicitly construct a homogeneous output-feedback stabilizer under the certainty equivalence principle. At last, we propose a scaled controller which guarantees global asymptotic stability in probability of the closed-loop system.

The outline of this paper is as follows. Sections 2 and 3 offer some preliminary results and problem formulation, respectively. The output-feedback controller is designed and analyzed in Section 4. This paper is concluded in Section 5.

Notations. The following standard notations are used throughout this paper. R^n denotes the real n-dimensional space. $|x| = (\sum_{i=1}^{n} x_i^2)^{1/2}$ for any $x \in R^n$. For any matrix $Q \in R^n \times R^m$, $|Q|$ denotes the Frobenius norm $|Q| = (\sum_{i,j} q_{ij}^2)^{1/2}$ and $|Q|_\infty = \max_{1\le i\le n}(\sum_{j=1}^{m} q_{ij}^2)^{1/2}$. For the sake of simplicity, sometimes function $x(t)$ is denoted by x; x_i represents the ith element of vector x and $\overline{x}_i = (x_1, \dots, x_i)^T$. C^i denotes the set of all functions with continuous ith partial derivative. $\mathscr{K}$ denotes the set of all functions: $R^+ \to R^+$ which are continuous, strictly increasing, and vanishing at zero; $\mathscr{K}_\infty$ denotes the set of all functions which are of class $\mathscr{K}$ and unbounded.

2. Preliminary Results

Consider the following stochastic time-delay system

$$dx(t) = f(x(t), x(t-\sigma(t)))\,dt + g(x(t), x(t-\sigma(t)))^T d\omega \tag{1}$$

with an initial condition $\{x(s) : -\sigma_0 \le s \le 0\} = \xi \in C_{F_0}^b \times ([-\sigma_0, 0], R^n)$, where $\sigma(t) : R^+ \to [0, \sigma_0]$ is a Borel measurable function; $x(t) \in R^n$ denotes the state vector and $x(t-\sigma(t))$ is the state vectors with time-delay; ω is an m-dimensional standard Wiener process defined on the complete probability space $(\Omega, F, \{F_t\}_{t\ge0}, P)$ with Ω being a sample space, F being a σ-field, $\{F_t\}_{t\ge0}$ being a filtration, and P being a probability measure; $f(\cdot) \in R^n$, $g(\cdot) \in R^{n\times m}$ are locally Lipschitz functions and satisfy $f(0,0) = 0$, $g(0,0) = 0$.

In the following, we borrow some definitions and lemmas which play an important role in stabilizing and analyzing the stochastic time-delay systems for later development in this paper.

Definition 1 (see [26]). Define the infinitesimal generator $\mathscr{L}$ of function $V(x) \in C^2$ along system (1) as follows:

$$\mathscr{L}V(x) = \frac{\partial V}{\partial z} f(x, x(t-\sigma(t))) + \frac{1}{2}\operatorname{Tr}\left\{ g(x, x(t-\sigma(t)))\frac{\partial^2 V}{\partial x^2} g \times (x, x(t-\sigma(t)))^T \right\}, \tag{2}$$

in which $(1/2)\operatorname{Tr}\{g(x, x(t-\sigma(t)))(\partial^2 V/\partial x^2) g(x, x(t-\sigma(t)))^T\}$ is called the Hessian term of $\mathscr{L}$.

Definition 2 (see [26]). The equilibrium $x = 0$ of system (1) is globally asymptotically stable (GAS) in probability if for any $\epsilon > 0$, there exists a function $\gamma(\cdot,\cdot) \in \mathscr{KL}$ such that $P\{|x| \le \gamma(\|\zeta\|, t)\} \ge 1 - \epsilon$ for any $t \ge 0$, $\zeta \in C_{F_0}^b \times ([-\tau, 0], R^n) \setminus \{0\}$, where $\|\zeta\| = \sup_{\theta\in[-\tau,0]}|\zeta(\theta)|$.

Definition 3 (see [16]). For fixed coordinates $(x_1, \dots, x_n)$ and real numbers $r_i > 0$, $i = 1, \dots, n$, one has the following.

(i) The dilation $\Delta_\varepsilon(x)$ is defined by $\Delta_\varepsilon(x) = (\varepsilon^{r_1}x_1, \dots, \varepsilon^{r_n}x_n)$, for all $\varepsilon > 0$, with r_i being called as the weights of the coordinates. For simplicity of notation, we define dilation weight $\Delta = (r_1, \dots, r_n)$.

(ii) A function $V \in C(R^n, R)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $V(\Delta_\varepsilon(x)) = \varepsilon^\tau V(x_1, \dots, x_n)$, for $\forall x \in R^n \setminus \{0\}$, $\varepsilon > 0$.

(iii) A vector field $f \in C(R^n, R^n)$ is said to be homogeneous of degree τ if there is a real number $\tau \in R$ such that $f_i(\Delta_\varepsilon(x)) = \varepsilon^{\tau+r_i} f_i(x_1, \dots, x_n)$, for $\forall x \in R^n \setminus \{0\}$, $\varepsilon > 0$, $i = 1, \dots, n$.

(iv) A homogeneous p-norm is defined as $\|x\|_{\Delta,p} = (\sum_{i=1}^{n} |x_i|^{p/r_i})^{1/p}$, for all $x \in R^n$, for a constant $p \ge 1$. For the simplicity, we choose $p = 2$ and write $\|x\|_\Delta$ for $\|x\|_{\Delta,2}$ in this paper.

