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Research Article

Disturbance Attenuation via Output Feedback for Uncertain Nonlinear Systems with Output and Input Depending Growth Rate

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The problem of output feedback disturbance attenuation is investigated for a class of uncertain nonlinear systems. The uncertainties of the considered systems are bounded by unmeasured states with growth rate function of output and input multiplying an unknown constant. Based on a dynamic gain observer, an adaptive output feedback controller is proposed such that the states of the closed-loop system are globally bounded, and the disturbance attenuation is achieved in the L_2-gain sense. An example is provided to demonstrate the effectiveness of the proposed design scheme.

1. Introduction

The problem of output feedback stabilization is one of the important problems in the field of nonlinear control and has attracted much attention [1–3]. In particular, the output feedback control design has received great attention for nonlinear systems with linearly bounded unmeasurable states in [4–10] recently. For the class of systems that are bounded by a low-triangular-type condition, when the growth rate is a polynomial function of output multiplying an unknown constant, the design of output feedback controller was proposed in [7]. Furthermore, when the growth function depends polynomially on input and output, the problem of global output feedback regulation was investigated in [9]. For feedforward nonlinear time-delay systems satisfying linear growth condition, the problem of global stabilization by state feedback and output feedback was studied in [10].

On the other hand, disturbance attenuation of nonlinear systems is a very meaningful problem for both control theory and applications. And the problem of almost disturbance decoupling for nonlinear systems has received considerable attention during the past decades. Several researchers have presented various approaches for the problems of disturbance attenuation of nonlinear systems with different forms and assumptions in [11–15]. For a class of nonlinear systems depending on unmeasured states with an unknown constant or polynomial-of-output growth rate, the problems of adaptive disturbance attenuation via output feedback were considered in [13, 15]. However, up to now, for a class of feedforward uncertain nonlinear systems with linearly bounded unmeasurable states, the problem of global disturbance attenuation by output feedback has seldom been studied.

Motivated by [7, 9, 10, 13, 15], in this paper, we consider the problem of output feedback disturbance attenuation for a class of uncertain nonlinear systems. The main contribution of this paper lies in the following. (i) For a class of feedforward uncertain nonlinear systems with linearly bounded unmeasurable states, an adaptive output feedback controller is proposed such that the states of the closed-loop system are globally bounded, and the disturbance attenuation is achieved in the L_2-gain sense. (ii) The assumptions in [8, 10] are relaxed; see Remark 2.

Notation 1. In this paper, $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{R}^n$ denote the set of real numbers, the set of nonnegative real numbers, and the set of real n-dimensional column vectors, respectively. I denotes the identify matrix with appropriate dimension. For a vector

or a matrix X, X^T denotes its transpose, and $\|\cdot\|$ denotes the Euclidean norm of a vector or the corresponding induced norm of a matrix. For a symmetric matrix P, $\lambda_{\min}(P)$ denotes its smallest eigenvalue. $L_2[0,T)$ and $L_\infty[0,T)$ denote the appropriate dimension space of square integrable functions on $[0,T)$ and the appropriate dimension space of uniformly bounded functions on $[0,T)$, respectively, where $0 < T \leq +\infty$. We define $\sum_{j=k}^{i} x_j = 0$, for $\forall k \geq i+1$.

2. Problem Statement and Preliminaries

Consider a class of nonlinear systems that can be written in the following form:

$$\begin{aligned}
\dot{x}_1 &= x_2 + f_1(t,x,u) + g_1^T(t,x,u)\,w,\\
&\vdots\\
\dot{x}_{n-1} &= x_n + f_{n-1}(t,x,u) + g_{n-1}^T(t,x,u)\,w,\\
\dot{x}_n &= u + f_n(t,x,u) + g_n^T(t,x,u)\,w,\\
y &= x_1,
\end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, $u \in \mathbb{R}$, and $y \in \mathbb{R}$ are the system state, control input, and output, respectively. $w \in \mathbb{R}^s$ is disturbance satisfying $w(t) \in L_2[0,+\infty)$. The functions $f_i : \mathbb{R}^+ \times \mathbb{R}^{n+1} \to \mathbb{R}$, and $g_i : \mathbb{R}^+ \times \mathbb{R}^{n+1} \to \mathbb{R}^s$, $i = 1, \ldots, n$, are locally Lipschitz continuous with respect to all the variables; $f_n(t,x,u) = 0$. And $f_i(t,x,u)$, $g_i(t,x,u)$ satisfy the following assumption.

Assumption 1. There exist an unknown constant $\theta > 0$, a known constant $p \in [0, 1/(2n))$, and a known continuous function $h : \mathbb{R} \to \mathbb{R}^+$ such that

$$\begin{aligned}
&|f_i(t,x,u)| \leq \theta\left(1 + |y|^p\right) h(u)\left(\sum_{j=i+2}^{n} |x_j| + |u|\right),\\
&|g_i(t,x,u)| \leq \theta\left(1 + |y|^p\right) h(u), \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{2}$$

Remark 2. An adaptive output feedback controller to globally stabilise system (1) satisfying (2) with $h(u) = 1$ and $p = w(t) = 0$ was proposed in [8]. The problem of output feedback stabilization for system (1) satisfying (2), where $\theta > 0$ is a known constant and $p = w(t) = 0$, was solved in [10]. However, to the best of our knowledge, the problem of output feedback disturbance attenuation has not been investigated for system (1) satisfying (2).

In this paper, our objective is to design, under Assumption 1, an adaptive output feedback controller for system (1), such that

(i) when $w(t) = 0$, the state of system (1) converges to zero, and the other signals of the closed-loop system are bounded on $[0,+\infty)$;

(ii) for every disturbance $w(t) \in L_2[0,+\infty)$ and any pregiven small real number $\gamma > 0$, the output y has the following property,

$$\int_0^t y^2(s)\,ds \leq \int_0^t \gamma^2 \|w(s)\|^2\,ds + \varrho(\cdot), \tag{3}$$

where $\varrho(\cdot)$ is a nonnegative bounded function.

