

## Research Article

# A Method of Data Recovery Based on Compressive Sensing in Wireless Structural Health Monitoring

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In practical structural health monitoring (SHM) process based on wireless sensor network (WSN), data loss often occurs during the data transmission between sensor nodes and the base station, which will affect the structural data analysis and subsequent decision making. In this paper, a method of recovering lost data in WSN based on compressive sensing (CS) is proposed. Compared with the existing methods, it is a simple and stable data recovery method and can obtain lower recovery data error for one-dimensional SHM's data loss. First, response signal  $x$  is measured onto the measurement data vector  $y$  through inner products with random vectors. Note that  $y$  is the linear projection of  $x$  and  $y$  is permitted to be lost in part during the transmission. Next, when the base station receives the incomplete data, the response signal  $x$  can be reconstructed from the data vector  $y$  using the CS method. Finally, the test of active structural damage identification on LF-21M aviation antirust aluminum plate is proposed. The response signal gathered from the aluminum plate is used to verify the data recovery ability of the proposed method.

## 1. Introduction

In wireless sensor networks for structural health monitoring [1–3], a large number of sensor nodes are deployed in the monitoring area [4] to implement data sensing and data acquisition in real time. Such information of structural health status is sent to the base station for users to make decisions. However, data loss in wireless sensor networks is common and it is heavily affected by hardware (such as faulty sensors), network communication interference (such as noise, collision and unreliable link during the communication), wireless conditions (such as WSN's scale), and so on. In particular, in SHM, complex structures and harsh environments often lead to continuous data loss or random data loss during the data transmission. Such imperfect data will affect the accuracy of identification of structural damage and thus will lead to wrong decisions. For the influence of lost data on structural analysis, Nagayama et al. [5] carried out an experimental study on the imote2 based SHM\_A platform. Their experimental results show that the loss of 0.5 percent

of data affects the coherence function in a similar way as 5 to 10 percent measurement noise addition. They also explain that a loss of 0.5 percent data might be acceptable, considering that corresponding 5 to 10 percent observation noise is unexceptional in SHM. However, due to limited resources and geographical location, data loss rate during transmission is up to 20% even reaches and 86%. Now, data loss is a critical problem in wireless sensor network based structural health monitoring.

To solve the problem, some data recovery methods [6–8] were proposed. Aktan et al. [6] used linear regression method and average method to realize the lost data recovery. These methods have a large data recovery error, as well as impractical. Hu et al. [7] presented a method of radical basis function (RBF) neural networks to restore the bridge deflection data loss. Zhao et al. [8] proposed the data restoring method by using back propagation (BP) neural networks to solve the problem of strain monitoring data loss in performance monitoring of large-span steel sky bridge. Although RBF or

BP neural networks can predict unknown lost data, it is difficult to choose the appropriate neural network model. Even for the same monitoring area, the approach to establish a neural network model is not the same from different angles. To overcome the disadvantages of the above methods, a powerful and generic technique for estimating missing data based on compressive sensing is proposed. The existing methods based on CS [9–11] can recover an entire dataset from only a small fraction of data. Kadhe et al. [9] integrated the emerging framework of CS with real expander codes for reliably transmitting image data in multimedia sensor networks. Pudlewski et al. [10] presented a system which uses CS to encode, compress, and protect an image from channel errors and packet losses. Although they can realize the reliable data transmission for two-dimensional image data, the two methods can not be directly applied to SHM's data loss. Since the collected data from SHM is real-time one-dimensional data by high-frequency sampling and is different from image data, a new method for estimating one-dimensional lost data should be studied in SHM. Charbiwala et al. [11] explored the application of CS to handle data loss from erasure channels by viewing it as a low encoding-cost, proactive, and erasure correction scheme. But the method has a relatively large recovery data error and can not satisfy the requirements of SHM. In this paper, we proposed a simple and stable data recovery approach based on CS which can obtain lower recovery data error for one-dimensional SHM's data loss. Instead of transmitting response signal, the CS method transfers the linear measurement data between sensor nodes and base station in WSN. The linear measurement data, which is allowed to be lost in part during the transmission, can be reconstructed into response signal in the base station.

The rest of this paper is organized as follows. In Section 2, related works of the CS and introduction of the sparse representation in data recovery method are presented. In Section 3, the procedure of lost data recovery is introduced. Experiments on perforated LF-21M aluminum plate are provided in Section 4 to verify the effectiveness of the proposed method. Summaries are covered in Section 5.