Lemma 4 (see [26]). *Consider the stochastic time-delay system (1); if there exist a function $V(x) \in C^2([-\sigma_0, \infty] \times R^n)$ and class $\mathscr{K}_\infty$ functions α_1, α_2 satisfying the following inequalities:*

$$\alpha_1(|x(t)|) \le V(x) \le \alpha_2\left(\sup_{(-\sigma_0\le s\le 0)} |x(t+s)|\right), \qquad \mathscr{L}V(x) \le -W(x(t)), \tag{3}$$

where $W(x(t))$ is continuous and positive definite, then

(1) *there exists a unique solution on $[-\sigma_0, \infty)$,*

(2) *the equilibrium $x = 0$ of the system (1) is GAS in probability and $P\{\lim_{t\to\infty}|x(t)| = 0\} = 1$.*

Lemma 5 (see [16]). *Given a dilation weight $\Delta = (r_1, \dots, r_n)$, suppose $V_1(x)$ and $V_2(x)$ are homogeneous functions of degree τ_1 and τ_2, respectively. Then, $V_1(x) \cdot V_2(x)$ is also homogeneous with respect to the same dilation weight Δ. Moreover, the homogeneous degree of $V_1(x) \cdot V_2(x)$ is $\tau_1 + \tau_2$.*

Lemma 6 (see [16]). *Suppose $V : R^n \to R$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then, the following holds.*

(i) $\partial V/\partial x_i$ *is homogeneous of degree* $\tau - r_i$ *with* r_i *being the homogeneous weight of* x_i.

(ii) *There is a constant c such that* $V(x) \le c\|x\|_\Delta^\tau$. *Moreover, if* $V(x)$ *is positive definite, then*

$$\underline{c}\|x\|_\Delta^\tau \le V(x), \quad \text{for a constant } \underline{c} > 0. \tag{4}$$

Lemma 7 (see [16]). *For any* $x \in R$, $y \in R$, *and* $p \ge 1$, *the following inequalities hold:*

$$\begin{aligned} |x+y|^p &\le 2^{p-1}|x^p + y^p|, \\ (|x| + |y|)^{1/p} &\le |x|^{1/p} + |y|^{1/p}. \end{aligned} \tag{5}$$

Moreover, when $p \in R_{odd}^{\ge 1}$,

$$\begin{aligned} |x-y|^p &\le 2^{p-1}|x^p - y^p|, \\ |x^p - y^p| &\le p|x-y||x^{p-1} + y^{p-1}| \\ &\le c|x-y||(x-y)^{p-1} + y^{p-1}|, \end{aligned} \tag{6}$$

where $c > 0$ *is a constant.*

Lemma 8 (see [27]). *Let x and y be any real numbers and* $p \in R_{odd}^{\ge 1}$; *then*

$$-(x-y)(x^p - y^p) \le -\frac{1}{2^{p-1}}(x-y)^{p+1}. \tag{7}$$

Lemma 9 (see [27]). *Let c and d be two positive real numbers. For any positive number* $\gamma > 0$, *the following inequality holds*

$$|x|^c|y|^d \le \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}|y|^{c+d}. \tag{8}$$

3. Problem Formulation

In this paper, we will consider the following stochastic high-order feedforward nonlinear system with time-varying delay in the form

$$\begin{aligned} dz_1 &= z_2^{p_1}dt + f_1(z, z(t-\sigma(t))) \\ &\quad + g_1^T(z, z(t-\sigma(t)))\,d\omega, \\ dz_2 &= z_3^{p_2}dt + f_2(z, z(t-\sigma(t))) \\ &\quad + g_2^T(z, z(t-\sigma(t)))\,d\omega, \\ &\ \ \vdots \\ dz_n &= v^{p_n} + f_n(z, z(t-\sigma(t))) \\ &\quad + g_n^T(z, z(t-\sigma(t)))\,d\omega, \\ y &= z_1, \end{aligned} \tag{9}$$

where $z = [z_1, \ldots, z_n]^T \in R^n$, $y \in R$, $v \in R$ denote the system state, output, and input, respectively; $z(t-\sigma(t)) = [z_1(t-\sigma(t)), \ldots, z_n(t-\sigma(t))]$ is the state vector with time-delay; $\sigma(t) : R^+ \to [0, \sigma_0]$ is a time-varying delay. ω is an m-dimensional standard Wiener process. For $i = 1, \ldots, n$, $p_i \in R_{\text{odd}}^{\ge 3} \triangleq \{p \in R : p \ge 3$ and p is a ratio of odd integers$\}$. The drift functions $f_i : R^n \times R^n \to R$ and the diffusion functions $g_i : R^n \times R^n \to R^m$ are locally Lipschitz in $(z, z(t-\sigma(t)))$ and vanish at the origin.

In order to obtain the main results of this paper, the following assumptions are needed.

Assumption 10. For $i = 1, \ldots, n-1$, there are constants $a_i > 0$, $b_i > 0$, such that

$$\begin{aligned} &|f_i(z, z(t-\sigma(t)))| \\ &\quad \le a_i \sum_{j=i+2}^{n+1}\left(|z_j(t)|^{(r_i+\tau)/r_j} + |z_j(t-\sigma(t))|^{(r_i+\tau)/r_j}\right), \\ &|g_i(z, z(t-\sigma(t)))| \\ &\quad \le b_i \sum_{j=i+2}^{n+1}\left(|z_j(t)|^{(2r_i+\tau)/2r_j} + |z_j(t-\sigma(t))|^{(2r_i+\tau)/2r_j}\right), \end{aligned} \tag{10}$$

where $p_i \in R_{\text{odd}}^{\ge 3}$, $\tau \in [d_0, \infty)$, $r_1 = 1$, $r_{i+1} = (r_i + \tau)/p_i$, $d_1 = (2 - 1/(p_1 \cdots p_{n-1}))/(1 + \sum_{s=1}^{n-1}(1/(p_s \cdots p_{n-1})))$, $d_{i+1} = (2/(p_1 \cdots p_i) - 1/(p_1 \cdots p_{n-1}))/(1 + \sum_{s=1}^{n-1}(1/(p_s \cdots p_{n-1})) - \sum_{s=1}^{i}(2/(p_s \cdots p_i)))$, and $d_0 = \max_{j=1,\ldots,n}\{d_j\}$.

