Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2014, Article ID 638259, 14 pages
http://dx.doi.org/10.1155/2014/638259

Research Article

Multipurpose Water Reservoir Management: An Evolutionary Multiobjective Optimization Approach

Luís A. Scola,[1] Ricardo H. C. Takahashi,[2] and Sérgio A. A. G. Cerqueira[3]

[1] *Department of Thermal and Fluid Sciences, Federal University of São João del-Rei, 36307-352 São João del Rei, MG, Brazil*
[2] *Department of Mathematics, Federal University of Minas Gerais, 31270-901 Belo Horizonte, MG, Brazil*
[3] *Department of Mechanical Engineering, Federal University of São João del-Rei, 36307-352 São João del Rei, MG, Brazil*

Correspondence should be addressed to Ricardo H. C. Takahashi; taka@mat.ufmg.br

Received 23 January 2014; Revised 14 July 2014; Accepted 16 July 2014; Published 15 September 2014

Academic Editor: Xingsheng Gu

The reservoirs that feed large hydropower plants should be managed in order to provide other uses for the water resources. Those uses include, for instance, flood control and avoidance, irrigation, navigability in the rivers, and other ones. This work presents an evolutionary multiobjective optimization approach for the study of multiple water usages in multiple interlinked reservoirs, including both power generation objectives and other objectives not related to energy generation. The classical evolutionary algorithm NSGA-II is employed as the basic multiobjective optimization machinery, being modified in order to cope with specific problem features. The case studies, which include the analysis of a problem which involves an objective of navigability on the river, are tailored in order to illustrate the usefulness of the data generated by the proposed methodology for decision-making on the problem of operation planning of multiple reservoirs with multiple usages. It is shown that it is even possible to use the generated data in order to determine the cost of any new usage of the water, in terms of the opportunity cost that can be measured on the revenues related to electric energy sales.

1. Introduction

The optimal use of water resources is increasingly being recognized as a strategic issue for the nations [1]. In several countries, a significant share of the energetic matrix is supported by hydropower plants. The large reservoirs that feed the operation of those plants should be managed in order to provide the economic operation of the power generation, considering also the other multiple uses that the water resources should allow. Those uses include, for instance, flood control and avoidance, irrigation, navigability in the rivers, and other ones. It is necessary to focus on improving the operational effectiveness of reservoir systems for maximizing the benefits of its usage.

The optimal planning of the operation of single and multiple reservoir systems involves essentially the decision about how much electricity should be generated in each plant on each time period. This decision determines how much water is available for electricity generation in the next time periods, interacting with the effect of reservoir filling due to seasonal rainfall. Other usages for the water will rely on the availability of water, either in the reservoir (measured by the reservoir level) or in the river watercourse (measured by the water flow). For instance, the flood control is better if there is less water in the reservoir, allowing the impoundment of a large additional volume of water, while more water in the reservoir is better for irrigation purposes. For allowing navigability, there is a need for a minimum water flow in the watercourse, which imposes a minimum level of electricity generation in order to liberate such a flow.

The underlying optimization problem can be characterized as (i) a multiobjective optimization problem, since the possible alternative usages of the water are conflicting with each other and with the power generation. (ii) It is a dynamic optimization due to the interdependence of the time stages with any decision being constrained by the decisions taken in the former stages. (iii) The problem is nonlinear, mainly due to the nonlinear relationship between the water flow through the reservoirs and the water level in them.

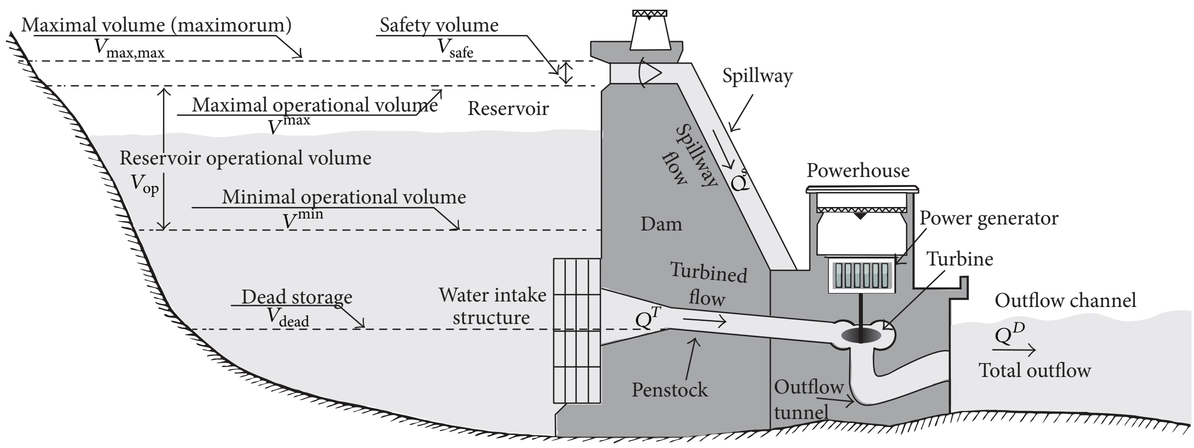

Figure 1: Schematic representation of a hydroelectric central.

These nonlinear functions are also nonconvex, in general. The objective functions may also be defined as nonlinear functions. (iv) There are constraints related to the minimum and maximum admissible levels of water in the reservoirs and to the minimum and maximum water flow that can be sent to power generation.

Several methods, both deterministic and stochastic, have been used for tackling simplified instances of this general problem. Some of them are reviewed in [2, 3]. Over the last decade, single objective evolutionary methods, in particular several instances of the genetic algorithms, were applied to single reservoir systems [4–6]. The nonlinear model of the complete Brazilian hydropower system was solved by linear programming techniques, after linearization, considering the weighted sum of six objectives [7]. The multiobjective evolutionary algorithm NSGA-II was employed for optimizing the operation of a multipurpose reservoir, considering two objectives [8]. In another work, the same authors also employed an elitist-mutated particle swarm algorithm, considering the weighted sum of two objectives [9]. The loss of diversity in the population was an important concern in several works. For instance, in [10], a new macroevolutionary multiobjective genetic algorithm was proposed for the optimization of the rule curves of a multipurpose reservoir system.

In this work, a multiobjective genetic algorithm is employed to optimize the operation of a set of five existing Brazilian hydropower plants through the course of one year for a typical year and for a dryer than usual year. Two objectives related to energy generation are considered: the maximization of the minimal power generation along the year (which is related to the power delivery which can be ensured by the system on any time) and the maximization of the total energy generation along the year. Those objectives represent a typical trade-off to be examined by decision-makers that work under the viewpoint of the electricity system planning. Another objective, not related to energy generation, is also included: the navigability in the river downstream the reservoir. This objective is expressed as the need to maintain the water flow above a given minimum, which allows the operation of ships above a given draft. The inclusion of this objective exemplifies the employment of the proposed methodology for the generation of data for the decision-making process considering several stakeholders, in addition to the electric power system viewpoint.

The proposed algorithm is a variation of the NSGA-II algorithm, which has been modified in order to include a new representation for the individuals, allowing a better constraint handling and new problem-specific mutation and crossover operators, which enhance the algorithm computational efficiency [11]. Preliminary results of the proposed algorithm were presented in the conference paper [12].

2. System Mathematical Model

A hydroelectric central is shown schematically in Figure 1. It is composed of a dam that impounds the river, making the reservoir. A duct system leads the water to the powerhouse where the generator turbines are driven and the water is restored to the river by an outflow channel. If the reservoir becomes full and the capacity of the outflow channel is exceeded by the water inflow, it becomes necessary to release water by the spillway, which means that this water will not generate electricity.

The difference between the water level in the reservoir and its level in the outflow channel is called the head. This difference determines the potential energy that can be transformed into electric power. The water level in the reservoir presents a nonlinear relationship with the stored volume, while the level in the outflow channel also depends nonlinearly on the sum of flow through the turbines and the spillway flow.