## 2. Related Work

*2.1. A Summary for CS.* Compressive sensing (CS) provides an alternative to Shannon/Nyquist sampling when signal under acquisition is known to be sparse or compressible [12, 13]. Mainly, CS theory includes three parts: the sparse representation of the signal, the sensing matrix ensuring the data minimal information loss, and the reconstruction algorithm using the no-distortion observed value to reconstruct signals.

In the process of sparse representation, signals are measured through inner products with random vectors and thus fewer measurements than periodic samples are needed. Suppose that  $x$  is original signal,  $y$  is measurement signal, and  $\hat{x}$  is reconstructed signal from  $y$ . In particular,  $x$  in SHM is also called response signal. For any  $N$ -dimensional response signal  $x$ , its measurements  $y$  is taken as follows:

$$y = \Phi x, \quad (1)$$

where  $x \in R^N$  is  $N$ -dimensional response signal,  $y \in R^M$  is  $M$ -dimensional linear measurement data, and  $\Phi \in R^{M \times N}$  ( $M \ll N$ ) is the sensing matrix. Usually, the response signal is not absolutely sparse. If it can be an approximate sparse signal in some transform domains such as Fourier domain or wavelet domain, we considered that it is compressible signal. So, through one of the orthogonal transformations  $\Psi$ , let  $x = \Psi\alpha$ ; we can achieve sparse representation as follows:

$$y = \Phi x = \Phi\Psi\alpha = \Theta\alpha, \quad (2)$$

where  $\Psi \in R^{N \times N}$  is the orthogonal transformations matrix and  $\alpha$  is the  $K$ -sparse decomposition coefficients in the  $\Psi$  transform domain. Note that  $K$  is the number of nonzero values in  $\alpha$  and  $K$  should be a small value. Without loss of generality, we denote the matrix multiplication  $\Phi\Psi$  as a single sensing matrix  $\Theta$ . So, formula (2) can be regarded as the linear projection of original signal  $x$  with  $\Phi$ , and it could be also viewed as the linear projection of transform decomposition coefficients  $\alpha$  in  $\Theta$ . If  $y$  and  $\Theta = \Phi\Psi$  meet with the restricted isometry property (RIP) [14],  $K$ -sparse decomposition coefficients  $\alpha$  can be reconstructed by solving the  $l_0$  norm [15] from  $y$  as follows:

$$\hat{\alpha} = \arg \min \|\alpha\|_0 \quad \text{s.t. } \Theta\alpha = y, \quad (3)$$

where  $\hat{\alpha}$  is the only exact solution of decomposition coefficients  $\alpha$ . Then, the exact solution  $\hat{x}$  can be obtained by reconstructing  $\hat{\alpha}$  under the orthogonal transform basis  $\Psi$  shown as follows:

$$\hat{x} = \Psi\hat{\alpha}. \quad (4)$$

To further demonstrate the intrinsic relationship between  $\Psi$  and  $\Phi$ , while  $\Theta = \Phi\Psi$  met with the RIP, Baraniuk [16] proposed that the equivalence condition of the RIP which is  $\Phi$  irrelevant with  $\Psi$ ; that is, the  $\Theta$  row vector cannot be represented by  $\Psi$  column vector, and the  $\Psi$  row vector cannot be represented by  $\Theta$  column vector. Therefore, we select tectonic  $\Phi$  sensing matrix, such as Gaussian random matrix, in the orthogonal base matrix  $\Psi$  which is fixed to make  $\Theta = \Phi\Psi$  satisfy with RIP in this paper.

In the process of signal reconstruction, the algorithm is mainly divided into three categories which are greedy algorithm [17], convex optimization algorithms [18, 19], and the sparse Bayesian statistical optimization algorithm [20]. The most typical algorithm is matching pursuit (MP) algorithm [18] and orthogonal matching pursuit (OMP) algorithm [19]. Among them, OMP is an efficient reconstruction algorithm in CS for recovering sparse signals despite its high computational cost for solving large scale problems. It is a simple, stable, and fast reconstruction algorithm. In this paper, we used OMP algorithm as the stabled reconstruction method.