To prove our main result, we need the following lemma.

Lemma 3 (see [5, 10]). *There exist a constant $\alpha > 0$, two constant symmetric matrices $P > 0$ and $Q > 0$ and two vectors $a = (a_1, \ldots, a_n)^T$, $b = (b_1, \ldots, b_n)^T$ such that*

$$\begin{aligned}
&A^TP + PA \leq -I, \qquad DP + PD \geq \alpha I,\\
&B^TQ + QB \leq -2I, \qquad DQ + QD \geq \alpha I,
\end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} -a_1 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ -a_{n-1} & 0 & \cdots & 1\\ -a_n & 0 & \cdots & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 1\\ -b_1 & -b_2 & \cdots & -b_n \end{bmatrix},$$
$$D = \operatorname{diag}\{n, n-1, \ldots, 1\}. \tag{5}$$

3. Main Result

Theorem 4. *Considering system (1) satisfying Assumption 1, we design the output feedback controller as follows:*

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \frac{a_1}{LM}(y - \hat{x}_1),\\
&\vdots\\
\dot{\hat{x}}_n &= u + \frac{a_n}{(LM)^n}(y - \hat{x}_1),
\end{aligned} \tag{6}$$

$$u = -\left[b_1\frac{\hat{x}_1}{(LM)^n} + b_2\frac{\hat{x}_2}{(LM)^{n-1}} + \cdots + b_n\frac{\hat{x}_n}{LM}\right], \tag{7}$$

$$\dot{M} = \frac{1}{\alpha M}\max\left\{\left(1 + |y|^p\right)^2 h^2(u) - \frac{M}{2}, 0\right\}, \quad \text{with } M(0) = 1, \tag{8}$$

$$\dot{L} = \frac{M}{(LM)^2}\left[\frac{y - \hat{x}_1}{(LM)^n}\right]^2, \quad \text{with } L(0) = 1, \tag{9}$$

where α, a_i, and b_i, $i = 1, 2, \ldots, n$ are the appropriately chosen parameters such that Lemma 3 holds. Then, the closed-loop system (1) and (6)–(9) achieve global disturbance in the L_2-gain sense. Furthermore, if $w(t) \in L_2[0,+\infty) \cap L_\infty[0,+\infty)$, then $\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty} \hat{x}(t) = 0$.

Proof. The proof process can be separated into the following three steps.

Step 1. The change of coordinates and Lyapunov functions.

Note that the dynamic gains L and M have the following properties from (8) and (9):

$$\begin{aligned} \dot{L} &\geq 0, \qquad L \geq 1, \\ \dot{M} &\geq 0, \qquad M \geq 1, \\ \frac{M}{2} + \alpha M\dot{M} &\geq \left(1 + |y|^p\right)^2 h^2(u). \end{aligned} \tag{10}$$

Define the state transformation

$$\begin{aligned} \varepsilon_i &= \frac{x_i - \widehat{x}_i}{(LM)^{n-i+1}}, \qquad z_i = \frac{\widehat{x}_i}{(LM)^{n-i+1}}, \\ & \qquad\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{11}$$

Then in the rescaled coordinates, the dynamics of ε and z can be written as the following compact form:

$$\begin{aligned} \dot{\varepsilon} &= \frac{1}{LM} A\varepsilon + \Phi(\cdot) - \left(\frac{\dot{L}}{L} + \frac{\dot{M}}{M}\right) D\varepsilon, \\ \dot{z} &= \frac{1}{LM} Bz + \frac{1}{LM} a\varepsilon_1 - \left(\frac{\dot{L}}{L} + \frac{\dot{M}}{M}\right) Dz, \end{aligned} \tag{12}$$

where a, A, B, and D are given by Lemma 3, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)^T$, $z = (z_1, \ldots, z_n)^T$, $\Phi(\cdot) = [(f_1 + g_1^T w)/(LM)^n, (f_2 + g_2^T w)/(LM)^{n-1}, \ldots, (f_{n-1} + g_{n-1}^T w)/(LM)^2, g_n^T w/(LM)]^T$, and we have $u = -b^T z$.

Let $V_\varepsilon = (\mu_1 + 1)\varepsilon^T P\varepsilon$ and $V_z = z^T Qz$, where P and Q are chosen as Lemma 3; $\mu_1 = \|Qa\|^2$ is a known positive constant.

Then, by (4), (10), and (12), the derivatives of V_ε and V_z can be bounded as

$$\begin{aligned} \dot{V}_\varepsilon \leq & -\frac{(\mu_1 + 1)}{LM} \|\varepsilon\|^2 + 2(\mu_1 + 1)\varepsilon^T P\Phi(\cdot) \\ & - \alpha(\mu_1 + 1)\frac{\dot{M}}{M}\|\varepsilon\|^2, \end{aligned} \tag{13}$$

$$\dot{V}_z \leq -\frac{2}{LM}\|z\|^2 + 2\frac{1}{LM} z^T Qa\varepsilon_1. \tag{14}$$

For $i = 1, \ldots, n-1$, it is easily deduced from (2), (11), and $u = -b^T z$ that

$$\begin{aligned} \left|\frac{f_i + g_i^T w}{(LM)^{n-i+1}}\right| &\leq \theta\left(1 + |y|^p\right) h(u) \frac{\left(\sum_{j=i+2}^n |x_j| + |u|\right)}{(LM)^{n-i+1}} \\ &\quad + \frac{\theta\left(1 + |y|^p\right) h(u)}{(LM)^{n-i+1}} \|w\| \\ &\leq \frac{\theta_1\left(1 + |y|^p\right) h(u)}{(LM)^2} (\|\varepsilon\| + \|z\| + \|w\|), \\ \left|\frac{g_n^T w}{(LM)}\right| &\leq \frac{\theta_1\left(1 + |y|^p\right) h(u)}{LM} \|w\|, \end{aligned} \tag{15}$$