The following variables are involved in the model of a hydroelectric central:

(i) V: volume stored in the reservoir;

(ii) $V^{\max}$: maximal operational volume of the reservoir;

(iii) $V^{\min}$: minimal operational volume of the reservoir;

(iv) $V_{op} = V^{\max} - V^{\min}$: reservoir operational volume;

(v) $\dot{W}$: generated power;

(vi) $\dot{W}^{\max}$: maximum power that can be generated;
(vii) Q^F: water inflow into the reservoir;
(viii) Q^T: turbined flow;
(ix) Q^S: spillway flow;
(x) $Q^D = Q^T + Q^S$: total outflow;
(xi) $Q^{D,\min}$: minimum allowed total outflow;
(xii) $Q^{T,\max}$: maximum turbined flow allowed;
(xiii) $\phi(V)$: height of the reservoir elevation, as a function of the stored volume;
(xiv) $\theta(Q^D)$: height of the outflow channel level, as a function of the total outflow;
(xv) $h = \phi(V) - \theta(Q^D)$: head.

In all cases, N stands for the number of reservoirs and M stands for the number of months in the optimization horizon. A variable represented by V stands for water volume; Q stands for water flow; W stands for energy; $\dot{W}$ stands for power. The considered time unit is represented by Δt.

2.1. Power Generation. The power generated by a hydropower plant i in the tth time interval can be determined by the expression

$$\dot{W}_{t,i} = \xi_{t,i} Q^T_{t,i}. \tag{1}$$

In this expression $\xi_{t,i}$ is the energy production function which depends on the average volume of the water stored in the reservoir during the tth time period for the ith power plant, and $Q^T_{t,i}$ is the average volumetric flow rate of water through the turbines of that plant in the same time period.

2.2. Constraints. There are both physical and operational constraints for the operation of hydroelectric power plants.

2.2.1. Conservation of Mass. The conservation of mass constraint, over a time interval, imposes that the increase of stored water volume in each one of the reservoirs must be equal to the amount of water flowing into it ($Q^F_{t,i}$) minus the amount of water discharged through the turbine ($Q^T_{t,i}$) and spilled ($Q^S_{t,i}$),

$$V_{t+1,i} - V_{t,i} = \left(Q^F_{t,i} - Q^T_{t,i} - Q^S_{t,i}\right)\Delta t. \tag{2}$$

If the reservoir is the first one on a river, the affluent flow is determined by nature. If not, the total volume coming from the upstream reservoir must be added to the incremental volume—that of water from other tributaries, added between the two reservoirs:

$$Q^F_{t,i} = \sum_{k=1,k\in U}^{N}\left(Q^T_{t,k} + Q^s_{t,k}\right) + Q^I_{t,i}. \tag{3}$$

Here U is the set of upstream reservoirs and $Q^I_{t,i}$ denotes the incremental flow into the river between reservoirs, on the tth month, upstream of the ith reservoir.

The mass conservation establishes a direct relationship between the water outflow and the volume of water in the reservoir, as the affluent flow is known.

2.2.2. Operational Limits. The volume of water in the reservoir is bounded below by the level of the forebay (intake channel) and above by the structural limit of the dam:

$$V^{\min}_j \leq V_{t,j} \leq V^{\max}_j. \tag{4}$$

The operational parameters of the turbines also impose constraints to the problem, in terms of both a maximum power and a maximum volumetric flow:

$$\begin{gathered}\dot{W}_{t,i} \leq \dot{W}^{\max}_i, \\ Q^T_{t,i} \leq Q^{T,\max}_i.\end{gathered} \tag{5}$$

If the volume variation in a given interval requires a flow through the turbine that exceeds the maximum allowed, it is assumed that some water was discharged through the spillway.

For ecological, sanitary, and economical reasons, a minimum volumetric flow to the river downstream of the reservoirs must also be ensured:

$$Q^{D,\min}_i \leq Q^T_{t,i}. \tag{6}$$

In this work the availability of the turbines is assumed. Therefore no spillage is allowed when the flow is smaller than the allowed maximum; that is, all the water passes through the turbines.

Finally, it is assumed here that every reservoir must be returned to its initial state at the end. Therefore,

$$V_{0,i} = V_{M,i}. \tag{7}$$

This constraint can be hard-coded on the algorithm by excluding the volume at the end of the last interval from the decision variable set.

3. System Optimization

The optimization of the reservoir system usage is defined by the choice of the objective functions to be maximized or minimized. In this section, several formulations of different objective functions are presented.

3.1. Energy Generation Objectives. Some possible objective functions related to energy generation are shown in this subsection. In all cases, the index $i \in \{1,\ldots,N\}$ indicates the specific power plant, with its reservoir, among all power plants that compose the system, and the index $t \in \{1,\ldots,M\}$ indicates the considered time interval, with the time discretized in intervals of Δt. $W_{t,i}$ represents the total energy generated by plant i in the time interval indexed by t.

3.1.1. Minimize the Usage of Complementary Sources. A usual expression [7, 13] for the objective to be pursued in the optimization procedure is given by W_{ce}:

$$\min\left[W_{ce} = \sum_{t=1}^{M}\left(D_t - \sum_{i=1}^{N} W_{t,i}\right)^2\right], \tag{8}$$

in which D_t represents the total demand of energy in time period t. This expression defines a differentiable function, which is suitable for being optimized by conventional gradient-based methods. The variable W_{ce} represents the total energy from complementary power sources that is necessary for supplying the demand, which should be minimized. The complementary power to the system is usually generated by thermal fossil fuel or nuclear power plants, with higher costs.

3.1.2. Maximize the Energy Generation. The maximization of the total generated energy is represented by

$$\max \left(W_G = \sum_{t=1}^{M} \sum_{i=1}^{N} W_{t,i} \right). \tag{9}$$

This expression makes sense under the assumption that the system to be optimized has a generation capacity, that is, always less than the demand, that is, supposing that all energy that the plant can produce will be sold. In such a case, the maximization of W_G will lead to an exact minimization of the total amount of complementary energy that is necessary along the whole optimization period. In this case, the same would not happen with the minimization of W_{ce}, which would not achieve the exact minimization of the complementary energy demand.

3.1.3. Maximize the Minimum Power Generated. Another objective is the maximization of the minimum monthly power generation:

$$\max \min_{t} \sum_{i=1}^{N} W_{t,i}. \tag{10}$$

Under the viewpoint of an independent company of electric power generation which sells energy in a market, this objective corresponds to the maximization of the assured energy, which is the share of the total energy produced by the company that should be guaranteed to be furnished under any circumstance. This share is sold by a price that is defined in a long term contract which, in average, provides a better price for the energy, with less volatility. Under the viewpoint of the whole system, this objective corresponds to the minimization of the need of installation of backup power plants that would operate only in case of low availability of hydroelectric energy. Notice that this objective is different both from $\max W_G$ and from $\min W_{ce}$, since those objectives perform the minimization of the need of energy from complementary power sources, but they do not perform the minimization of the need of installation of those power plants. This objective becomes particularly important when the rainfall is subject to large variations which, without a large reservoir, would lead to corresponding variations in the power generation.

3.1.4. Maximize the Profit from Secondary Energy. After a share of the total energy produced by a company is compromised by a long term contract (the assured energy), the remaining energy is sold in a spot market. It is possible to maximize the profit that comes from the sale of that energy, by selling the energy in the moments in which the price is expected to be higher. This objective can be expressed as

$$\max \sum_{t=1}^{M} \left(p_t \left(\sum_{i=1}^{N} W_{t,i} - D_c \right) \right), \tag{11}$$

in which p_t is the expected price of energy on period t and D_c is the assured energy, which is not available for sale in the spot market.

3.2. Other Objectives

3.2.1. Water Supply for Cities, Industry, and Irrigation. Several works [14] employ the following expression in order to represent an objective related to the extraction of water from reservoirs, for several uses:

$$\min \sum_{t=1}^{M} \sum_{i=1}^{N} \left(Q_{t,i}^{s} \Delta t - D_{t,i}^{s} \right)^2, \tag{12}$$

in which $Q_{t,i}^{s} \Delta t$ represents the volume of water withdrawn from the reservoir i for the other usages in time period t and $D_{t,i}^{s}$ represents the corresponding demand for water supply on that time and from that reservoir. It should be noticed that the demand for water is specific for each reservoir, differently from the energy demand, which is for the whole system.