*2.2. Lost Data Recovery Method Based on CS.* On the basis of the above CS theory, the lost data recovery method based on CS also contains three parts. The last two parts of the sensing matrix selection and the reconstruction algorithm selection



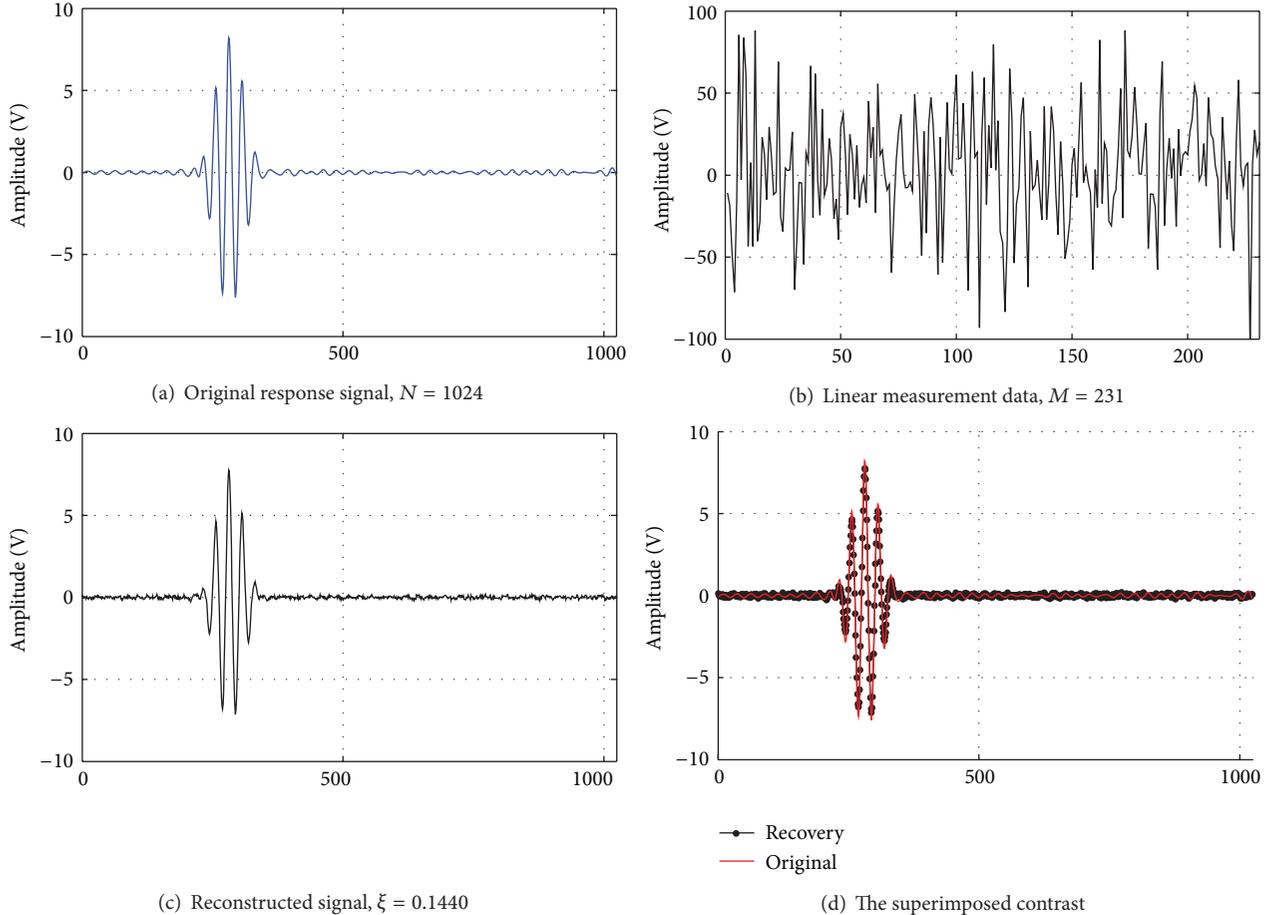


FIGURE 4: The analysis for signal reconstruction based on CS.

$\pm 10$  V and the peak number is five. We used the polling method for data collection. Each of the eight piezoelectric patches is selected as the driver in turn, while the rest of the piezoelectric patches are used as the receiver. Each receiver should collect the data of reflection wave on the direction of  $0^\circ \sim 180^\circ$ . Note that the data of reflection wave is also called original response signal.