where θ_1 is a suitable unknown constant depending on θ. Thus we get

$$\begin{aligned} &\left|2(\mu_1 + 1)\varepsilon^T P\Phi(\cdot)\right| \\ &\quad \leq 2(\mu_1 + 1)\|\varepsilon\|\|P\| \\ &\qquad \times \left[\left|\frac{f_1 + g_1^T w}{(LM)^n}\right| + \cdots + \left|\frac{f_{n-1} + g_{n-1}^T w}{(LM)^2}\right| + \left|\frac{g_n^T w}{(LM)}\right|\right] \\ &\quad \leq \frac{\theta_2\left(1 + |y|^p\right) h(u)}{(LM)^2} \|\varepsilon\| (\|\varepsilon\| + \|z\|) \\ &\qquad + \frac{\theta_2\left(1 + |y|^p\right) h(u)}{LM} \|\varepsilon\| \|w\| \\ &\quad \leq \frac{\theta_3 M}{(LM)^2}\left(\|\varepsilon\|^2 + \|z\|^2\right) + \frac{\left(1 + |y|^p\right)^2 h^2(u)}{(LM)^2}\|\varepsilon\|^2 \\ &\qquad + \theta_3\|w\|^2, \end{aligned} \tag{16}$$

where θ_3 is an unknown positive constant.

On the other hand, it is obvious that

$$\begin{aligned} \left|2\frac{1}{LM} z^T Qa\varepsilon_1\right| &\leq \frac{1}{LM}\|z\|^2 + \frac{1}{LM}\|Qa\|^2\varepsilon_1^2 \\ &= \frac{1}{LM}\|z\|^2 + \frac{1}{LM}\mu_1\varepsilon_1^2 \\ &\leq \frac{1}{LM}\|z\|^2 + \frac{1}{LM}\mu_1\|\varepsilon\|^2. \end{aligned} \tag{17}$$

(18)

Choose the Lyapunov function $V_0 = V_\varepsilon + V_z$. Then, using (13)-(14), (16), (18), and (10), we have the following inequality:

$$\begin{aligned} \dot{V}_0 \leq & -\frac{1}{LM}\left(\|\varepsilon\|^2 + \|z\|^2\right) - \alpha(\mu_1 + 1)\frac{\dot{M}}{M}\|\varepsilon\|^2 \\ & + \frac{\theta_3 M}{(LM)^2}\left(\|\varepsilon\|^2 + \|z\|^2\right) + \frac{\left(1 + |y|^p\right)^2 h^2(u)}{(LM)^2}\|\varepsilon\|^2 \\ & + \theta_3\|w\|^2 \\ \leq & -\frac{1}{LM}\left(\|\varepsilon\|^2 + \|z\|^2\right) - \alpha\frac{M\dot{M}}{(LM)^2}\|\varepsilon\|^2 \\ & + \frac{\theta_3 M}{(LM)^2}\left(\|\varepsilon\|^2 + \|z\|^2\right) + \frac{\left(1 + |y|^p\right)^2 h^2(u)}{(LM)^2}\|\varepsilon\|^2 \\ & + \theta_3\|w\|^2 \\ \leq & -\frac{1}{LM}\left(\|\varepsilon\|^2 + \|z\|^2\right) + \frac{\theta_3 M}{(LM)^2}\left(\|\varepsilon\|^2 + \|z\|^2\right) \\ & + \frac{M}{(LM)^2}\left(\|\varepsilon\|^2 + \|z\|^2\right) - \frac{1}{(LM)^2} \end{aligned}$$

$$\times \left[\frac{M}{2} + \alpha M\dot{M} - \left(1 + |y|^p\right)^2 h^2(u)\right] \|\varepsilon\|^2 + \theta_3 \|w\|^2$$
$$= -\frac{M}{(LM)^2}(L - \theta_3 - 1)\left(\|\varepsilon\|^2 + \|z\|^2\right) + \theta_3 \|w\|^2. \tag{19}$$

Step 2. Now, we prove the boundedness of the closed-loop system by (19).

Since the solution of the closed-loop system exists and is unique on a small time interval $[0, T)$ for any initial condition $(\varepsilon(0), z(0)) \in \mathbb{R}^n \times \mathbb{R}^n$ and $L(0) = M(0) = 1$ (see [15, 16]), without loss of generality, we assume that this solution can be extended to the maximal interval $[0, T_f)$ for some T_f satisfying $0 < T_f \leq +\infty$ (see [15, 16]). We claim that $T_f = +\infty$. To prove this claim, we will first prove that the states variables (L, z, ε, M) are bounded on $[0, T_f)$.

Claim 1 ($L \in L_\infty[0, T_f)$ and $\int_0^{T_f}(\varepsilon_1^2(t)/M(t))dt < +\infty$). If not, according to (10), we obtain $\lim_{t\to T_f} L(t) = +\infty$. Thus, there exists a finite time $t_1 \in (0, T_f)$ such that

$$L(t) \geq \theta_3 + 2, \quad \text{for } \forall t \in \left[t_1, T_f\right). \tag{20}$$

With this in mind, it follows from (19) that for $\forall t \in [t_1, T_f)$

$$\dot{V}_0 \leq -\frac{M}{(LM)^2}\left(\|\varepsilon\|^2 + \|z\|^2\right) + \theta_3 \|w\|^2. \tag{21}$$

From $\dot{L} = (M/(LM)^2)[(y - \hat{x}_1)/(LM)^n]^2 = (M/(LM)^2)\varepsilon_1^2$ and $w(t) \in L_2[0, +\infty)$, it is deduced that

$$+\infty = L\left(T_f\right) - L\left(t_1\right) = \int_{t_1}^{T_f} \dot{L}(t)\,dt$$
$$= \int_{t_1}^{T_f} \frac{M(t)\,\varepsilon_1^2(t)}{[L(t)\,M(t)]^2} dt \leq V_0\left(t_1\right) + \theta_3 \int_{t_1}^{T_f} \|w(t)\|^2\,dt$$
$$< +\infty, \tag{22}$$

which is a contradiction. Hence, $L \in L_\infty[0, T_f)$ and $\lim_{t\to T_f} L(t) < +\infty$. Moreover, we get $\int_0^{T_f}(\varepsilon_1^2(t)/L^2(t)M(t))dt < +\infty$. Then, $\int_0^{T_f}(\varepsilon_1^2(t)/M(t))dt < +\infty$.