Assumption 11. The time-varying delay $\sigma(t)$ satisfies $\dot{\sigma}(t) \le \sigma < 1$ for a constant σ.

In this paper, we aim to constructively design a homogeneous output-feedback controller

$$\begin{aligned} \dot{\chi}(t) &= \nu(\chi(t), y(t)), \\ u(t) &= \omega(\chi(t), y(t)), \end{aligned} \tag{11}$$

for system (9) under Assumptions 10-11, such that the closed-loop system is GAS in probability.

4. Controller Design and Stability Analysis

In this section, an output-feedback controller will be explicitly constructed for the nonlinear system (9). We will combine the adding one power integrator technique with the homogeneous domination approach for output-feedback stabilization. The design procedure can be divided into three steps: (i) we first design a state-feedback controller for the nominal system without the drift and diffusion functions using the adding one power integrator technique; (ii) then, by designing a homogeneous observer for the nominal system, we explicitly construct a homogeneous output-feedback stabilizer under the certainty equivalence principle; (iii) in the end, we propose a scaled controller which guarantees global asymptotic stability in probability of the closed-loop system.

For simplicity, we assume $\tau = m/n$ in this paper with m being an even integer and n being an odd integer. Based on this assumption, it is obvious that r_i is a ratio of odd integers.

4.1. Homogeneous State-Feedback Control of Nominal Nonlinear System. In this subsection, we firstly introduce a key lemma, which avoids the zero-division problem and serves as a basis in the following design procedure. The proof is similar to that in [32].

Lemma 12. *If $p_i \in R^{\geq 3}_{odd}$, $\tau \in [d_0, +\infty)$ are guaranteed, then we can find $l_0, \mu_0 \in R^{+}_{odd}$, such that*

$$\max_{1\leq i\leq n}\left\{2r_i, \frac{r_i+\tau}{l_0}\right\} \leq \mu_0 \leq r_n + \tau \tag{12}$$

holds.

Now, we design a homogeneous state-feedback controller for the nominal system

$$\begin{aligned} dx_1 &= x_2^{p_1}dt, \\ dx_2 &= x_3^{p_2}dt, \\ &\vdots \\ dx_n &= u^{p_n}dt. \end{aligned} \tag{13}$$

Lemma 13. *For system* (13), *there are positive definite, proper, and C^2 Lyapunov function V_n, a state-feedback controller u^*, and two positive constants, such that*

$$\mathscr{L}V_n \leq -\sum_{j=1}^{n}\pi_{n,j}\eta_j^{4l_0} + \eta_n^{q_n/\mu_0}\left(u^{p_n} - u^{*p_n}\right), \tag{14}$$

where V_n, u^* are defined in the following form:

$$\begin{aligned} V_n(\overline{x}_n) &= \sum_{i=1}^{n}\int_{x_i^*}^{x_i}\left(s^{\mu_0/r_i} - x_i^{*\mu_0/r_i}\right)^{q_i/\mu_0}ds, \\ x_1^* &= 0, \qquad \eta_1 = x_1^{\mu_0/r_1}, \\ x_2^* &= -\beta_1^{r_2/\mu_0}\eta_1^{r_2/\mu_0}, \qquad \eta_2 = x_2^{\mu_0/r_2} - x_2^{*\mu_0/r_2}, \\ &\vdots \qquad\qquad \vdots \\ x_n^* &= -\beta_{n-1}^{r_n/\mu_0}\eta_{n-1}^{r_n/\mu_0}, \qquad \eta_n = x_n^{\mu_0/r_n} - x_n^{*\mu_0/r_n}, \\ u^* &= -\left(\pi_{nn} + b_{n1} + b_{n2}\right)^{1/p_n}\eta_n^{r_{n+1}/\mu_0} \\ &\triangleq -\beta_n^{r_{n+1}/\mu_0}\eta_n^{r_{n+1}/\mu_0} \\ &= -\left(\overline{\beta}_n x_n^{\mu_0/r_n} + \overline{\beta}_{n-1}x_{n-1}^{\mu_0/r_{n-1}} + \cdots + \overline{\beta}_1 x_1^{\mu_0/r_1}\right)^{r_{n+1}/\mu_0}, \\ \overline{\beta}_j &= \beta_n\cdots\beta_j, \qquad j = 1,\ldots,n. \end{aligned} \tag{15}$$

Remark 14. For the deterministic work, it is well known that there only needs a positive definite, proper, and C^1 Lyapunov function for the controller design and analysis. However, for the stochastic system, the Lyapunov function must be positive definite, proper, and at least C^2 because of the appearance of the Hessian term.

4.2. Construction of a Homogeneous Observer. Since $x_2,\ldots,x_n$ are unmeasurable, we design a reduced-order homogeneous observer

$$\begin{aligned} \dot{\widehat{\eta}}_2 &= -k_2\widehat{x}_2^{p_1}, \qquad \widehat{x}_2 = \left[\widehat{\eta}_2 + k_2\widehat{x}_1\right]^{r_2/r_1}, \\ \dot{\widehat{\eta}}_3 &= -k_3\widehat{x}_3^{p_2}, \qquad \widehat{x}_3 = \left[\widehat{\eta}_3 + k_3\widehat{x}_2\right]^{r_3/r_2}, \\ &\vdots \qquad\qquad \vdots \\ \dot{\widehat{\eta}}_n^* &= -k_n\widehat{x}_n^{p_{n-1}}, \qquad \widehat{x}_n = \left[\widehat{\eta}_n + k_n\widehat{x}_{n-1}\right]^{r_n/r_{n-1}}, \end{aligned} \tag{16}$$

where $k_2 > 0,\ldots,k_n > 0$ are constant gains to be determined later and $\widehat{x}_1 = x_1$.