A typical original response signal gathered from aluminum plate by sensors is shown in Figure 4(a). The sampling frequency is 1 MHz and collected 1024 points. It can be seen that the response signals are nearly sparse because part of the point is near to zero. Using Gaussian random matrix as sensing matrix, Figure 4(b) is the linear measurement data from original response signal. The reconstructed signal by OMP algorithm is shown in Figure 4(c), and the relative error of the reconstructed signal is  $\xi = 0.1440$ . The superimposed contrast between reconstructed signal and the original one is in Figure 4(d), and the result shows the well reconstruction.

## 4.2. The Analysis of Lost Data Recovery Results

**4.2.1. Reconstruction Error Definition.** In order to evaluate the performance of data recovery method, we define the parameter of the reconstruction error. Reconstruction error ( $\xi$ ) is

on behalf of the similarity degree of the reconstructed signal and the original one. It is an important indicator to measure the effects of data decompression which is written as formula (7), where  $\hat{x}$ ,  $x$  separately indicated the reconstructed signal and the original one. The smaller the reconstruction error is, the higher the data recovery accuracy of the compressed sensing reconstruction algorithm is.

Consider

$$\xi = \frac{\|\hat{x} - x\|_2}{\|x\|_2}. \quad (7)$$

**4.2.2. Loss Data Set and Packet Loss Probability Definition.** In this example, an original response data sequence is defined as  $x(n)$ ,  $n = 1, 2, 3, \dots, N$ . In the practical application of SHM, the performance of structural damage identification may be affected when the length of the collected data is less than 1024. So, the acquisition data in SHM is usually more than 1024; that is,  $N \geq 1024$ . With the increase of the length of  $N$  (such as  $N = 2048$ ), the number of signal's zero value increases and the performance of the proposed method based on CS is getting better. Therefore, in order to satisfy the requirements of SHM, we take the lower limit of the length  $N = 1024$ . Keep

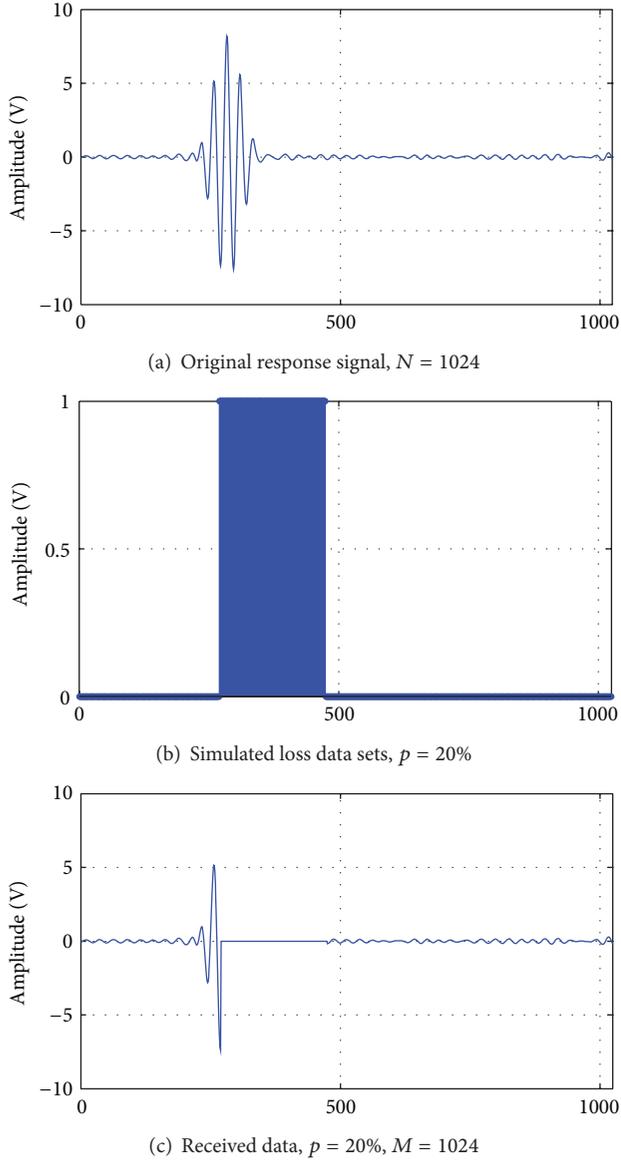


FIGURE 5: Response data with 20% continuous data loss in transmission.

the length of linear measurement data same as the original data; that is,  $M = N = 1024$ . Therefore, there is no any increasing data acquisition cost for the proposed loss data recovery method.