Claim 2 ($z \in L_\infty[0, T_f)$ and $\int_0^{T_f}(\|z(t)\|^2/M(t))dt < +\infty$). From (14), (17), and (10), it yields

$$\dot{V}_z \leq -\frac{1}{LM}\|z\|^2 + \frac{1}{LM}\mu_1\varepsilon_1^2 = -\frac{1}{LM}\|z\|^2 + \mu_1 L\dot{L}, \quad \text{for } \forall t \in \left[0, T_f\right). \tag{23}$$

Integrating the above inequality, we obtain

$$\lambda_{\min}(Q)\|z(t)\|^2 - z^T(0)Qz(0)$$
$$\leq -\int_0^t \frac{1}{L(s)\,M(s)}\|z(s)\|^2\,ds + \frac{\mu_1}{2}\left[L^2(t) - 1\right]$$
$$\leq \frac{\mu_1}{2}\left[L^2(t) - 1\right], \quad \text{for } \forall t \in \left[0, T_f\right),$$
$$\int_0^t \frac{1}{L(s)\,M(s)}\|z(s)\|^2\,ds \leq z^T(0)Qz(0) + \frac{\mu_1}{2}\left[L^2(t) - 1\right], \quad \text{for } \forall t \in \left[0, T_f\right). \tag{24}$$

Since $L \in L_\infty[0, T_f)$ and $\lim_{t\to T_f} L(t) < +\infty$, from (24), we have $z \in L_\infty[0, T_f)$, $\lim_{t\to T_f} z(t) < +\infty$, and $\int_0^{T_f}(\|z(t)\|^2/M(t))dt < +\infty$.

Claim 3 ($\varepsilon \in L_\infty[0, T_f)$ and $\int_0^{T_f}(\|\varepsilon(t)\|^2/M(t))dt < +\infty$). To this end, we redefine the scaled error as follows:

$$\eta_i = \frac{x_i - \hat{x}_i}{(L^*M)^{n-i+1}}, \quad i = 1, 2, \ldots, n, \tag{25}$$

where the constant $L^* \geq \max\{L(T_f), \theta_4 + 4\}$; θ_4 is also a suitable unknown positive constant depending on θ. The dynamic of η is given by

$$\dot{\eta} = \frac{1}{L^*M}A\eta + \frac{1}{L^*M}a\eta_1 - \frac{1}{L^*M}\Gamma a\eta_1 + \Phi^*(\cdot) - \frac{\dot{M}}{M}D\eta, \tag{26}$$

where a and A are defined by Lemma 3, $\eta = (\eta_1, \ldots, \eta_n)^T$, $\Gamma = \text{diag}\{L^*/L, (L^*/L)^2, \ldots, (L^*/L)^n\}$, and $\Phi^*(\cdot) = [(f_1 + g_1^T w)/(L^*M)^n, (f_2 + g_2^T w)/(L^*M)^{n-1}, \ldots, (f_{n-1} + g_{n-1}^T w)/(L^*M)^2, g_n^T w/(L^*M)]^T$.

Consider the Lyapunov function $V_\eta = \eta^T P\eta$. Thus, the derivative V_η of (26) is bounded as

$$\dot{V}_\eta \leq -\frac{1}{L^*M}\|\eta\|^2 + 2\frac{1}{L^*M}\eta^T Pa\eta_1 - 2\frac{1}{L^*M}\eta^T P\Gamma a\eta_1 + 2\eta^T P\Phi^*(\cdot) - \alpha\frac{\dot{M}}{M}\|\eta\|^2. \tag{27}$$

By completing the squares, it is clear that

$$\left|2\frac{1}{L^*M}\eta^T Pa\eta_1\right| \leq \frac{M}{(L^*M)^2}\|\eta\|^2 + \|Pa\|^2\frac{\eta_1^2}{M},$$
$$\left|2\frac{1}{L^*M}\eta^T P\Gamma a\eta_1\right| \leq \frac{M}{(L^*M)^2}\|\eta\|^2 + \|P\Gamma a\|^2\frac{\eta_1^2}{M}. \tag{28}$$

Moreover, similar to (16), we get from (2) and (10) that

$$\left|2\eta^T P\Phi^*(\cdot)\right| \leq \frac{\theta_4 M}{(L^*M)^2}\left(\|\eta\|^2 + \|z\|^2\right) + \frac{\left(1 + |y|^p\right)^2 h^2(u)}{(L^*M)^2}\|\eta\|^2 + \theta_4\|w\|^2. \tag{29}$$

Substituting the estimations (28)-(29) into (27) and noticing that $|\eta_1| \le |\varepsilon_1|$, we deduce from (10) that

$$\begin{aligned}
\dot{V}_\eta &\le -\frac{1}{L^* M}\|\eta\|^2 + \frac{\theta_4+2}{(L^*)^2 M}\|\eta\|^2 + \frac{\theta_4}{(L^*)^2 M}\|z\|^2 \\
&\quad + \left(\|Pa\|^2 + \|P\Gamma a\|^2\right)\frac{\eta_1^2}{M} - \alpha\frac{M\dot{M}}{(L^*M)^2}\|\eta\|^2 \\
&\quad + \frac{\left(1+|y|^p\right)^2 h^2(u)}{(L^*M)^2}\|\eta\|^2 + \theta_4\|w\|^2 \\
&\le -\frac{1}{L^* M}\|\eta\|^2 + \frac{\theta_4+3}{(L^*)^2 M}\|\eta\|^2 + \frac{\theta_4}{(L^*)^2 M}\|z\|^2 \\
&\quad + \left(\|Pa\|^2 + \|P\Gamma a\|^2\right)\frac{\eta_1^2}{M} - \frac{1}{(L^*M)^2} \\
&\quad \times \left[\frac{M}{2} + \alpha M\dot{M} - \left(1+|y|^p\right)^2 h^2(u)\right]\|\eta\|^2 + \theta_4\|w\|^2 \\
&\le -\frac{1}{(L^*)^2 M}\left(L^* - \theta_4 - 3\right)\|\eta\|^2 \\
&\quad + \frac{\theta_4}{(L^*)^2 M}\|z\|^2 + \left(\|Pa\|^2 + \|P\Gamma a\|^2\right)\frac{\eta_1^2}{M} + \theta_4\|w\|^2 \\
&\le -\frac{1}{(L^*)^2 M}\|\eta\|^2 + \theta_4\frac{\|z\|^2}{M} + \mu_2\frac{\varepsilon_1^2}{M} + \theta_4\|w\|^2,
\end{aligned} \tag{30}$$