3.2.2. Flood Control. The control of water flow downstream a reservoir can contribute to avoid the occurrence of floods in urban or agricultural areas. Such an objective is sometimes expressed as [14]

$$\min \sum_{t=1}^{M} \sum_{i=1}^{N} \left(Q_{t,i}^{D} - Q_{i}^{fc} \right)^2, \tag{13}$$

in which Q_i^{fc} represents a threshold above which a flood can occur downstream the reservoir i. This expression is constructed such that both negative and positive flow differences in relation to the threshold are avoided. The reasoning behind this expression is that if one tries to keep the water flow always near the threshold (and not below it), the reservoirs will be driven to their minimum possible level in the end of the optimization period, and there will be the largest capacity for flood control in the future in any moment of maximum rainfall.

A different formulation which takes into account only the positive violations of the threshold may be stated as

$$\min \sum_{t=1}^{M} \sum_{i=1}^{N} \max \left\{ Q_{t,i}^{D} - Q_{i}^{fc}, 0 \right\}. \tag{14}$$

This formulation is less conservative, avoiding the floods while allowing the preservation of water in the reservoirs for other usages.

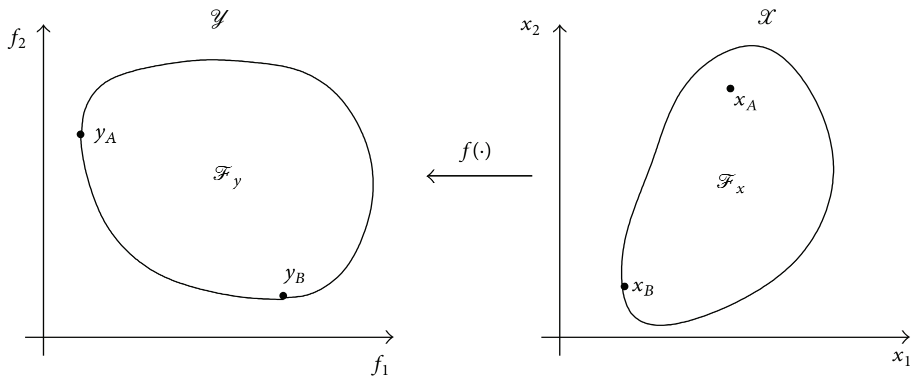

Figure 2: Representation of the Pareto-set, in the space $\mathscr{X}$ of decision variables, as a set starting in the point x_A and ending in the point x_B. The image of such a set is represented in the objective space $\mathscr{Y}$ as the curve starting in the point y_A and ending in the point y_B. The feasible set, in the decision variable space, is represented by the closed set $\mathscr{F}_x$, and its image in the objective space is represented by the closed set $\mathscr{F}_y$. The image of the Pareto-set is part of the boundary of the set $\mathscr{F}_y$, which motivates it being called the *Pareto-front* of the problem. It should be clear that any point inside $\mathscr{F}_y$, except the points belonging to the Pareto-front, will have some other points belonging to $\mathscr{F}_y$ that dominates it.

3.2.3. Navigability. Several rivers have large seasonal variations of their water flows along the year, which makes them be navigable by large ships during part of the year only. The flow in those rivers can be regulated by the reservoirs, making them navigable throughout the year. A larger minimum water flow means that a larger ship is allowed to navigate the river, which means that the maximization of the minimum water flow leads to the maximization of the allowed ship's draft:

$$\max \min_t Q^D_{t,i}. \tag{15}$$

The optimization of this expression leads to the maximization of the minimum flow downstream the reservoir i, where the navigability will be guaranteed.

4. Multiobjective Optimization

The problem of planning the usage of the water stored in hydroelectric plant reservoirs was shown to be related to several different objective functions, which may be conflicting with each other. Therefore, the suitable framework for performing such a planning is the multiobjective optimization. A brief explanation about the subject of multiobjective optimization is presented in this section.

A conventional single-objective optimization problem is stated as the problem of finding the point, in a space of decision variables, in which an objective function reaches its minimum value. The multiobjective optimization problem, instead, searches for a set of points, the *Pareto-optimal set*, which constitutes the set of optimal solutions of a problem with more than one objective function [15]. The Pareto-optimal set, $\mathscr{X}^*$, is defined in the following.

Consider the minimization of a vector function $f(\cdot) : \mathscr{F} \mapsto \mathbb{R}^m$ (the vector of m objective functions), in which the set $\mathscr{F}$ represents the problem feasible set. There is not, in general, any single point $x \in \mathscr{F}$ in which $f(\cdot)$ reaches the minimum value for all its components. Then

$$\begin{aligned}\mathscr{X}^* = \{x^* \in \mathscr{F} \mid \nexists z \in \mathscr{F} \text{ such that } f(z) \\ \leq f(x^*),\ f(z) \neq f(x^*)\},\end{aligned} \tag{16}$$

in which the relational operators $\leq$ and $\neq$ are defined for vectors $u, v \in \mathbb{R}^m$ as

$$\begin{aligned}u \leq v &\Longleftrightarrow u_i \leq v_i \quad \forall i = 1, \ldots, m, \\ u \neq v &\Longleftrightarrow u_i \neq v_i \quad \text{for some } i = 1, \ldots, m.\end{aligned} \tag{17}$$

The points $x \in \mathscr{F}$ that do not belong to the set $\mathscr{X}$ are said to be *dominated*, since there is at least one other point, $z \in \mathscr{F}$, such that $f(z) \leq f(x)$ and $f(z) \neq f(x)$, meaning that $f(z)$ is better than $f(x)$ in at least one coordinate, without being worse in any other coordinate. In this case, *z dominates x*. The solutions x^* that belong to the set $\mathscr{X}^*$ are the *efficient solutions*, which are not dominated by any other solution. Notice that the efficient points have all objective coordinates not worse than the coordinates of the nonefficient points that *they dominate*, but not necessarily in comparison with the coordinates of all nonefficient points. The feature that determines that a point is nonefficient is that it is dominated by *some* efficient points, but not necessarily by *all* efficient points. The multiobjective optimization looks for the set of efficient solutions of a problem of vector optimization. The concept of a Pareto set is illustrated in Figure 2.

Finding the set $\mathscr{X}^*$ is a useful analysis tool in a system design procedure, since the relative position of the elements of this set represents the information about existing *trade-offs* among the problem objectives. This means that the designer can evaluate the effect of replacing a solution by another one, in terms of loss in some objective with simultaneous enhancement in another one. The single-objective approach does not allow for such an analysis. In the case of problems in which the decision variables are continuous, as is the case here, the set $\mathscr{X}^*$ becomes a continuous object, and therefore

(1) $P_1 \leftarrow$ **InitialPopulationGenerator**
(2) $P_0 \leftarrow \emptyset$
(3) **for** $t \leftarrow 1$: maxgen **do**
(4) $R_t \leftarrow P_t \bigcup P_{t-1}$
(5) Evaluate **fitness**(R_t) using **fast non-dominated sorting** and **crowding-distance-assignment**
(6) $R_t \leftarrow$ **StochasticTournament**(R_t)
(7) $R_t \leftarrow$ **Crossover**(R_t)
(8) $R_t \leftarrow$ **Mutation**(R_t)
(9) $P_{t+1} \leftarrow R_t$
(10) **end for**
(11) $O_{\text{maxgen}} \leftarrow$ **ParetoExtraction**($P_t \bigcup P_{t+1}$)

Algorithm 1: Multiobjective genetic algorithm-NSGA-II.

it is not possible to determine all its elements. In this case, the usual procedure is to find a set of *representative samples* of $\mathcal{X}^*$, covering all its extension—this discretized set can be used for the same purpose of performing the multicriteria analysis. This approach will be employed here.

5. Problem-Specific Multiobjective Genetic Algorithm

Although general purpose shelf routines devoted to evolutionary multiobjective optimization are usually quite user-friendly, it has been recognized recently that in several cases some problem-specific adaptations become necessary in order to achieve reasonable levels of algorithm performance [11]. This is the case of the problem considered here, for which no consistent result has been achieved with shelf routines only.

This section describes the problem-specific evolutionary multiobjective optimization algorithm that was developed here. The computational tool employed in this work in order to estimate the Pareto-set solutions for the problem of multiple reservoir operation is based on the classical NSGA-II algorithm [16], modified in some aspects in order to enhance its suitability to the specific problem. The basic structure of the algorithm is presented first, and the specific features introduced in this work are presented after.

5.1. Basic NSGA-II. The basic structure of the NSGA-II is presented in Algorithm 1, according to Deb et al. [16].

Details about the *fast-nondominated sorting*, *crowding-distance-assignment,* and *stochastic tournament* procedures can be found in [16]. The set O_{maxgen} is the approximation of the Pareto-set of the problem.