In the actual transmission, the packet loss probability of data can not be accurately controlled. Therefore, a simulation data loss is proposed to verify the feasibility of the data recovery method. To simulate the data loss process in data transmission, including continuous data loss and random data loss, we design a loss data set  $q(n)$ ,  $n = 1, 2, \dots, N$ , which is shown in Figures 5(b) and 6(b). Among them, Figure 5(b) is a continuous loss data set and Figure 6(b) is a random one. The value of  $q(n)$  is always equal to zero or one. Replacing the original response signal with zero value in the data loss position, then the received data  $x_L(n)$  on base

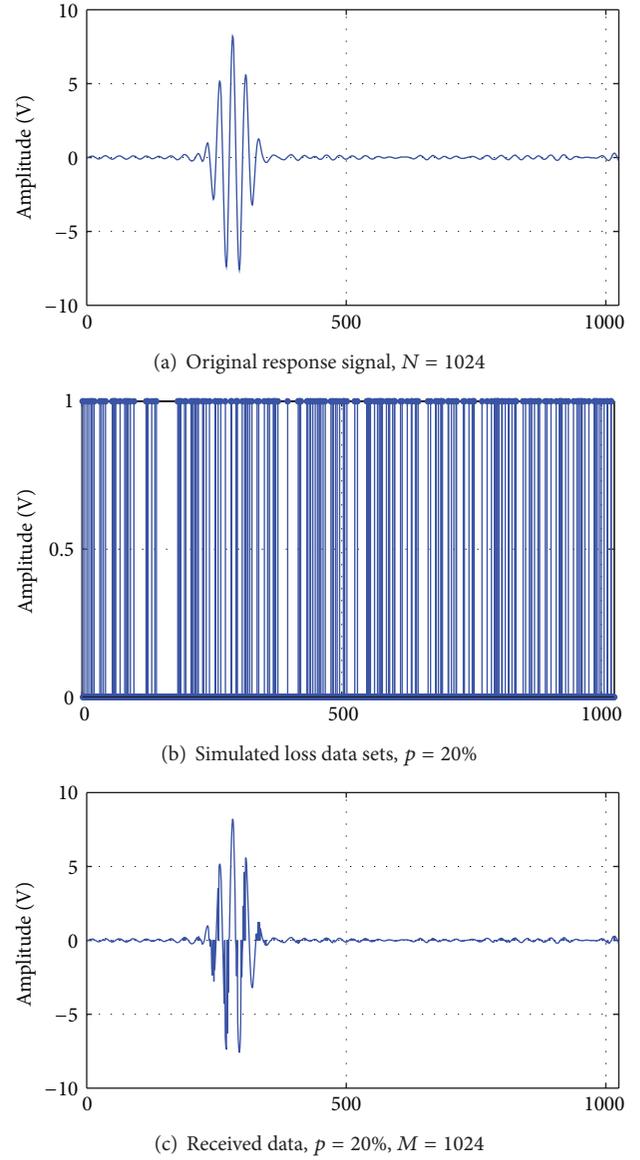


FIGURE 6: Response data with 20% random data loss in transmission.

station are shown in Figures 5(c) and 6(c). Such process of simulation data loss can be presented as formula (8), where  $x(n)$  is the original response signal and  $x_L(n)$  is the received data with data loss in part during transmission:

$$x_L(n) = x(n) \times (1 - q(n)), \quad n = 1, 2, 3, \dots, N. \quad (8)$$

According to the loss data set  $q(n)$ , the packet loss probability  $p$  can be defined as the ratio of loss data number and response data length, which is written as formula (8), where  $N$  is the length of original response data sequence, and here,  $N = 1024$ . The formula  $\sum_{n=1}^N q(n)$  is the number of sequence points whose value is equal to one in loss data set. To investigate the loss data recovery ability, the packet loss probability is  $p = (0.05, 0.1, 0.15, 0.2, \dots, 0.4)$  in the simulation.