According to the certainty equivalence principle, we use the estimated states $\widehat{x}_2,\ldots,\widehat{x}_n$ to construct the implementable controller

$$u(\widehat{x}) = -\left(\overline{\beta}_n\widehat{x}_n^{\mu_0/r_n} + \overline{\beta}_{n-1}\widehat{x}_{n-1}^{\mu_0/r_{n-1}} + \cdots + \overline{\beta}_1\widehat{x}_1^{\mu_0/r_1}\right)^{r_{n+1}/\mu_0}, \tag{17}$$

where $\widehat{x} = (\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_n)$.

In order to prove the global stability for the closed-loop system (13)–(16)-(17), we define

$$U_j = \int_{\lambda_j^{q_{j-1}/r_{j-1}}}^{x_j^{q_{j-1}/r_j}}\left(s^{r_{j-1}/q_{j-1}} - \lambda_j\right)ds, \quad \lambda_j = \widehat{\eta}_j + k_j\widehat{x}_{j-1}, \tag{18}$$

and let

$$e_j = \left(x_j^{p_{j-1}} - \widehat{x}_j^{p_{j-1}}\right)^{\mu_0/r_jp_{j-1}}, \tag{19}$$

for $j = 2,\ldots,n$.

Therefore, the differential operator $\mathscr{L}$ of U_j is

$$\begin{aligned} \mathscr{L}\left(U_j\right) &= \frac{q_{j-1}}{r_j}x_j^{q_{j-1}/r_j-1}\left(x_j^{r_{j-1}/r_j} - \lambda_j\right)x_{j+1}^{p_j} \\ &\quad - k_j\left(x_j^{q_{j-1}/r_j} - \lambda_j^{q_{j-1}/r_{j-1}}\right)x_j^{p_{j-1}} \\ &\quad + k_j\left(x_j^{q_{j-1}/r_j} - \lambda_j^{q_{j-1}/r_{j-1}}\right)\widehat{x}_j^{p_{j-1}} \\ &= \frac{q_{j-1}}{r_j}x_j^{q_{j-1}/r_j-1}\left(x_j^{r_{j-1}/r_j} - \lambda_j\right)x_{j+1}^{p_j} \\ &\quad - k_j\left[\left(x_j^{q_{j-1}/r_j} - \widehat{x}_j^{q_{j-1}/r_j}\right)\right. \\ &\qquad \left. + \left(\widehat{x}_j^{q_{j-1}/r_j} - \lambda_j^{q_{j-1}/r_{j-1}}\right)\right]e_j^{r_jp_{j-1}/\mu_0}, \end{aligned} \tag{20}$$

where $x_{n+1} = u(\widehat{x})$.

Just as in [16], each term of the right-hand side of (20) can be estimated by the following propositions, whose proofs are omitted here.

Proposition 15. *For* $j = 2, \dots, n-1$,

$$\frac{q_{j-1}}{r_j} x_j^{q_{j-1}/r_j - 1} \left(x_j^{r_{j-1}/r_j} - \lambda_j \right) x_{j+1}^{p_j} \leq \sum_{k=j-1}^{j+1} \delta_{j,k,3} \eta_k^{4l_0} + g_{j1}\left(k_j\right) e_{j-1}^{4l_0} + b_{j3} e_j^{4l_0}, \quad (21)$$

where $\delta_{j,k,3} > 0$, $b_{j3} > 0$, $g_{j1}(k_j)$ *is a continuous function with* $g_{21}(k_2) = 0$.

Proposition 16. *For the implementable controller* $u(\hat{x})$ *in* (17), *one obtains*

$$\frac{q_{j-1}}{r_j} x_j^{q_{j-1}/r_j - 1} \left(x_j^{r_{j-1}/r_j} - \lambda_j \right) u(\hat{x})^{p_n} \leq \sum_{j=1}^{n} \delta_{n,j,4} \eta_j^{4l_0} + g_{n1}\left(k_n\right) e_{n-1}^{4l_0} + \sum_{j=2}^{n} b_{j4} e_j^{4l_0} \quad (22)$$

with positive constants $\delta_{n,j,4}$, b_{j4} *and with* $g_{n1}(k_n)$ *being a continuous function.*

Proposition 17. *For* $j = 2, \dots, n$, *there exists a constant* $\varpi_j = 2^{1 - q_{j-1}/r_j p_{j-1}} > 0$ *such that*

$$-k_j \left(x_j^{q_{j-1}/r_j} - \hat{x}_j^{q_{j-1}/r_j} \right) e_j^{r_j p_{j-1}/\mu_0} \leq -k_j \varpi_j e_j^{4l_0}. \quad (23)$$

Proposition 18. *For* $j = 3, \dots, n$,

$$-k_j \left(\hat{x}_j^{q_{j-1}/r_j} - \lambda_j^{q_{j-1}/r_{j-1}} \right) e_j^{r_j p_{j-1}/\mu_0} \leq \sum_{k=j-1}^{j} \delta_{j,k,5} \eta_k^{4l_0} + g_{j2}\left(k_j\right) e_{j-1}^{4l_0} + b_{j5} e_j^{4l_0} \quad (24)$$

with positive constants $\delta_{j,k,5}$, b_{j5} *and with* $g_{j2}(k_j)$ *being a continuous function.*

Considering the Lyapunov function $U = \sum_{j=2}^{n} U_j$ and substituting (21)–(24) into (20), one concludes

$$\begin{aligned} \mathscr{L}U \leq& \sum_{j=1}^{n} \epsilon_{j1} \eta_j^{4l_0} + e_2^{4l_0} \\ &\times \left[-k_2 \varpi_2 + b_{23} + b_{24} + g_{31}\left(k_3\right) + g_{32}\left(k_3\right) \right] \\ &+ \sum_{j=3}^{n-1} e_j^{4l_0} \left[-k_j \varpi_j + b_{j3} + b_{j4} + b_{j5} \right. \\ &\qquad \left. + g_{j+1,1}\left(k_{j+1}\right) + g_{j+1,2}\left(k_{j+1}\right) \right] \\ &+ e_n^{4l_0} \left[-k_n \varpi_n + b_{n4} + b_{n5} \right]. \end{aligned} \quad (25)$$

To determinate the observer gain k_j in (16), we have the following proposition to deal with the redundant term $\eta_n^{q_n/\mu_0}(u^{p_n}(\hat{x}) - u^{*p_n})$ in (14).