The nonstandard components of the algorithm which are proposed here are described next.

5.2. Variable Encoding and Constraint Handling. The authors proposed, in a previous work [12], a new variable encoding that implicitly provides most of the constraint-handling that is necessary in the problem. The same formulation is employed here, as described below.

First, a modified lower bound to the volume of the reservoir is imposed at the end of each interval, ensuring that the reservoir can be replenished at the end interval with the available upstream affluent flow and discounting the minimum downstream volume:

$$V_{i-1,j}^{\min} = V_{ij}^{\min} - \left(Q_{ij}^{a} + Q_{j}^{t,\min}\right)\Delta t_i \quad \forall i \in \{1,2,\ldots,M-1\}, \quad \forall j \in \{1,2,\ldots,N\}. \tag{18}$$

A modification to the upper bound constraint was also introduced in order to avoid cases in which the affluent flow minus the minimum downstream flow becomes insufficient to allow for the desired increase in volume:

$$V_{i+1,j}^{\max} = \begin{cases} V_{ij} + \left(Q_{ij}^{a} - Q_{j}^{t,\min}\right)\Delta t_i, & \text{if} < V_j^{\max}, \\ V_j^{\max}, & \text{otherwise}, \end{cases} \quad \forall i \in \{1,2,\ldots,M-1\}, \quad \forall j \in \{1,2,\ldots,N\}. \tag{19}$$

The decision variables were also changed from the monthly volume of water stored in the reservoir to the fractions of the monthly allowable volumes:

$$V_{ij} = V_{ij}^{\min} + \alpha_{ij}\left(V_{ij}^{\max} - V_{ij}^{\min}\right), \quad \alpha_{ij} \in [0,1]. \tag{20}$$

Within those limits, the volumetric flow through the turbines is always greater than the minimum, though it can still lead to a calculated power generation which is greater than the maximum allowed or it can be itself greater than the maximum allowed volumetric flow. To deal with those cases, it is assumed that some water was spilled, and the true flow through the turbine is one that satisfies both conditions (5).

5.3. Crossover. The crossover adopted in the proposed algorithm is described in Algorithm 2. This crossover is an adaptation of the *real-biased crossover* which was presented in [17]. In Algorithm 2, P represents the population that will perform crossover, with $P(i, j)$ representing the variable j of individual i and N_c represents the number of individuals in P. The probability of crossover per pair of individuals is given by p_{cross}. The function $\mathcal{F}(P(i,:))$ furnishes the ranking position of the fitness of individual $P(i,:)$ in the population, after

```
Inputs: P, p_cross, N_c and F(P)
Output: G
P ← randpermP
P1 ← P(1 : N_c/2, :)
P2 ← P(N_c/2 + 1 : N_c, :)
for i ← 1 : N_c/2 do
    cross ← rand(1)
    if cross < p_cross then
        if F(P1(i, :)) > F(P2(i, :)) then
            AUX(:) ← P1(i, :)
            P1(i, :) ← P2(i, :)
            P2(i, :) ← AUX
        end if
        β_1 ← rand(1)
        β_2 ← rand(1)
        α ← 1.4β_1β_2 − 0.2
        G(i, :) ← αP1(i, :) + (1 − α)P2(i, :)
    else
        sel ← rand(1)
        if sel < 0.5 then
            G(i, :) ← P1(i, :)
        else
            G(i, :) ← P2(i, :)
        end if
    end if
end for
```

Algorithm 2: Crossover.

the application of nondominated sorting and crowding distance, with the best individual assigned with $\mathcal{F} = 1$. The offspring individuals are stored in matrix G, with $G(i, j)$ representing the variable j of individual i.

In Algorithm 2 the function *rand* returns a random number drawn from a uniform distribution $\mathcal{U}(0, 1)$, and the function *randperm* returns a random permutation, with uniform probability, of the argument set. An important difference of Algorithm 2 in relation to the one presented in [17] is that there is a low probability of Algorithm 2 to generate an offspring which is too different from one of the parent individuals. This feature was verified to promote the algorithm efficiency in the problem under study here.

5.4. Mutation. The mutation employed in the algorithm is performed on each variable, independently of the individual, as shown in Algorithm 3. In this algorithm, P represents the population to be mutated, with $P(i, j)$ representing the variable j of individual i, and p_{mut} represents the probability of mutation per variable. N_p represents the number of individuals in P. The offspring mutated individual is stored in matrix G, in which $G(i, j)$ represents the variable j of individual i.

In Algorithm 3 the function *rand* works as in Algorithm 2 and the function *randn* returns a random Gaussian number with zero mean and unitary standard deviation. The parameter f_{mut} controls the relative range of the mutation. The algorithm was represented as Algorithm 3 for clarity;

```
Inputs: P and p_mut
Output: G
VR(j) ← max(P(i, j)) − min(P(i, j))    ∀i = 1, ..., n
for i ← 1 : N_p do
    for j ← 1 : n do
        mut ← rand(1)
        if mut < p_mut then
            G(i, j) ← P(i, j) + VR(j) × randn(1) × f_mut
        else
            G(i, j) ← P(i, j)
        end if
    end for
end for
```

Algorithm 3: Mutation.

however, it was actually implemented with matrix operations for the sake of computational efficiency.

6. Algorithm Validation

The proposed constraint-handling procedure (the unconstrained-encoding formulation) constitutes the main modification in the conventional NSGA-II which leads to a much enhanced algorithm performance, in terms of the solution set quality. In order to assess the importance of that procedure, a preliminary study of its effect was performed on a single-reservoir system, considering a series of comparisons of the proposed procedure with (i) a naive approach, in which the conventional constraint-handling procedure of NSGA-II is employed [16], and (ii) a more sophisticated approach which would constitute an obvious enhancement of the naive approach, employing a variable reduced-range constraint box.

This reduced-range constraint management is defined as follows. It can be observed that most of the infeasible individuals generated in the naive approach are related to the need for replenishing the reservoir, if at the end of any interval its volume becomes significantly lower than the initial one. If the affluent flow becomes insufficient, the conservation of mass leads to an infeasible flow through the turbines. To deal with this situation, instead of imposing a single lower bound to the volume of the reservoir, a lower bound can be imposed at the end of each interval:

$$V_{i-1}^{\min} = V_i^{\min} - \left(Q_i^a + Q_t^{\min}\right) \Delta t_i, \qquad \forall i \in \{1, 2, \ldots, N - 1\}. \tag{21}$$

The lower bound volume at the end of a period becomes equal to the upstream affluent water minus the minimum downstream volume.

The experiments were run using data for a typical year (from May, 1976 to April, 1977), with initial and final volumes of the reservoir set to 95 percent of the maximum. The multiobjective optimization problem that was solved considered

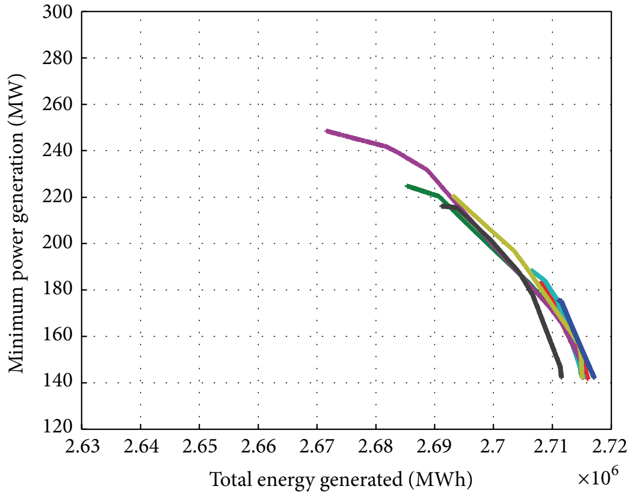

FIGURE 3: Efficient sets for 8 runs of the basic formulation, with 400 individuals and 5000 generations.