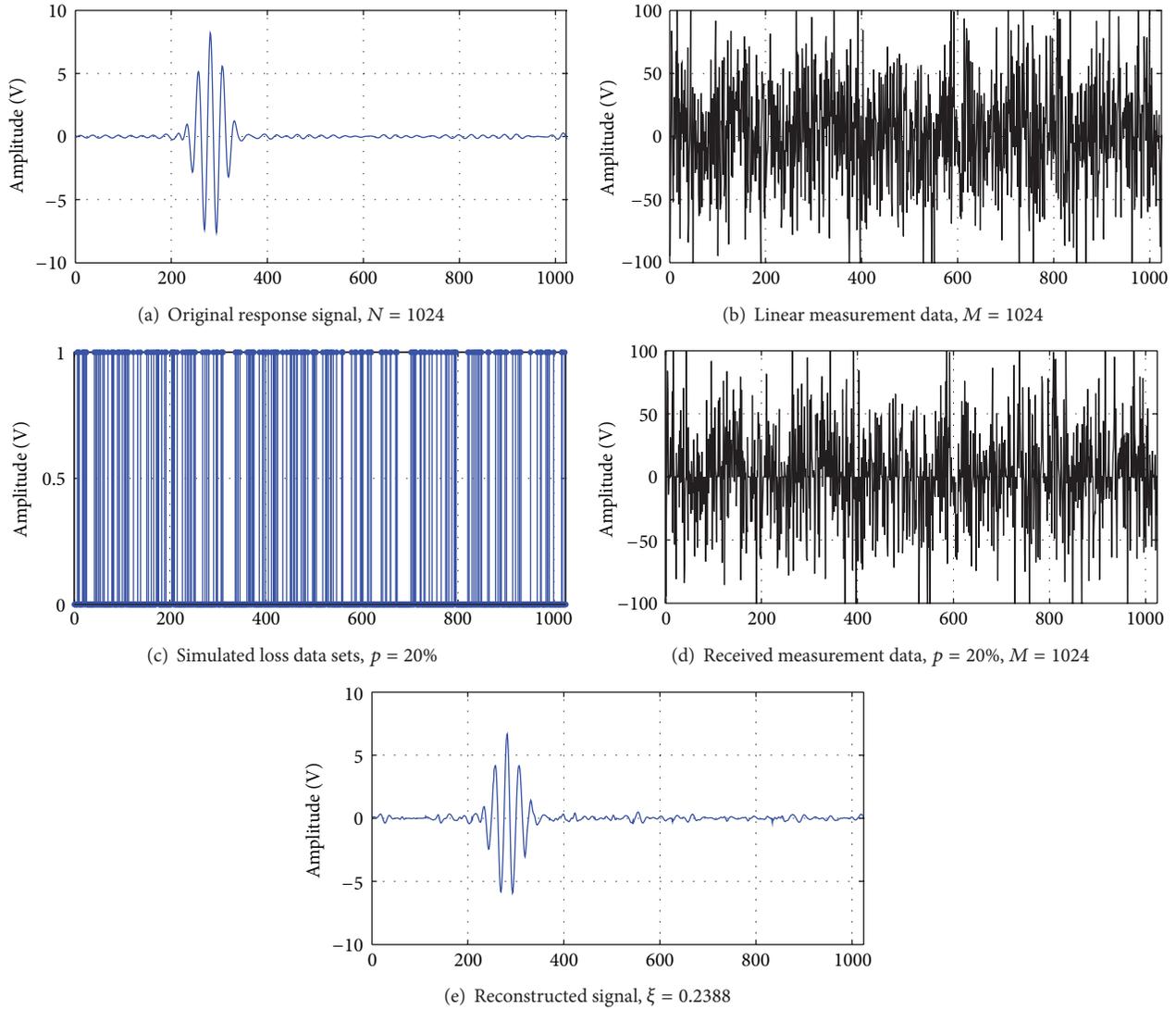


FIGURE 7: Data recovery with 20% random data loss.

Consider

$$p = \frac{\sum_{n=1}^N q(n)}{N}. \quad (9)$$

The simulated data loss process is shown in Figures 5 and 6. Figure 5 describes the process of response signal continuous loss 20% data in data transmission, where Figure 5(a) is the original response signal and its length is 1024. When there is a response signal with 1024 length continuous loss 20% data, the length of loss data is  $M_L = \lfloor 1024 \times 20\% \rfloor = 205$ , where the  $\lfloor \cdot \rfloor$  is the function that can round up the value to integer. Figure 5(b) is the location of 20% selected lost data in loss data set; the data sequence number from 270 to 475 is lost. Figure 5(c) is the received data with 20% continuous data loss on base station. Figure 6 is the process of response data with 20% random data loss and its process is similar as in Figure 5. Without loss of generality,

we will choose the random data loss in the next experiment analysis.