Proposition 19. *There are constants* $\epsilon_{j2} > 0, b_{j6} > 0$ *such that*

$$\eta_n^{q_n/\mu_0} \left(u^{p_n}(\hat{x}) - u^{*p_n} \right) \leq \sum_{j=1}^{n} \epsilon_{j2} \eta_j^{4l_0} + \sum_{j=2}^{n} b_{j6} e_j^{4l_0}. \quad (26)$$

Defining the Lyapunov function $V = V_n + U$ and combining (14), (25), and (26) together yield

$$\begin{aligned} \mathscr{L}V \leq& -\sum_{j=1}^{n} \pi_{n,j} \eta_j^{4l_0} + \sum_{j=1}^{n} \epsilon_{j2} \eta_j^{4l_0} + \sum_{j=1}^{n} \epsilon_{j1} \eta_j^{4l_0} \\ &+ \sum_{j=2}^{n} b_{j6} e_j^{4l_0} + e_2^{4l_0} \\ &\times \left[-k_2 \varpi_2 + b_{23} + b_{24} + g_{31}\left(k_3\right) + g_{32}\left(k_3\right) \right] \\ &+ \sum_{j=3}^{n-1} e_j^{4l_0} \left[-k_j \varpi_j + b_{j3} + b_{j4} + b_{j5} \right. \\ &\qquad \left. + g_{j+1,1}\left(k_{j+1}\right) + g_{j+1,2}\left(k_{j+1}\right) \right] \\ &+ e_n^{4l_0} \left[-k_n \varpi_n + b_{n4} + b_{n5} \right]. \end{aligned} \quad (27)$$

It is clear that by choosing

$$\begin{aligned} l_j &\triangleq \pi_{n,j} - \epsilon_{j1} - \epsilon_{j2} > 0, \quad (j = 1, \dots, n) \\ k_n &\geq \frac{1}{\varpi_n} \left(\varphi_n + b_{n4} + b_{n5} + b_{n6} \right), \\ k_{n-1} &\geq \frac{1}{\varpi_{n-1}} \left(\varphi_{n-1} + b_{n-1,3} + b_{n-1,4} + b_{n-1,5} \right. \\ &\qquad \left. + b_{n-1,6} + g_{n,1}\left(k_n\right) + g_{n,2}\left(k_n\right) \right), \\ &\vdots \\ k_2 &\geq \frac{1}{\varpi_2} \left(\varphi_2 + b_{2,3} + b_{2,4} + b_{2,6} + g_{3,1}\left(k_3\right) + g_{3,2}\left(k_3\right) \right), \end{aligned} \quad (28)$$

one leads to

$$\mathscr{L}V \leq -\sum_{j=1}^{n} l_j \eta_j^{4l_0} - \sum_{j=2}^{n} \varphi_j e_j^{4l_0} \quad (29)$$

with positive constants $\varphi_2, \dots, \varphi_n$.

By denoting $\Phi = (x_1, \dots, x_n, \hat{\eta}_2, \dots, \hat{\eta}_n)^T$, the closed-loop system (13), (16), and (17) can be rewritten as

$$d\Phi = E(\Phi)\, dt = \left[x_2^{p_1}, \dots, x_{n-1}^{p_{n-2}}, u^{p_n}(\hat{x}), \dot{\hat{\eta}}_2, \dots, \dot{\hat{\eta}}_n \right] dt, \quad (30)$$

which is homogeneous of degree τ with respect to $\Delta = (\underbrace{r_1, \dots, r_n}_{\text{for } x_1,\dots,x_n}, \underbrace{r_1, \dots, r_{n-1}}_{\text{for } \hat{\eta}_2,\dots,\hat{\eta}_n})$. Following a similar way in [16], it can be shown that

$$V(\Phi) = \sum_{j=1}^{n} \int_{x_j^*}^{x_j} \left(s^{\mu_0/r_j} - x_i^{*\mu_0/r_j}\right)^{q_j/\mu_0} ds + \sum_{j=2}^{n} \int_{\lambda_j^{q_{j-1}/r_{j-1}}}^{x_j^{q_{j-1}/r_j}} \left(s^{r_{j-1}/q_{j-1}} - \lambda_j\right) ds \quad (31)$$

is homogeneous of degree $4l_0\mu_0 - \tau$ about Δ.

4.3. Stability Analysis. To state the main result in this paper, we first introduce the following coordinate transformation:

$$x_1 = z_1, \qquad x_i = \frac{z_i}{L^{s_i}}, \qquad u^{p_n} = \frac{v^{p_n}}{L^{s_n+1}}, \quad (32)$$

where $s_1 = 0$, $s_i = (s_{i-1} + 1)/p_{i-1}$, $i = 2, \dots, n$, $0 < L < 1$ is a constant to be determined later.