where μ_2 is a suitable constant satisfying $\mu_2 \ge \|Pa\|^2 + \|P\Gamma a\|^2$.

Integrating (30), it is easy to obtain that for any $t \in [0, T_f)$

$$\begin{aligned}
\lambda_{\min}(P)\|\eta(t)\|^2 &\le V_\eta(0) + \theta_4\int_0^t \frac{\|z(s)\|^2}{M(s)}\mathrm{d}s \\
&\quad + \mu_2\int_0^t \frac{\varepsilon_1^2(s)}{M(s)}\mathrm{d}s + \theta_4\int_0^t \|w(s)\|^2\,\mathrm{d}s, \\
\frac{1}{(L^*)^2}\int_0^t \frac{\|\eta(s)\|^2}{M(s)}\mathrm{d}s &\le V_\eta(0) + \theta_4\int_0^t \frac{\|z(s)\|^2}{M(s)}\mathrm{d}s \\
&\quad + \mu_2\int_0^t \frac{\varepsilon_1^2(s)}{M(s)}\mathrm{d}s + \theta_4\int_0^t \|w(s)\|^2\,\mathrm{d}s.
\end{aligned} \tag{31}$$

Using the fact that $\int_0^{T_f}(\varepsilon_1^2(t)/M(t))\mathrm{d}t < +\infty$, $\int_0^{T_f}(\|z(t)\|^2/M(t))\mathrm{d}t < +\infty$, and $w(t) \in L_2[0,+\infty)$, we deduce from (31) that $\eta \in L_\infty[0,T_f)$, $\lim_{t\to T_f}\|\eta(t)\| < +\infty$, and $\int_0^{T_f}(\|\eta(t)\|^2/M(t))\mathrm{d}t < +\infty$. Accordingly, by (25) and (11), we have that $\varepsilon \in L_\infty[0,T_f)$, $\lim_{t\to T_f}\|\varepsilon(t)\| < +\infty$, and $\int_0^{T_f}(\|\varepsilon(t)\|^2/M(t))\mathrm{d}t < +\infty$.

Claim 4 ($M \in L_\infty[0,T_f)$). This claim can be done by a contradiction argument; suppose $\lim_{t\to T_f} M(t) = +\infty$. Recalling that $L, z, \varepsilon \in L_\infty[0,T_f)$ and $u = -b^T z$, as $h(u)$ is a continuous function on $\mathbb{R}$, we can find a positive constant C_0 from (11) such that for any $t \in [0,T_f)$

$$\frac{x_1}{M^n} \le L^n\left(|z_1| + |\varepsilon_1|\right) \le C_0, \qquad h(u) \le C_0. \tag{32}$$

Then we can find a positive constant C_1 such that

$$\left(1+|y|^p\right)^2 h^2(u) \le C_1\left(1+M^{2np}\right) < C_1(1+M). \tag{33}$$

Hence,

$$\lim_{M\to+\infty}\frac{\left(1+|y|^p\right)^2 h^2(u)}{M/2} = 0. \tag{34}$$

Thus, by $\lim_{t\to T_f} M(t) = +\infty$, we know that there exists a finite time $t_2 \in (0,T_f)$ such that

$$\left(1+|y|^p\right)^2 h^2(u) < \frac{M}{2}, \quad \text{for any } t \in \left[t_2, T_f\right). \tag{35}$$

As a consequence, for any $t \in [t_2,T_f)$,

$$\dot{M} = \frac{1}{\alpha M}\max\left\{\left(1+|y|^p\right)^2 h^2(u) - \frac{M}{2}, 0\right\} \equiv 0. \tag{36}$$

Therefore, for any $t \in [0,T_f)$, $M(t) \le M(t_2)$, which is in contradiction with $\lim_{t\to T_f} M(t) = +\infty$. Then $M \in L_\infty[0,T_f)$. Hence, we get $z, \varepsilon \in L_2[0,T_f)$.

Step 3. In this step, we proof that the closed-loop system (1) and (6)–(9) achieve global disturbance in the L_2-gain sense.

Since L, z, ε, and M are all bounded on the maximal interval $[0,T_f)$, we have $T_f = +\infty$ (see [15, 16]). Consequently, L, z, M, and ε are well defined on $[0,+\infty)$, and $L, z, \varepsilon, M \in L_\infty[0,+\infty)$, $z, \varepsilon \in L_2[0,+\infty)$.

Using (19), for any pregiven small real number $\gamma > 0$, we obtain

$$\dot{V}_0 + \frac{\theta_3}{\gamma^2}|y|^2 \le \theta_5\left(\|\varepsilon\|^2 + \|z\|^2\right) + \theta_3\|w\|^2 + \frac{\theta_3}{\gamma^2}|y|^2, \tag{37}$$

where θ_5 is a suitable constant.