**4.2.3. Random Data Loss in Part with Fixed  $p = 0.20$ .** To illustrate the procedure of the loss data recovery, an example of data recovery based on CS with 20% data random loss is shown in Figure 7. Figure 7(a) shows the original response data  $x$  with 1024 sequence points. The linear measurement data  $y$  is calculated by  $y = \Phi x$  and will be sent to base station, as shown in Figure 7(b), where the sensing matrix  $\Phi$  is a Gaussian random matrix with zero mean and unit variance. Figure 7(c) is a loss data set of location  $q(n)$  with 20% randomly data loss of  $y$ . After the base station received the data  $\hat{y}$  with random data loss in part, as shown in Figure 7(d), it can reconstruct and recover the response signal. The reconstructed data  $\hat{x}$  can be calculated by formula

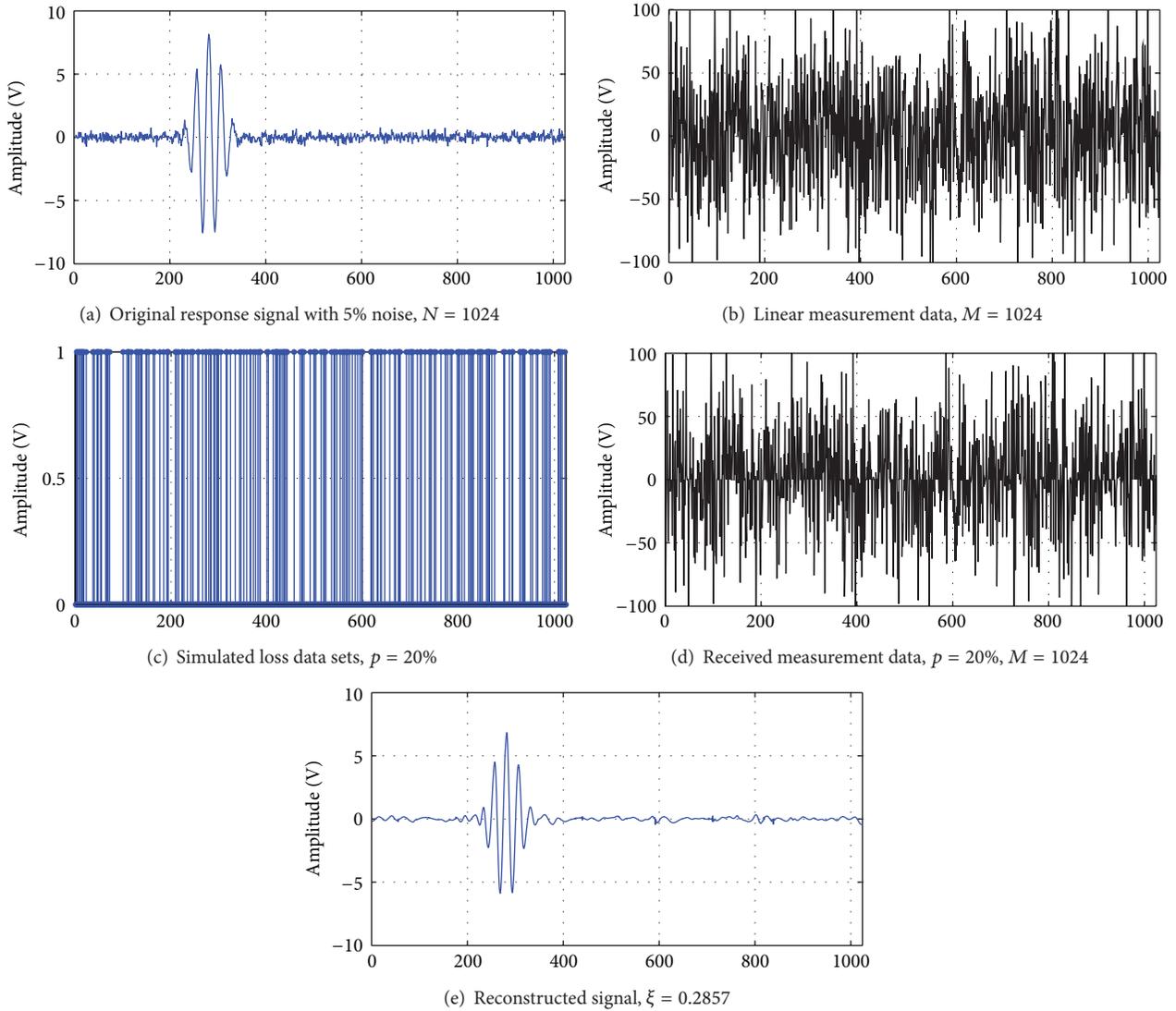


FIGURE 8: Data recovery with 20% random data loss and with 5% noise.

(4) and the result with reconstruction error  $\xi = 0.2388$  is shown in Figure 7(e).

During the experiment of our SHM, 5% or less than 5% noise corruption may be found. To verify immunity and robustness of the proposed method to noise, the noise corruption on the transform data  $y$  also be considered besides considering the data random loss. Considering the additional 5% noise corruption on the original response data, the result is shown in Figure 8 and the reconstruction error  $\xi$  is 0.2857.

The results of the two experiments show that when the packet loss probability is fixed at 20%, the proposed method has good effect in random lost data recovery.

**4.2.4. Random Data Loss in Part with Different  $p$ .** The investigation of data recovery above is in the fixed packet loss probability, but different parameter of  $p$  will produce different

effect on the recovery. To further analyze the performance of the proposed method, we change the range of  $p$  value from 0.05 to 0.4 and verify the ability of data recovery method at different  $p$ .