It follows from (32) that the system (9) can be rewritten as

$$\begin{aligned} dx_1 &= Lx_2^{p_1} dt + f_1(x, x(t-\sigma(t))) + g_1^T(x, x(t-\sigma(t)))\, d\omega, \\ dx_2 &= Lx_3^{p_2} dt + \frac{f_2(x, x(t-\sigma(t)))}{L^{s_2}} + \frac{g_2^T(x, x(t-\sigma(t)))}{L^{s_2}} d\omega, \\ &\vdots \\ dx_n &= Lu^{p_n} + \frac{f_n(x, x(t-\sigma(t)))}{L^{s_n}} + \frac{g_n^T(x, x(t-\sigma(t)))}{L^{s_n}} d\omega, \\ y &= z_1. \end{aligned} \quad (33)$$

Next, we construct a scaled homogeneous observer

$$\begin{aligned} \dot{\hat{\eta}}_2 &= -Lk_2\hat{x}_2^{p_1}, & \hat{x}_2 &= [\hat{\eta}_2 + k_2\hat{x}_1]^{r_2/r_1}, \\ \dot{\hat{\eta}}_3 &= -Lk_3\hat{x}_3^{p_2}, & \hat{x}_3 &= [\hat{\eta}_3 + k_3\hat{x}_2]^{r_3/r_2}, \\ &\vdots & &\vdots \\ \dot{\hat{\eta}}_n^* &= -Lk_n\hat{x}_n^{p_{n-1}}, & \hat{x}_n &= [\hat{\eta}_n + k_n\hat{x}_{n-1}]^{r_n/r_{n-1}}, \end{aligned} \quad (34)$$

where $k_2 > 0, \dots, k_n > 0$ are the gains selected in (28) and the controller u is designed with the same construction of (17). The closed-loop system (33), (34), and (17) can be expressed as

$$d\Phi = LE(\Phi)\, dt + F dt + G^T d\omega, \quad (35)$$

where $F = (f_1, f_2/L^{s_2}, \dots, f_n/L^{s_n}, 0, \dots, 0)$, $G = (g_1, g_2/L^{s_2}, \dots, g_n/L^{s_n}, 0, \dots, 0)$. Based on the above discussion, we are ready to prove the main result of this paper.

Theorem 20. *Under Assumptions 10 and 11, there is an output-feedback controller $v^{p_n} = u^{p_n}L^{s_n+1}$ such that the closed-loop system consisting of (9), (16), and (17) has a global unique solution and the equilibrium $x = 0$ is GAS in probability.*

Proof. We prove Theorem 20 as follows.

Step 1. From (17), for $j = 1, \dots, n$, one obtains

$$\frac{\partial u^{p_n}(\hat{x})}{\partial \hat{x}_j} = -\frac{r_n + \tau}{r_j}\overline{\beta}_j \hat{x}_j^{\mu_0/r_j - 1} \times \left(\overline{\beta}_n \hat{x}_n^{\mu_0/r_n} + \overline{\beta}_{n-1}\hat{x}_{n-1}^{\mu_0/r_{n-1}} + \cdots + \overline{\beta}_1 \hat{x}_1^{\mu_0/r_1}\right)^{(r_n+\tau)/\mu_0 - 1}. \quad (36)$$

By Lemma 12, we know $\mu_0/r_j - 1 \ge 0$, $(r_n + \tau)/\mu_0 - 1 \ge 0$, which implies that $u^{p_n}(\hat{x})$ is C^1. Therefore, the closed-loop system satisfies the locally Lipschitz condition.

Step 2. Construct a Lyapunov-Krasovskii functional

$$W(\Phi) = V(\Phi) + \frac{L^{1+\varepsilon_0}(a_0 + b_0)}{1 - \sigma} \int_{t-\sigma(t)}^{t} \|\Phi(s)\|_{\Delta}^{4l_0\mu_0} ds, \quad (37)$$

which is positive definite, proper, and C^2 on Φ, where ε_0, a_0, b_0 are positive parameters to be determined later. According to Lemma 4.3 in [33], there are two $\mathcal{K}_\infty$ functions α_1 and α_{21} such that

$$\alpha_1(|\Phi(t)|) \le V(\Phi) \le \alpha_{21}(|\Phi(t)|). \quad (38)$$

Then, with the notation in [26], it is easy to verify that

$$\frac{L^{1+\varepsilon_0}(a_0 + b_0)}{1 - \sigma} \int_{t-\sigma(t)}^{t} \|\Phi(s)\|_{\Delta}^{4l_0\mu_0} ds \le \alpha_{22}\left(\sup_{-\sigma_0 \le s \le 0} |\Phi(t+s)|\right), \quad (39)$$

where α_{22} is a $\mathcal{K}_\infty$ function. Because $\alpha_{21}(|\Phi(t)|) \le \alpha_{21}(\sup_{-\sigma_0 \le s \le 0}|\Phi(t+s)|)$, then

$$\alpha_1(|\Phi(t)|) \le W(\Phi) \le \alpha_2\left(\sup_{-\sigma_0 \le s \le 0} |\Phi(t+s)|\right) \quad (40)$$

with $\alpha_2 = \alpha_{21} + \alpha_{22}$.

Step 3. Since $V(\Phi)$ and $E(\Phi)$ are homogenous of degree $4l_0\mu_0 - \tau$ and τ, respectively, by Lemmas 5 and 6, one has

$$\frac{\partial W(\Phi)}{\partial X} LE(\Phi) = \frac{\partial V(\Phi)}{\partial X} LE(X) \le -L\gamma_1 \|\Phi(s)\|_{\Delta}^{4l_0\mu_0}, \quad (41)$$

where $\gamma_1 > 0$ is a constant.