Integrating (37), it is easy to see that for any $t \in [0,+\infty)$

$$\begin{aligned}
&\int_0^t |y(s)|^2\,\mathrm{d}s \\
&\le \gamma^2\int_0^t \|w(s)\|^2\,\mathrm{d}s + \frac{\gamma^2}{\theta_3}V_0(\varepsilon(0), z(0)) \\
&\quad + \int_0^t |y(s)|^2\,\mathrm{d}s + \frac{\theta_5\gamma^2}{\theta_3}\int_0^t \left(\|\varepsilon(s)\|^2 + \|z(s)\|^2\right)\mathrm{d}s \\
&= \gamma^2\int_0^t \|w(s)\|^2\,\mathrm{d}s + \varrho(\cdot),
\end{aligned} \tag{38}$$

where $\varrho(\cdot)$ is a nonnegative bounded function. Then, global disturbance attenuation of the system is achieved in the L_2-gain sense.

Furthermore, if $w(t) \in L_2[0,+\infty) \cap L_\infty[0,+\infty)$, we get from $\varepsilon, z \in L_\infty[0,+\infty)$ that

$$\begin{aligned} &\varepsilon \in L_2[0,+\infty), \qquad \dot{\varepsilon} \in L_\infty[0,+\infty), \\ &z \in L_2[0,+\infty), \qquad \dot{z} \in L_\infty[0,+\infty). \end{aligned} \tag{39}$$

By Barbalat's Lemma, we arrive at

$$\lim_{t\to+\infty} z(t) = \lim_{t\to+\infty} \varepsilon(t) = 0, \tag{40}$$

which together with (11) and $L, M \in L_\infty[0,+\infty)$ yields

$$\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty} \widehat{x}(t) = 0. \tag{41}$$

□

Remark 5. It is worth mentioning that, since system (1) has the feature of feedforward system and we adopt a low-gain adaptive controller, $\varrho(\cdot) \neq 0$ when the initial state $x(0) = \widehat{x}(0) = 0$, which is different from the existing results on disturbance attention [11–13] but is similar with [14, 15].

Remark 6. Since $\dot{L} \geq 0, \dot{M} \geq 0$, and $L, M \in L_\infty[0,+\infty)$, there exist constants $\overline{L} > 0$, $\overline{M} > 0$ such that $\lim_{t\to+\infty} L(t) = \overline{L}$, $\lim_{t\to+\infty} M(t) = \overline{M}$. That is, the dynamic gains L and M are time-invariant in nature.

Remark 7. From the proof procedure of Theorem 4, we see that the dynamic gains L and M are introduced to deal with the unknown growth rate θ and the function $h(u)$, respectively, and both are required.

Remark 8. It is worth pointing out that, for any known constant $N > 0$ and any known continuous function $\varpi(u)$ satisfying $\varpi(u) \geq h(u)$, if we define $\dot{M} = \max\left\{(1+|y|^p)^2\varpi(u)/N - M/2, 0\right\}/(\alpha M)$ in (8), Theorem 4 also holds. Moreover, when N is a sufficiently large constant, we can get the better state properties of the closed-loop system; that is, the values of x, $\widehat{x}$ in the transient phase are getting smaller, and the convergence to zero of x and $\widehat{x}$ is getting faster when $w(t) \in L_2[0,+\infty) \cap L_\infty[0,+\infty)$.

4. Simulation Example

Consider the following uncertain nonlinear system:

$$\begin{aligned} \dot{x}_1 &= x_2 + c_1 x_3 + d_1(t)\ln\left(1+\left(u^2\right)^{c_2}\right)\sqrt[7]{\ln\left(1+\left(x_1^2\right)^{c_3}\right)}x_3, \\ \dot{x}_2 &= x_3 + d_2(t)\,u^2, \\ \dot{x}_3 &= u + d_3(t)\,u\sqrt[8]{\ln\left(1+\left(x_1^2\right)^{c_4}\right)}w(t), \\ y &= x_1, \end{aligned} \tag{42}$$

where $|d_i(t)| \leq c_{4+i}$, $i = 1,2,3$, are unknown continuous functions, $c_1, c_2 \geq 1$, $c_j > 4$, $j = 3,4$, $c_k > 0$, and $k = 5,6,7$ are unknown constants. The system disturbance $w(t) = t/(1+t^2)$. Apparently, $w(t) \in L_2[0,+\infty) \cap L_\infty[0,+\infty)$. It is easy to prove that the uncertain system (42) satisfies Assumption 1 with $p = 1/7$ and $h(u) = |u|$.

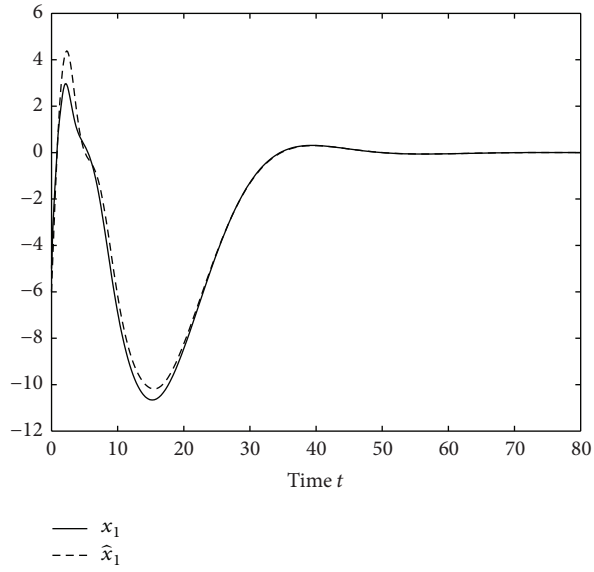

Figure 1: Trajectories of x_1 and $\widehat{x}_1$.

Remark 9. In [17], Jo et al. have shown that the nonlinear LLC resonant circuit system, through appropriate transformation, can be changed into system (42) with $d_1(t) = d_2(t) = d_3(t) = 0$. Considering that disturbance and the uncertainty, which are unavoidable, are frequently encountered in real engineering systems, we decide to adopt system (42) to verify our theoretical analysis.