The results of two experiments, including with 5% noise method and without noise method, are shown in Figure 9. The trend lines of reconstruction error in Figure 9 show that with the increase of  $p$ , the reconstruction errors are rising steadily. Overall, the error values of with 5% noise method are higher than the method without noise. Within a range of  $p \in [0.05, 0.4]$ , the former method has an error value range between 0.1893 and 0.4859, while the latter method has a minimum error value of 0.1185 and a maximum of 0.4999. The experimental results show that the proposed method has good recovery performance on random data loss at different packet loss probability. In the practical application of SHM, the reconstruction errors  $\xi$  should be less than 0.3 so as to satisfy the engineering requirement. Therefore, the  $p$  must be

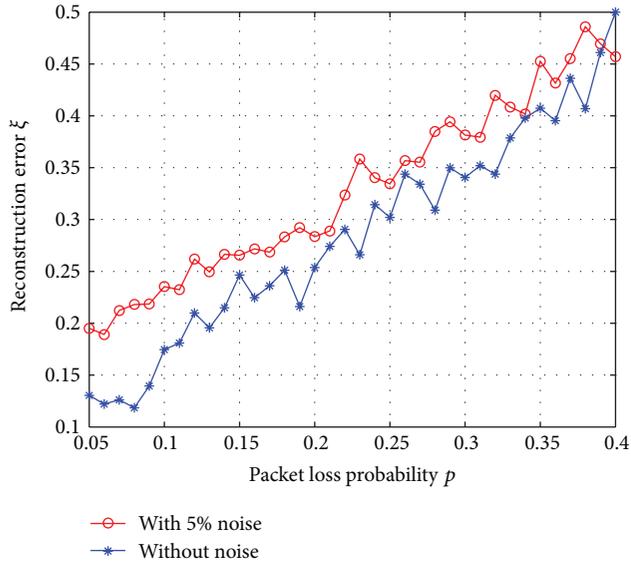


FIGURE 9: Reconstruction error at different packet loss probability in random data loss in part.

less than 0.23 in the method without noise and must be less than 0.21 in the method with 5% noise, which is shown in Figure 9.

## 5. Conclusions

This paper proposed a novel method based on CS for loss data recovery in wireless structural health monitoring. First, the original response signal was measured by a random Gaussian sensing matrix so as to generate a linear measurement data vector, where data loss in part is allowed in wireless data transmission. Secondly, the response signal is reconstructed by linear measurement data with part loss by OMP reconstruction algorithm. Finally, an example of the wireless sensor data measured from a real LF-21M aluminum plate is collected so as to illustrate the data recovery ability of the proposed method. Experiments results show that the proposed data recovery method can recover signals with data loss in part and resist to the additional noise corruption during the data transmission.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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