Using Assumption 10 and the new coordinates in (32), we have

$$\left|\frac{f_i}{L^{s_i}}\right| \le a_i \sum_{j=i+2}^{n+1} \left(\frac{\left|L^{s_j} x_j(t)\right|^{(r_i+\tau)/r_j} + \left|L^{s_j} x_j(t-\sigma(t))\right|^{(r_i+\tau)/r_j}}{L^{s_i}} \right) \tag{42}$$

and the power of L in (42) is

$$\begin{aligned} s_j \frac{r_i+\tau}{r_j} - s_i &= 1 + \frac{s_j(r_i+\tau) - (s_i+1) r_j}{r_j} \\ &= 1 + \frac{p_i}{r_j}\left(s_j r_{i+1} - s_{i+1} r_j\right). \end{aligned} \tag{43}$$

Owing to $s_1 = 0$, $s_i = (s_{i-1}+1)/p_{i-1}$, one has

$$s_i = \sum_{s=1}^{i-1} \frac{1}{p_s \cdots p_{i-1}}, \quad i = 2, \ldots, n. \tag{44}$$

For $j \ge i+2$, (44) gives

$$\begin{aligned} s_j r_{i+1} - s_{i+1} r_j &= s_j \left(\frac{1}{p_1 \cdots p_i} + s_{i+1}\tau \right) - s_{i+1}\left(\frac{1}{p_1 \cdots p_{j-1}} + s_j \tau \right) \\ &= \frac{s_j}{p_1 \cdots p_i} - \frac{s_{i+1}}{p_1 \cdots p_{j-1}} > 0. \end{aligned} \tag{45}$$

In summary, it can be concluded that there exists a positive constant

$$\varepsilon_1 = \min\left\{ \frac{p_i}{r_j}\left(s_j r_{i+1} - s_{i+1} r_j\right) : 1 \le i \le n-1, i+2 \le j \le n \right\} \tag{46}$$

such that

$$\begin{aligned} \left|\frac{f_i}{L^{s_i}}\right| &\le a_i L^{1+\varepsilon_1} \sum_{j=i+2}^{n+1} \left(\left|x_j(t)\right|^{(r_i+\tau)/r_j} + \left|x_j(t-\sigma(t))\right|^{(r_i+\tau)/r_j} \right) \\ &\le \widehat{a}_i L^{1+\varepsilon_1} \left(\|\Phi(t)\|_\Delta^{r_i+\tau} + \|\Phi(t-\sigma(t))\|_\Delta^{r_i+\tau} \right) \end{aligned} \tag{47}$$

with $\widehat{a}_i > 0$. According to Lemma 6, we deduce that

$$\begin{aligned} \left|\frac{\partial V(\Phi)}{\partial \Phi} F\right| &\le \sum_{i=1}^{n-1} \left|\frac{\partial V(\Phi)}{\partial \Phi_i}\right| \left|\frac{f_i}{L^{s_i}}\right| \\ &\le \sum_{i=1}^{n-1} \widehat{a}_i L^{1+\varepsilon_1} \|\Phi(t)\|_\Delta^{4l_0\mu_0 - r_i - \tau} \\ &\quad \times \left(\|\Phi(t)\|_\Delta^{r_i+\tau} + \|\Phi(t-\sigma(t))\|_\Delta^{r_i+\tau} \right) \\ &\le a_0 L^{1+\varepsilon_1} \left(\|\Phi(t)\|_\Delta^{4l_0\mu_0} + \|\Phi(t-\sigma(t))\|_\Delta^{4l_0\mu_0} \right), \end{aligned} \tag{48}$$

where a_0 is a positive constant.

Similarly, there is a positive constant

$$\varepsilon_2 = \min\left\{ \frac{s_j(2r_i+\tau) - (2s_i+1) r_j}{2r_j} : 1 \le i \le n-1, \; i+2 \le j \le n \right\}, \tag{49}$$

such that

$$\begin{aligned} \left|\frac{g_i}{L^{s_i}}\right| &\le b_i L^{1/2+\varepsilon_2} \\ &\quad \times \sum_{j=i+2}^{n+1} \left(\left|\Phi_j(t)\right|^{(2r_i+\tau)/2r_j} + \left|\Phi_j(t-\sigma(t))\right|^{(2r_i+\tau)/2r_j} \right) \\ &\le \widehat{b}_i L^{1/2+\varepsilon_2} \left(\|\Phi(t)\|_\Delta^{r_i+\tau/2} + \|\Phi(t-\sigma(t))\|_\Delta^{r_i+\tau/2} \right) \end{aligned} \tag{50}$$

with $\widehat{b}_i > 0$. Then,

$$\begin{aligned} &\frac{1}{2} \operatorname{Tr}\left\{ G \frac{\partial^2 V(\Phi)}{\partial \Phi^2} G^T \right\} \\ &\le \frac{1}{2} m\sqrt{m} \sum_{k,l=1}^{n-1} \left|\frac{\partial^2 V(\Phi)}{\partial \Phi_k \partial \Phi_l}\right| \left|\frac{g_k}{L^{s_k}}\right| \left|\frac{g_l}{L^{s_l}}\right| \\ &\le \widetilde{b} L^{1+2\varepsilon_2} \\ &\quad \times \sum_{k,l=1}^{n-1} \|\Phi(t)\|^{4l_0\mu_0 - \tau - r_k - r_l} \\ &\quad \times \left(\|\Phi(t)\|_\Delta^{r_k+\tau/2} + \|\Phi(t-\sigma(t))\|_\Delta^{r_k+\tau/2} \right) \\ &\quad \times \left(\|\Phi(t)\|_\Delta^{r_l+\tau/2} + \|\Phi(t-\sigma(t))\|_\Delta^{r_l+\tau/2} \right) \\ &\le b_0 L^{1+2\varepsilon_2} \left(\|\Phi(t)\|_\Delta^{4l_0\mu_0} + \|\Phi(t-\sigma(t))\|_\Delta^{4l_0\mu_0} \right), \end{aligned} \tag{51}$$

where $\widetilde{b}$, b_0 are positive constants.