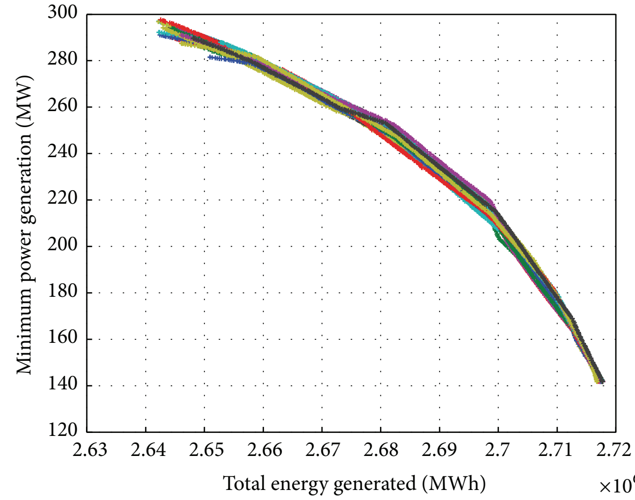

FIGURE 5: Efficient sets for 21 runs of the unconstrained-encoding formulation, with 400 individuals and 2500 generations.

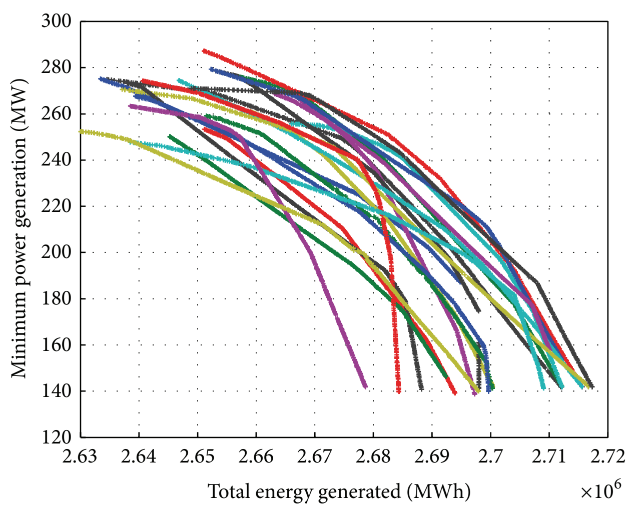

FIGURE 4: Efficient sets for 21 runs of the reduced-range formulation, with 400 individuals and 2500 generations.

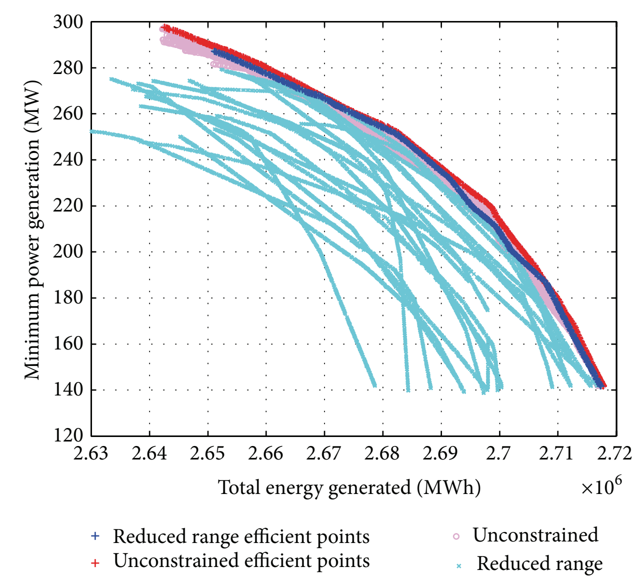

FIGURE 6: Efficient sets for every run of the reduced-range and unconstrained-encoding formulations, with the combined Pareto fronts.

only the Nova Ponte hydropower plant in the Araguari River. The problem was formulated as follows:

$$\begin{aligned} &\max \quad \{(9), (10)\} \\ &\text{subject to: } (3), (4), (5), (6), (7). \end{aligned} \tag{22}$$

The crossover and mutation probabilities were studied in preliminary experiments and finally set at 85% and 35%, respectively, in order to improve the exploration of the domain. This mutation probability is for each individual, since a Gaussian operator is employed.

Only some few experiments that were run with the naive formulation yielded a population with feasible individuals, even when the initial population was hand-tailored to include only near-feasible individuals. The Pareto fronts for 8 of those runs, with 400 individuals and 5000 generations, are shown in Figure 3. The first objective, the total energy generated (in MWh), is shown in the horizontal axis. The second objective, the minimum power generated (in MW), is shown in the vertical axis. In all those experiments, the Pareto front is short extending more in the direction of the first objective.

The proposed approach and the reduced-range constraint formulation were able to find feasible solutions in all runs. For those algorithms, 21 experiments were run with 400 individuals and 2500 generations. The reduced-range formulation was able to generate a fully populated Pareto front, but with large variation in solution quality between runs, as can be seen in Figure 4. Most of the runs generated solution sets that were dominated by other solution sets, in those experiments.

Every run of the new proposed formulation yielded a well spread Pareto front, as shown in Figure 5. In Figure 6,

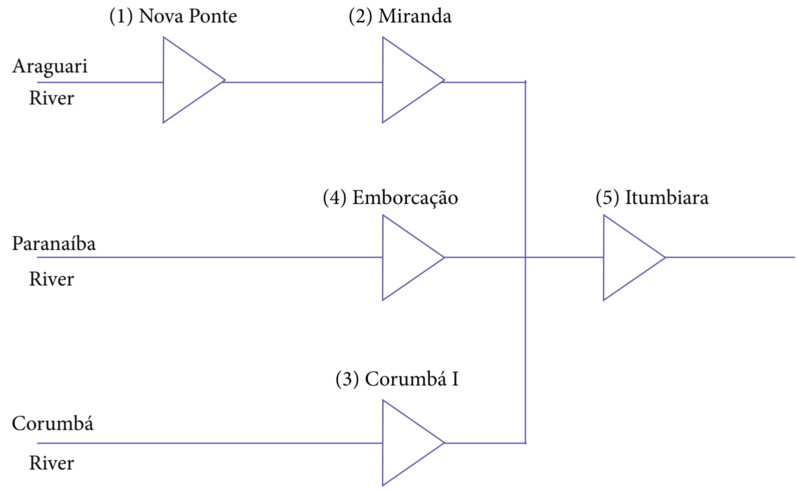

Figure 7: Schematic representation of the subsystem composed of five power plants.

every efficient solution and the combined Pareto front for both formulations are presented. It becomes clear that the proposed formulation leads to better results, with a more spread front that consistently dominates even the combined front of all runs of the reduced-range formulation algorithm. Those results support the conclusion that the proposed formulation is suitable for being included in the algorithm that will perform the studies to be conducted here.

7. Case Studies

In this section, the results of two case studies are presented. The first one deals with power generation objectives only, and the second one includes an objective that is not related to power generation, the river navigability.

The case studies consider a small subset of the Brazilian electric generation system. This subsystem is composed of five hydroelectric power plants, in the Paranaíba river basin, on the southeastern region of Brazil, with a total installed power of 4,765 MW (see Figure 7).

Three of the considered power plants are installed on affluents of the Paranaíba River: two on the Araguari River and the Nova Ponte dam, hereafter referred to as (1), with 510 MW, and downstream the Miranda dam (2), with 408 MW of installed power. A single plant is on the Corumbá River: the Corumbá I dam (3) with 375 MW. On the Paranaíba River itself, the Emborcação dam (4), with 1,192 MW, is located upstream of the mouths of the Araguari and Corumbá Rivers. The largest power station of the subsystem, the Itumbiara Dam (5), with 2,280 MW of total installed power, is located further downstream.

The climate in the region features two very well defined seasons: a dry winter, from April to September, and a rainy summer, from October to March. This determines the volumetric affluent flow to the reservoirs, for which data is available from 1931. The power generated by each plant is given by (1), where the energy production function is represented by a fifth order polynomial of the average volume in the reservoir, whose coefficients are presented in Table 1.

Table 1: Energy production function polynomial coefficients.

Coefficient	Power plant 1	2	3	4	5
$a_0(\times 10^1)$	7.261600	5.774166	4.555294	7.985410	5.244398
$a_1(\times 10^5)$	6.84174	18.1900	34.9788	5.7208	3.1731
$a_2(\times 10^9)$	−7.87711	0.0	196.0	5.0	2.0
$a_3(\times 10^{13})$	9.11797	0.0	0.0	0.0	0.0
$a_4(\times 10^{17})$	−6.49561	0.0	0.0	0.0	0.0
$a_5(\times 10^{21})$	1.95365	0.0	0.0	0.0	0.0

Table 2: Operational parameters and constraints for the five hydropower plants and their reservoirs.