Then, according to Remark 8 and Theorem 4, we design the observer dynamics and the output feedback controller for (42) as follows:

$$\begin{aligned} \dot{\widehat{x}}_1 &= \widehat{x}_2 + \frac{3}{LM}(y - \widehat{x}_1), \\ \dot{\widehat{x}}_2 &= \widehat{x}_3 + \frac{3}{(LM)^2}(y - \widehat{x}_1), \\ \dot{\widehat{x}}_3 &= u + \frac{1}{(LM)^3}(y - \widehat{x}_1), \end{aligned}$$

$$u = -\left[\frac{\widehat{x}_1}{(LM)^3} + 3\frac{\widehat{x}_2}{(LM)^2} + 3\frac{\widehat{x}_3}{LM}\right],$$

$$\dot{M} = \frac{1}{0.4M}\max\left\{\frac{\left(1+|y|^{1/7}\right)^2 u^2}{100} - \frac{M}{2}, 0\right\}, \quad \text{with } M(0) = 1,$$

$$\dot{L} = \frac{M}{(LM)^2}\left[\frac{y - \widehat{x}_1}{(LM)^3}\right]^2, \quad \text{with } L(0) = 1. \tag{43}$$

Let $c_1 = c_2 = 1$, $c_3 = c_4 = 4.1$, $d_1(t) = 0.3$, $d_2(t) = 0.2$, and $d_3(t) = 0.1$. We choose the initial condition

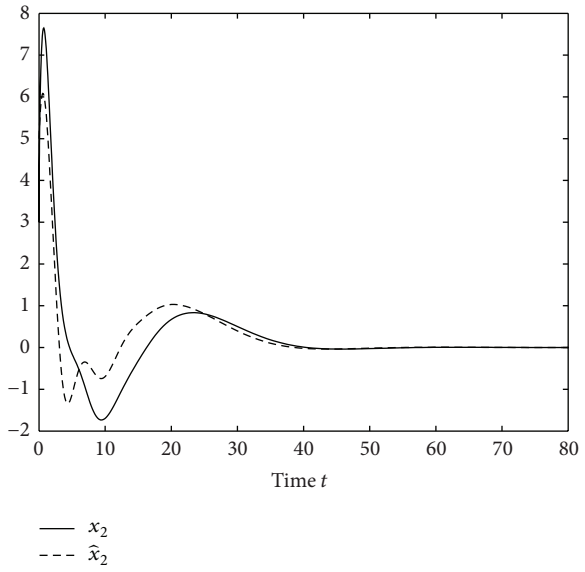

Figure 2: Trajectories of x_2 and $\hat{x}_2$.

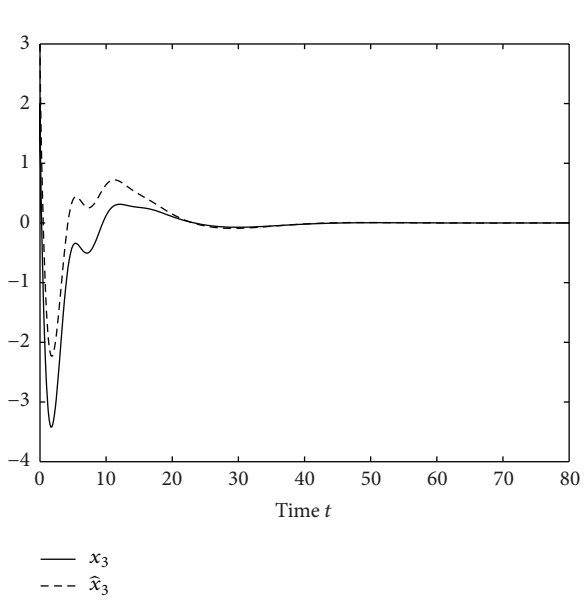

Figure 3: Trajectories of x_3 and $\hat{x}_3$.

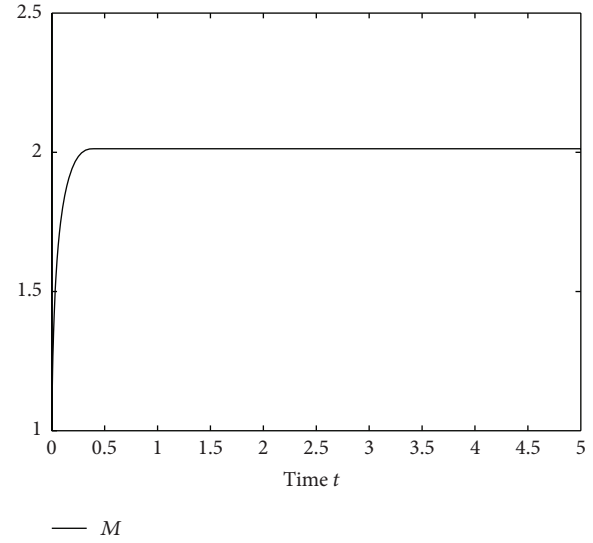

Figure 4: The dynamic gain M.

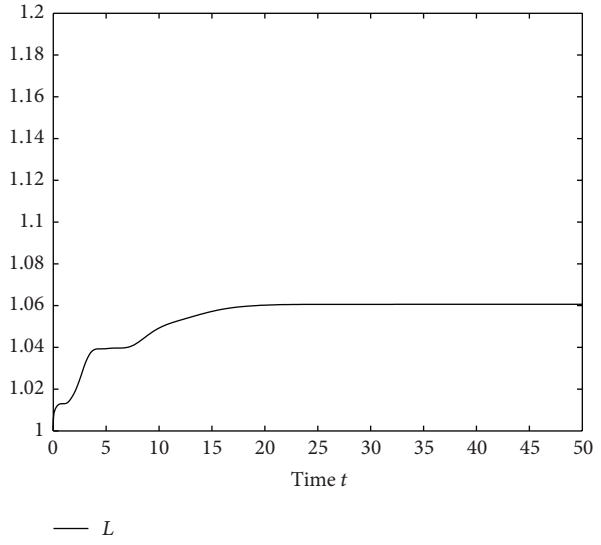

Figure 5: The dynamic gain L.