By choosing $\varepsilon_0 = \min\{\varepsilon_1, 2\varepsilon_2\}$ and Definition 1, Assumption 11, (41), (48), and (51), one can obtain

$$\begin{aligned} \mathscr{L}W(\Phi) &\le \frac{\partial W(\Phi)}{\partial X} LE(\Phi) + \frac{\partial V(\Phi)}{\partial X} F \\ &\quad + \frac{1}{2} \operatorname{Tr}\left\{ G \frac{\partial^2 V(\Phi)}{\partial \Phi^2} G^T \right\} + L^{1+\varepsilon_0}(a_0 + b_0) \\ &\quad \times \left(\frac{1}{1-\sigma} \|\Phi(t)\|_\Delta^{4l_0\mu_0} - \|\Phi(t-\sigma(t))\|_\Delta^{4l_0\mu_0} \right) \\ &\le -L\left(\gamma_1 - L^{\varepsilon_0}\left(a_0 + b_0 + \frac{a_0+b_0}{1-\sigma} \right) \right) \|\Phi(t)\|_\Delta^{4l_0\mu_0}, \end{aligned} \tag{52}$$

where $0 < L < 1$. Apparently, by choosing $0 < L < \min\{(\gamma_1/(a_0+b_0+(a_0+b_0)/(1-\sigma)))^{1/\varepsilon_0}, 1\}$, there is a constant δ, such that $\mathscr{L}W(\Phi) \le -\delta\|\Phi(t)\|_\Delta^{4l_0\mu_0}$.

From Steps 1 to 3 and Lemma 4, it is obtained that the closed-loop system (33), (34), and (17) has a global unique solution and the equilibrium $\Phi = 0$ is GAS in probability.

Step 4. Because of the equivalence in (32), then there is a global unique solution for the closed-loop system consisting of (9), (16), and (17) and $v^{p_n} = u^{p_n} L^{s_n+1}$ and the equilibrium $z = 0$ is GAS in probability. □

Remark 21. Using the adding one power integrator method and the homogeneous domination technique, [34] has designed a state-feedback stabilizer for a class of stochastic high-order feedforward nonlinear systems. However, to deal with the unmeasurable states and time-delay terms in this paper, we choose a reduced-order homogeneous observer and an appropriate Lyapunov-Krasovskii functional, which is not an easy work.

5. Conclusions

For a class of stochastic high-order feedforward nonlinear systems with different power orders and time-varying delay, an output-feedback stabilizer has been designed by virtue of the adding one power integrator technique and homogeneous domination approach. It globally stabilizes the origin of the closed-loop system. The proof of stability we have adopted depends on the construction of a Lyapunov-Krasovskii functional.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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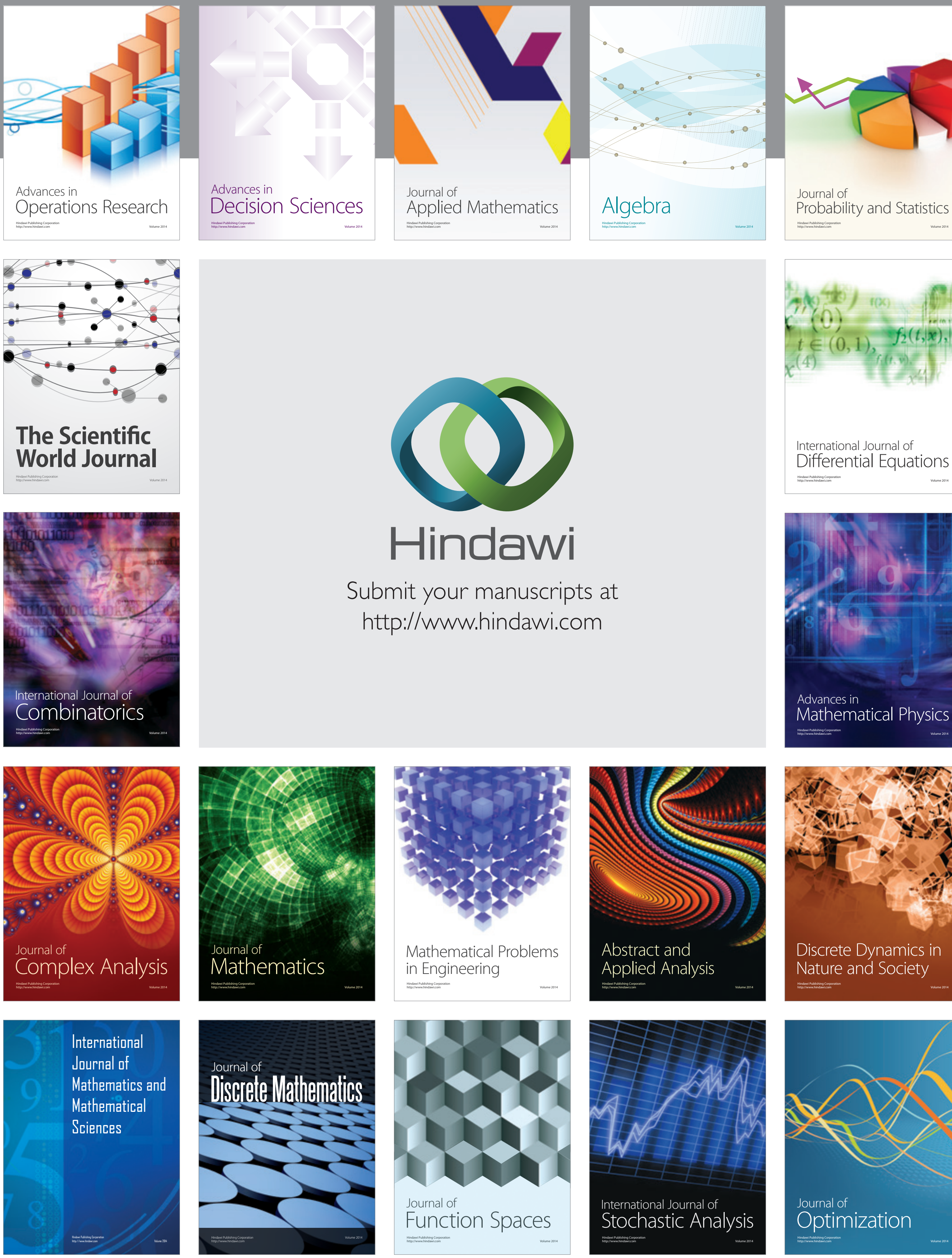
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