Parameter	Power plant 1	2	3	4	5
Minimum volume (hm^3)	2,412	974	470	4,669	4,573
Maximum volume (hm^3)	12,792	1,120	1,500	17,725	17,027
Maximum power (MW)	510	408	375	1,192	2,280
Minimum flow (m^3/s)	125	080	070	170	190
Maximum flow (m^3/s)	510	675	570	1,048	3,222

The operational parameters and constraints for each hydropower plant, listed in Table 2, as well as the polynomial coefficients, were provided by ONS, the National Electrical System Operator.

7.1. Case Study 1. The first case study was performed using as objectives the expressions (9) and (10), which, respectively, mean the maximization of total power generation in the system and the maximization of the minimum power provided by the system along the year. Formally, the multiobjective optimization problem becomes

$$\begin{aligned} \max \quad & \{(9), (10)\} \\ \text{subject to:} \quad & (3), (4), (5), (6), (7). \end{aligned} \tag{23}$$

Those objectives are conflicting, as can be inferred from a simple reasoning: the situation in which the reservoir has a higher level of water generates more power than a situation of lower water level for the same volume of water passing through the turbines due to the difference of gravitational potential energy in the water. Therefore, given a certain water inflow regime in the reservoir, the best policy for reaching a maximum total generated energy would be to avoid the generation when the reservoir is with low level, in order to make the level as high as possible in the future, in this way leading to the extraction of more power in that future for the same total water volume. This policy would lead to some periods of very small power generation. On the other hand, a policy which tried to avoid the moments of low power generation would spend the water more uniformly and consequently would not be able to take so much profit of the power peaks that would be obtained in the moments in which the reservoir would be almost full, because those moments would occur less frequently under such a policy.

The optimization was performed with the modified NSGA-II algorithm as described previously. As indicated by previous experience, the crossover and mutation probabilities were initially set at 85% and 35%, respectively, in order to improve the exploration of the domain. The mutation probability is per individual, for a Gaussian mutation operator. This value becomes similar to the mutation probability per individual for a simulated binary crossover, employed in reference [16]. The crossover probability is per pair of individuals.

The experiments were run with data for both a typical year (from May, 1976 to April, 1977), with initial and final volumes of all the reservoirs set to 95 percent of the maximum and a dryer than usual year (from May, 2000 to April, 2001), with initial and final volumes of the reservoirs set to 85 percent of the maximum.

The efficient solutions sets that resulted from preliminary experiments with the 1976-77 data were widely spread. Extending the number of generations to 50,000 reduced this problem somewhat, as can be observed in Figure 8. Attempts to further improve the quality of the results were made by increasing the mutation probability to 50% and 65%. They led to solution sets that lied between the best and the worst that resulted from the experiments with the basic 35% mutation probability and could not, visually, be judged neither better nor worse than these.

Shown in Figure 9, the results from experiments with the more difficult conditions of the 2000-01 years, at the end of a dryer than usual period, were more consistent. As could be expected from the operational conditions, they were worse than the ones in the 1976-77 case but spanned a wider range of values of the objectives, in special of the minimum power generated.

There was a clear difference in the results from the algorithm in the two cases. An explanation was found in the analysis of the decision variables for the overall most efficient solutions. In the 1976-77 case, though those solutions originated from four different experiments throughout the solution set the fractions of the allowable volumes were nearly constant for the two largest power plants, Emborcação and Itumbiara (see Figure 10), and their spread was very narrow for the third largest one, Nova Ponte.

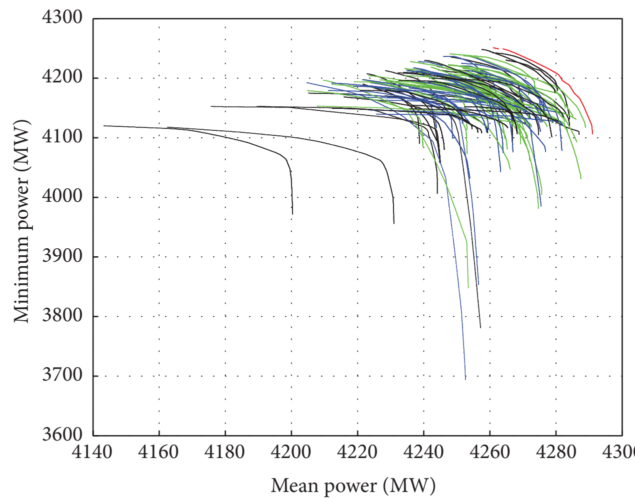

Figure 8: Results for the 1976-77 case. Efficient sets from three sets of 30 experiments, with 400 individuals and 50,000 generations with mutation probabilities of 35% in black, 50% in blue, and 65% in green. In red, the efficient set for a single run initialized with the combined efficient set, with 400 individuals and 25000 generations.

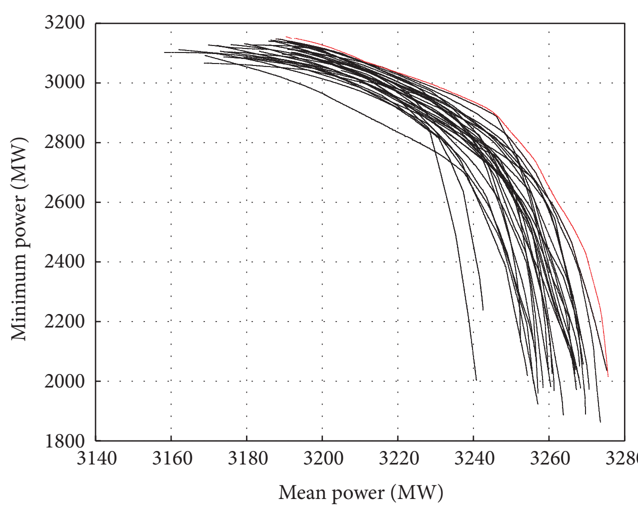

Figure 9: Results for the 2000-01 case. In black, the efficient sets for 30 experiments, with 400 individuals and 50,000 generations. In red, the efficient set for a single run initialized with the combined efficient set, with 400 individuals and 25000 generations.

In the 2000-01 case, the spreads were significant for every power plant, except Itumbiara (which, being the furthest downstream, receives a steadier inflow of water) (see Figure 11). The analysis of this behavior and the apparent insensitivity to the mutation probability suggests the mutation operator as being the culprit for the observed problem. Since a mutation corresponds to a change in every variable of the individual and since, in the 1976-77 case, the values of almost half of the variables are nearly fixed in the most

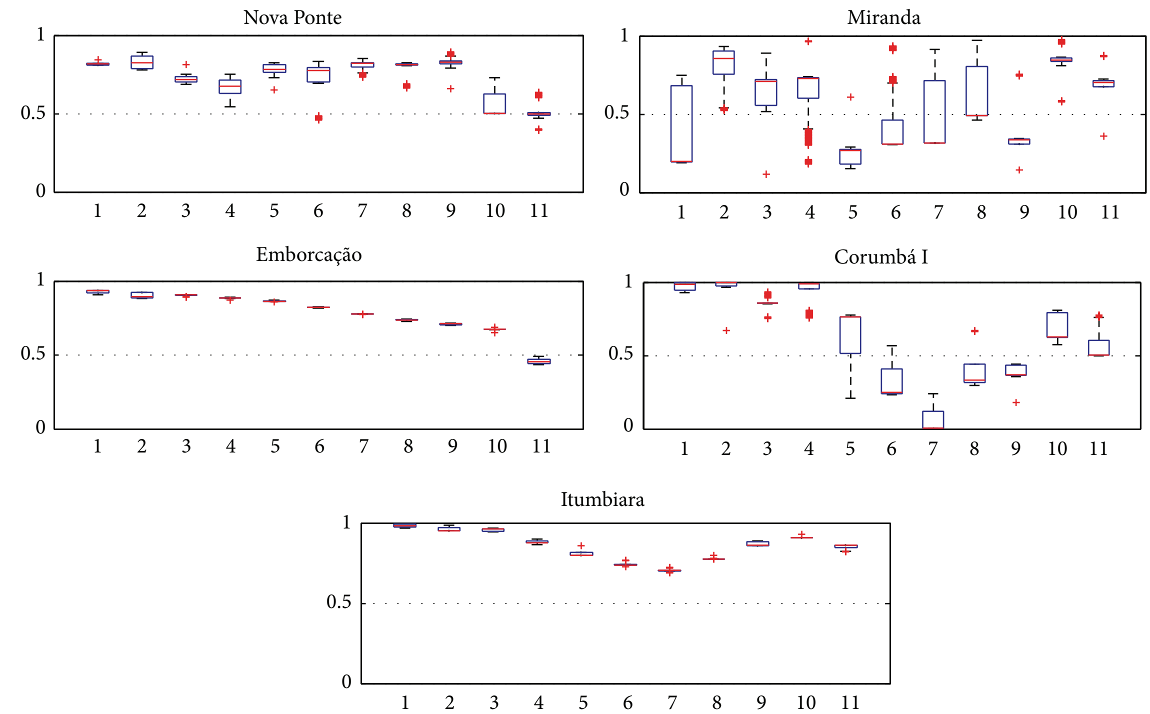

FIGURE 10: Results for the 1976-77 case. Boxplot of the decision variables (α) for each month and power plant for the set of overall most efficient solutions.