$[x_1(0), x_2(0), x_3(0), \hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0), L(0), M(0)] = [-5, 3, 2, -6, 5, 3, 1, 1]$. The simulation results are shown in Figures 1, 2, 3, 4, 5, and 6 for the closed-loop system consisting of (42) and (43).

5. Conclusion

In this paper, we have studied the problem of output feedback disturbance attenuation for a class of uncertain nonlinear systems. By using a linear observer with two dynamic gains and introducing the transformation of coordinates, we propose an adaptive output feedback controller such that the states of the closed-loop system are globally bounded, and the disturbance attenuation is achieved in the sense of L_2-gain. Furthermore, the system is globally asymptotically regulated when the system disturbance $w(t) \in L_2[0, +\infty) \cap L_\infty[0, +\infty)$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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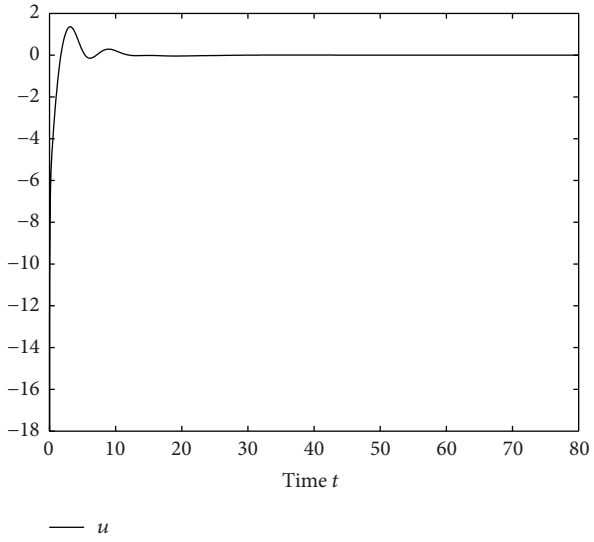

Figure 6: The control input u.

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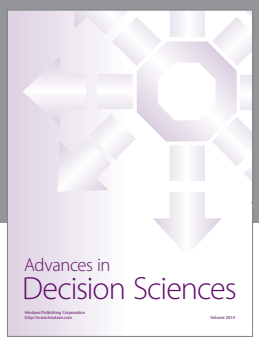

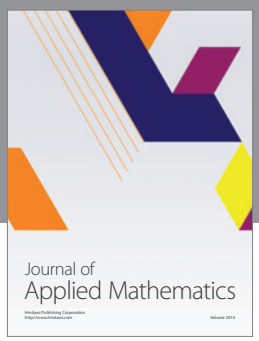

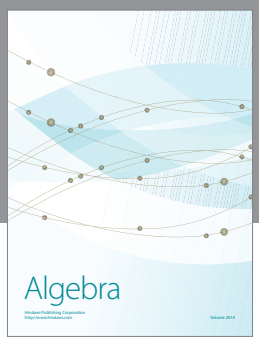

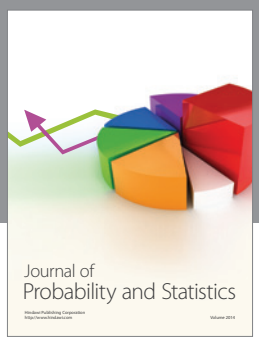

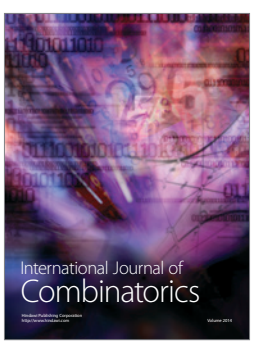

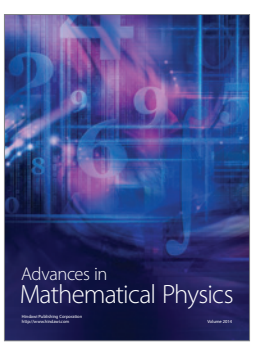

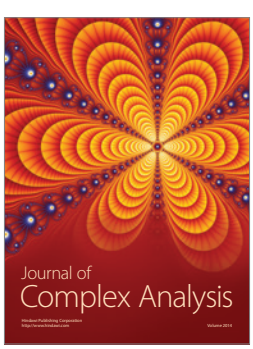

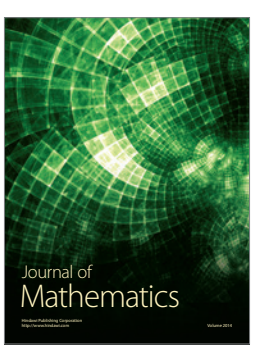

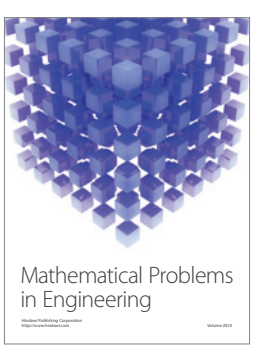

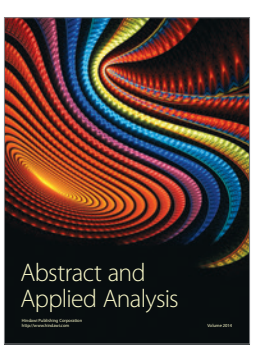

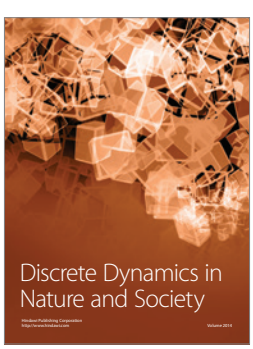

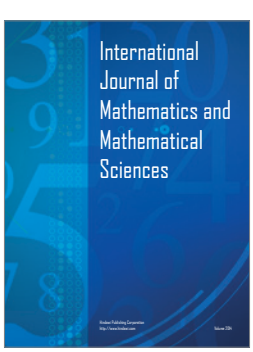

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