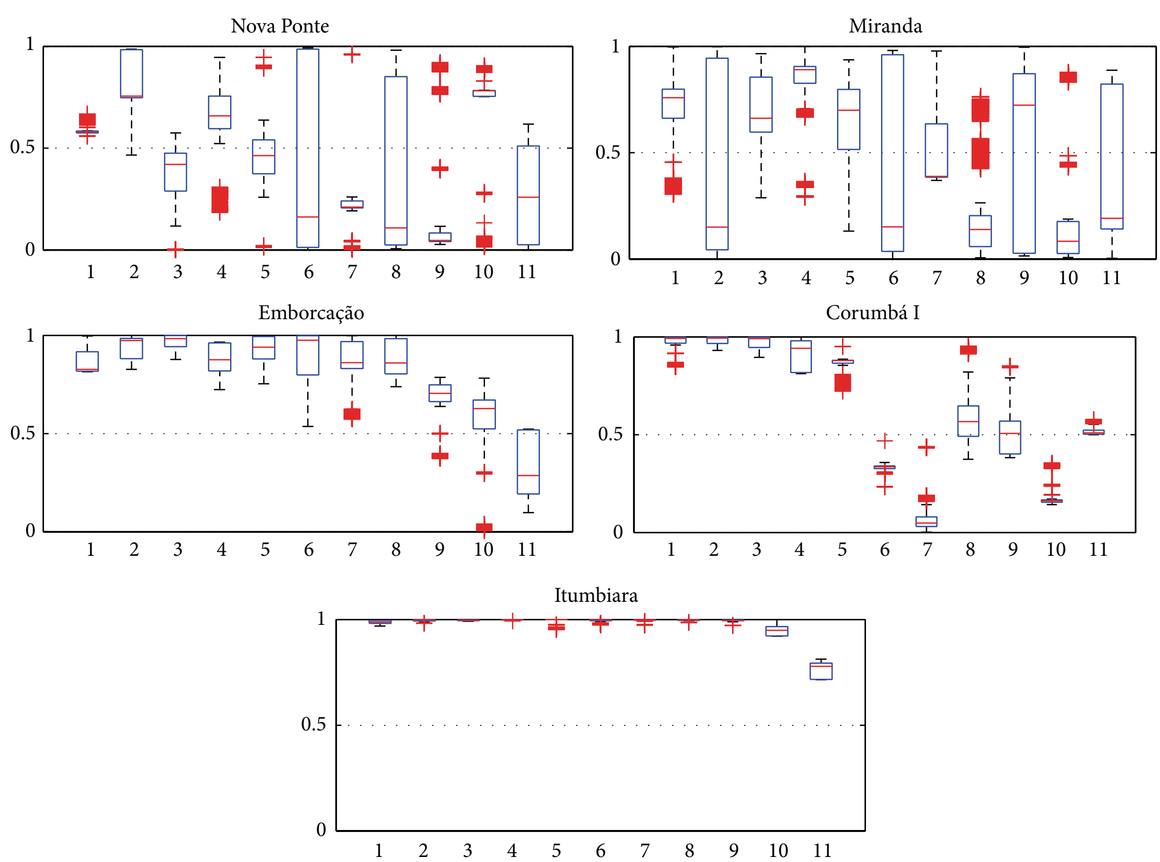

FIGURE 11: Results for the 2000-01 case. Boxplot of the decision variables (α) for each month and power plant for the set of overall most efficient solutions.

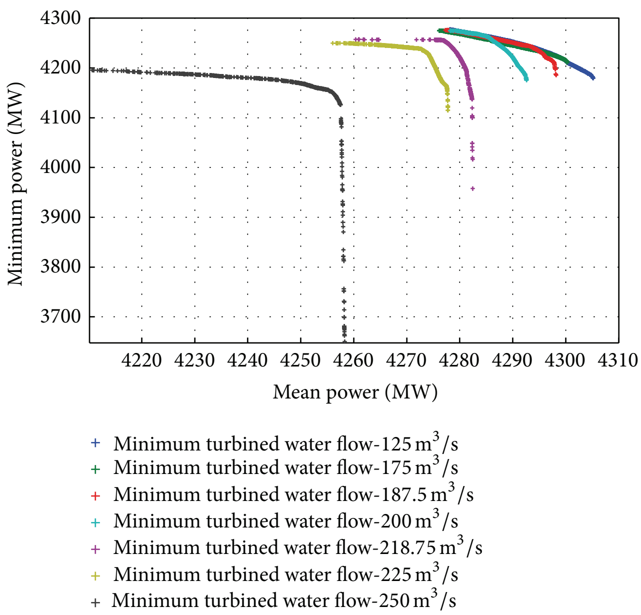

Figure 12: Pareto-set estimates for 10 runs.

efficient set found, a mutation had a very low probability of producing a better individual.

7.2. Case Study 2. The second case study considered the problem of guaranteeing the navigability in the segment of the Araguari River between Nova Ponte dam and Miranda Dam, according to objective function (15), in conjunction with the same power generation objectives (9) and (10) that were employed in Case Study 1, considering the data for the year from May 1976 to April 1977. The multiobjective optimization problem becomes formulated as

$$\begin{aligned} \max \quad & \{(9), (10), (15)\} \\ \text{subject to:} \quad & (3), (4), (5), (6), (7). \end{aligned} \tag{24}$$

An adaptation of the ϵ-constraint formulation [15] was employed, such that the Pareto-optimal solutions of the problem (9) × (10) are found for fixed values of function (15), which leads to a planar graphical representation of the three-objective Pareto-front. Those results are presented in Figure 12.

As expected, the minimum power that will be generated is much less affected than the mean power when the navigability objective is considered. However, even the minimum power objective has some conflict with the navigability objective because a given navigability index requires a given water flow, while a given minimum power can be obtained with different turbined water flows, provided that the reservoir level has different values—therefore making those objectives different.

Another visualization of the same results can be obtained considering the prices for assured energy and variable energy. In order to exemplify this, an example was built considering the price of 120/MWh for the assured energy and 80/MW for the variable energy. Figure 13 represents, in its horizontal axis, the assured energy, and in its vertical axis, the total revenue

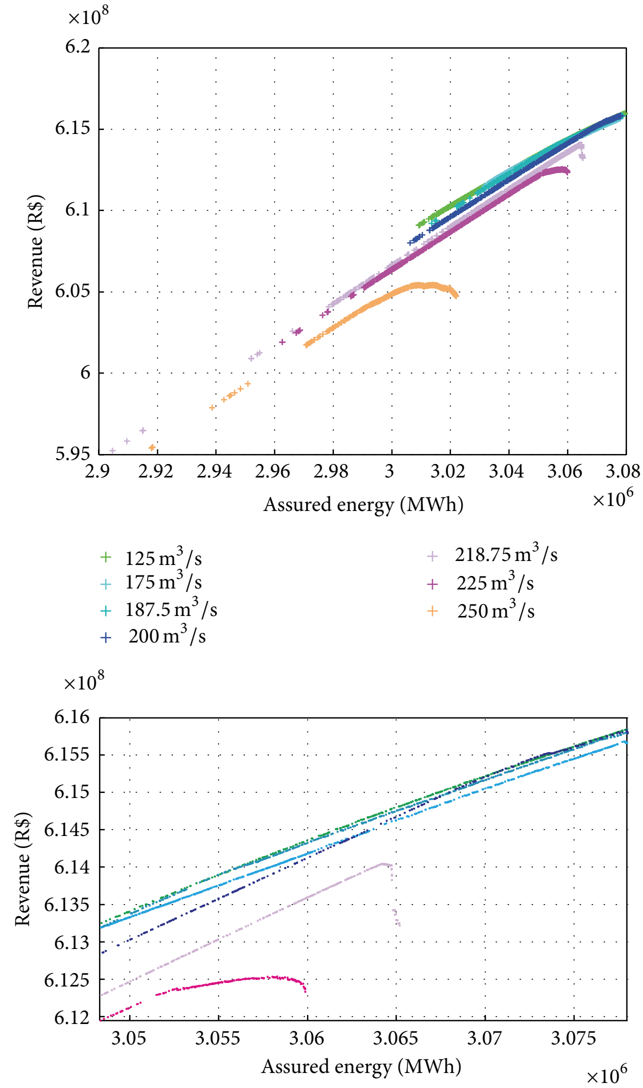

Figure 13: Total revenue versus assured energy for different navigability conditions. A zoom view of the region in which the maximum values occur is presented in the bottom.

from both assured energy and variable energy. It becomes clear, from this figure, that the maximum revenue for this situation of relative prices occurs nearby the maximum of the assured energy. The maximum revenue, however, does not coincide exactly with the maximum assured energy and deviates more from it for higher navigability indices.

Figure 14 shows the points in which the maximum revenue obtained from energy sale occurs for that relative price condition. This figure allows the analysis of the economic trade-off between power generation and navigability. It is interesting to notice that changing the navigability index, from 125 m^3/s to 225 m^3/s, may be obtained almost "for free," with a change of only 0.4% in the revenue that would be obtained without considering the navigability objective. However, changing the water flow from 225 m^3/s to 275 m^3/s would cost nearly 2.5% of the total revenue. It becomes clear that the analysis of this trade-off defines the cost of

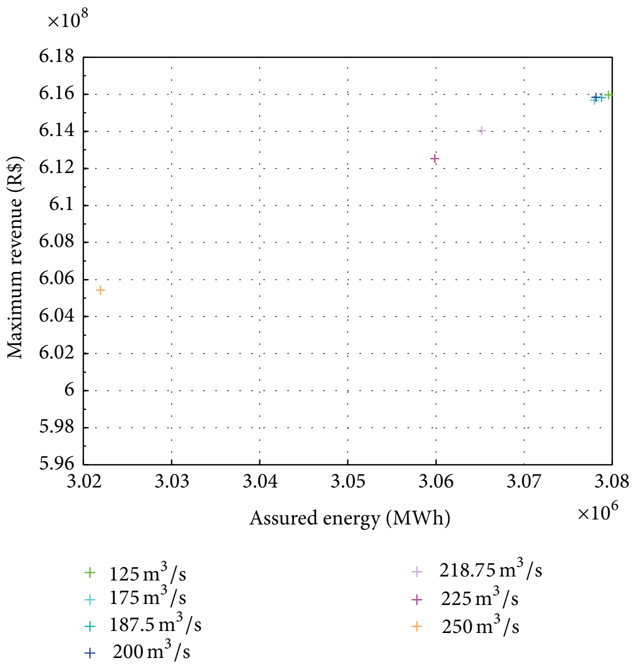

Figure 14: Point of maximum revenue versus assured energy for different navigability conditions.

navigability, in terms of an opportunity cost related to a tradable commodity (the electric energy). A similar approach can be used in order to evaluate the cost of other usages for the water.

8. Conclusions

This work presented an evolutionary multiobjective optimization approach for the study of multiple water usages in multiple interlinked reservoirs, including both power generation objectives and other objectives not related to energy generation.

The classical algorithm NSGA-II was employed as the basic multiobjective optimization machinery. This algorithm was modified in order to cope with specific problem features. The main modification, which caused the major enhancement in the algorithm performance, was the new encoding scheme. This encoding procedure allowed the implicit handling of most of the constraints involved in the problem, in this way removing an important computational bottleneck.

The case studies, which included the analysis of a problem involving energy generation objectives only and of another problem which involved also an objective of navigability on the river, were tailored in order to illustrate the usefulness of the data generated by the proposed methodology for decision-making on the problem of operation planning of multiple reservoirs with multiple usages. It was shown that it is even possible to use the generated data in order to determine the cost of any new usage of the water in terms of the opportunity cost that can be measured on the revenues related to electric energy sales.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors would like to thank the support provided by the Brazilian agencies CAPES, CNPq, and FAPEMIG. The essential collaboration of their colleague Oriane M. Neto *(in memoriam)* in the development of this work is also acknowledged.

References

[1] M. S. Hossain and A. El-shafie, "Intelligent systems in optimizing reservoir operation policy: a review," *Water Resources Management*, vol. 27, no. 9, pp. 3387–3407, 2013.

[2] J. W. Labadie, "Optimal operation of multireservoir systems: state-of-the-art review," *Journal of Water Resources Planning and Management*, vol. 130, no. 2, pp. 93–111, 2004.

[3] W. W.-G. Yeh, "Reservoir management and operations models: a state-of-the-art review," *Water Resources Research*, vol. 21, no. 12, pp. 1797–1818, 1985.

[4] F.-J. Chang, L. Chen, and L.-C. Chang, "Optimizing the reservoir operating rule curves by genetic algorithms," *Hydrological Processes*, vol. 19, no. 11, pp. 2277–2289, 2005.

[5] V. Jothiprakash and G. Shanthi, "Single reservoir operating policies using genetic algorithm," *Water Resources Management*, vol. 20, no. 6, pp. 917–929, 2006.

[6] C.-T. Cheng, W.-C. Wang, D.-M. Xu, and K. W. Chau, "Optimizing hydropower reservoir operation using hybrid genetic algorithm and chaos," *Water Resources Management*, vol. 22, no. 7, pp. 895–909, 2008.

[7] M. T. L. Barros, F. T.-C. Tsai, S.-L. Yang, J. E. G. Lopes, and W. W.-G. Yeh, "Optimization of large-scale hydropower system operations," *Journal of Water Resources Planning and Management*, vol. 129, no. 3, pp. 178–188, 2003.

[8] M. J. Reddy and D. N. Kumar, "Optimal reservoir operation using multi-objective evolutionary algorithm," *Water Resources Management*, vol. 20, no. 6, pp. 861–878, 2006.

[9] D. N. Kumar and M. J. Reddy, "Multipurpose reservoir operation using particle swarm optimization," *Journal of Water Resources Planning and Management*, vol. 133, no. 3, pp. 192–201, 2007.

[10] L. Chen, J. McPhee, and W. W.-G. Yeh, "A diversified multiobjective GA for optimizing reservoir rule curves," *Advances in Water Resources*, vol. 30, no. 5, pp. 1082–1093, 2007.

[11] E. G. Carrano, L. A. E. Soares, R. H. C. Takahashi, R. R. Saldanha, and O. M. Neto, "Electric distribution network multiobjective design using a problem-specific genetic algorithm," *IEEE Transactions on Power Delivery*, vol. 21, no. 2, pp. 995–1005, 2006.

[12] L. A. Scola, O. M. Neto, R. H. C. Takahashi, and S. A. A. G. Cerqueira, "Multi objective optimal reservoir operation," in *Proceedings of the IEEE World Congress on Computational Intelligence*, 2010.

[13] V. Sharma, R. Jha, and R. Naresh, "Optimal multi-reservoir network control by two-phase neural network," *Electric Power Systems Research*, vol. 68, no. 3, pp. 221–228, 2004.

[14] J. L. B. Brandao, *Modelo para operação de sistemas de reservatórios com usos múltiplos [Ph.D. thesis]*, Universidade de Sao Paulo, 2004, (Portuguese).

[15] V. Chankong and Y. Y. Haimes, *Multiobjective Decision Making: Theory and Methodology*, Elsevier, 1983.

[16] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[17] R. M. Ramos, R. R. Saldanha, R. H. C. Takahashi, and F. J. S. Moreira, "The real-biased multiobjective genetic algorithm and its application to the design of wire antennas," *IEEE Transactions on Magnetics*, vol. 39, no. 3, pp. 1329–1332, 2003.